MoMEMta, a software for the computation of the Matrix Element weights

Florian Bury

Supervisor : Christophe Delaere

When the M meets the P

UCLouvain
Institut de recherche
en mathématique et physique

fnrs
LA LIBERTÉ DE CHERCHER

CMS

December 13, 2018

Matrix Element Method (MEM)

Objective

$P(x|\alpha)$ = Probability to observe an experimental event x, given a specific theoretical hypothesis α

Matrix Element Method (MEM)

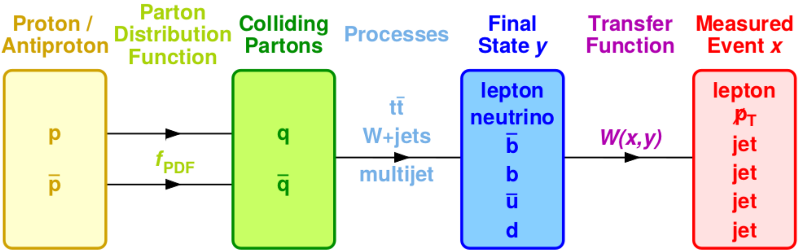
Objective

$P(x|\alpha)$ = Probability to observe an experimental event x, given a specific theoretical hypothesis α

Computation

$$P(x|\alpha) = \frac{1}{\sigma_\alpha^{vis}} \int_y d\phi(y) \int_{q_1,q_2} dq_1 dq_2 \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2) |M_\alpha(q_1,q_2,y)|^2 W(x|y)$$

Where $d\phi(y) = \left(\prod_{i=3}^{N} \frac{d^3 P_i}{2E_i(2\pi)^3}\right)(2\pi)^4 \delta^4(P_1 + P_2 - \sum_{j=3}^{N} P_j)$



arXiv:1003.1316

Matrix Element Method (MEM)

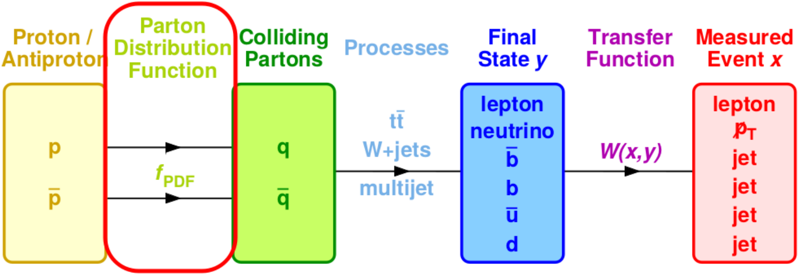
Objective

$P(x|\alpha)$ = Probability to observe an experimental event x, given a specific theoretical hypothesis α

Computation

$$P(x|\alpha) = \frac{1}{\sigma_\alpha^{vis}} \int_y d\phi(y) \int_{q_1,q_2} dq_1 dq_2 \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2) |M_\alpha(q_1,q_2,y)|^2 W(x|y)$$

Where $d\phi(y) = \left(\prod_{i=3}^{N} \frac{d^3 P_i}{2E_i(2\pi)^3}\right) (2\pi)^4 \delta^4(P_1 + P_2 - \sum_{j=3}^{N} P_j)$



arXiv:1003.1316

Matrix Element Method (MEM)

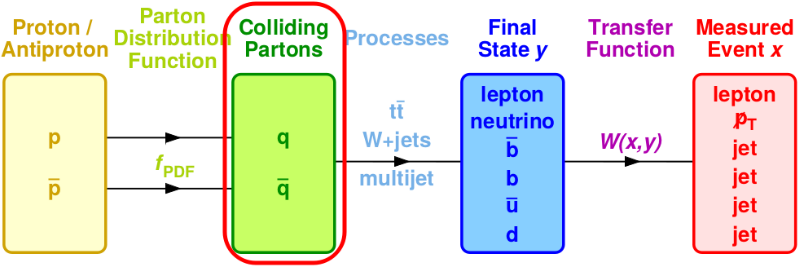
Objective

$P(x|\alpha)$ = Probability to observe an experimental event x, given a specific theoretical hypothesis α

