Bipartite fidelity of critical dense polymers

Gilles Parez

Joint work with **Alexi Morin-Duchesne** and **Philippe Ruelle**

When the $\mathcal{M}$ meets the $\mathcal{P}$

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- Thermodynamical entropy: **Statistical mechanics**

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What about quantum mechanics?

Quantum entanglement

Two subsystems **A** and **B** in an overall **pure state**

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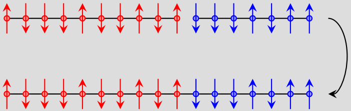
Quantifying entanglement is crucial in quantum information theory and in condensed matter physics

Bipartite fidelity of critical dense polymers

Definition

$\mathbf{A} \otimes \mathbf{B}$

$\mathbf{A} \cup \mathbf{B}$



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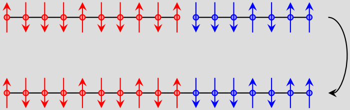

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[Dubail, Stéphan, 2011]

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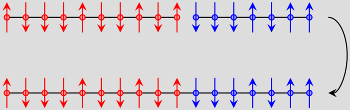

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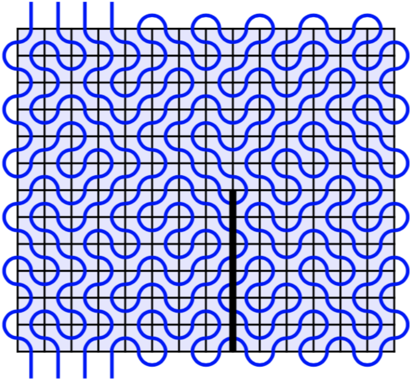
From $1d$ quantum to $2d$ statistical models

$1d$ quantum spin chain

$\Longleftrightarrow$

$2d$ statistical model [Baxter, Lieb, 70's]

Results



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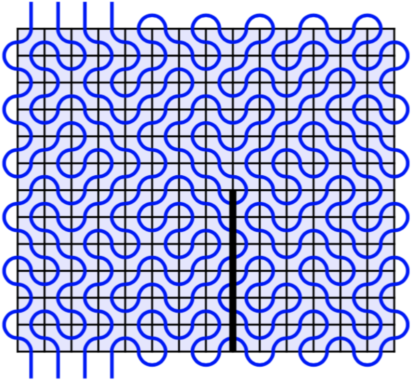

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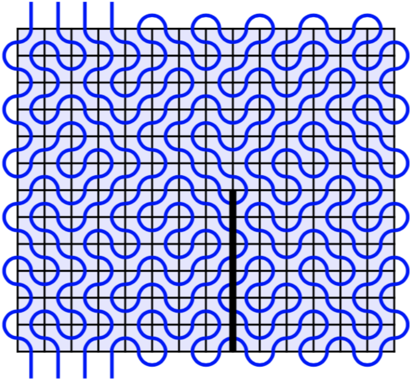

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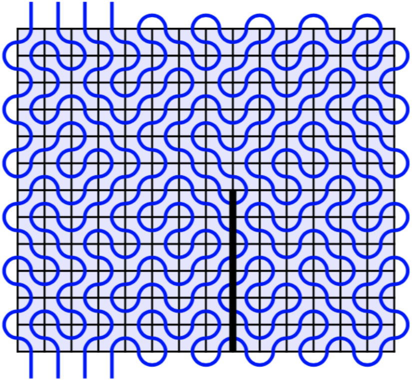

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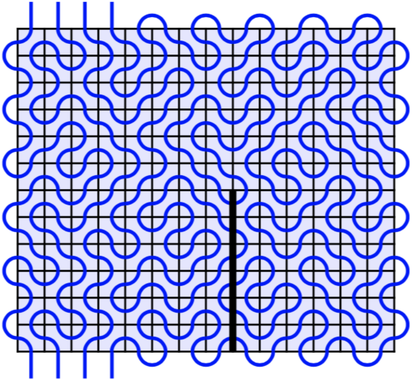

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We were also able to generalize this result to **logarithmic CFT**, both with lattice and CFT computations

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Thank you!