Using MHV amplitudes in the VINCIA Helicity Shower

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In collaboration with Peter Skands, Nadine Fischer

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Work Done at Monash University

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Overview

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- VIrtual Numerical Collider with Interleaved Antennae
- Began in 2007 as proof of concept by P. Z. Skands, W. T. Giele and D. A. Kosower [1]
- Plugin to Pythia, replaces its parton shower
- Had 2 main goals in mind
  - Include systematic uncertainty estimates
  - Allow matching to any LO or NLO matrix element
- 2 main versions:
  - Vincia 1: e^+e collisions
  - Vincia 2: e^+e and pp collisions [2]
- Recently released Vincia 2.204

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Comparing Vincia's and Pythia's Parton Showers

Pythia

- Markovian, no history
- Parton shower with spectator parton
- Corrects only 1st emission with full ME
- Minimal spin correlations

Vincia

- Non-Markovian, historical MEs
- Antenna shower, radiate off 2 partons in one splitting function
- Can correct up to 3 emissions with full ME
- Helicity shower, spin correlations in MEC

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Why Helicity Shower?

- $M_i \;\; 1^{h_1};:::;n^{h_n} \;\; = C_i(t^1;:::;t^n)A_i \;\; p_1^{h_1};:::;p_n^{h_n}$
- Helicity-amplitudes easier than helicity-summed amplitudes
- Organise processes based on the number of opposite helicities (all particles outgoing)
  - $M[n \;\;]$, $M[(n \;\; 1) \;\; ;1 \;\;] = 0$
  - $M[(n \;\; 2) \;\; ;2 \;\;] = MHV$
  - $M[(n \;\; 3) \;\; ;3 \;\;] = NMHV$
  - etc.
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Feynman Diagrams vs Recursion Relations: The All-Gluon MHV Case

No. of External Gluons	No. of Feynman Diagrams	Relative Growth
4	4	-
5	25	6.3
6	220	8.8
7	2485	11.3
8	34300	13.8
9	559405	16.3
10	10525900	18.8

All-gluon Feynman Diagram numbers calculated by Kleiss and Kuijf [3]

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Recursion relation for n 4 Gluons, $(n$ 2$) +$ hel; 2 hel

$$A_{\;i}(i\;\;;j\;\;) = i\frac{hiji^4}{h12ih23i:::hn1i} \qquad hiji \quad \bar{u}\;\;(p_i)u_+(p_j)$$

Speed Testing MHV Amplitudes

$gg \rightarrow ng$ MHV amplitudes, micro-seconds per calculation

nParticles	RAMBO	MadGraph4	MHV	Ratio
4	0.671	1.868	1.496	1.451
5	0.806	7.716	2.546	3.966
6	0.931	76.434	7.940	10.771

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nParticles	RAMBO	MadGraph4	MHV	Ratio
4	0.855	1.551	1.596	0.927
5	0.822	3.216	2.669	1.296
6	0.935	18.579	3.447	7.024
7	1.088	236.183	14.355	17.720

MHV Amplitudes within Vincia

- Swaps incoming particles to outgoing, checks it has process
- Base class calculates all relevant spinor products
- Uses MHV wherever possible for MEC and setting polarisations
- Can also use Vincia to calculate MHV amplitudes as standalone
- The following can be calculated

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Type of Process	Number of Particles
All-Gluon	4 6
Single $q\bar{q}$ Pair + Gluons	4 7
Two $q\bar{q}$ Pairs + Gluons	$4; 5$
$q\bar{q}$ and $l\bar{l}$ Pairs + Gluons (Z-Boson Exchange)	4 9

Setting up the Shower

Reminder

$$M_{i}\left(1^{h_1},\ldots,n^{h_n}\right) = C_i(t^1,\ldots,t^n)A_i\left(p_1^{h_1},\ldots,p_n^{h_n}\right)$$

