Using MHV amplitudes in the VINCIA Helicity Shower

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In collaboration with Peter Skands, Nadine Fischer

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Work Done at Monash University

January 25^{th} 2019

Overview

1. Summarising the Vincia Antenna Shower
2. Helicity Amplitudes
3. Vincia's Helicity Shower - The Details
4. Shower Validation Tests
5. Summary

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- Began in 2007 as proof of concept by P. Z. Skands, W. T. Giele and D. A. Kosower [1]
- Plugin to Pythia, replaces its parton shower
- Had 2 main goals in mind
  - Include systematic uncertainty estimates
  - Allow matching to any LO or NLO matrix element
- 2 main versions:
  - Vincia 1: e^+e^- collisions
  - Vincia 2: e^+e^- and pp collisions [2]
- Recently released Vincia 2.204

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Comparing Vincia's and Pythia's Parton Showers

Pythia

- Markovian, no history
- Parton shower with spectator parton
- Corrects only 1st emission with full ME
- Minimal spin correlations

Vincia

- Non-Markovian, historical MEs
- Antenna shower, radiate off 2 partons in one splitting function
- Can correct up to 3 emissions with full ME
- Helicity shower, spin correlations in MEC

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Why Helicity Shower?

- $\mathcal{M}_i\left(1^{h_1},\ldots,n^{h_n}\right) = C_i(t^1,\ldots,t^n)A_i\left(p_1^{h_1},\ldots,p_n^{h_n}\right)$
- Helicity-amplitudes easier than helicity-summed amplitudes
- Organise processes based on the number of opposite helicities (all particles outgoing)
  - $\mathcal{M}[n^{\pm}]$, $\mathcal{M}[(n-1)^{\pm},1^{\mp}]=0$
  - $\mathcal{M}[(n-2)^{\pm},2^{\mp}]=MHV$
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  - etc.
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Feynman Diagrams vs Recursion Relations: The All-Gluon MHV Case

No. of External Gluons	No. of Feynman Diagrams	Relative Growth
4	4	-
5	25	6.3
6	220	8.8
7	2485	11.3
8	34300	13.8
9	559405	16.3
10	10525900	18.8

All-gluon Feynman Diagram numbers calculated by Kleiss and Kuijf [3]

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Recursion relation for $n \geq 4$ Gluons, $(n-2)+\text{hel}, 2-\text{hel}$

$$A_{\sigma_i}(i^-, j^-) = i\frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle} \qquad \langle ij \rangle \equiv \bar{u}_-(p_i)u_+(p_j)$$

Speed Testing MHV Amplitudes

$gg \to ng$ MHV amplitudes, micro-seconds per calculation

nParticles	RAMBO	MadGraph4	MHV	Ratio
4	0.671	1.868	1.496	1.451
5	0.806	7.716	2.546	3.966
6	0.931	76.434	7.940	10.771

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$q\bar{q} \to ng$ MHV amplitudes, micro-seconds per calculation

nParticles	RAMBO	MadGraph4	MHV	Ratio
4	0.855	1.551	1.596	0.927
5	0.822	3.216	2.669	1.296
6	0.935	18.579	3.447	7.024
7	1.088	236.183	14.355	17.720

MHV Amplitudes within Vincia

- Swaps incoming particles to outgoing, checks it has process
- Base class calculates all relevant spinor products
- Uses MHV wherever possible for MEC and setting polarisations
- Can also use Vincia to calculate MHV amplitudes as standalone
- The following can be calculated

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Type of Process	Number of Particles
All-Gluon	$4-6$
Single $q\bar{q}$ Pair $+$ Gluons	$4-7$
Two $q\bar{q}$ Pairs $+$ Gluons	$4, 5$
$q\bar{q}$ and $l\bar{l}$ Pairs $+$ Gluons (Z-Boson Exchange)	$4-9$

Setting up the Shower

Reminder

$$\mathcal{M}_i\left(1^{h_1},\ldots,n^{h_n}\right)=C_i(t^1,\ldots,t^n)A_i\left(p_1^{h_1},\ldots,p_n^{h_n}\right)$$

- Requires both a colour flow and a polarisation
- To understand, first need definitions:
  - $LC_i=\mathcal{M}_i^*\mathcal{M}_i$, $FC=\sum_{ij}\mathcal{M}_i^*\mathcal{M}_j$, $VC_i=FC\frac{LC_i}{\sum_{i'}LC_{i'}}$
- If no colour flow in hard process:
  - $P(h,i)=\underbrace{\frac{\mathrm{FC}^h}{\sum_{h'}\mathrm{FC}^{h'}}}_{\text{Helicity-Selection Factor}}\times\underbrace{\frac{\mathrm{LC}_i^h}{\sum_j\mathrm{LC}_j^h}}_{\text{Colour-Flow Selection Factor}}$
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Polarising the Shower with MHV Amplitudes

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- Usually polarise $2 \to 2$ or $2 \to 3$, i.e. MHV
- MHV kinematics can* be factorised into helicity and colour parts

$$\mathrm{FC}^h = |A_n^h(1,\dots,n)|^2 \left| \sum_\sigma \frac{1}{\langle \sigma(1)\sigma(2) \rangle \dots \langle \sigma(n)\sigma(1) \rangle} \mathsf{C}(\sigma(t^1),\dots,\sigma(t^n)) \right|^2$$

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Match to Full (LO) Matrix Element

- MEC:
  $\mathcal{M}_{PS}^{n+1} \to \mathcal{M}_{PS}^{n+1} \times \mathcal{R}\ , \quad \left(\mathcal{M}_{PS}^{n+1} = \mathcal{A} \times \mathcal{M}^{n}\ , \quad \mathcal{R} \sim \mathcal{M}_{\mathrm{ex}}^{n+1}/\mathcal{M}_{PS}^{n+1}\right)$

