A new tool for mastering the Singular Limits

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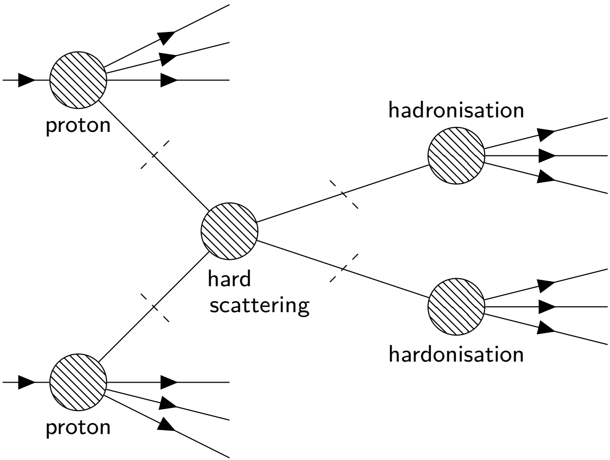
18th MCnet Meeting, Louvain, January 24th, 2019

LHC Event Factorisation



NLO calculations

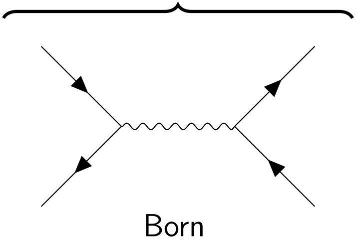
$$|\mathcal{M}|^2 = \mathcal{A}_1 + \alpha_S \mathcal{A}_2 + \alpha_S^2 \mathcal{A}_3 + \ldots$$

NLO calculations

$$|\mathcal{M}|^2 = \mathcal{A}_1 + \alpha_S \mathcal{A}_2$$

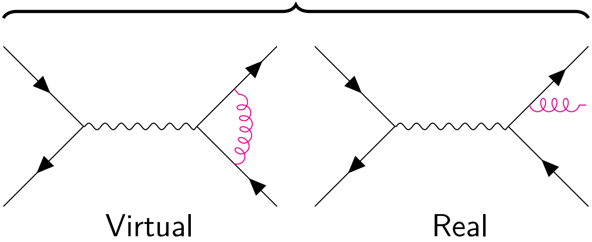
NLO calculations

$$|\mathcal{M}|^2 = \color{red}{\mathcal{A}_1} + \alpha_S \mathcal{A}_2$$



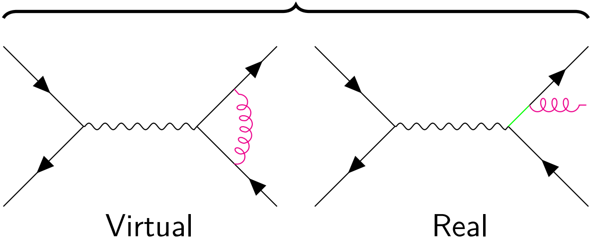
NLO calculations

$$|\mathcal{M}|^2 = \mathcal{A}_1 + \alpha_S \mathcal{A}_2$$



Problem!

$$|\mathcal{M}|^2 = \mathcal{A}_1 + \alpha_S \mathcal{A}_2$$



- Propagator introduces factor: $\frac{1}{(p_g+p_q)^2} = \frac{1}{2E_g E_q(1-\cos\theta_{qg})}$
- Goes singular as gluon or quark go soft or collinear
- Resolve this using symmetry between virtual and real

Subtraction terms

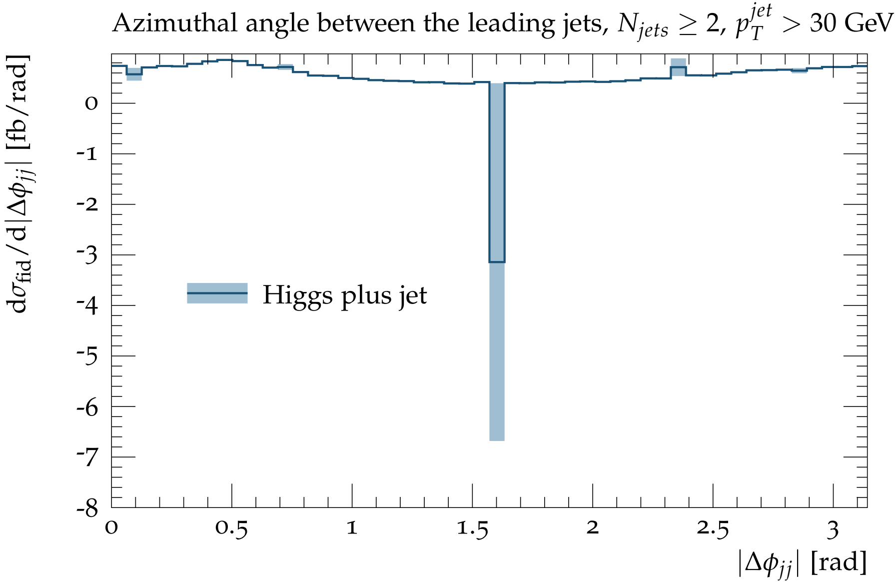
$$\mathrm{d}\sigma^{NLO} = \mathrm{d}\phi_n \, [B + V] + \mathrm{d}\phi_{n+1} \, R$$

$$= \mathrm{d}\phi_n \, [B + V] + \mathrm{d}\phi_{n+1} \, R + \mathrm{d}\phi_{n+1} \, S - \mathrm{d}\phi_{n+1} \, S$$

$$= \mathrm{d}\phi_n \left[B + V + \int_{\phi_1} S \right] + \mathrm{d}\phi_{n+1} \, [R - S]$$