- Requires both a colour flow and a polarisation
- To understand, first need definitions:
  - $LC_i = M_i^* M_i$; $FC = \sum_{ij} M_i^* M_j$; $VC_i = FC \frac{LC_i}{\sum_{i'} LC_{i'}}$
- If no colour flow in hard process:
  - $P(h;i) = \underbrace{\frac{FC^h}{\sum_{h'} FC^{h'}}}_{\text{Helicity-Selection Factor}} \underbrace{\frac{LC_i^h}{\sum_j LC_j^h}}_{\text{Colour-Flow Selection Factor}}$
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Polarising the Shower with MHV Amplitudes

$$P(h|i) = \frac{\mathrm{VC}_i^h}{\sum_{h'} \mathrm{VC}_i^{h'}} = \frac{\mathrm{FC}^h \mathrm{LC}_i^h}{\sum_j \mathrm{LC}_j^h} \left(\sum_{h'} \frac{\mathrm{FC}^{h'} \mathrm{LC}_i^{h'}}{\sum_k \mathrm{LC}_k^{h'}} \right)^{1}$$

- Usually polarise $2 \to 2$ or $2 \to 3$, i.e. MHV
- MHV kinematics can be factorised into helicity and colour parts

$$\mathrm{FC}^h = |A_n^h(1, \ldots, n)|^2 \left| \sum \frac{1}{\langle (1)\ (2) \rangle \ldots \langle (n)\ (1) \rangle} C(\ (t^1), \ldots, \ (t^n)) \right|^2$$

$$= |A_n^h|^2 \left| \sum F(\ ) \right|^2$$

$$\mathrm{LC}_i^h = |A_n^h|^2 \, |F(\ _i)|^2$$

Polarising the Shower with MHV Amplitudes

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Match to Full (LO) Matrix Element

- MEC:

M_{PS}^{n+1} ! M_{PS}^{n+1} R ; $M_{PS}^{n+1} = A$ M^{n} ; R M_{ex}^{n+1}=M_{PS}^{n+1}

Match to Full (LO) Matrix Element

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MEC factor

$$R(\Phi_{n+1}) = j\, M(\Phi_{n+1}) j^{2}$$

$$\mathrm{h} \sum_{\Phi_n^0} A(\Phi_{n+1}/\Phi_n^0) \; R(\Phi_n^0) \sum_{\Phi_{n\;1}^0} (t(\Phi_n^0/\Phi_{n\;\;1}^0) \quad t(\Phi_{n+1}/\Phi_n^0)) \; A \;\; \Phi_n^0/\Phi_{n\;\;1}^0 \quad R(\Phi_{n\;\;1}^0)$$

$$\prod_{k=n\;\;2}^{1} @ \sum_{\Phi_k^0} (t(\Phi_{k+1}^0/\Phi_k^0) \quad t(\Phi_{k+2}^0/\Phi_{k+1}^0)) \; A(\Phi_{k+1}^0/\Phi_k^0) \; R(\Phi_k^0) \, \mathrm{A}$$

$$0 \qquad\qquad 1$$

$$\sum_{\Phi_0^0} (t(\Phi_1^0/\Phi_0^0) \quad t(\Phi_2^0/\Phi_1^0)) \; A(\Phi_1^0/\Phi_0^0) \quad (t(\Phi_0^0) \quad t(\Phi_1^0/\Phi_0^0)) \; j\, M(\Phi_0^0) j^{2} \; \mathrm{i} \;\; {}_1$$

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Full Matrix Element — Antenna Function — Ensure Correct Shower Scale

MEC factor

$$R(\Phi_{n+1}) = j M(\Phi_{n+1}) j^2 \ \text{h} \sum_{\Phi^0_n} A(\Phi_{n+1}/\Phi^0_n) \ R(\Phi^0_n) \sum_{\Phi^0_{n-1}} (t(\Phi^0_n/\Phi^0_{n-1}) \quad t(\Phi_{n+1}/\Phi^0_n)) \ A \ \ \Phi^0_n/\Phi^0_{n-1} \quad R(\Phi^0_{n-1})$$

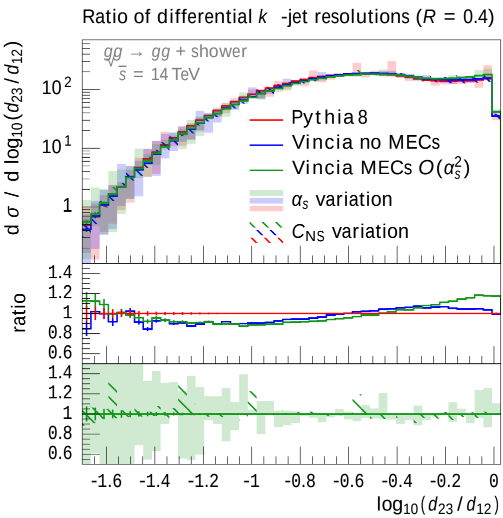
$$\prod_{k=n-2}^{1} \text{@} \sum_{\Phi^0_k} (t(\Phi^0_{k+1}/\Phi^0_k) \quad t(\Phi^0_{k+2}/\Phi^0_{k+1})) \ A(\Phi^0_{k+1}/\Phi^0_k) \ R(\Phi^0_k) \text{A}$$

