

Some Details of The Minimal Higgsless Model

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We give a detailed derivation of the Minimal Higgsless Model, its masses, wavefunction and couplings. We also discuss a subtlety in its implementation in MadGraph.

I. THE HIGGSLESS SM (HSM)

We have discovered all of the particles of the SM except the Higgs. Maybe we will find the Higgs in the near future and maybe we won't. For the moment then, let's just consider the fields that we have discovered. Figure 1 is a schematic diagram of the Higgsless SM (HSM). It contains all the fields of the SM except the Higgs in a gauge invariant way. Everything is exactly the same as in the SM, except that the Higgs scalar fields (with four degrees of freedom) are replaced with nonlinear sigma fields (with three degrees of freedom):

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_r^+ + i\phi_i^+ \\ \phi_r^0 + i\phi_i^0 \end{pmatrix} \longrightarrow \Sigma = e^{i2\pi/f} \quad (1)$$

where

$$\pi = \begin{pmatrix} \frac{1}{2}\pi^0 & \frac{1}{\sqrt{2}}\pi^+ \\ \frac{1}{\sqrt{2}}\pi^- & -\frac{1}{2}\pi^0 \end{pmatrix} \quad (2)$$

are the goldstone bosons eaten by the W and Z gauge bosons, and f is the goldstone boson decay constant.

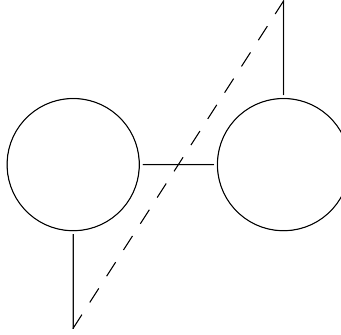


FIG. 1: Schematic "Moose" diagram of the Higgsless SM (or the 2-site model). It contains all the fields of the SM except the Higgs in a gauge invariant way. The circles represent the gauge groups. The circle on the left represents the $SU(2)_W$ gauge group while the circle on the right represents the $U(1)_Y$ gauge group. The horizontal line represents the Goldstone bosons that are eaten by the massive gauge bosons. They are in the form of a nonlinear sigma model. The vertical lines represent the fermions. The vertical line below the circle represents the left chiral fermions while the vertical line above the circle represents the right chiral fermions. The diagonal line represents the mass term that connects the fermions. The fermions attached to nonabelian gauge groups are fundamental representations of that group and singlets under all other nonabelian gauge groups. All fermions are charged under the $U(1)$, however. The lepton doublets have charge -1 under the $U(1)$. The charged leptons attached to the $U(1)$ have charge -2 , while the neutrinos at the same site have charge 0 . The quark doublets have charge $1/6$, the charged $2/3$ quarks attached to the $U(1)$ have charge $4/3$ and the charged $-1/3$ quarks at the same site have charge $-2/3$ under the $U(1)$.

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A. Gauge Sector

The interactions of the goldstone bosons and gauge bosons are given by

$$\mathcal{L}_{D\Sigma} = \frac{f^2}{4} \text{Tr} \left[(D_\mu \Sigma)^\dagger D^\mu \Sigma \right] \quad (3)$$

where the covariant derivative is given by

$$D_\mu \Sigma = \partial_\mu \Sigma + ig W_{0\mu} \Sigma - ig' \Sigma W_{1\mu} \quad (4)$$

where

$$W_0 = \begin{pmatrix} \frac{1}{2} W_0^0 & \frac{1}{\sqrt{2}} W_0^+ \\ \frac{1}{\sqrt{2}} W_0^- & -\frac{1}{2} W_0^0 \end{pmatrix} \quad (5)$$

and

$$W_1 = \begin{pmatrix} \frac{1}{2} W_1^0 & 0 \\ 0 & -\frac{1}{2} W_1^0 \end{pmatrix} \quad (6)$$

are the gauge bosons at sites 0 and 1. The masses of the gauge bosons are easily obtained by going to unitary gauge (or expanding the nonlinear sigma fields to zeroth order)

$$\mathcal{L}_{DI} = \frac{f^2}{4} \text{Tr} \left[(D_\mu I)^\dagger D^\mu I \right] = \frac{f^2}{8} (g W_0^0 - g' W_1^0)^2 + \frac{f^2}{4} g^2 W_0^+ W_0^-. \quad (7)$$

The charged gauge boson mass can be read off directly

$$M_W^2 = \frac{g^2 f^2}{4} \quad (8)$$

while the neutral gauge bosons mass matrix is

$$M_N^2 = \frac{f^2}{4} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \quad (9)$$

and can be diagonalized to give the photon wave function

$$v_\gamma = e \left(\frac{1}{g}, \frac{1}{g'} \right) = (s_W, c_W) \quad (10)$$

where the normalization constant e (also known as the electric charge) is given by

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}. \quad (11)$$

The Z wavefunction is then given by the orthogonal combination

$$v_Z = e \left(\frac{1}{g'}, -\frac{1}{g} \right) = (c_W, -s_W). \quad (12)$$

The mass of the Z is given by

$$M_Z^2 = \frac{(g^2 + g'^2) f^2}{4} \quad (13)$$

making the ratio of W and Z masses

$$\frac{M_W^2}{M_Z^2} = c_W^2. \quad (14)$$

B. Couplings

The interactions of the gauge bosons are given by the overlap of the wavefunctions. For example, the γWW vertex coupling is given by:

$$g_{\gamma WW} = g v_\gamma^0 (v_W^0)^2 = g \frac{e}{g} = e \quad (15)$$

and the ZWW coupling by

$$g_{ZWW} = g v_Z^0 (v_W^0)^2 = g \frac{e}{g'} = \frac{e c_W}{s_W}. \quad (16)$$

The four-point vertices are similar. The $\gamma\gamma WW$ coupling is

$$g_{\gamma\gamma WW} = g^2 (v_\gamma^0)^2 (v_W^0)^2 = g \left(\frac{e}{g} \right)^2 = e^2, \quad (17)$$

the γZWW coupling is

$$g_{\gamma ZWW} = g^2 v_\gamma^0 v_Z^0 (v_W^0)^2 = g^2 \frac{e}{g} \frac{e}{g'} = \frac{e^2 c_W}{s_W}, \quad (18)$$

and finally, the $ZZWW$ coupling is

$$g_{ZZWW} = g^2 (v_Z^0)^2 (v_W^0)^2 = g^2 \left(\frac{e}{g'} \right)^2 = \frac{e^2 c_W^2}{s_W^2}. \quad (19)$$