Computation

$$P(x|\alpha) = \frac{1}{\sigma_\alpha^{vis}} \int_y d\phi(y) \int_{q_1,q_2} dq_1 dq_2 \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2) |M_\alpha(q_1,q_2,y)|^2 W(x|y)$$

$$\text{Where } d\phi(y) = \left(\prod_{i=3}^N \frac{d^3 P_i}{2E_i(2\pi)^3}\right)(2\pi)^4 \delta^4(P_1 + P_2 - \sum_{j=3}^N P_j)$$



arXiv:1003.1316

Matrix Element Method (MEM)

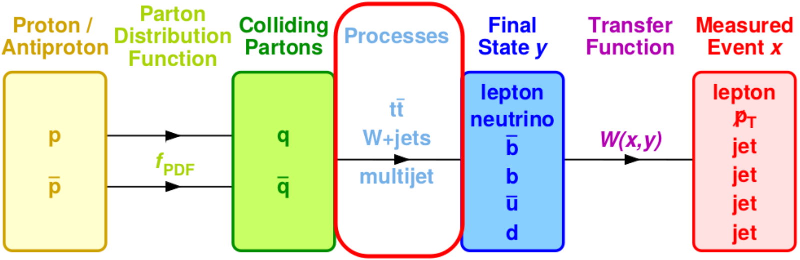
Objective

$P(x|\alpha)$ = Probability to observe an experimental event x, given a specific theoretical hypothesis α

Computation

$$P(x|\alpha) = \frac{1}{\sigma_\alpha^{vis}} \int_y d\phi(y) \int_{q_1,q_2} dq_1 dq_2 \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2) |M_\alpha(q_1, q_2, y)|^2 W(x|y)$$

Where $d\phi(y) = \left(\prod_{i=3}^{N} \frac{d^3 P_i}{2E_i(2\pi)^3}\right) (2\pi)^4 \delta^4(P_1 + P_2 - \sum_{j=3}^{N} P_j)$



arXiv:1003.1316

Matrix Element Method (MEM)

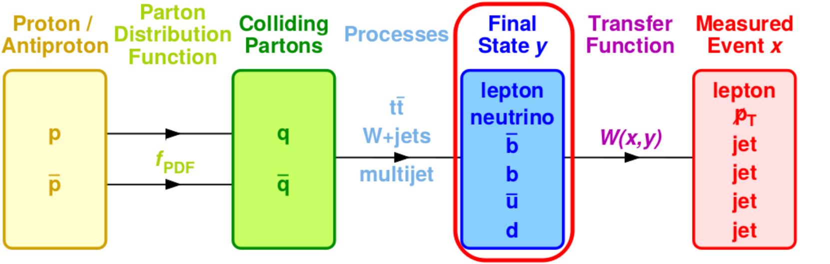
Objective

$P(x|\alpha)$ = Probability to observe an experimental event x, given a specific theoretical hypothesis α

Computation

$$P(x|\alpha) = \frac{1}{\sigma_\alpha^{vis}} \int_y d\phi(y) \int_{q_1,q_2} dq_1 dq_2 \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2) |M_\alpha(q_1,q_2,y)|^2 W(x|y)$$

Where $d\phi(y) = \left(\prod_{i=3}^{N} \frac{d^3 P_i}{2E_i(2\pi)^3}\right)(2\pi)^4 \delta^4(P_1 + P_2 - \sum_{j=3}^{N} P_j)$



arXiv:1003.1316

Matrix Element Method (MEM)

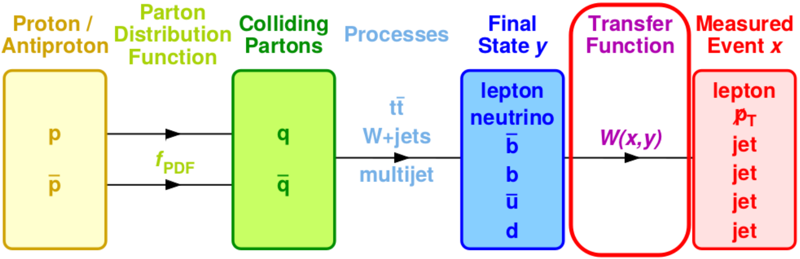
Objective

$P(x|\alpha)$ = Probability to observe an experimental event x, given a specific theoretical hypothesis α