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MEC factor

$$
\begin{aligned}
\mathcal{R}(\Phi_{n+1}) = &\left|\mathcal{M}(\Phi_{n+1})\right|^2 \times \\
&\Bigg[\sum_{\Phi'_n} \mathcal{A}\left(\Phi_{n+1}/\Phi'_n\right)\ \mathcal{R}(\Phi'_n) \sum_{\Phi'_{n-1}} \Theta(t(\Phi'_n/\Phi'_{n-1}) - t(\Phi_{n+1}/\Phi'_n))\ \mathcal{A}\left(\Phi'_n/\Phi'_{n-1}\right)\ \mathcal{R}(\Phi'_{n-1}) \\
&\prod_{k=n-2}^{k\leq 1} \left(\sum_{\Phi'_k} \Theta(t(\Phi'_{k+1}/\Phi'_k) - t(\Phi'_{k+2}/\Phi'_{k+1}))\ \mathcal{A}\left(\Phi'_{k+1}/\Phi'_k\right)\ \mathcal{R}(\Phi'_k) \right) \\
&\sum_{\Phi'_0} \Theta(t(\Phi'_1/\Phi'_0) - t(\Phi'_2/\Phi'_1))\ \mathcal{A}\left(\Phi'_1/\Phi'_0\right)\ \Theta(t(\Phi'_0) - t(\Phi'_1/\Phi'_0))\ \left|\mathcal{M}(\Phi'_0)\right|^2 \Bigg]^{-1}
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Full Matrix Element   Antenna Function   Ensure Correct Shower Scale

MEC factor

$$\mathcal{R}(\Phi_{n+1}) = |\mathcal{M}(\Phi_{n+1})|^2 \times$$

$$\Bigg[\sum_{\Phi'_n} \mathcal{A}\left(\Phi_{n+1}/\Phi'_n\right)\ \mathcal{R}(\Phi'_n) \sum_{\Phi'_{n-1}} \Theta\big(t(\Phi'_n/\Phi'_{n-1}) - t(\Phi_{n+1}/\Phi'_n)\big)\ \mathcal{A}\left(\Phi'_n/\Phi'_{n-1}\right)\ \mathcal{R}(\Phi'_{n-1})$$

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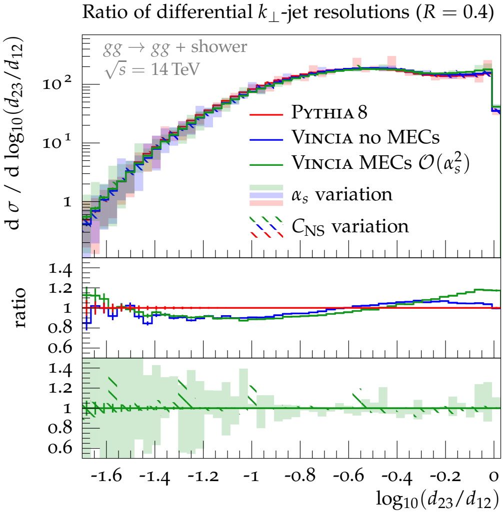
$$\sum_{\Phi'_0} \Theta\big(t(\Phi'_1/\Phi'_0) - t(\Phi'_2/\Phi'_1)\big)\ \mathcal{A}\left(\Phi'_1/\Phi'_0\right)\ \Theta\big(t(\Phi'_0) - t(\Phi'_1/\Phi'_0)\big)\ |\mathcal{M}(\Phi'_0)|^2 \Bigg]^{-1}$$

Sum All Shower Histories   Possible Born-Level Processes

Effects on Early Branchings, $gg \to gg$

- Corrected first 2 emissions
- Large d_{23} (i.e. $\log_{10}(d_{23}/d_{12}) \sim 0$) expect MECs important
- Pythia has no MECs
- Vincia and Pythia showers intrinsically different



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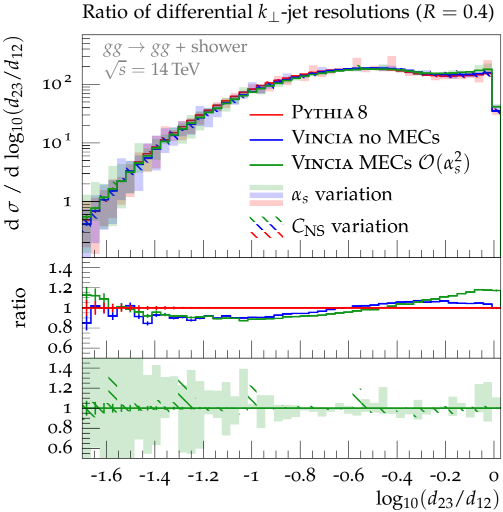

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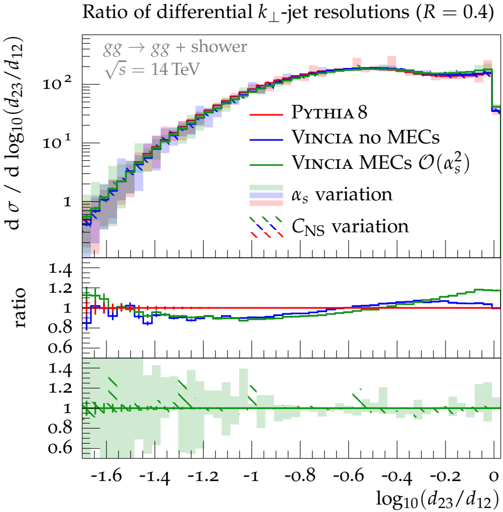

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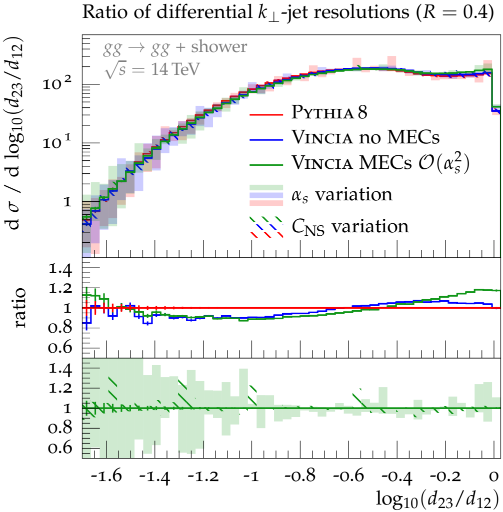