$$\sum_{\Phi^0_0} (t(\Phi^0_1/\Phi^0_0) \quad t(\Phi^0_2/\Phi^0_1)) \ A(\Phi^0_1/\Phi^0_0) \quad (t(\Phi^0_0) \quad t(\Phi^0_1/\Phi^0_0)) \ j M(\Phi^0_0) j^2 \ \text{i}^{-1}$$

Sum All Shower Histories — Possible Born-Level Processes

Effects on Early Branchings, $gg \to gg$

- Corrected first 2 emissions
- Large d_{23} (i.e. $\log_{10}(d_{23}/d_{12}) \to 0$) expect MECs important
- Pythia has no MECs
- Vincia and Pythia showers intrinsically different



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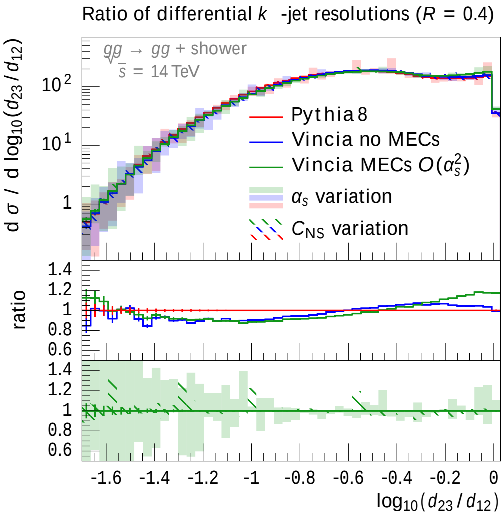

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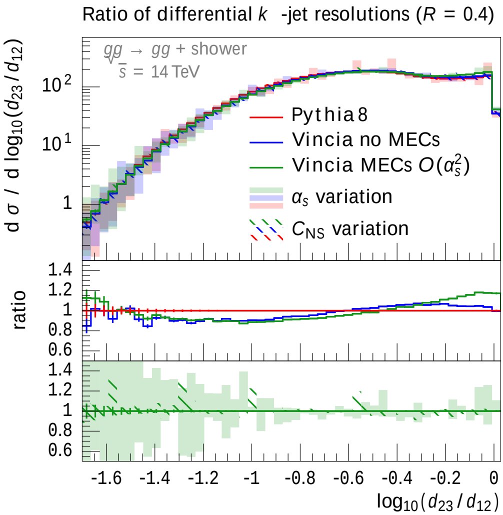

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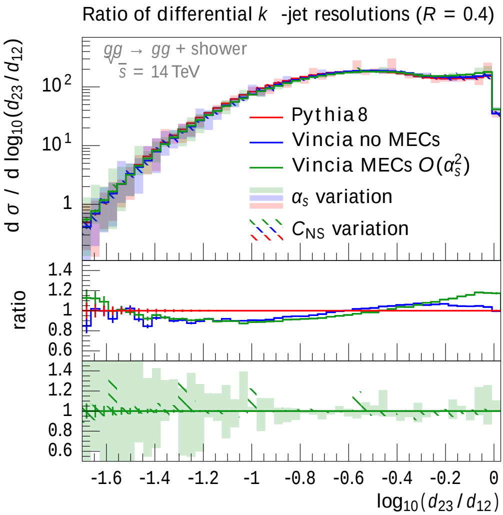


Polarisation Effects, $gg\ !\ gg$, $g\ !\ b\bar{b}$

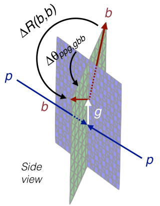
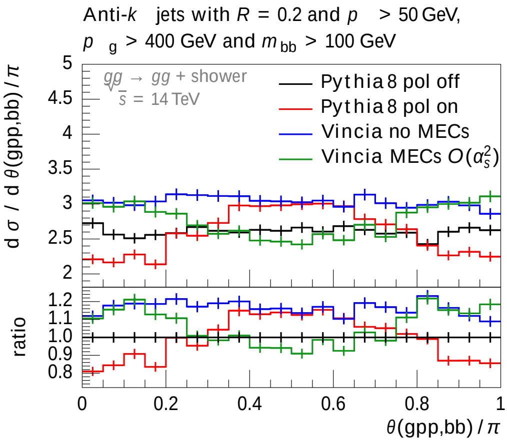


Image from arxiv:1812.09283

- $(gpp; bb)$ angle between planes
- Both showers flat
- Vincia MECs, Pythia azymuthal Asym give preferred directions
- But, opposite!?