Computation

$$P(x|\alpha) = \frac{1}{\sigma_\alpha^{vis}} \int_y d\phi(y) \int_{q_1,q_2} dq_1 dq_2 \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2) |M_\alpha(q_1,q_2,y)|^2 W(x|y)$$

$$\text{Where } d\phi(y) = \left(\prod_{i=3}^{N} \frac{d^3 P_i}{2E_i(2\pi)^3} \right) (2\pi)^4 \delta^4(P_1 + P_2 - \sum_{j=3}^{N} P_j)$$



arXiv:1003.1316

Matrix Element Method (MEM)

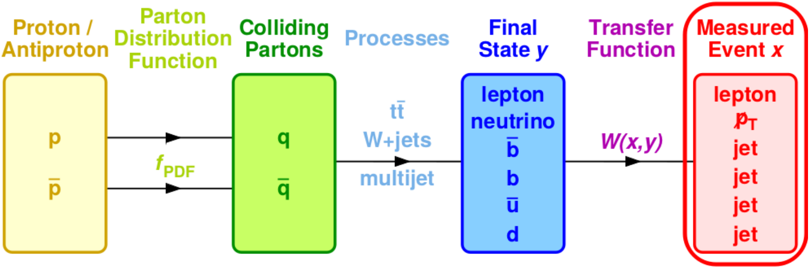
Objective

$P(x|\alpha)$ = Probability to observe an experimental event x, given a specific theoretical hypothesis α

Computation

$$P(x|\alpha) = \frac{1}{\sigma_\alpha^{vis}} \int_y d\phi(y) \int_{q_1,q_2} dq_1 dq_2 \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2) |M_\alpha(q_1,q_2,y)|^2 W(x|y)$$

Where $d\phi(y) = \left(\prod_{i=3}^N \frac{d^3 P_i}{2E_i(2\pi)^3}\right)(2\pi)^4 \delta^4(P_1 + P_2 - \sum_{j=3}^N P_j)$



arXiv:1003.1316

Matrix Element Method (MEM)

Objective

$P(x|\alpha)$ = Probability to observe an experimental event x, given a specific theoretical hypothesis α

Computation

$$P(x|\alpha) = \frac{1}{\sigma_\alpha^{vis}} \int_y d\phi(y) \int_{q_1,q_2} dq_1 dq_2 \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2) |M_\alpha(q_1,q_2,y)|^2 W(x|y)$$

Where $d\phi(y) = \left(\prod_{i=3}^{N} \frac{d^3P_i}{2E_i(2\pi)^3}\right)(2\pi)^4 \delta^4(P_1 + P_2 - \sum_{j=3}^{N} P_j)$

Applications

- Likelihood : $-ln(L) = -\sum_{i=1}^{N} ln\left(P(x_i|\alpha)\right)$
- Dicriminant between two hypotheses α and β (eg sig and bkgd) : $\mathcal{D}(x) = \left(1 + \frac{P(x|\alpha)}{P(x|\beta)}\right)^{-1}$

Matrix Element Method (MEM)

Objective

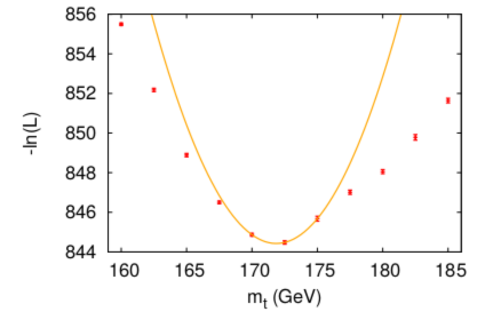
$P(x|\alpha)$ = Proba … , given a specific