The $WWWW$ vertex is very simple

$$g_{WWWW} = g^2 (v_W^0)^4 = g^2 = \frac{e^2}{s_W^2} \quad (20)$$

C. Unitarity

The HSM is gauge invariant, it gives masses to the W and Z gauge bosons, but it is an effective theory with a cutoff. We can understand this cutoff by studying unitarity of WW scattering. If we compute the amplitude of the process $WW \rightarrow WW$ and $WW \rightarrow ZZ$ at tree level in the Higgsless SM and then expand them to leading order in M_W^2/s (where s is the center of mass energy squared), we get:

$$\mathcal{M}_{WW \rightarrow WW} = \frac{e^2 s (1 + \cos \theta_{Exp})}{8 s_W^2 M_W^2} \quad (21)$$

and

$$\mathcal{M}_{WW \rightarrow ZZ} = \frac{e^2 s}{4 s_W^2 M_W^2} \quad (22)$$

The bound from unitarity is on a partial wave amplitude. The real part of a partial wave amplitude cannot exceed $1/2$. The strongest bound comes from the $J = 0$ channel, so we obtain the $J = 0$ partial wave amplitude as:

$$a_0 = \frac{1}{32\pi} \int_{-1}^1 d \cos \theta_{Exp} \mathcal{M} \quad (23)$$

Doing this, we get the two partial wave amplitudes:

$$a_0 \text{ } WW \rightarrow WW = \frac{\alpha s}{32 s_W^2 M_W^2} \quad (24)$$

$$a_0 \text{ } WW \rightarrow ZZ = 2 \frac{\alpha s}{32 s_W^2 M_W^2} \quad (25)$$

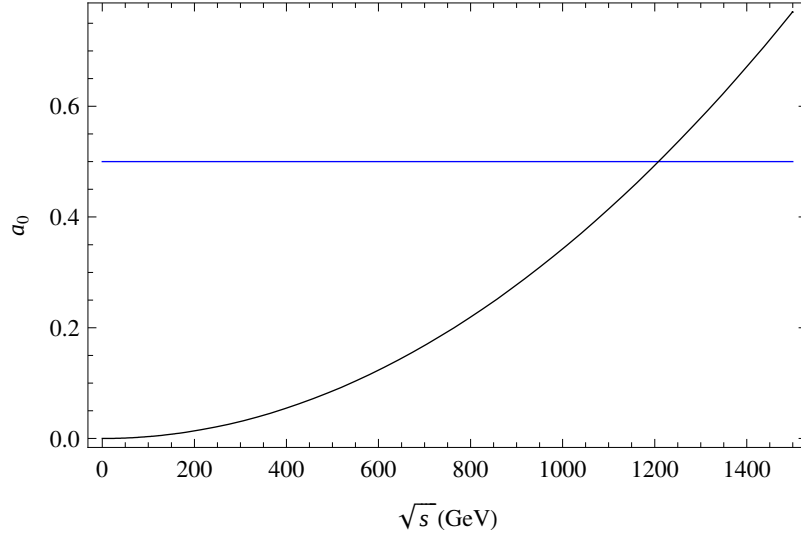


FIG. 2: A plot of the real part of the $J = 0$ partial wave amplitude of WW scattering in the Higgsless SM (HSM) at tree level. The blue line is the unitarity bound of $1/2$. The HSM overcomes this bound at 1.2TeV .

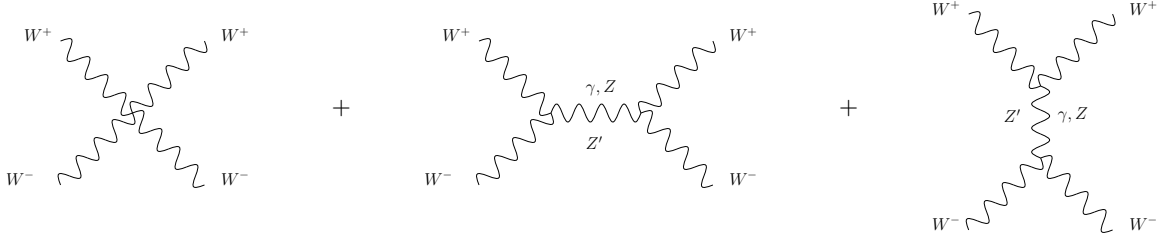


FIG. 3: The diagrams involved in WW scattering when a new Z' gauge boson is added to the HSM. The Z' contributes in the S and T channels and reduces the amplitude, thus improving unitarity.

We can strengthen the bound by considering the coupled channel:

$$|f\rangle = \frac{1}{\sqrt{5}} |WW\rangle + \sqrt{\frac{4}{5}} |ZZ\rangle \quad (26)$$

which maximizes the amplitude and maximally strengthens the bound, giving

$$a_0 = \frac{\alpha s(1 + 2\sqrt{4})}{32\sqrt{5} s_W^2 M_W^2} \quad (27)$$

This is plotted in Figure 2 along with the unitarity bound of $1/2$. The HSM overcomes the unitarity bound at 1.2TeV . We know that something new beyond the HSM must appear before 1TeV to unitarize WW scattering. It might be a scalar field as in the SM, but it could also be gauge bosons. For example, a Z' gauge boson could contribute in the process $WW \rightarrow WW$ in the S and T channel as in Figure 3. The partial wave amplitude of this process is given by

$$a_{0 \text{ } WW \rightarrow WW} = \frac{s}{128\pi M_W^4} \left[\frac{e^2}{s_W^2} M_W^2 + g_{Z'WW}^2 (4M_W^2 - 3M_{Z'}^2) \right]. \quad (28)$$

The first term is the contribution from the HSM. The second term is the contribution from Z' exchange. Since the Z' is much heavier than the W gauge boson, the second term is negative and reduces the quadratic growth and allows unitarity to be valid to higher energies.