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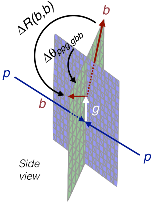

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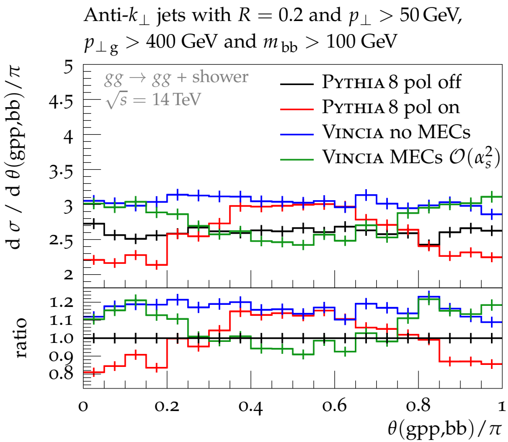

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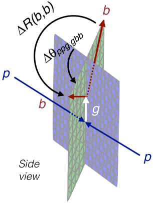

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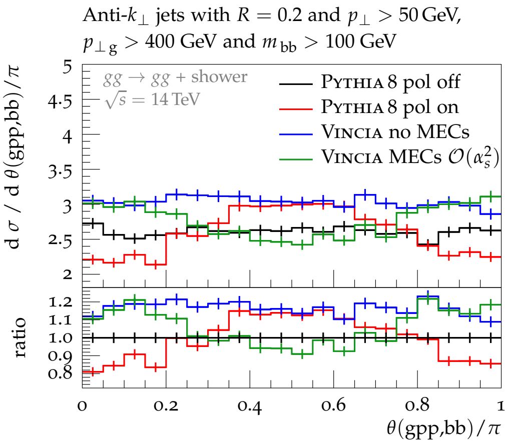

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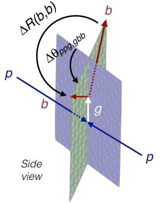

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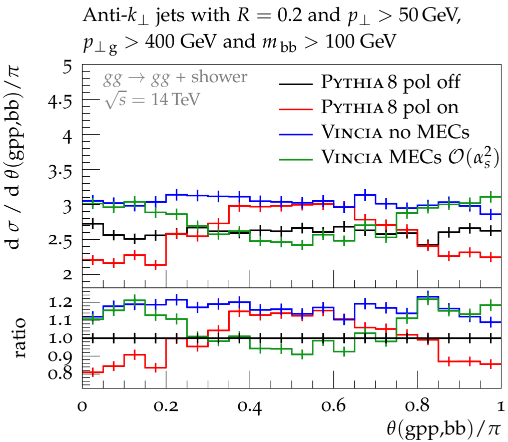

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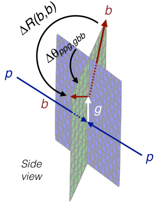

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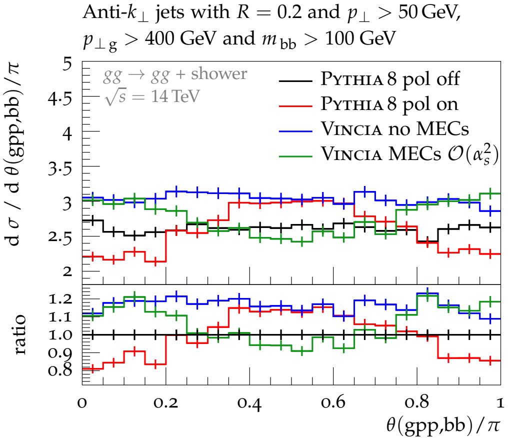
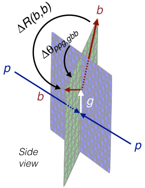
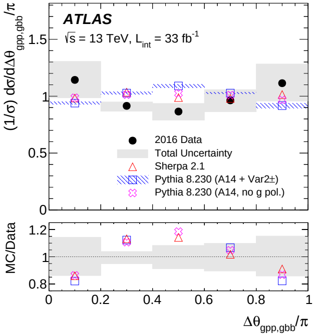


Polarisation Effects: a New ATLAS Measurement

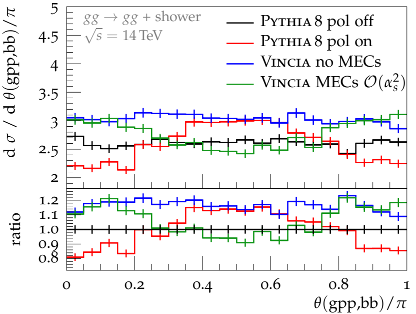
- Recent measurement of gluon splitting at small opening angle (arxiv:1812.09283)
- Sherpa $2 \to n$ + PS is flat, Pythia opposite shape, Vincia correct shape



Anti-$k_\perp$ jets with $R = 0.2$ and $p_\perp > 10$ GeV, $p_{\perp j} > 450$ GeV and m_{bb} unrestricted



Anti-$k_\perp$ jets with $R = 0.2$ and $p_\perp > 50$ GeV, $p_{\perp \mathrm{g}} > 400$ GeV and $m_{\mathrm{bb}} > 100$ GeV



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- Vincia adds recursive MECs, giving better predictions in hard, wide angle limits
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Spinor-Helicity Formalism

