

Collinear Splitting Functions in QCD

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When the M meets the P
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Context

- QCD: interactions of gluons and (massless) quarks

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- QCD: interactions of gluons (spin 1) and quarks (spin 1/2)

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- Amplitudes \mathcal{A} : $|\mathcal{A}|^2$ is a probability density

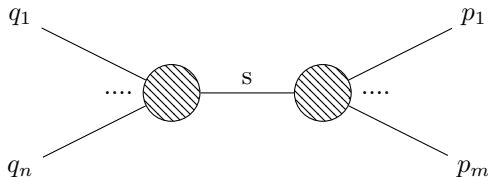
Context

- QCD: interactions of gluons and (massless) quarks
- Amplitudes \mathcal{A} : $|\mathcal{A}|^2$ is a probability density
- Collinear limit: $p_i \rightarrow p \forall i = 1, \dots, m$

Factorization and Universality

$$|\mathcal{A}_{a_1, \dots, a_m, \dots}(p_1, \dots, p_m, \{q_j\})|^2 \propto \mathcal{T}_{a, \dots}^{ss'}(p, \{q_j\}) P_{a_1, \dots, a_m}^{ss'}$$

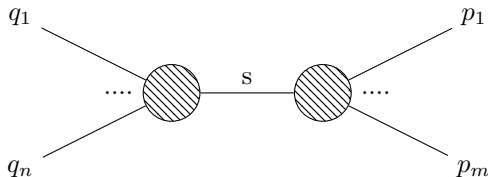
if the particles a_1, \dots, a_m become collinear.



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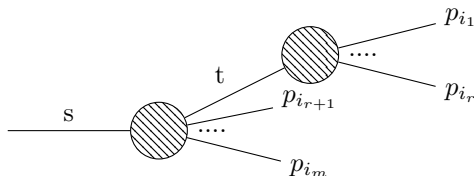


One step further

What about collinear limit of Splitting Functions ?

$$P_{a_1, \dots, a_m}^{ss'} \propto Q_{a_{i_r+1}, \dots, a_{i_m}}^{ss'tt'} P_{a_{i_1}, \dots, a_{i_r}}^{tt'}$$

if the particles a_{i_1}, \dots, a_{i_r} become collinear.



Remove spin dependence, by averaging:

$$\langle M \rangle = M^{ss'} \frac{d^{ss'}}{n_s}$$

Factorization of squared amplitudes:

$$\begin{aligned} |\mathcal{A}_{a_1, \dots, a_m, \dots}(p_1, \dots, p_m, \{q_j\})|^2 &\propto \mathcal{T}_{a, \dots}^{ss'}(p, \{q_j\}) P_{a_1, \dots, a_m}^{ss'} \\ &\Downarrow \\ |\mathcal{A}_{a_1, \dots, a_m, \dots}(p_1, \dots, p_m, \{q_j\})|^2 &\propto |\mathcal{A}_{a, \dots}(p, \{q_j\})|^2 \langle P_{a_1, \dots, a_m} \rangle \end{aligned}$$

Remove spin dependence, by averaging:

$$\langle M \rangle = M^{ss'} \frac{d^{ss'}}{n_s}$$

Factorization of splitting functions (quark case):

$$P_{a_1, \dots, a_m}^{ss'} \approx Q_{a_{i_r+1}, \dots, a_{i_m}}^{ss' tt'} P_{a_{i_1}, \dots, a_{i_r}}^{tt'}$$

⇓

$$\langle P_{a_1, \dots, a_m} \rangle \approx \langle P_{a_{i_r+1}, \dots, a_{i_m}} \rangle \langle P_{a_{i_1}, \dots, a_{i_r}} \rangle$$

Remove spin dependence, by averaging:

$$\langle M \rangle = M^{ss'} \frac{d^{ss'}}{n_s}$$

Factorization of splitting functions (gluon case):

$$P_{a_1, \dots, a_m}^{ss'} \approx Q_{a_{i_r+1}, \dots, a_{i_m}}^{ss' tt'} P_{a_{i_1}, \dots, a_{i_r}}^{tt'}$$

$$\Downarrow$$

$$\langle P_{a_1, \dots, a_m} \rangle \approx \langle P_{a_{i_r+1}, \dots, a_{i_m}} \rangle \langle P_{a_{i_1}, \dots, a_{i_r}} \rangle + \text{other terms}$$

Questions ?