We don't know what the next new physics will be. It might be a new scalar field, it might be new gauge bosons, it might even be a combination. For the moment, let's consider the case that the next new physics is a pair of gauge

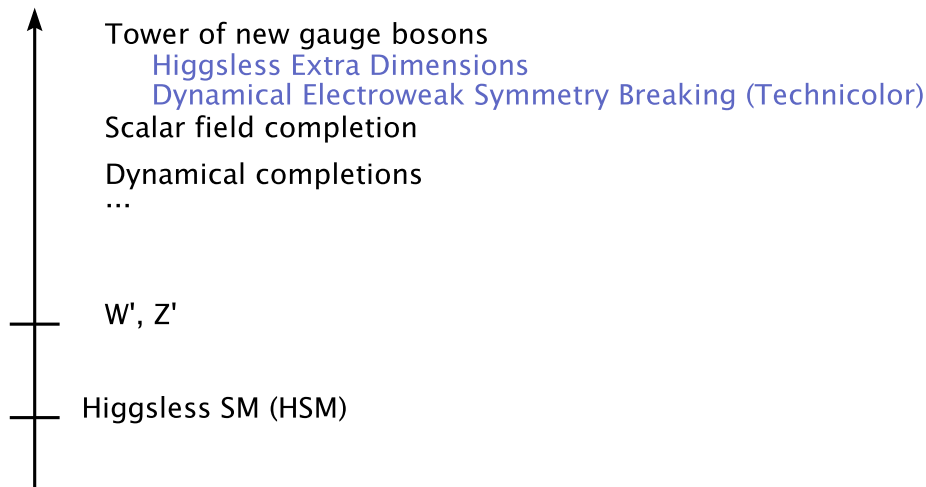


FIG. 4: A diagram of some possible completions for the MHM.

bosons that are partners to the W and Z gauge bosons and contribute to unitarity. This theory will also be an effective theory with a cutoff, so let's further assume that whatever new physics lies above this W' and Z' , is much more weakly coupled to the SM fields, so that we can write an effective theory with just the fields that we have so far discovered (contained in the HSM) and the additional W' and Z' . This is the Minimal Higgsless Model (MHM), also sometimes called the Three-Site Model (TSM).

Before we describe the MHM in detail, let's pause to think a little about what the UV completion of this model might be (see Figure 4). It is possible that this W' and Z' are the first in a tower of new gauge bosons. In this scenario, the W' and Z' are just the first KK modes of a Higgsless Extra Dimensional theory, where typically, the low energy phenomenology is dominated by the lowest KK modes, namely the W' and Z' . So, this MHM is a good candidate for studying the phenomenology of a Higgsless ED. Also, since warped ED are dual to walking technicolor models, this theory gives us insight into a realistic technicolor theory (one where S is zero (see section II E)). We also might consider a more pedestrian completion involving two scalar fields (one for each set of goldstone bosons eaten by the massive gauge bosons). Or, we might consider a completion involving two new strong dynamics groups (one for each set of goldstone bosons). We think that this model gives us insight into a large class of interesting new physics scenarios.

II. THE MINIMAL HIGGSLESS MODEL OR THREE SITE MODEL

The Minimal Higgsless Model (MHM) or Three-Site Model is a gauge invariant combination of the fields of the HSM and a new pair of gauge bosons W' and Z' which are partners of the W and Z . It can be shown schematically by a “Moose” diagram as in Figure 5. In subsection II A we discuss the gauge boson sector of the theory, in subsection II E we discuss the fermionic sector and in subsection II G we discuss the couplings that result in this theory.

A. Gauge Boson Sector

The gauge group of the Three Site Model is

$$G = SU(2)_0 \times SU(2)_1 \times U(1)_2 \quad (29)$$

where $SU(2)_0$ is represented by the leftmost circle in Figure 5 and has coupling g , $SU(2)_1$ is represented by the center circle in Figure 5 and has coupling $\tilde{g}$ and $U(1)_2$ is represented by the rightmost dashed circle in Figure 5 and has coupling g' . It is sometimes useful to define

$$x = \frac{g}{\tilde{g}}. \quad (30)$$

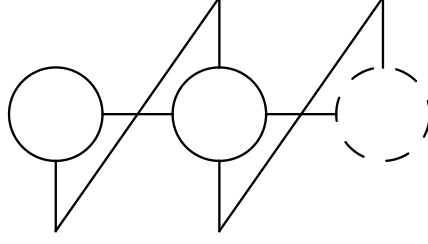


FIG. 5: A schematic “Moose” diagram of the MHM. The circles represent gauge groups. The two circles on the left are $SU(2)$ gauge groups while the one on the right is a $U(1)$ gauge group. The horizontal lines represent the goldstone bosons eaten by the massive gauge bosons. They are in the form of nonlinear sigma fields. The physical vector bosons (mass eigenstates) are linear combinations of the gauge bosons at each site. The W gauge boson is a linear combination of the charged gauge bosons at sites 0 and 1. It lives mostly at site 0 with a small leakage into site 1. The W' boson is the orthogonal linear combination of charged gauge bosons at the first two sites. It lives mostly at site 1. The photon and Z are linear combinations of the neutral gauge bosons at all three sites, but live mostly at the edges with a small leakage into site 1. The Z' lives mostly at site 1. The vertical lines represent the fermions in the theory. The vertical lines below the circles represent the left chiral fields while those above the circles represent the right chiral fermions. The diagonal lines represent the mass terms connecting the fermions. The fermions attached to nonabelian gauge groups are fundamental representations of that group and singlets under all other nonabelian gauge groups. All fermions are charged under the $U(1)$, however. The lepton doublets have charge -1 under the $U(1)$. The charged leptons attached to the $U(1)$ have charge -2 , while the neutrinos at the same site have charge 0 . The quark doublets have charge $1/6$, the charged $2/3$ quarks attached to the $U(1)$ have charge $4/3$ and the charged $-1/3$ quarks at the same site have charge $-2/3$ under the $U(1)$.

We also find it convenient to define the parameters

$$t = \frac{g'}{g} = \frac{s}{c} \quad (31)$$

where $s^2 + c^2 = 1$.