Spinors

- $v_{\mp}(p) = u_{\pm}(p) = \frac{1}{2}\left(1 \pm \gamma^5\right) u(p)$
- $\bar{v}_{\mp}(p) = \bar{u}_{\pm}(p) = \bar{u}(p)\frac{1}{2}\left(1 \mp \gamma^5\right)$
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- $[ij] \equiv \bar{u}_+(i) u_-(j) = \langle ji \rangle^*$
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- Generate unpolarised antenna branching
  - Means shower without MECs has no spin correlations
- Then choose a polarisation for i, j, k
  - $P(h_A, h_B; h_i, h_j, h_k) = \frac{\mathcal{A}(h_A, h_B; h_i, h_j, h_k)}{\sum\limits_{h_i, h_j, h_k} \mathcal{A}(h_A, h_B; h_i, h_j, h_k)}$
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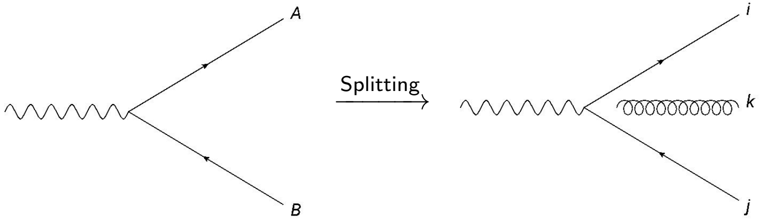

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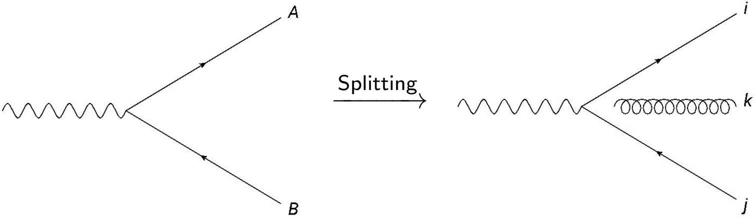

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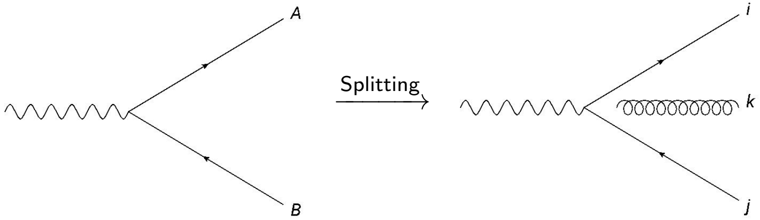

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$$A_{\sigma_i}(q^-, i^-, \bar{q}^+) = \frac{\langle qi\rangle^3\langle \bar{q}i\rangle}{\langle \bar{q}q\rangle\langle q1\rangle\langle 12\rangle\ldots\langle (n-2)\bar{q}\rangle}$$

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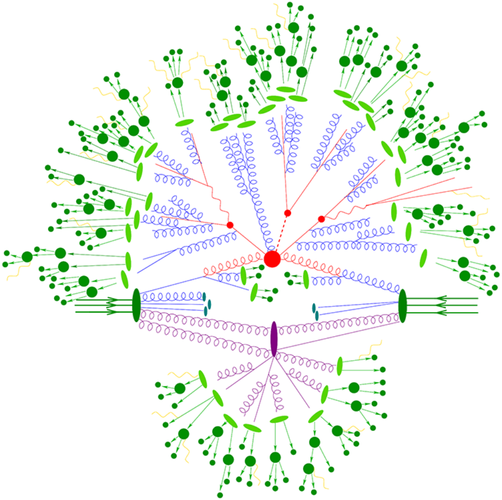

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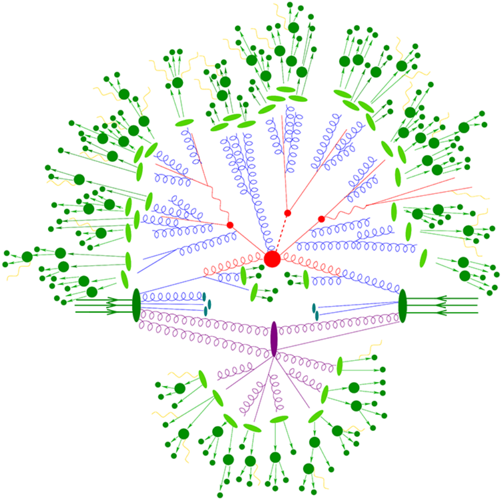

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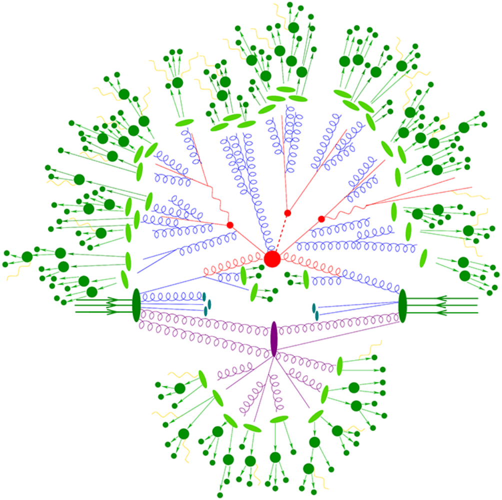

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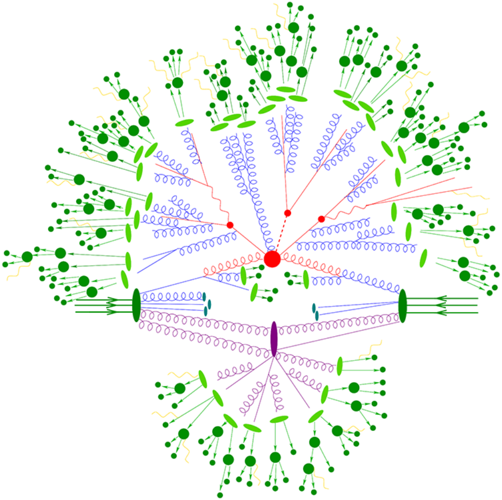

