Multiple Emission Kernels for Parton Showers

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What does a parton shower do?

- Bridge between hard matrix element and non-perturbative physics
- Facilitates resummation of leading logarithmic contributions
- Uses emission kernels to describe rate of emission for a propagating quark/gluon to emit an additional quark/gluon
- Contains mapping to factorise emissions within the phase space

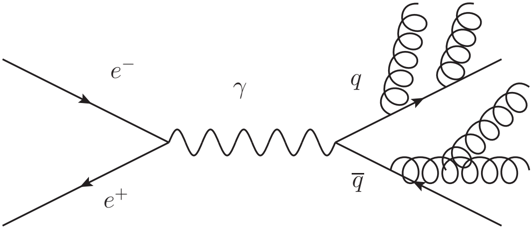


Diagram of an e^+e^- collision and possible parton shower

The future of parton showers

Push to higher orders and improved accuracy :

- Design new parton shower, beyond $1 \rightarrow 2$ branching
- Global recoil scheme for one and more emissions
- Include more spin and colour interferences
- Use higher-order emission kernels

Will be central to address:

- Singularity structure for NNLO matching, increasingly important
- Lack of systematic expansion of uncertainties at higher orders
- Issues with local recoil schemes[2]

[2] Dasgupta et al. 2018.

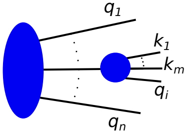
What do we need to build the parton shower?

- Start from hard process with n external legs, momenta q_i is emitter with m emissions k_l:

Diagrams with emission from external leg q_i:

$$\boldsymbol{Sp}(q_i, k_1 \ldots k_m) | \mathcal{M}(q_1, \ldots, q_{i-1}, q_i + \sum_{l=1}^{m} k_l, q_{i+1}, \ldots, q_n) \rangle$$

- Where ***Sp*** is an operator in spin and colour space
- For full matrix element need to sum over all emitters $\sum_i q_i$
- Develop a new mapping in order to:
  - Distribute recoils globally
  - Factorise without explicitly taking soft or collinear limits



Mapping for multiple emissions with global recoil

Mapping with both soft and collinear cases included:

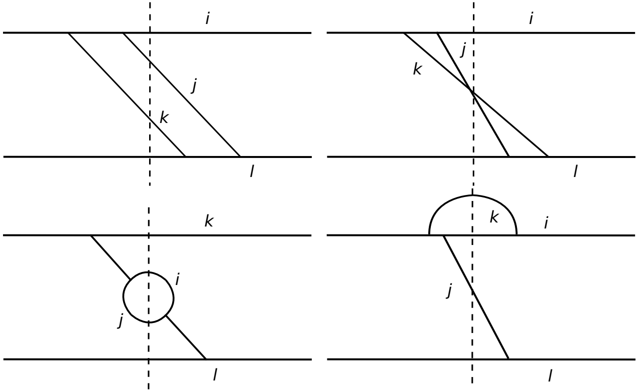
$$q_i^\mu = (1 - \sum_{l=1}^{m} \alpha_l)\, \alpha \Lambda^\mu{}_\nu p_i^\nu + y(1 - \sum_{l=1}^{m} \beta_l) n^\mu - \sum_{l=1}^{m} \sqrt{y\alpha_l\beta_l}\, n_{\perp,l}^\mu \,,$$

$$k_l^\mu = \alpha_l\, \alpha \Lambda^\mu{}_\nu p_i^\nu + y\beta_l n^\mu + \sqrt{y\alpha_l\beta_l}\, n_{\perp,l}^\mu \,,$$

$$q_k^\mu = \alpha \Lambda^\mu{}_\nu p_k^\nu \,, \qquad (k = 1, .., n \qquad k \neq i)$$

- Above case is single emitter q_i with m emissions k_l, multiple emitter case possible
- Includes soft limit ($\alpha_l, y \to 0$) and collinear limit ($y \to 0$)
- $n_\perp$ represents the transverse component
- Includes global treatment of recoil via Lorentz transformation $\Lambda^\mu{}_\nu$

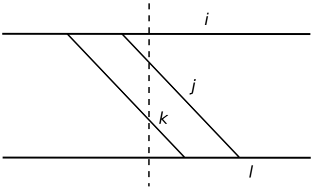
Focus on two emission case



Examples of 2 emission diagrams

- Complicated interaction of soft and collinear singularities for two emissions
- Devise framework for separating singularities

Example: Separating singularities

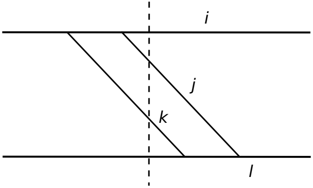


Singularities in the above diagram:

$$\frac{1}{S_{ijk}}\frac{1}{S_{ij}}\frac{1}{S_{kl}}\frac{1}{S_{jkl}}$$

- Where $S_{ijk} = S_{ij} + S_{ik} + S_{jk}$ and $S_{ij} = (q_i + q_j)^2$
- Aim to partition singularities so can be expressed as a sum