The horizontal bars in Figure 5 represent nonlinear sigma models Σ_j which come from unspecified physics at a higher scale and which give mass to the 6 gauge bosons other than the photon. This is encoded in the leading order effective Lagrangian term

$$\mathcal{L}_{D\Sigma} = \frac{f^2}{4} \text{Tr} \left[(D_\mu \Sigma_0)^\dagger D^\mu \Sigma_0 + (D_\mu \Sigma_1)^\dagger D^\mu \Sigma_1 \right] \quad (32)$$

where

$$D_\mu \Sigma_0 = \partial_\mu \Sigma_0 + ig W_{0\mu} \Sigma_0 - i\tilde{g} \Sigma_0 W_{1,\mu} \quad (33)$$

$$D_\mu \Sigma_1 = \partial_\mu \Sigma_1 + i\tilde{g} W_{1\mu} \Sigma_1 - ig' \Sigma_1 W_{2,\mu} \quad (34)$$

The nonlinear sigma models can be written in exponential form

$$\Sigma_j = e^{i2\pi_j/f} \quad (35)$$

which exposes the Goldstone bosons which become the longitudinal components of the massive gauge bosons. π_j and W_j are written in matrix form and are

$$\pi_j = \begin{pmatrix} \frac{1}{2}\pi_j^0 & \frac{1}{\sqrt{2}}\pi_j^+ \\ \frac{1}{\sqrt{2}}\pi_j^- & -\frac{1}{2}\pi_j^0 \end{pmatrix} \quad (36)$$

$$W_j = \begin{pmatrix} \frac{1}{2}W_j^0 & \frac{1}{\sqrt{2}}W_j^+ \\ \frac{1}{\sqrt{2}}W_j^- & -\frac{1}{2}W_j^0 \end{pmatrix} \quad (37)$$

$$W_2 = \begin{pmatrix} \frac{1}{2}W_2^0 & 0 \\ 0 & -\frac{1}{2}W_2^0 \end{pmatrix} \quad (38)$$

where j is 1 or 2.

The mass matrix of the gauge bosons can be obtained by going to unitary gauge ($\Sigma_j \rightarrow 1$) and is

$$M_{\pm}^2 = \frac{M_G^2}{2} \begin{pmatrix} x^2 & -x \\ -x & 2 \end{pmatrix} \quad (39)$$

for the charged gauge bosons and

$$M_n^2 = \frac{M_G^2}{2} \begin{pmatrix} x^2 & -x & 0 \\ -x & 2 & -xt \\ 0 & -xt & x^2 t^2 \end{pmatrix} \quad (40)$$

for the neutral gauge bosons where

$$M_G^2 = \frac{\tilde{g}^2 f^2}{2}. \quad (41)$$

The photon is massless and given by the exact wavefunction

$$v_{\gamma} = e \left(\frac{1}{g}, \frac{1}{\tilde{g}}, \frac{1}{g'} \right) \quad (42)$$

where

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{\tilde{g}^2} + \frac{1}{g'^2}. \quad (43)$$

After diagonalizing the gauge boson mass matrices, we find that the other masses and wavefunctions are given by

$$M_W = \frac{M_G}{2} \sqrt{2 + x^2 - \sqrt{4 + x^4}} \simeq \frac{g^2 f^2}{8} \left(1 - \frac{x^2}{4} + \dots \right) \quad (44)$$

$$v_W^0 = \frac{1}{N_W} \simeq 1 - \frac{x^2}{8} + \dots \quad (45)$$

$$v_W^1 = \frac{1}{N_W} \frac{2x}{2 - x^2 + \sqrt{4 + x^4}} \simeq \frac{x}{2} + \frac{x^3}{16} + \dots \quad (46)$$

$$M_{W'} = \frac{M_G}{2} \sqrt{2 + x^2 + \sqrt{4 + x^4}} \simeq \frac{\tilde{g}^2 f^2}{2} \left(1 + \frac{x^2}{4} + \dots \right) \quad (47)$$

$$v_{W'}^0 = \frac{1}{N_{W'}} \frac{2 - x^2 - \sqrt{4 + x^4}}{2x} \simeq -\frac{x}{2} - \frac{x^3}{16} + \dots \quad (48)$$

$$v_{W'}^1 = \frac{1}{N_{W'}} \simeq 1 - \frac{x^2}{8} + \dots \quad (49)$$

for the charged gauge bosons (where N_W and $N_{W'}$ are normalization constants) and

$$M_Z = \frac{M_G}{2} \sqrt{2 + x^2(1 + t^2) - \sqrt{4 + x^4(1 - t^2)^2}} \simeq \frac{g^2 f^2}{8c^2} \left(1 - \frac{x^2}{4} \frac{(c^2 - s^2)^2}{c^2} + \dots \right) \quad (50)$$

$$v_Z^0 = \frac{1}{N_Z} \left[x^2 t + \frac{1}{t} \left(x^2 - \sqrt{4 + x^4(1 - t^2)^2} \right) \right] \simeq c - \frac{x^2 c^3 (1 + 2t^2 - 3t^4)}{8} + \dots \quad (51)$$

$$v_Z^1 = \frac{1}{N_Z} \frac{-1}{xt} \left[2 + x^2(1 - t^2) - \sqrt{4 + x^4(1 - t^2)^2} \right] \simeq \frac{xc(1 - t^2)}{2} + \frac{x^3 c^3 (1 - t^2)^3}{16} + \dots \quad (52)$$

$$v_Z^2 = \frac{1}{N_Z} 2 \simeq -s - \frac{x^2 s c^2 (3 - st^2 - t^4)}{8} + \dots \quad (53)$$

$$M_{Z'} = \frac{M_G}{2} \sqrt{2 + x^2(1 + t^2) + \sqrt{4 + x^4(1 - t^2)^2}} \simeq \frac{\tilde{g}^2 f^2}{2} \left(1 + \frac{x^2}{4c^2} + \dots \right) \quad (54)$$

$$v_{Z'}^0 = \frac{1}{N_{Z'}} \left[-x^2 t + \frac{1}{t} \left(x^2 + \sqrt{4 + x^4(1 - t^2)^2} \right) \right] \simeq -\frac{x}{2} - \frac{x^3 (1 - 3t^2)}{16} + \dots \quad (55)$$

$$v_{Z'}^1 = \frac{1}{N_{Z'}} \frac{-1}{xt} \left[2 + x^2(1 - t^2) + \sqrt{4 + x^4(1 - t^2)^2} \right] \simeq 1 - \frac{x^2 (1 + t^2)}{8} + \dots \quad (56)$$

$$v_{Z'}^2 = \frac{1}{N_{Z'}} 2 \simeq -\frac{xt}{2} + \frac{x^3 t (3 - t^2)}{16} \quad (57)$$

for the neutral gauge bosons (where N_Z and $N_{Z'}$ are normalization constants). We note that x can be obtained in terms of the masses M_W and $M_{W'}$.