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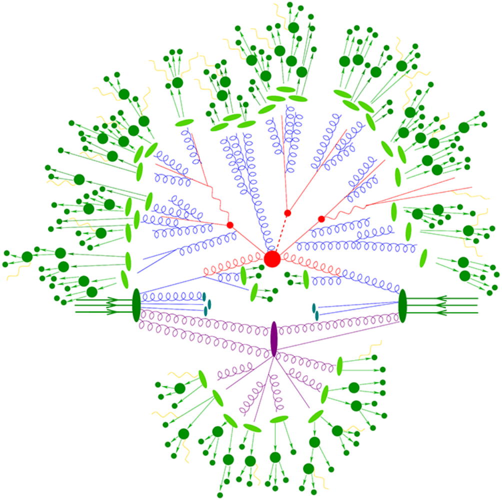

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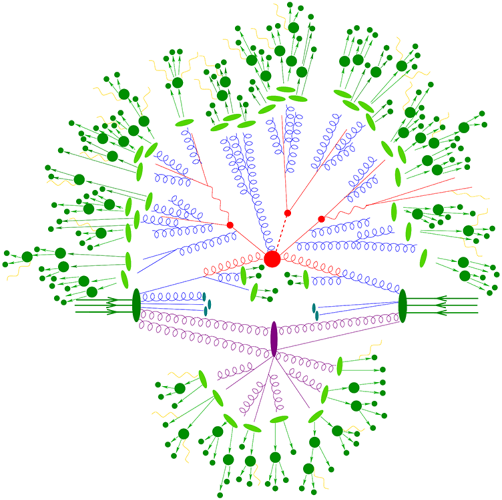

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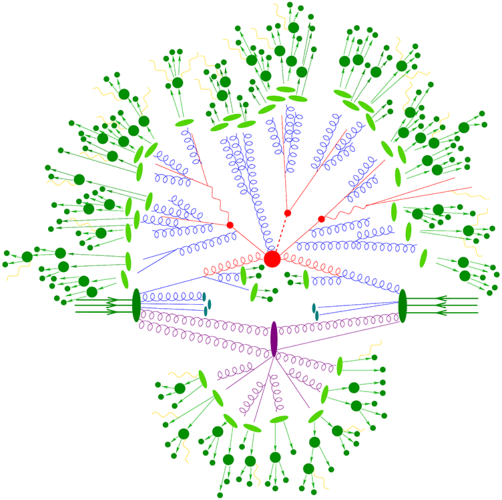

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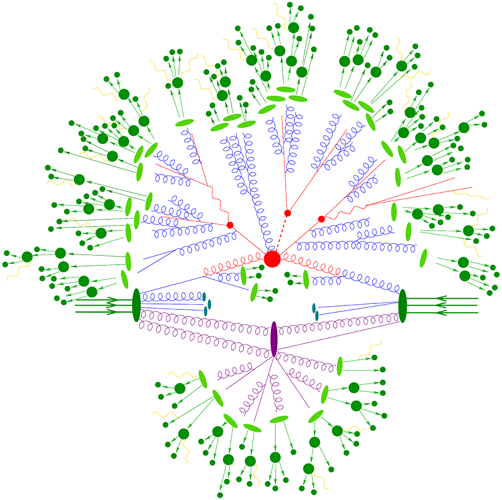

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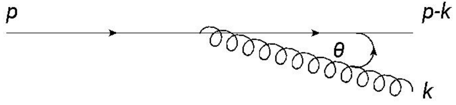
What a Parton Shower Does

- Bremsstrahlung occurs in initial- (ISR) and final- (FSR) state radiation
- Recursively generates emissions off a parton
- Assumes radiation to be soft and/or collinear
  - Logarithmic enhancement in emission probability
  - Describes soft and/or collinear radiation very well
- Poor job of describing hard, wide-angle emissions



What a Parton Shower Does

- Bremsstrahlung occurs in initial- (ISR) and final- (FSR) state radiation
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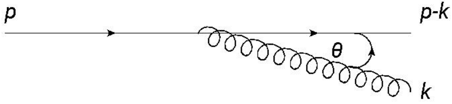

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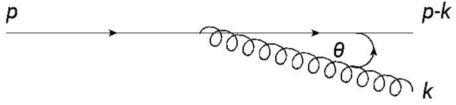

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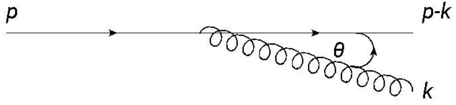

Pythia's Parton Shower

- Markov chain of collinear emissions off single partons
  - No concept of history
  - Soft and wide-angled emissions artificially separated
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- Dipole shower. Spectator parton for momentum conservation
- Unpolarised partons, minimal spin correlations
- Uses the full matrix element to correct first emission (MEC)
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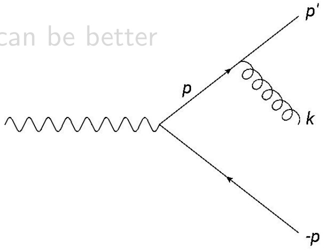
- Conclusion: is good, can be better



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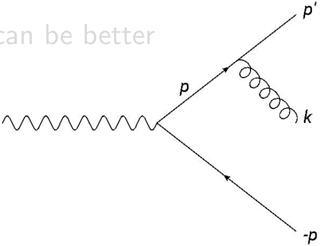

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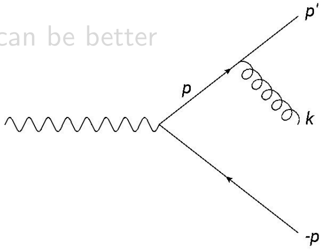

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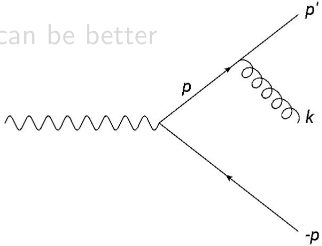

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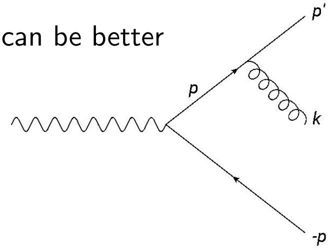