Computation

$P(x|\alpha) = \frac{1}{\sigma_\alpha^{vis}}$ … $, q_2, y)|^2 W(x|y)$

Wher … $\sum_{j=3}^{N} P_j)$

1



Applications

- Likelihood : $-ln(L) = -\sum_{i=1}^{N} ln\left(P(x_i|\alpha)\right)$
- Dicriminant between two hypotheses α and β (eg sig and bkgd) :
  $\mathcal{D}(x) = \left(1 + \frac{P(x|\alpha)}{P(x|\beta)}\right)^{-1}$

[1] arXiv:1007.3300

Matrix Element Method (MEM)

Objective

$P(x|\alpha) =$ Probabili

nt x, given a specific

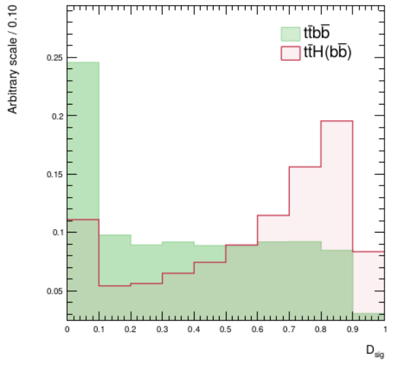
Computation

$P(x|\alpha) = \frac{1}{\sigma_\alpha^{vis}} \int_y d\varphi$

$_\alpha(q_1, q_2, y)|^2 W(x|y)$

Where d

$- \sum_{j=3}^{N} P_j)$



Applications

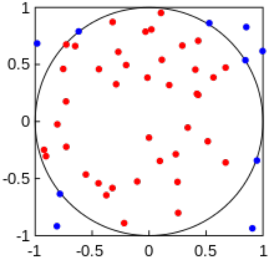
2

- Likelihood : $-ln(L) = -\sum_{i=1}^{N} ln\left(P(x_i|\alpha)\right)$
- Dicriminant between two hypotheses α and β (eg sig and bkgd) :

$$\mathcal{D}(x) = \left(1 + \frac{P(x|\alpha)}{P(x|\beta)}\right)^{-1}$$

[2] arXiv:1805.08555

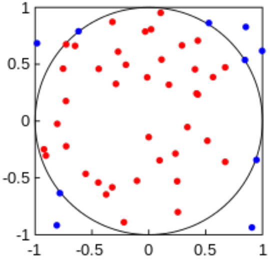
Monte Carlo Integration



[1]

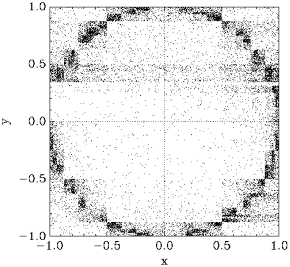
[1]wikipedia.org/wiki/Monte_Carlo_integration

Monte Carlo Integration



⇒

Stratified sampling

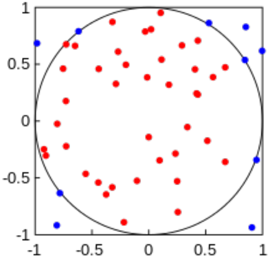


[2]

---

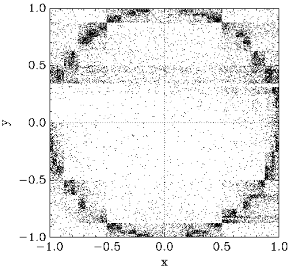
[2] wikipedia.org/wiki/Monte_Carlo_integration