$$R_M^2 = \left(\frac{M_W}{M_{W'}} \right)^2 = \frac{2 + x^2 - \sqrt{4 + x^4}}{2 + x^2 + \sqrt{4 + x^4}} \simeq \frac{x^2}{4} - \frac{x^4}{8} + \dots \quad (58)$$

which can be inverted to give

$$x = \frac{1 + R_M^2 - \sqrt{1 - 6 R_M^2 + R_M^4}}{2 R_M} \quad (59)$$

$$x^2 \simeq 4 R_M^2 + 8 R_M^4 + \dots \quad (60)$$

The ratio of the masses M_W and M_Z can be taken

$$c_M^2 = \frac{M_W^2}{M_Z^2} = \frac{2 + x^2 - \sqrt{4 + x^4}}{2 + x^2(1 + t^2) - \sqrt{4 + x^4}(1 - t^2)^2} \simeq c^2 \left(1 - \frac{x^2}{4} \left[1 - \frac{(c^2 - s^2)^2}{c^2} \right] \right) \quad (61)$$

which can be inverted to give:

$$t^2 = \frac{-\left((-1 + c_M^2)\left(x^2\left(2 + x^2 - \sqrt{4 + x^4}\right) + \left(-x^6 + 2x^2\left(-4 + \sqrt{4 + x^4}\right) + x^4\left(-4 + \sqrt{4 + x^4}\right) + 2\left(-2 + \sqrt{4 + x^4}\right)\right)c_M^2 + 2\left(x^2 + x^4\right)c_M^4\right)}{2x^2c_M^2\left(x^2 - \left(2 + 3x^2 + x^4\right)c_M^2 + \left(1 + x^2\right)^2c_M^4\right)} \quad (62)$$

Not very pretty, but useful for studying $x \sim 1$. The approximate formula is much nicer:

$$c^2 = c_M^2 \left(1 + \frac{x^2}{4} [1 - c_M^2(1 - t_M^2)^2] \right) \quad (63)$$

Now the couplings can be determined in terms of the electric charge e and x and t .

$$g^2 = e^2 \left(1 + x^2 + \frac{1}{t^2} \right) \simeq \frac{e^2}{s_M^2} \left[1 + x^2 \left(s_M^2 + \frac{c_M^2}{4s_M^2} [1 - c_M^2(1 - t_M^2)^2] \right) + \dots \right] \quad (64)$$

$$\tilde{g}^2 = e^2 \left(1 + \frac{1}{x^2} + \frac{1}{x^2 t^2} \right) \simeq \frac{e^2}{x^2 s_M^2} \left[1 + x^2 \left(s_M^2 + \frac{c_M^2}{4s_M^2} [1 - c_M^2(1 - t_M^2)^2] \right) + \dots \right] \quad (65)$$

$$(g')^2 = e^2 (1 + t^2 + x^2 t^2) \simeq \frac{e^2}{c_M^2} \left[1 + x^2 \left(-\frac{1}{4} + s_M^2 - \frac{1}{4} c_M^2 (1 - t_M^2)^2 \right) + \dots \right] \quad (66)$$

$$(67)$$

which gives them in terms of the physical parameters.

Finally, the constant f can be obtained by setting the mass M_W

$$f = \frac{2\sqrt{2}M_W}{\tilde{g}\sqrt{2 + x^2 - \sqrt{4 + x^4}}} \simeq \frac{2\sqrt{2}M_W}{g} \left(1 + \frac{x^2}{8} \right) \quad (68)$$

We now have all the parameters defined in terms of physical parameters.

B. Goldstone Boson Sector

As we mentioned in the previous subsection, the horizontal lines in Figure 5 represent nonlinear sigma models. Although tree level calculations can be done in unitary gauge, there are times when a different gauge is useful. Many calculations with gauge bosons in the external states can be computed more simply using the equivalence theorem and replacing the massive gauge bosons with the goldstone bosons that they eat. Another case that another gauge is advantageous is in calchep, where the time of computation of certain processes is dramatically decreased. For this reason, we have calculated the interactions with goldstone bosons and with ghosts. In this subsection, we outline the goldstone bosons, while in the next we determine the ghost terms.

We must first determine the goldstone bosons that are eaten by the gauge bosons. We do this using the lagrangian of equation 32. Expanding the nonlinear sigma field, we obtain the kinetic term for the goldstone bosons, the masses

for the gauge bosons, the mixing of the gauge bosons and the goldstone bosons and the interactions of the gauge bosons and goldstone bosons. To determine the mixing, we expand and keep terms linear in the goldstone bosons and gauge bosons. These are

$$\mathcal{L}_{\pi W} = \frac{1}{2} \tilde{g} f \left(\{ \partial_\mu \pi_0, x W_0^\mu - W_1^\mu \} + \{ \partial_\mu \pi_1, W_1^\mu - x t W_2^\mu \} \right) \quad (69)$$

By inserting the eigenwave functions of these fields, we obtain the mixing of the eigenstates

$$\mathcal{L}_{\pi W} = \frac{1}{2} \tilde{g} f \left(v_\pi^0 (x v_W^0 - v_W^1) + v_\pi^1 (v_W^1 - \delta x t v_W^2) \right) \{ \partial_\mu \pi, W^\mu \} \quad (70)$$

$$+ \frac{1}{2} \tilde{g} f \left(v_{\pi'}^0 (x v_{W'}^0 - v_{W'}^1) + v_{\pi'}^1 (v_{W'}^1 - \delta x t v_{W'}^2) \right) \{ \partial_\mu \pi', W'^\mu \} \quad (71)$$

where δ is 1 if the gauge boson is neutral but 0 otherwise. Since π' does not mix with W , we obtain

$$v_{\pi'}^0 (x v_W^0 - v_W^1) + v_{\pi'}^1 (v_W^1 - \delta x t v_W^2) = 0 \quad (72)$$

Or

$$v_{\pi'}^0 = -\frac{1}{N_\pi} (v_W^1 - \delta x t v_W^2) \quad (73)$$

$$v_{\pi'}^1 = \frac{1}{N_\pi} (x v_W^0 - v_W^1) \quad (74)$$

and since v_π is orthornormal to $v_{\pi'}$ we get

$$v_\pi^0 = \frac{1}{N_\pi} (x v_W^0 - v_W^1) \quad (75)$$

$$v_\pi^1 = \frac{1}{N_\pi} (v_W^1 - \delta x t v_W^2) \quad (76)$$

Further expansion of equation 32 and insertion of the wavefunctions gives the interactions of gauge bosons and goldstone bosons.