Vincia's Parton Shower

- Uses colour antennae, not emission off single partons
  - Soft and wide-angle emissions described more naturally
- Can use full matrix element to correct up to 3rd emission
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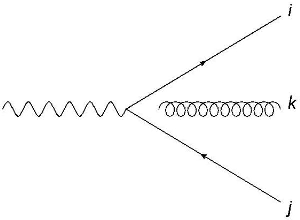
- Helicity shower
  - Many spin correlations automatically accounted for
  - Quicker calculation of MECs



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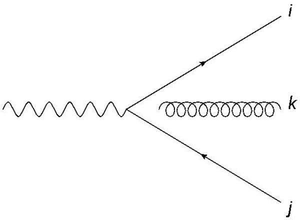

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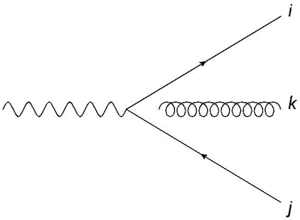

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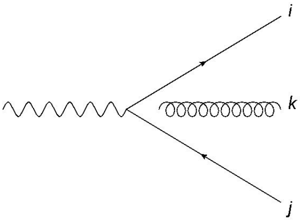

Why Quicker MECs in a Helicity Shower?

- In normal Amplitudes we sum and average spins/helicities
- Here we only need a single helicity configuration for each amplitude
- Helicity amplitudes are often remarkably simple to compute
- Most simple is called Maximally Helicity Violating (MHV)
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A Solution: Recursion Relations

- Recursively generate a compact form for the matrix element
- Often use spinor-helicity formalism
- Assume all particles are outgoing
  - Use crossing symmetry for initial-state partons
- Organises processes based on the number of opposite helicities
  - $\mathcal{M}[n^{\pm}]$, $\mathcal{M}[(n-1)^{\pm}, 1^{\mp}] = 0$
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Colour Ordering

- Can easily separate colour and kinematics in a process
- $\mathcal{M}_i\left(1^{h_1},\ldots,n^{h_n}\right) = C_i(t^1,\ldots,t^n)A_i\left(p_1^{h_1},\ldots,p_n^{h_n}\right)$
- $|\mathcal{M}|^2 = \sum_{i,j}\mathcal{M}_i^*\mathcal{M}_j = \sum_{i,j}A_i^*C_{ij}A_j$
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- Most common is so-called trace basis
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All-Gluon Amplitude Structure

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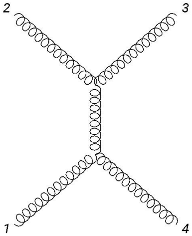

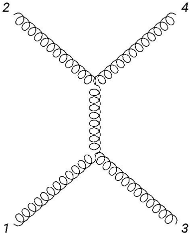

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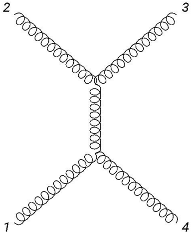

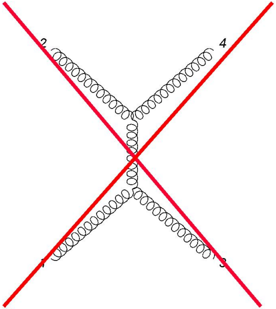

Colour Order in the Trace Basis: 1 $q\bar{q}$ Pair, $n-2$ Gluons

1 Quark Pair QCD Amplitude Structure

$$\mathcal{M}_i(q, g_1, \ldots, g_{n-2}, \bar{q}) = g_s^{n-2}(t^{a_1} \ldots t^{a_{n-2}})_{q\bar{q}} A_i(q^{h_q}, p_1^{h_1}, \ldots, p_{n-2}^{h_{n-2}}, \bar{q}^{h_{\bar{q}}})$$

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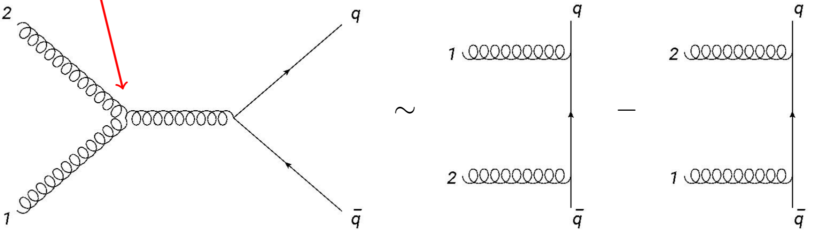

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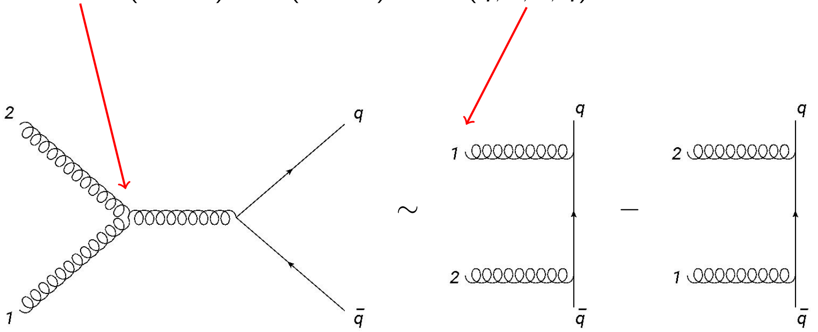

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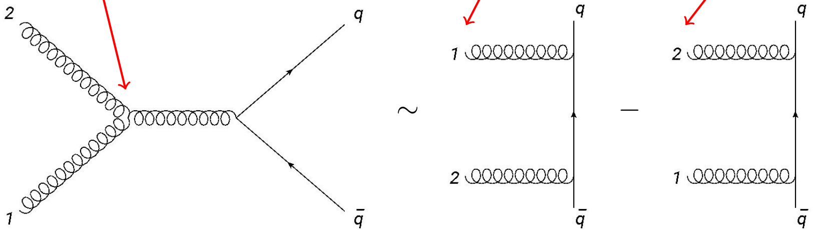
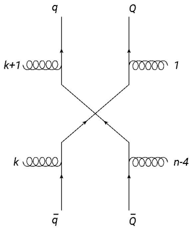


