Functional relations in a lattice model of dilute loops

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When the M meets the P

Louvain-la-Neuve

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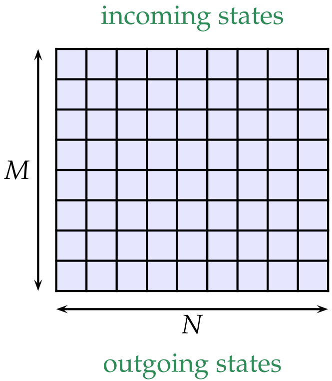
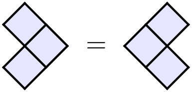
Joint work with Paul A. Pearce and Jørgen Rasmussen

arXiv:1809.07868

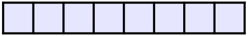
Integrable lattice models



- **Yang-Baxter equation:**



- Transfer matrix:

$T(u) \sim$ 

- The **spectral parameter** u measures the anisotropy
- Partition function:

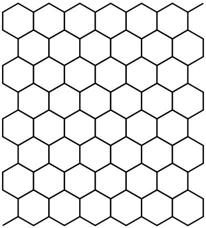
$$Z = \mathrm{Tr}(\boldsymbol{T}(u)^M)$$

- **Integrability:**

$$[\boldsymbol{T}(u), \boldsymbol{T}(v)] = 0$$

Dimers on the honeycomb lattice

- Bijection between **dimer coverings** and configurations of the **fully packed loop model:** (Kondev, de Gier, Nienhuis '96)



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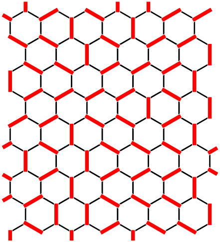

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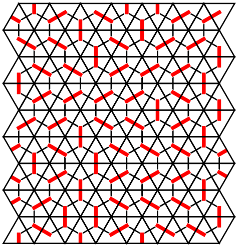

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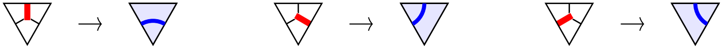

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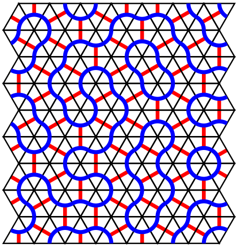

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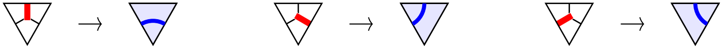

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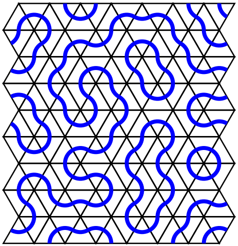

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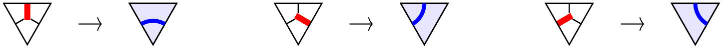

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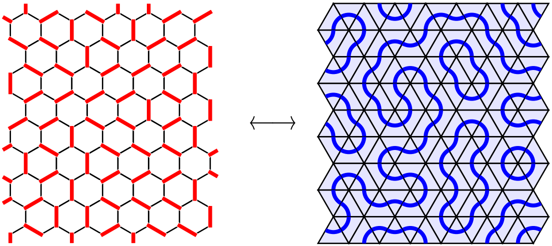

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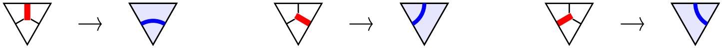

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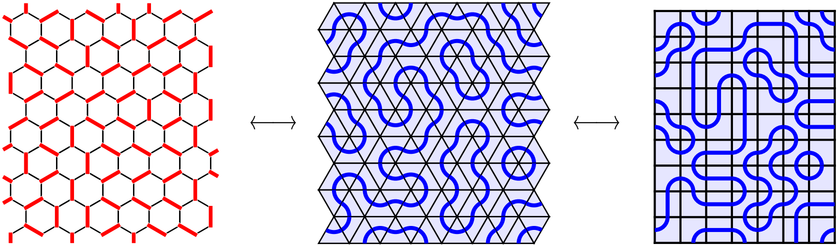

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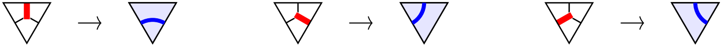
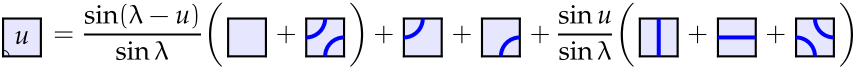


- Equivalent to a dilute loop model

The dilute loop model

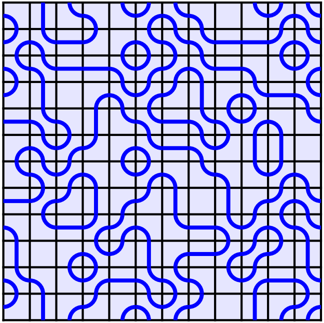
- The elementary face operator: (Nienhuis, Warnaar '93)



- u is the **spectral parameter**
- λ is the **crossing parameter**
- Loop fugacity: $\beta = 2\cos\lambda$
- Weights and partition functions:

$$W_\sigma = \beta^n \prod_f w_f \qquad Z = \sum_\sigma W_\sigma$$

- **Dimers:** $u = \lambda = \frac{\pi}{3} \qquad \beta = 1$



A loop configuration σ on the 12×12 torus

Transfer matrices and functional relations

- Two transfer matrices: $\boldsymbol{T}^{1,0}(u)$ and $\boldsymbol{T}^{0,1}(u)$

$$Z = \mathrm{Tr}\left[\left(\boldsymbol{T}^{1,0}(u)\boldsymbol{T}^{0,1}(u)\right)^M\right]$$

- **Functional relations**: $\left[f_k = \left(\frac{\sin(u+k\lambda)}{\sin\lambda}\right)^N \right]$
  - For $\lambda = \frac{\pi}{2}$ ($\beta = 0$):

$$\boldsymbol{T}^{1,0}(u)\boldsymbol{T}^{1,0}(u+\tfrac{\pi}{2}) = f_1\boldsymbol{T}^{0,1}(u) + f_0\boldsymbol{T}^{0,1}(u+\tfrac{\pi}{2}) + f_0 f_1\,\boldsymbol{I}$$

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Outlook

Overview:

- Statistical mechanics on the lattice
- Functional equations satisfied by the transfer matrices
- Underlying $s\ell(3)$ structure

Future work:

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Thank you!