Monte Carlo Integration



$\Longrightarrow$

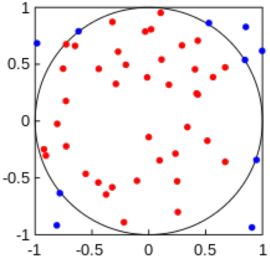
Stratified sampling



... But takes a lot of computation efforts in high dimensions

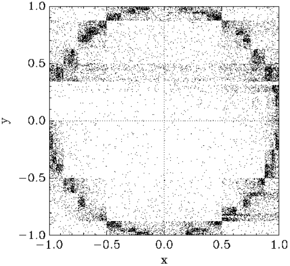
Alternative : **Importance sampling**

Monte Carlo Integration



$\Rightarrow$

Stratified sampling

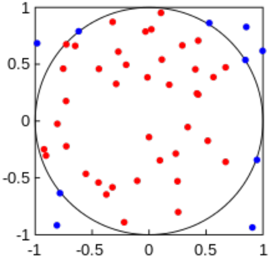


... But takes a lot of computation efforts in high dimensions

Alternative : **Importance sampling**

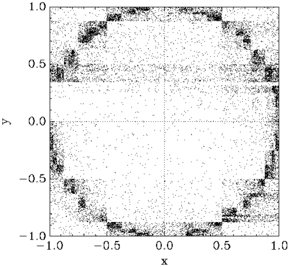
1. $S_\alpha = \frac{1}{N}\sum_{i=1}^{N}\frac{f(\vec{x}_i)}{p(\vec{x}_i)}$ and $\sigma_\alpha^2 = \frac{1}{N-1}\left(\frac{1}{N}\sum_{i=1}^{N}\frac{f^2(\vec{x}_i)}{p(\vec{x}_i)} - S_\alpha^2\right)$
2. $p(\vec{x}) \to p'(\vec{x})$ to reduce σ_α^2
3. $\alpha = \alpha + 1$ and back to step **1** unless σ_α^2 stops decreasing

Final result : $\textit{Integral} = \bar{S} = \bar{\sigma}^2 \sum_\alpha \frac{S_\alpha}{\sigma_\alpha^2}$ and $\frac{1}{\bar{\sigma}^2} = \sum_\alpha \frac{1}{\sigma_\alpha^2}$



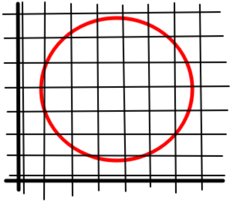
$\Longrightarrow$

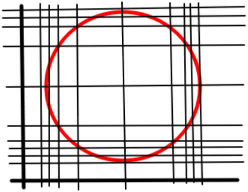
Stratified sampling



... But takes a lot of computation efforts in high dimensions

Alternative : **Importance sampling**





MoMEMta

A Modular toolkit for the Matrix Element Method at the LHC

Reminder

$$P(x|\alpha) = \frac{1}{\sigma_\alpha^{vis}} \int_y d\phi(y) \int_{q_1,q_2} dq_1 dq_2 \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2) |M_\alpha(q_1, q_2, y)|^2 W(x|y)$$

MoMEMta

A Modular toolkit for the Matrix Element Method at the LHC

Reminder

$$P(x|\alpha) = \frac{1}{\sigma_\alpha^{vis}} \int_y d\phi(y) \int_{q_1,q_2} dq_1 dq_2 \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2) |M_\alpha(q_1,q_2,y)|^2 W(x|y)$$

Approximation : factorization of W

$$W(x|y) = \prod_{i=1}^{n} W(x^i|y^i) \text{ for each particle } i$$

$$W(x^i|y^i) = W^E(x^i|y^i)\ W^\eta(x^i|y^i)\ W^\phi(x^i|y^i)$$

MoMEMta

A Modular toolkit for the Matrix Element Method at the LHC

Reminder

$$P(x|\alpha) = \frac{1}{\sigma_\alpha^{vis}} \int_y d\phi(y) \int_{q_1,q_2} dq_1 dq_2 \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2) |M_\alpha(q_1,q_2,y)|^2 W(x|y)$$

Approximation : factorization of W

$$W(x|y) = \prod_{i=1}^{n} W(x^i|y^i) \text{ for each particle } i$$

$$W(x^i|y^i) = W^E(x^i|y^i)\ W^\eta(x^i|y^i)\ W^\phi(x^i|y^i)$$

The integrant has sharp peaks, either in

- $|M_\alpha|^2$: due to propagator enhancements
- $W(x|y)$: resolutions of the different detected variables

Reminder

$$P(x|\alpha) = \frac{1}{\sigma_\alpha^{vis}} \int_y d\phi(y) \int_{q_1,q_2} dq_1 dq_2 \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2) |M_\alpha(q_1,q_2,y)|^2 W(x|y)$$