C. Gauge Fixing

The gauge fixing function is constructed to do two things. First of all, it fixes the gauge. Secondly, it cancels the mixing of the goldstone bosons and gauge bosons. For each site, this gauge fixing term is

$$G_0 = \partial \cdot W_0 - \frac{\xi}{2} g f(\pi_0) \quad (77)$$

$$G_1 = \partial \cdot W_1 - \frac{\xi}{2} \tilde{g} f(\pi_1 - \pi_0) \quad (78)$$

$$G_2 = \partial \cdot W_2 - \frac{\xi}{2} g' f(-\pi_1^{ns}) \quad (79)$$

where π_1^{ns} we mean just the neutral sector of π_1 , namely

$$\pi_1^{ns} = \frac{1}{2} \pi_1^0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (80)$$

With this definition, the gauge fixing function is

$$\mathcal{L}_{GF} = -\frac{1}{\xi} \text{Tr} \left(G_0^2 + G_1^2 + G_2^2 \right) \quad (81)$$

where $\xi = 1$ corresponds to the Feynman gauge (which we implemented in calchep).

In addition to fixing the gauge and cancelling the mixing between gauge bosons and goldstone bosons, this term gives the goldstone bosons a gauge dependent mass.

$$M_{\pi^\pm} = \sqrt{\xi} M_{W^\pm} \quad (82)$$

$$M_{\pi^0} = \sqrt{\xi} M_Z \quad (83)$$

$$M_{\pi'} = \sqrt{\xi} M_{W'} \quad (84)$$

D. Ghosts

The ghost lagrangian terms are obtained by multiplying the BRST transformation of the gauge fixing term on the left with the antighost. To do this, we must find the BRST transformations of the gauge fixing terms. To do this, we begin by writing the infinitesimal BRST transformation of the fields in the gauge fixing term.

$$\delta_{BRST} W_{\mu j} = -\left(\partial_\mu c_j + i g_j [W_{\mu j}, c_j]\right) \quad (85)$$

for the gauge bosons where c_j is the ghost for site j in matrix notation

$$c_j = \begin{pmatrix} \frac{1}{2}c_j^0 & \frac{1}{\sqrt{2}}c_j^+ \\ \frac{1}{\sqrt{2}}c_j^- & -\frac{1}{2}c_j^0 \end{pmatrix} \quad (86)$$

where j is 0 or 1, but

$$c_2 = \begin{pmatrix} \frac{1}{2}c_2^0 & 0 \\ 0 & -\frac{1}{2}c_2^0 \end{pmatrix} \quad (87)$$

The BRST transformations to quadratic order in the goldstone bosons are

$$\delta_{BRST} \pi_j = +\frac{1}{2}f(g_j c_j - g_{j+1} c_{j+1}) + \frac{i}{2}[g_j c_j + g_{j+1} c_{j+1}, \pi_j] - \frac{1}{6f}[\pi_j, [\pi_j, g_j c_j - g_{j+1} c_{j+1}]] \quad (88)$$

so that

$$\delta_{BRST} G_0 = \partial \cdot \delta_{BRST} W_0 - \frac{\xi}{2} g f(\delta_{BRST} \pi_0) \quad (89)$$

$$\delta_{BRST} G_1 = \partial \cdot \delta_{BRST} W_1 - \frac{\xi}{2} \tilde{g} f(\delta_{BRST} \pi_1 - \delta_{BRST} \pi_0) \quad (90)$$

$$\delta_{BRST} G_2 = \partial \cdot \delta_{BRST} W_2 - \frac{\xi}{2} g' f(-\delta_{BRST} \pi_1^{ns}) \quad (91)$$

Then the ghost terms are

$$\mathcal{L}_{ghost} = -\text{Tr}\left(\bar{c}_0 \delta_{BRST} G_0 + \bar{c}_1 \delta_{BRST} G_1 + \bar{c}_2 \delta_{BRST} G_2\right) + h.c. \quad (92)$$

E. Fermion Sector

The vertical lines in Figure 5 represent the fermionic fields in the theory. The vertical lines on the bottom of the circles represent the left chiral fermions while the vertical lines attached to the tops of the circles are the right chiral fermions. Each fermion is a fundamental representation of the gauge group to which it is attached and a singlet under all the other gauge groups except $U(1)_2$. The charges under $U(1)_2$ are as follows: If the fermion is attached to an $SU(2)$ then its charge is $1/6$ for quarks and $-1/2$ for leptons. If the fermion is attached to $U(1)_2$ its charge is the same as its electromagnetic charge: 0 for neutrinos, -1 for charged leptons, $2/3$ for up type quarks and $-1/3$ for down type quarks.

The fermions attached to the internal site ($SU(2)_1$) are vectorially coupled and are thus allowed Dirac masses. We have taken these masses to be M_F . The symmetries also allow various linkings of fermions via the nonlinear sigma fields. We have assumed a very simple form, inspired by an extra dimension and represented by the diagonal lines in Figure 5. The left chiral field at site j is linked to the right chiral field at site $j+1$ through the nonlinear sigma field at link j . The mass parameter for these diagonal links is taken to be $\epsilon_L M_F$ and $\epsilon_R M_F$ for the left and right links respectively. All together, the masses of the fermions and the leading order interactions of the fermions and nonlinear sigma fields is given by

$$\mathcal{L}_{\psi\Sigma} = -M_F \left[\epsilon_L \bar{\psi}_{L0} \Sigma_0 \psi_{R1} + \bar{\psi}_{L1} \psi_{R1} + \bar{\psi}_{L1} \epsilon_R \Sigma_1 \psi_{R2} \right] \quad (93)$$

where

$$M_F = \lambda f \quad (94)$$

and where ϵ_L is a number which is the same for all fermions but ϵ_R is a diagonal matrix which distinguishes flavors. For example for the top and bottom quark we have

$$\epsilon_R = \begin{pmatrix} \epsilon_{Rt} & 0 \\ 0 & \epsilon_{Rb} \end{pmatrix} \quad (95)$$