Colour Order in the Trace Basis: 2 $q\bar{q}$ Pairs, $n-4$ Gluons

2 Quark Pair QCD Amplitude Structure 1

$$\mathcal{M}_i(Q,1,\ldots,k,\bar{q},q,k+1,\ldots,n-4,\bar{Q}) = g_s^{n-2}\times$$
$$\mathcal{A}_i(h_q,h_Q,h_g)(t^{a_1}\ldots t^{a_k})_{Q\bar{q}}(t^{a_{k+1}}\ldots t^{a_{n-4}})_{q\bar{Q}}\times$$
$$A_i^{(1)}(Q,1,\ldots,k,\bar{q},q,k+1,\ldots,n-4,\bar{Q})$$



Colour Order in the Trace Basis: 2 $q\bar{q}$ Pairs, $n-4$ Gluons

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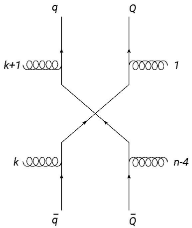
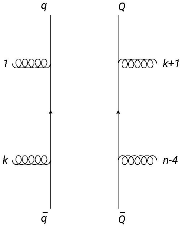
$$\mathcal{M}_i(Q,1,\ldots,k,\bar{q},q,k+1,\ldots,n-4,\bar{Q}) = g_s^{n-2}\times$$
$$\mathcal{A}_i(h_q,h_Q,h_g)(t^{a_1}\ldots t^{a_k})_{Q\bar{q}}(t^{a_{k+1}}\ldots t^{a_{n-4}})_{q\bar{Q}}\times$$
$$\mathcal{A}_i^{(1)}(Q,1,\ldots,k,\bar{q},q,k+1,\ldots,n-4,\bar{Q})$$



2 Quark Pair QCD Amplitude Structure 2

$$\mathcal{M}_i(q,1,\ldots,k,\bar{q},Q,k+1,\ldots,n-4,\bar{Q}) = -g_s^{n-2}\times$$
$$\frac{1}{N_C}\mathcal{A}_i(h_q,h_Q,h_g)(t^{a_1}\ldots t^{a_k})_{q\bar{q}}(t^{a_{k+1}}\ldots t^{a_{n-4}})_{Q\bar{Q}}\times$$
$$\mathcal{A}_i^{(2)}(q,1,\ldots,k,\bar{q},Q,k+1,\ldots,n-4,\bar{Q})$$



MHV Amplitudes: All-Gluon

Full Colour-Summed Amplitude (MHV $= \mathcal{M}[(n-2)^{\pm}, 2^{\mp}]$)

$$\sum_i \mathcal{M}_i(g_1, g_2, \ldots, g_n) = g_s^{n-2} \sum_{\sigma_i \in S_n/Z_n} \mathrm{Tr}(t^{a_{\sigma_i(1)}} \ldots t^{a_{\sigma_i(n)}}) A_{\sigma_i}(\sigma_i(p_1^{h_1}), \ldots, \sigma_i(p_n^{h_n}))$$

Kinematic Amplitude

$$A_{\sigma_i}(i^-, j^-) = i \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle}$$

$$A_{\sigma_i}(i^+, j^+) = i \frac{[ji]^4}{[1n][n(n-1)] \ldots [21]}$$

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- $|\mathcal{M}|_h^2 = |\mathcal{M}|_{-h}^2$

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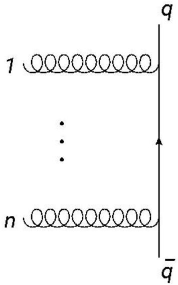

Kinematic Amplitude

$$A_{\sigma_i}(q^-, i^-, \bar{q}^+) = \frac{\langle qi\rangle^3 \langle \bar{q}i\rangle}{\langle \bar{q}q\rangle\langle q1\rangle\langle 12\rangle \ldots \langle (n-2)\bar{q}\rangle}$$
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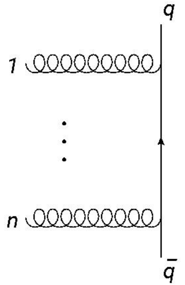

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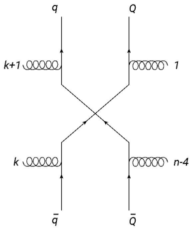
$$(t^{a_{\sigma_i(k+1)}}\ldots t^{a_{\sigma_i(n-4)}})_{q\bar{Q}}\times A_{\sigma_i}(Q,1,\ldots,k,\bar{q},q,k+1,\ldots,n-4,\bar{Q}) - \frac{1}{N_C}(\bar{q}\leftrightarrow\bar{Q})\Bigg]$$

Kinematic Amplitude (part 1)

(h_q,h_Q,h_g)	$A_0(h_q,h_Q,h_g)$
$(+,+,+)$	$\langle\bar{q}\bar{Q}\rangle^2$
$(+,+,-)$	$[qQ]^2$
$(+,-,+)$	$\langle\bar{q}Q\rangle^2$
$(+,-,-)$	$[q\bar{Q}]^2$



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$$A_{\sigma_i} = \frac{\{q\bar{Q}\}}{\{q1\}\{12\}\ldots\{k\bar{Q}\}}\frac{\{Q\bar{q}\}}{\{Q(k+1)\}\{(k+1)(k+2)\}\ldots\{(n-4)\bar{q}\}}$$

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- Same as 2 quark pairs with 1 pair not radiating
- Correct for coupling

Kinematic Amplitude

$$A_n(q, 1, \ldots, n-4, \bar{q}, l, \bar{l}) = \sum_{V=\gamma, Z, W^\pm} M_V^l(h_l, h_q, h_g) \frac{1}{\{q1\}\{12\}\ldots\{(n-4)\bar{q}\}}$$
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What is A_n^h??