Approximation : factorization of W

$$W(x|y) = \prod_{i=1}^{n} W(x^i|y^i) \text{ for each particle } i$$

$$W(x^i|y^i) = W^E(x^i|y^i)\ W^\eta(x^i|y^i)\ W^\phi(x^i|y^i)$$

The integrant has sharp peaks, either in

- $|M_\alpha|^2$: due to propagator enhancements
- $W(x|y)$: resolutions of the different detected variables

Objective : strength of each peak mapped onto a single variable of integration

For $W(x|y)$: $d\phi(y) = \left(\prod_{i=3}^{N} \frac{d^3 P_i}{2E_i (2\pi)^3}\right) (2\pi)^4 \delta^4(P_1 + P_2 - \sum_{j=3}^{N} P_j)$

Where MoMEMta appears

Integration variables $\longrightarrow$ Lorentz invariants (Breit-Wigner resonances)

Straightforward

Where MoMEMta appears

Integration variables $\longrightarrow$ Lorentz invariants (Breit-Wigner resonances)

Straightforward

Integration variables $\longleftarrow$ Lorentz invariants (Breit-Wigner resonances)

Hard and sometimes not analytical

+

Integration of the δ^4

Where MoMEMta appears

Integration variables $\longrightarrow$ Lorentz invariants (Breit-Wigner resonances)

Straightforward

Integration variables $\longleftarrow$ Lorentz invariants (Breit-Wigner resonances)

Hard and sometimes not analytical

$+$

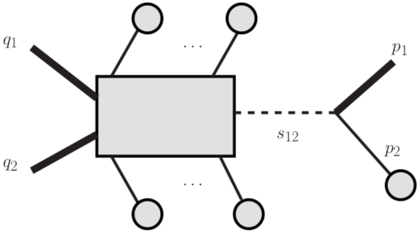
Integration of the δ^4

$\Downarrow$

MoMEMta

- Main Blocks : change of variables and integration of the δ^4
- Secondary Blocks : change of variables

Main Block B :



Main Block B :

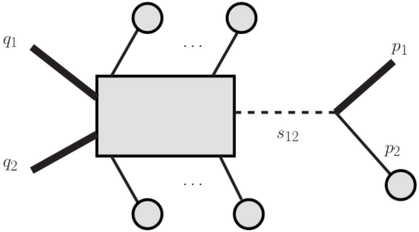
Removes q_1, q_2 and $\vec{P}_1$

Integrate over s_{12}

Part of the integrant concerned by this change :

$$dq_1 dq_2 \frac{d^3 P_1}{(2\pi)^3 2E_1} (2\pi)^4 \delta^4 (P_{in} - P_{fin})$$
$$\downarrow$$
$$\frac{1}{4\pi E_1} ds_{12} \times J$$

$$J = \frac{E_1}{s} |p2zE_1 - E_2 p_{1z}|^{-1}$$



MoMEMta Blocks examples

Main Block B :

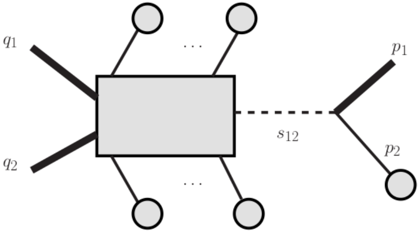
Removes q_1, q_2 and $\vec{P}_1$

Integrate over s_{12}

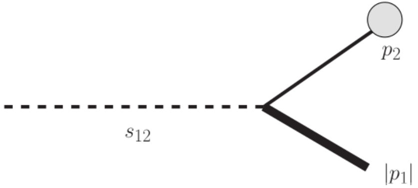
Part of the integrant concerned by this change :

$$dq_1 dq_2 \frac{d^3P_1}{(2\pi)^3 2E_1}(2\pi)^4\delta^4(P_{in} - P_{fin})$$
$$\downarrow$$
$$\frac{1}{4\pi E_1} ds_{12} \times J$$

$$J = \frac{E_1}{s}|p2zE_1 - E_2 p_{1z}|^{-1}$$



Secondary Block C/D :