The mass matrix can be obtained by going to unitary gauge and diagonalized by a biunitary transformation. By doing this, we find the following masses and wavefunctions

$$M_{f0} = \frac{M_F}{\sqrt{2}} \sqrt{1 + \epsilon_L^2 + \epsilon_R^2 - \sqrt{(1 + \epsilon_L^2 + \epsilon_R^2)^2 - 4\epsilon_L^2\epsilon_R^2}} \simeq M_F \epsilon_L \epsilon_R \left(1 - \frac{1}{2}(\epsilon_L^2 + \epsilon_R^2)\right) \quad (96)$$

$$v_{Lf0}^0 = \frac{-1}{N_{Lf0}} \simeq -1 + \frac{1}{2}\epsilon_L^2 + \dots \quad (97)$$

$$v_{Lf0}^1 = \frac{1}{N_{Lf0}} \frac{2\epsilon_L}{1 - \epsilon_L^2 + \epsilon_R^2 + \sqrt{(1 + \epsilon_L^2 + \epsilon_R^2)^2 - 4\epsilon_L^2\epsilon_R^2}} \simeq \epsilon_L \left(1 - \frac{1}{2}\epsilon_L^2 - \epsilon_R^2 + \dots\right) \quad (98)$$

$$v_{Rf0}^1 = \frac{1}{N_{Rf0}} \frac{1 + \epsilon_L^2 - \epsilon_R^2 - \sqrt{(1 + \epsilon_L^2 + \epsilon_R^2)^2 - 4\epsilon_L^2\epsilon_R^2}}{2\epsilon_R} \simeq -\epsilon_R \left(1 - \epsilon_L^2 - \frac{1}{2}\epsilon_R^2 + \dots\right) \quad (99)$$

$$v_{Rf0}^2 = \frac{1}{N_{Rf0}} \simeq 1 - \frac{1}{2}\epsilon_R^2 + \dots \quad (100)$$

$$M_{f1} = \frac{M_F}{\sqrt{2}} \sqrt{1 + \epsilon_L^2 + \epsilon_R^2 + \sqrt{(1 + \epsilon_L^2 + \epsilon_R^2)^2 - 4\epsilon_L^2\epsilon_R^2}} \simeq M_F \left(1 + \frac{1}{2}(\epsilon_L^2 + \epsilon_R^2) + \dots\right) \quad (101)$$

$$v_{Lf1}^0 = \frac{1}{N_{Lf1}} \frac{1 - \epsilon_L^2 + \epsilon_R^2 - \sqrt{(1 + \epsilon_L^2 + \epsilon_R^2)^2 - 4\epsilon_L^2\epsilon_R^2}}{2\epsilon_L} \simeq -\epsilon_L \left(1 - \frac{1}{2}\epsilon_L^2 - \epsilon_R^2 + \dots\right) \quad (102)$$

$$v_{Lf1}^1 = \frac{-1}{N_{Lf1}} \simeq -1 \left(1 - \frac{1}{2}\epsilon_L^2 + \dots\right) \quad (103)$$

$$v_{Rf1}^1 = \frac{-1}{N_{Rf1}} \simeq -1 + \frac{1}{2}\epsilon_R^2 + \dots \quad (104)$$

$$v_{Rf1}^2 = \frac{-1}{N_{Rf1}} \frac{2\epsilon_R}{1 + \epsilon_L^2 - \epsilon_R^2 + \sqrt{(1 + \epsilon_L^2 + \epsilon_R^2)^2 - 4\epsilon_L^2\epsilon_R^2}} \simeq -\epsilon_R \left(1 - \epsilon_L^2 - \frac{1}{2}\epsilon_R^2 + \dots\right) \quad (105)$$

F. Precision Electroweak Constraints

Precision electroweak measurements can be satisfied at tree level in this theory. Custodial symmetry protects the value of T while S requires tuning of ϵ_L . Although this theory accomodates a small S parameter, it does not explain it. The explanation lies in the UV completion.

S parametrizes deviations of couplings from those of the SM. Given some coupling in the SM (g_{SM}), the related coupling in this theory (g) is given by

$$g \simeq g_{SM} (1 + aS) \quad (106)$$

where a is some $O(\alpha)$ parameter (and other corrections are small). The coupling of light SM fermions to the W gauge boson is a particularly simple example of this.

$$g_{We\nu} = g_{We\nu_{SM}} (1 + \alpha S) \quad (107)$$

This coupling is a function of the parameter ϵ_L and can be set equal to the SM value by adjusting ϵ_L . By doing this, we will set S to zero. This adjustment has the consequence that the W' and Z' gauge bosons are fermiophobic; they have very small coupling to the light SM fermions. This can be seen in the following way. If we relate the left chiral fermionic wavefunction to the W wavefunction by

$$g_i v_{Le}^i v_{L\nu}^i = g_{We\nu_{SM}} v_W^i \quad (108)$$

where g_i is the coupling at site i , v_{Le}^i is the wavefunction of the left chiral electron at site i , $v_{L\nu}^i$ is the wavefunction of left chiral neutrino at site i and v_W^i is the wavefunction of the W gauge boson at site i . This arrangement is called

”ideal fermion delocalization” and is sufficient to set $g_{W e \nu} = g_{W e \nu \text{ SM}}$ (for small x).

$$\begin{aligned} g_{W e \nu} &= g v_{L e}^0 v_{L \nu}^0 v_W^0 + \tilde{g} v_{L e}^1 v_{L \nu}^1 v_W^1 \\ &= g_{W e \nu \text{ SM}} (v_W^0 v_W^0 + v_W^1 v_W^1) \\ &= g_{W e \nu \text{ SM}} \end{aligned} \quad (109)$$

where we have used the orthonormality of the W wavefunction. As we mentioned, the consequence of this is that the W' effectively decouples from the light fermions:

$$\begin{aligned} g_{L W' e \nu} &= g v_{L e}^0 v_{L \nu}^0 v_{W'}^0 + \tilde{g} v_{L e}^1 v_{L \nu}^1 v_{W'}^1 \\ &= g_{W e \nu \text{ SM}} (v_{W'}^0 v_{W'}^0 + v_{W'}^1 v_{W'}^1) \\ &= 0 \end{aligned} \quad (110)$$

where we have used the orthogonality of the W and W' wavefunctions in the last line. The S parameter constraint comes mainly from light fermions, so ϵ_R is tiny and the ideal delocalization condition is

$$\epsilon_L^2 = \frac{2x^2}{2 - x^2 + \sqrt{4 + x^4}} \simeq \frac{1}{2} x^2 \left(1 + \frac{1}{4} x^2 + \dots \right) \quad (111)$$

Finally, the parameter ϵ_{R_f} can be determined by taking the ratio of the masses of the light fermion and the massive partner.