Polarisation Effects, *gg* ! *gg*, *g* ! $b\bar{b}$

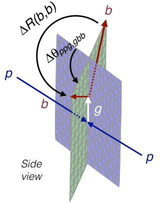


Image from arxiv:1812.09283

- $(gpp; bb)$ angle between planes
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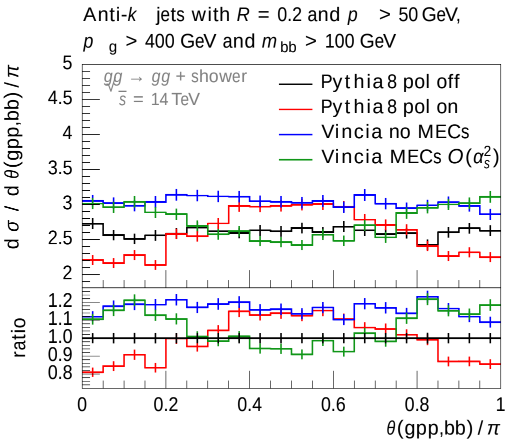

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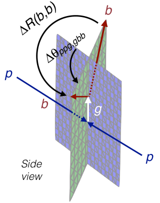

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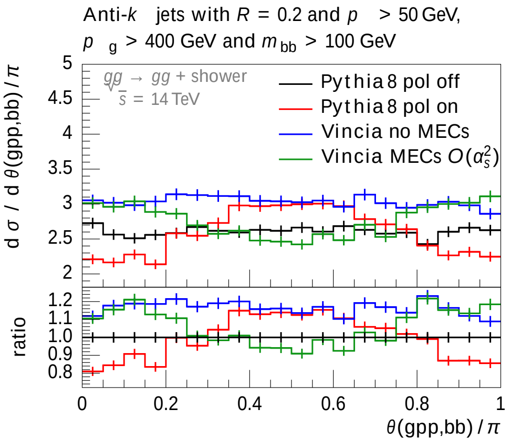

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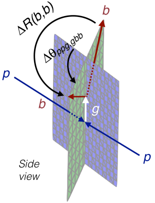

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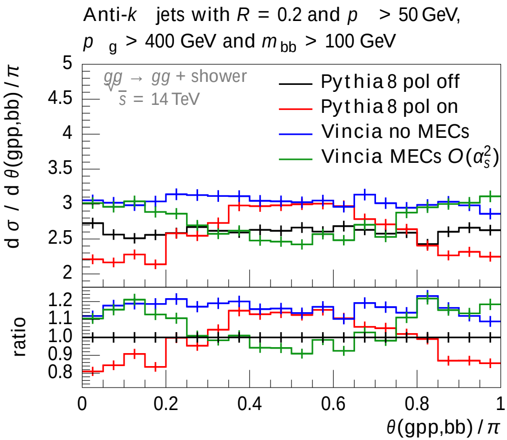
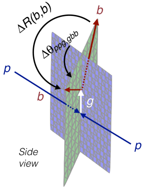
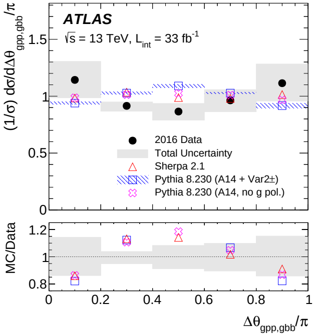