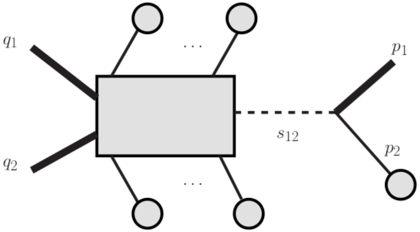


MoMEMta Blocks examples

Main Block B :
Removes q_1, q_2 and $\vec{P}_1$
Integrate over s_{12}
Part of the integrant concerned by this change :

$$dq_1 dq_2 \frac{d^3P_1}{(2\pi)^3 2E_1}(2\pi)^4\delta^4(P_{in} - P_{fin})$$
$$\downarrow$$
$$\frac{1}{4\pi E_1} ds_{12} \times J$$

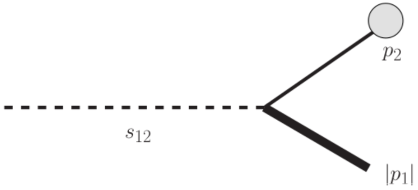
$$J = \frac{E_1}{s}|p2zE_1 - E_2 p_{1z}|^{-1}$$



Secondary Block C/D :

$$\frac{d^3P_1}{(2\pi)^3 2E_1}$$
$$\downarrow$$
$$\frac{1}{(2\pi)^3 2E_1} d\phi_1 d\theta_1 ds_{12} \times J$$

$$J = \frac{E_1}{2} sin\theta_1 |\vec{P}_1|^2| \ |\vec{P}_1|E_2 - E1\hat{P}_1.\vec{P}_2|^{-1}$$



MoMEMta + Deep Neural Network (DNN)

Main issue : slow computations ($\sim 5 - 10\ s/event$)

MoMEMta + Deep Neural Network (DNN)

Main issue : slow computations ($\sim 5 - 10\ s/event$)

$\downarrow$

Treat $P(x|\alpha)$ as a function of x ($= P_1, P_2, P_3, ..., MET$) and use a DNN a an approximation : $f_{DNN}(x) \simeq P(x|\alpha)$

MoMEMta + Deep Neural Network (DNN)

Main issue : slow computations ($\sim 5 - 10\ s/event$)

$\downarrow$

Treat $P(x|\alpha)$ as a function of x ($= P_1, P_2, P_3, ..., MET$) and use a DNN a an approximation : $f_{DNN}(x) \simeq P(x|\alpha)$

PRO : apart from training, $\sim 20\mu s/event$

CON : inevitable loss in performances

MoMEMta + Deep Neural Network (DNN)

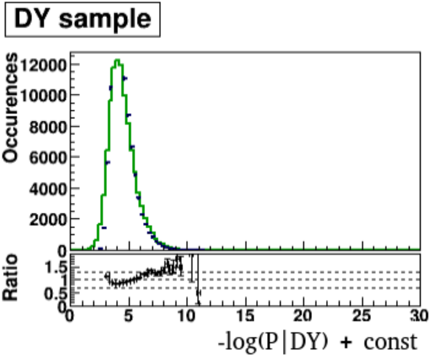
Main issue : slow computations ($\sim 5 - 10\ s/event$)

$\downarrow$

Treat $P(x|\alpha)$ as a function of x ($= P_1, P_2, P_3, ..., MET$) and use a DNN a an approximation : $f_{DNN}(x) \simeq P(x|\alpha)$

PRO : apart from training, $\sim 20\mu s/event$

CON : inevitable loss in performances



MoMEMta + Deep Neural Network (DNN)

Main issue : slow computations ($\sim 5 - 10\ s/event$)

$\downarrow$

Treat $P(x|\alpha)$ as a function of x ($= P_1, P_2, P_3, ..., MET$) and use a DNN a an approximation : $f_{DNN}(x) \simeq P(x|\alpha)$

PRO : apart from training, $\sim 20\mu s/event$

CON : inevitable loss in performances

$$D(x) = \frac{P(x|t\bar{t}\rightarrow llbb)}{P(x|t\bar{t}\rightarrow llbb)+P(x|DY\rightarrow llbb)}$$