Process	Negative-helicity particles	$A_n^h(1,\ldots,n)$
All-gluon	i,j	$\langle ij\rangle^4$
Single Quark Pair	q,i	$\langle qi\rangle^3\langle\bar{q}i\rangle$
Single Quark Pair	$\bar{q},i$	$\langle qi\rangle\langle\bar{q}i\rangle^3$
Quark and Lepton Pairs	–	$A_0(h_l,h_q,+)(g^l_{h_l})_V(g^q_{h_q})_V$

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Calculating the Antenna Functions

- All antennae have to be positive-definite
- Sum of all antennae must equal the unpolarised antenna
- In collinear and soft limits must reproduce DGLAP splitting
- Quarks cannot change helicity
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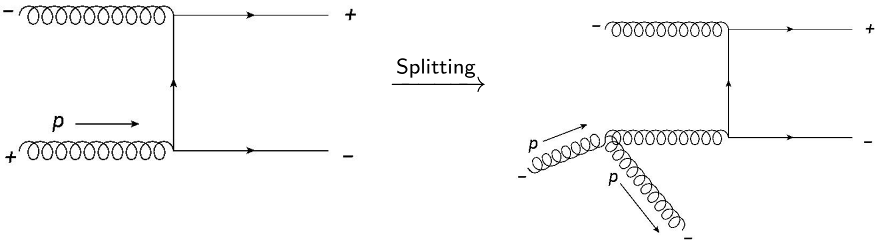
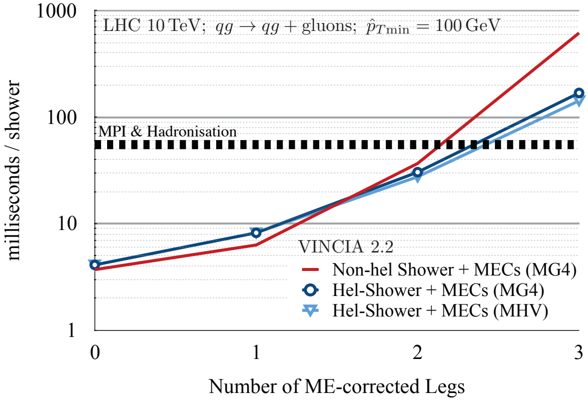


Speed Test of Shower



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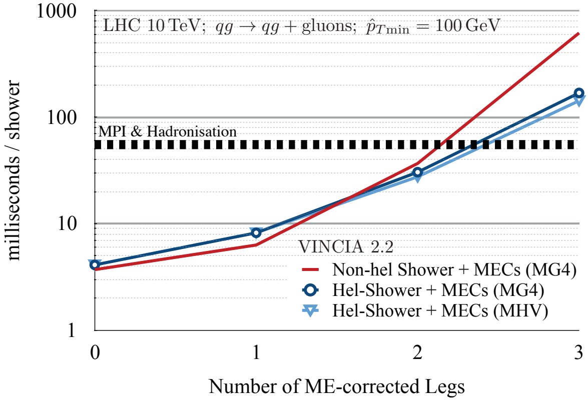

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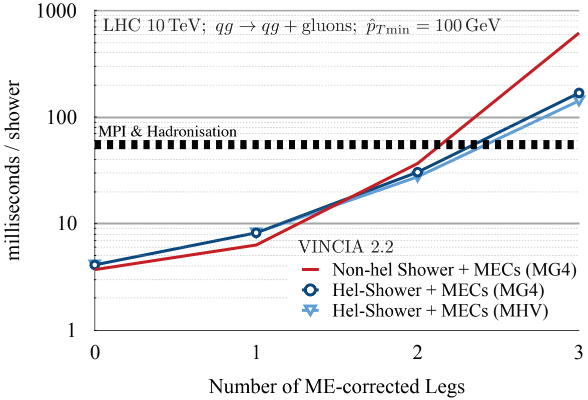

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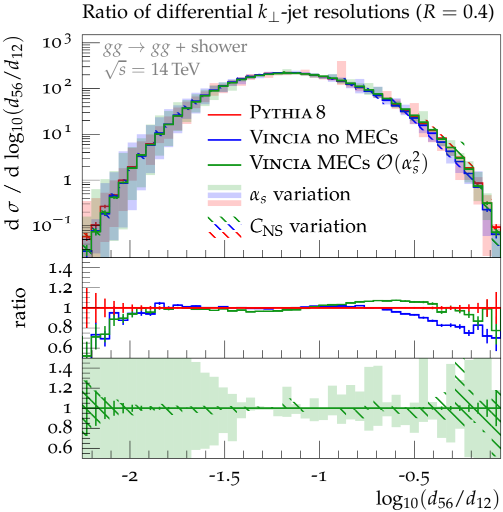
Effects on Late Branchings, $gg \to gg$

- Corrected first 2 emissions
- Large d_{56} (large $\log_{10}(d_{56}/d_{12}) \sim 0$) expect MECs important
- Pythia has no MECs
- Vincia and Pythia showers intrinsically different



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$$\mathrm{FC}^h = |A_n^h(1,\ldots,n)|^2 \left| \sum_\sigma \frac{1}{\langle \sigma(1)\sigma(2)\rangle \ldots \langle \sigma(n)\sigma(1)\rangle} \mathrm{C}(\sigma(t^1),\ldots,\sigma(t^n)) \right|^2$$

$$= |A_n^h|^2 \left| \sum_\sigma F(\sigma) \right|^2$$

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Match to Full (L0) Matrix Element

- Requires ME of all possible histories
- Many of these historical MEs are MHV ($(n-2)\pm, 2\mp$)
- If final-state is MHV, all historical states are either:
  - $(n-2)\pm, 1\mp$, i.e. unphysical
  - $(n-3)\pm, 2\mp$, i.e. MHV
- So MHV multi-parton states are recursively faster
- If not MHV, use MG4 (currently implementing MG5 interface)
  - May be able to speed up much more using NMHV

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  - $(n-3)\pm$, $2\mp$, i.e. MHV
- So MHV multi-parton states are recursively faster
- If not MHV, use MG4 (currently implementing MG5 interface)
  - May be able to speed up much more using NMHV