Polarisation Effects: a New ATLAS Measurement

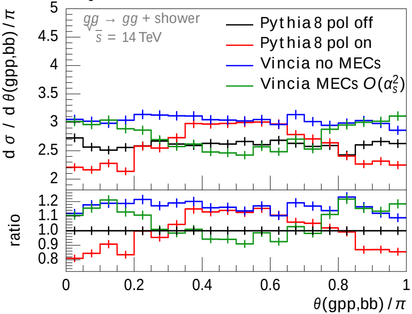
- Recent measurement of gluon splitting at small opening angle (arxiv:1812.09283)
- Sherpa 2 ! n + PS is flat,
  Pythia opposite shape, Vincia correct shape



Anti-$k_?$ jets with R = 0:2 and $p_?$ > 10 GeV,
$p_{?j}$ > 450 GeV and m_{bb} unrestricted



Anti-k jets with R = 0.2 and p > 50 GeV,
p g > 400 GeV and m_{bb} > 100 GeV



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- Vincia adds recursive MECs, giving better predictions in hard, wide angle limits
- Vincia uses helicity shower, giving more spin data, better angular information
- MECs slowed down by factorial-like growth of Feynman diagrams
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Spinor-Helicity Formalism

Spinors

- $v\ \ (p) = u\ \ (p) = \frac{1}{2}\ \ 1 \qquad {}^{5}\ \ u(p)$
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- $hiji \qquad \bar{u}\ \ (i)u_{+}(j) = \quad \mathrm{q}\,\overline{}\ p_j^{+} e^{i\ \ i} \qquad \mathrm{q}\,\overline{}\ p_i^{+} e^{i\ \ j} = \quad hjii$
- $[ij] \qquad \bar{u}_{+}(i)u\ \ (j) = hjii$
  - $p_i^{+} = p_i^0 + p_i^3\ ; \qquad e^{i\ \ i} = (p_i^1 + ip_i^2) = \ \mathrm{q}\,\overline{}\ p_i^{+}$
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- Generate unpolarised antenna branching
  - Means shower without MECs has no spin correlations
- Then choose a polarisation for $i; j; k$
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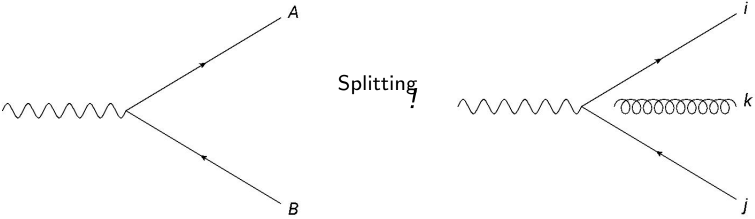
  - Reproduces correct limits in soft/collinear regions
  - Sum of polarised antennae = unpolarised antenna



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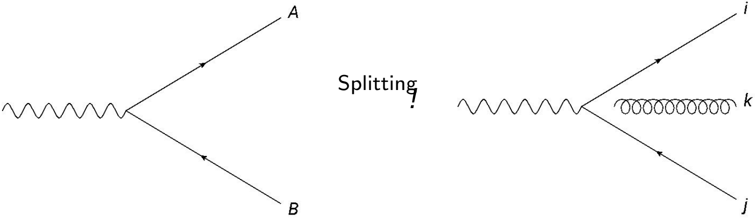

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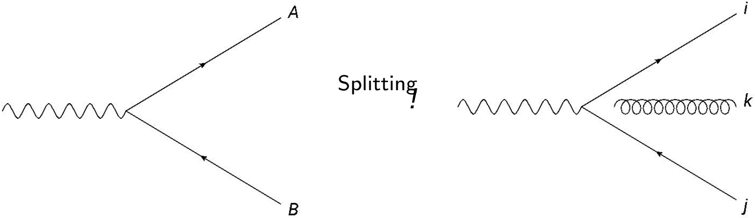

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1 Quark Pair QCD Amplitudes

$$A_{\,i}(q \;\; ; i \;\; ; \bar{q}^{+}) = \frac{\langle qi\rangle^3\langle \bar{q}i\rangle}{\langle \bar{q}q\rangle\langle q1\rangle\langle 12\rangle\ldots\langle (n \quad 2)\bar{q}\rangle}$$

$$A_{\,i}(q^{+}; i \;\; ; \bar{q} \;\;) = \frac{\langle qi\rangle\langle \bar{q}i\rangle^3}{\langle \bar{q}q\rangle\langle q1\rangle\langle 12\rangle\ldots\langle (n \quad 2)\bar{q}\rangle}$$