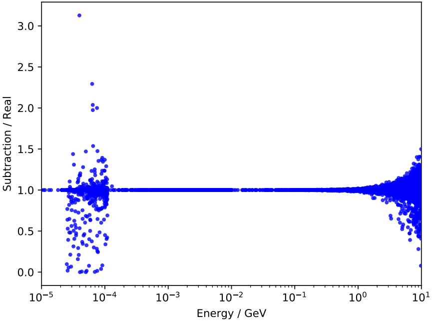
- The terms labelled S are called the Subtraction Dipoles
- They are defined to mimic the singular behaviour of R/V and be analyitcally integrable over ϕ_1
- Both integrands are now finite and can be Monte Carlo integrated 👌

Mismatch between singularities



Current diagnosis tools

Energy of softest gluon in event against S/R for that event:



- $\sim 10^{-4}$GeV is the problem area
- Difficult to work out much more though (other than problem is not numerical)
- Points are smeared because of randomised momenta

The SingularPhasespace approach

- Generate events with progressively lower gluon energy
- Only adjust other particles for the sake of momentum conservation
- Scatter plot should now trace out smooth line
- Allow user to choose other singular limits too

Conserving momentum

- 2 parton final state is easy: CoM frame, angles are constant, just vary energy
- In the >2 case though it is significantly more fiddly
- For example, $gg \to ggH_0$:

$$\begin{pmatrix} E_1 \\ E_1 \cos\phi_1 \sin\theta_1 \\ E_1 \sin\phi_1 \sin\theta_1 \\ E_1 \cos\theta_1 \end{pmatrix} + \begin{pmatrix} E_2 \\ E_2 \cos\phi_2 \sin\theta_2 \\ E_2 \sin\phi_2 \sin\theta_2 \\ E_2 \cos\theta_2 \end{pmatrix} + \begin{pmatrix} E_3 \\ \sqrt{E_3^2 - m_H^2} \cos\phi_3 \sin\theta_3 \\ \sqrt{E_3^2 - m_H^2} \sin\phi_3 \sin\theta_3 \\ \sqrt{E_3^2 - m_H^2} \cos\theta_3 \end{pmatrix} = \text{const}$$

- Thankfully though I can exploit someone else's hard work

Subtraction terms

Equation from before:

$$\mathrm{d}\sigma^{NLO} = \mathrm{d}\phi_n \left[B + V + \int_{\phi_1} S \right] + \mathrm{d}\phi_{n+1} \left[R - S \right]$$

- Do not want a new S for each new matrix element
- Thankfully they can be written in the form of $\mathrm{d}\sigma_B \otimes \mathrm{d}A$
- But we have just generated a phase space point of ϕ_{n+1} - what do we give to $\mathrm{d}\sigma_B\left(\phi_n\right)$?
- Hence, with the invention of Subtraction terms came a transformation $\{p_i, p_j, p_k\} \rightarrow \{\widetilde{p_{ij}}, \widetilde{p_k}\}$ that conserves momentum + virtuality

Conserving momentum

Massless case

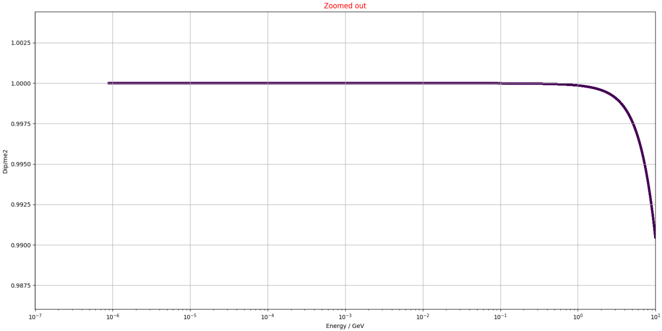
$$\widetilde{p}_k^\mu = \frac{1}{1 - y_{ij,k}} p_k^\mu$$

$$\widetilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - \frac{y_{ij,k}}{1 - y_{ij,k}} p_k^\mu$$

- Soft limit corresponds to $y_{ij,k} \to 0$
- Plan: $3 \to 2 \to 2' \to 3'$
- Similar but more complex equations for non-massive case and collinear limits

Summary

- Regulating NLO singularities is an important but subtle task
- SingularPhasespace will allow one the explore these limits in a controlled manner
- Momentum conservation will be achieved with some help from the Dipole Subtraction formalism
- Cheeky proof of concept:



Thanks!