$$R_{M_f}^2 = \frac{M_{f_0}^2}{M_{f_1}^2} = \frac{1 + \epsilon_L^2 + \epsilon_R^2 - \sqrt{(1 + \epsilon_L^2 + \epsilon_R^2)^2 - 4\epsilon_L^2 \epsilon_R^2}}{1 + \epsilon_L^2 + \epsilon_R^2 + \sqrt{(1 + \epsilon_L^2 + \epsilon_R^2)^2 - 4\epsilon_L^2 \epsilon_R^2}} \quad (112)$$

from which we get

$$\epsilon_R^2 = \frac{\epsilon_L^2 (1 + R_{M_f}^4) - 2R_{M_f}^2 - \epsilon_L (1 + R_{M_f}^2) \sqrt{\epsilon_L^2 (1 - R_{M_f}^2)^2 - 4R_{M_f}^2}}{2R_{M_f}^2} \quad (113)$$

G. Couplings

1. 3-Point Gauge Boson Vertices

$$g_{\gamma W W} = g v_{\gamma}^0 (v_W^0)^2 + \tilde{g} v_{\gamma}^1 (v_W^1)^2 = e [(v_W^0)^2 + (v_W^1)^2] = e \quad (114)$$

$$g_{\gamma W W'} = g v_{\gamma}^0 v_W^0 v_{W'}^0 + \tilde{g} v_{\gamma}^1 v_W^1 v_{W'}^1 = e [v_W^0 v_{W'}^0 + v_W^1 v_{W'}^1] = 0 \quad (115)$$

$$g_{\gamma W' W'} = g v_{\gamma}^0 (v_{W'}^0)^2 + \tilde{g} v_{\gamma}^1 (v_{W'}^1)^2 = e [(v_{W'}^0)^2 + (v_{W'}^1)^2] = e \quad (116)$$

$$g_{Z W W} = g v_Z^0 (v_W^0)^2 + \tilde{g} v_Z^1 (v_W^1)^2 \simeq e \frac{c_M}{s_M} \left(1 + \frac{x^2}{8c_M^2} + \dots \right) \quad (117)$$

We will come back to this coupling in section III H.

2. 4-Point Gauge Boson Vertices

$$g_{\gamma \gamma W W} = g (v_{\gamma}^0)^2 (v_W^0)^2 + \tilde{g} v_{\gamma}^1 (v_W^1)^2 = e^2 [(v_W^0)^2 + (v_W^1)^2] = e^2 \quad (118)$$

$$g_{\gamma\gamma WW'} = g(v_\gamma^0)^2 v_W^0 v_{W'}^0 + \tilde{g} v_\gamma^1 v_W^1 v_{W'}^1 = e^2 [v_W^0 v_{W'}^0 + v_W^1 v_{W'}^1] = 0 \quad (119)$$

$$g_{\gamma\gamma W'W'} = g(v_\gamma^0)^2 (v_{W'}^0)^2 + \tilde{g} v_\gamma^1 (v_{W'}^1)^2 = e^2 [(v_{W'}^0)^2 + (v_{W'}^1)^2] = e^2 \quad (120)$$

where we used the W and W' wavefunction normality conditions in the last steps. These couplings are exactly the same as in the SM and are protected by the $U(1)_{EM}$ gauge invariance.

$$g_{WWWW'} = g(v_W^0)^3 v_{W'}^0 + \tilde{g}(v_W^1)^3 v_{W'}^1 \simeq -\frac{3ex}{8s_M^2} \left[1 + x^2 \left(s_M^2 + \frac{1}{4} \frac{c_M^2}{s_M^2} - \frac{1}{4} \frac{c_M^4}{s_M^2} (1 - t_M^2)^2 - \frac{1}{4} \right) + \dots \right] \quad (121)$$

We will come back to this coupling in Section IV A where we will show that MadGraph has to be modified in order to implement this couplings.

H. g_{ZWW} and LEP

Equation 117 tells us that the deviation in the ZWW coupling is

$$\delta g_{ZWW} \simeq \frac{x^2}{8c_M^2} \simeq \frac{1}{2c_M^2} \frac{M_W^2}{M_{W'}^2} \quad (122)$$

LEP tells us that this is smaller than 0.028 at the 95% confidence level. From this, we obtain a lower bound on $M_{W'}$

$$M_{W'} > \sim \frac{M_W}{\sqrt{0.056} c_M} \simeq 380 \text{ GeV} \quad (123)$$

III. CALCHEP IMPLEMENTATION

We implemented the Minimal Higgsless Model in CalcHEP in the following way ...

IV. MADGRAPH IMPLEMENTATION

We implemented the Minimal Higgsless Model in MadGraph in the following way ...

A. $g_{WWWW'}$ and Modification of the HELAS Routines

As we pointed out in Equation 121, the coupling $g_{WWWW'}$ is given at leading order by:

$$g_{WWWW'} = -\frac{3ex}{8s_M^2} + \mathcal{O}(x^2) \quad (124)$$

which is negative for a large range of W' masses. To implement this vertex (and a few others similar to this one) required modification of the following HELAS routines in MadGraph.

- `wwwvxx.F`
- `jwwwxx.F`
- `w3w3xx.F`
- `jw3wxx.F`

As an example of the problem and the solution that we implemented, we write one of the final lines of `wwwvxx.F`.

$$\text{dvertx} = (\text{v12}^* \text{v34} + \text{v14}^* \text{v23} - \text{rTwo}^* \text{v13}^* \text{v24})^* (\text{gwwa}^{**2} + \text{gwwz}^{**2}) \quad (125)$$

For each 4-point vertex, two couplings are given by the user in their model implementation. In this case, they are called g_{ww} and $g_{ww'}$. These two couplings are then squared and added to get the overall coupling. This works for the SM, but not for the 3-site model. Since the $g_{WWWW'}$ coupling is negative, there is no way to reproduce this coupling using the HELAS routines as they stand. We modified this HELAS routine by accepting just one coupling for the 4-point vertex (we continued to call it ' g_{ww} ' to minimize the amount of code we modified) and then compute d_{vertx} like so:

$$d_{\text{vertx}} = (v_{12}^2 v_{34} + v_{14}^2 v_{23} - r_{\text{Two}}^2 v_{13}^2 v_{24}) (g_{ww}) \quad (126)$$

(More details here would be helpful.)

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