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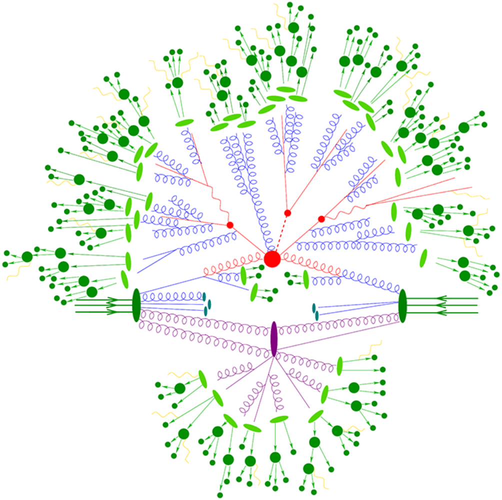

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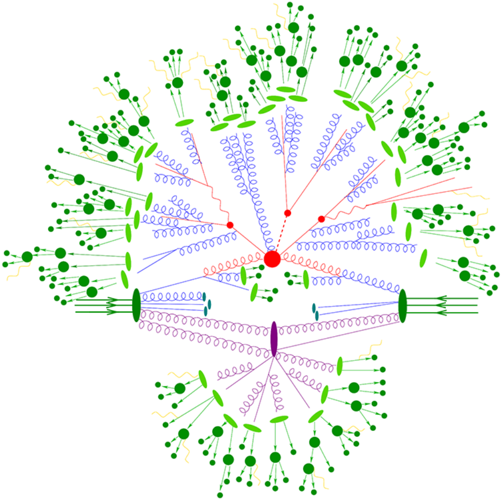

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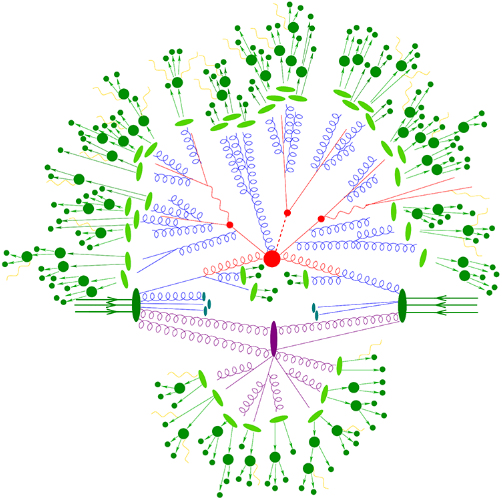

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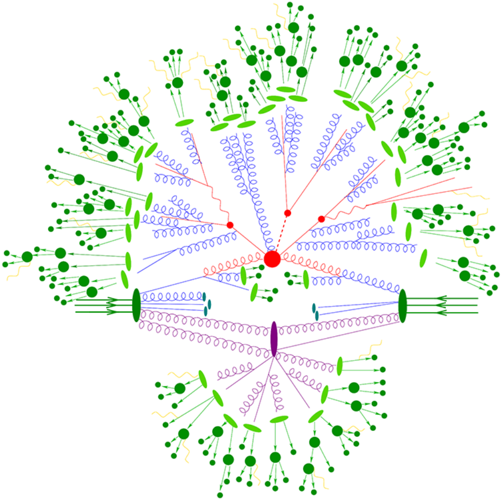

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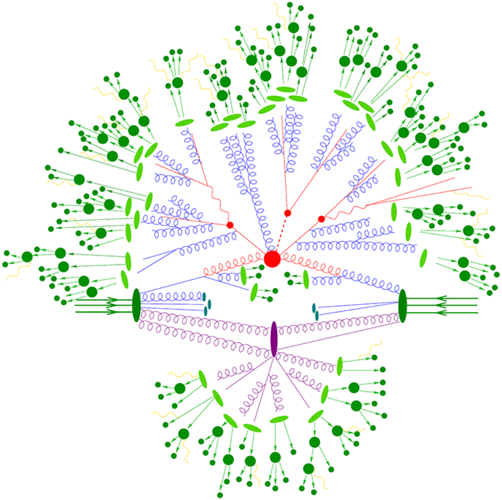