Example: Separating singularities



- Factors used for this case, can be generalised to other diagrams:

$$\frac{S_{ijk}+S_{kl}}{S_{ijk}+S_{kl}}\frac{S_{jkl}+S_{ij}}{S_{jkl}+S_{ij}} = \frac{S_{ijk}}{S_{ijk}+S_{kl}}\left[\frac{S_{jkl}}{S_{jkl}+S_{ij}}+\frac{S_{ij}}{S_{jkl}+S_{ij}}\right] + \frac{S_{kl}}{S_{ijk}+S_{kl}}\left[\frac{S_{jkl}}{S_{jkl}+S_{ij}}+\frac{S_{ij}}{S_{jkl}+S_{ij}}\right]$$

- The following limits are contained:
  - Triple collinear $S_{ijk} \to 0$, $S_{kl}, S_{jl} \not\to 0$
  - Triple collinear $S_{jkl} \to 0$, $S_{ij}, S_{ik} \not\to 0$
  - Two collinear pairs $S_{ij}S_{kl} \to 0$, $S_{ik}, S_{jk}, S_{jl} \not\to 0$
- The factorisation is safe, only one term contains triple collinear limit

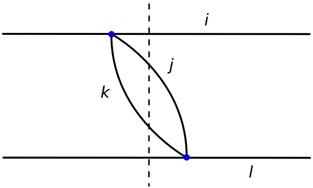
Partitioning framework

- For the two emissions from one parton case with one spectator, we have four possible triplets:

$$\sum_{i<j<k<l} \left(V_{ijk,l} + V_{ijl,k} + V_{ikl,j} + V_{jkl,i} \right)$$

- Each term contains sum of diagrams for that triplet with only the relevant singularities i.e. $V_{ijk,l}$ only contains S_{ijk} singular contributions
- Use partitioning factors, each term contributes to a different triplet:

$$1 = w_{ijk,jkl} + w_{jkl,ijk} = \frac{S_{jkl}}{S_{ijk} + S_{jkl}} + \frac{S_{ijk}}{S_{ijk} + S_{jkl}}$$



Fitting all the pieces together

Bulding blocks:

- Mapping for n-emission case
- Combinatorics to give all possible diagrams
- Partitioning framework to add together singular contributions
- Expansion to relevant order in α_s of matrix elements squared

Example: Triplet spitting for 2 emission case, $\mathcal{O}(\alpha_s)$

$$\langle n|\mathcal{M}(n)\rangle = g_s^2 \sum_{i<j<k}^{n} \sum_{\beta} \langle i,j,k|\boldsymbol{Sp}(i,j,k,i+j+k) \otimes \langle [n]_{\overline{ijk}}|\mathcal{M}([n]_{\overline{ijk}},(i+j+k))\rangle$$

- First test, emission kernel for 2 emission case:
  1. Evaluate all diagrams including self energies
  2. Combine according to partitioning framework
  3. Include mapping and phase space

Summary

Current status:

- New mapping which exposes soft and collinear singularities for up to n-emissions
- Comprehensive framework for organising the singularities up to 2-emission case
- Partial implementation in Mathematica

What's next?

- Implement mapping for 2-emission case
- Improve diagrammatic algorithmic procedure
- Test implementation in Herwig parton shower

Backup slides

Determination of emission kernels

- Need to take collinear and soft limits which allow factorisation[3]
- Where q_k is momentum of final state parton k:

  a) Soft limit $q_k = \lambda q$, $\lambda \rightarrow 0$, $|\mathcal{M}_{m+1,a...}|^2 \propto 1/\lambda^2$

  b) Collinear limit $q_k \rightarrow (1-z)q_i/z$, $|\mathcal{M}_{m+1,a...}|^2 \propto 1/q_i.q_k$

Dipole factorisation

- Consider $(m+1)$ partons, factorise out parton k to give $|\mathcal{M}_{m,a...}|^2$

$$|\mathcal{M}_{m+1,a...}|^2 \rightarrow |\mathcal{M}_{m,a...}|^2 \otimes V_{ik,j}$$

- $V_{ik,j}$ = singular factor including parton k and it's interaction with partons i and j from the m parton amplitude

[3]Catani and Seymour 1997.

Mapping in two emission case

$$
\begin{aligned}
q_i^\mu =&(1-(\alpha_1+\alpha_2))\,\alpha\Lambda^\mu{}_\nu p_i^\nu + y(1-(\beta_1+\beta_2)n^\mu \\
&-\sqrt{y\alpha_1\beta_1}n^\mu_{\perp,1} - \sqrt{y\alpha_2\beta_2}n^\mu_{\perp,2}\,, \\
k_1^\mu =&\alpha_1\,\alpha\Lambda^\mu{}_\nu p_i^\nu + y\beta_1 n^\mu + \sqrt{y\alpha_1\beta_1}n^\mu_{\perp,1}\,, \\
k_2^\mu =&\alpha_2\,\alpha\Lambda^\mu{}_\nu p_i^\nu + y\beta_2 n^\mu + \sqrt{y\alpha_2\beta_2}n^\mu_{\perp,2}\,, \\
q_k^\mu =&\alpha\Lambda^\mu{}_\nu p_k^\nu\,, \qquad (k=1,..,n \qquad k\neq i)
\end{aligned}
$$

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