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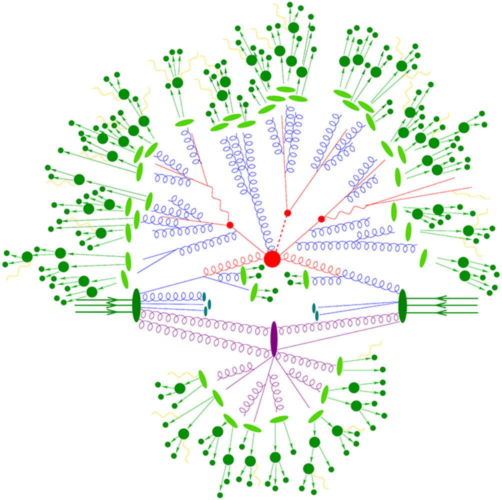

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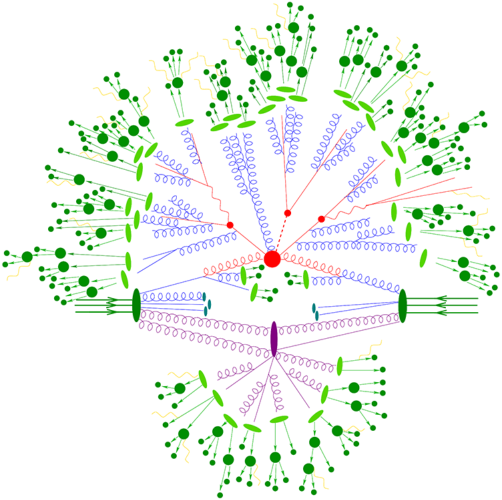

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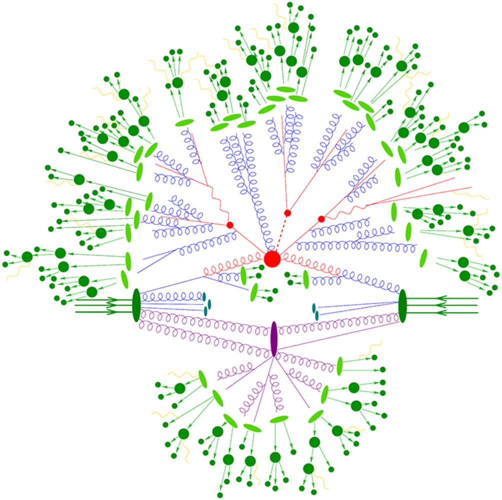

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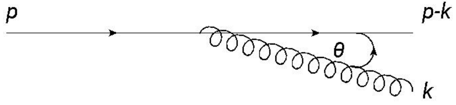
- Bremsstrahlung occurs in initial- (ISR) and final- (FSR) state radiation
- Recursively generates emissions off a parton
- Assumes radiation to be soft and/or collinear
  - Logarithmic enhancement in emission probability
  - Describes soft and/or collinear radiation very well
- Poor job of describing hard, wide-angle emissions



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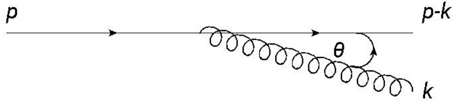

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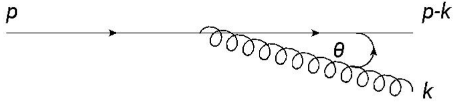

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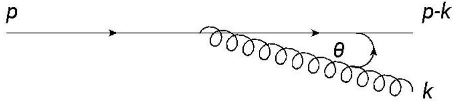
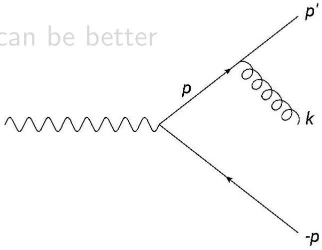


Pythia's Parton Shower

- Markov chain of collinear emissions off single partons
  - No concept of history
  - Soft and wide-angled emissions artificially separated
  - Angular distributions can be compromised
- Dipole shower. Spectator parton for momentum conservation
- Unpolarised partons, minimal spin correlations
- Uses the full matrix element to correct first emission (MEC)
  - All subsequent emissions only well-described in soft/collinear parts of phase space
- Conclusion: is good, can be better



Pythia's Parton Shower

- Markov chain of collinear emissions off single partons
  - No concept of history
  - Soft and wide-angled emissions artificially separated
  - Angular distributions can be compromised
- Dipole shower. Spectator parton for momentum conservation
- Unpolarised partons, minimal spin correlations
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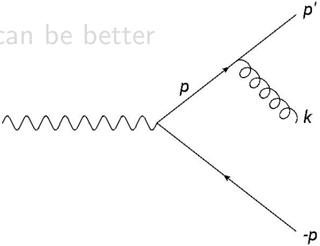

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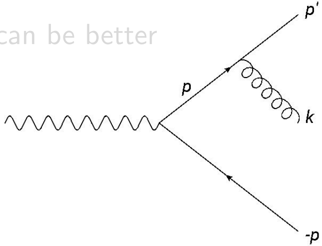

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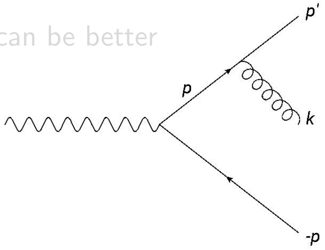

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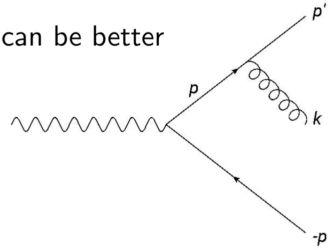
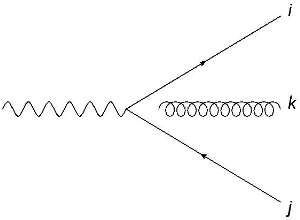


Vincia's Parton Shower

- Uses colour antennae, not emission off single partons
  - Soft and wide-angle emissions described more naturally
- Can use full matrix element to correct up to 3rd emission
- Non-Markovian emissions, remembers histories
- Helicity shower
  - Many spin correlations automatically accounted for
  - Quicker calculation of MECs



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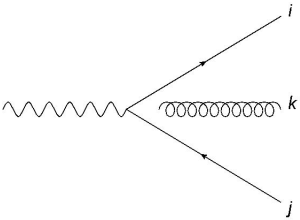

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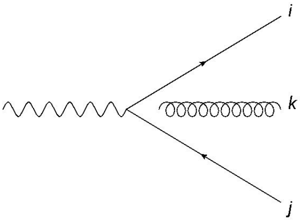

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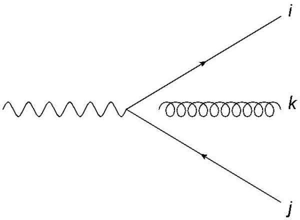

Why Quicker MECs in a Helicity Shower?

- In normal Amplitudes we sum and average spins/helicities
- Here we only need a single helicity configuration for each amplitude
- Helicity amplitudes are often remarkably simple to compute
- Most simple is called Maximally Helicity Violating (MHV)
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A Solution: Recursion Relations

- Recursively generate a compact form for the matrix element
- Often use spinor-helicity formalism
- Assume all particles are outgoing
  - Use crossing symmetry for initial-state partons
- Organises processes based on the number of opposite helicities
  - $M[n\ \ ],\ M[(n\quad 1)\ \ ;1\ \ ] = 0$
  - $M[(n\quad 2)\ \ ;2\ \ ] = MHV$
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Colour Ordering

- Can easily separate colour and kinematics in a process
- $M_i \left(1^{h_1}, \ldots, n^{h_n}\right) = C_i(t^1, \ldots, t^n) A_i \left(p_1^{h_1}, \ldots, p_n^{h_n}\right)$
- $|M|^2 = \sum_{i,j} M_i M_j = \sum_{i,j} A_i C_{ij} A_j$
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  - each give different kinematics
- Most common is so-called trace basis
  - Conceptually simple, but non-orthogonal and overcomplete

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Colour Order in the Trace Basis: All-Gluon Case

All-Gluon Amplitude Structure

$$M_i(g_1; g_2; :::; g_n) = g_s^{n \quad 2} \mathrm{Tr}(t^{a_1} ::: t^{a_n}) A_i(p_1^{h_1}; :::; p_n^{h_n})$$

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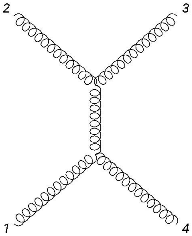

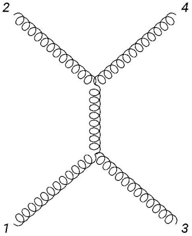

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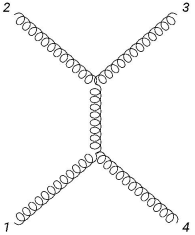

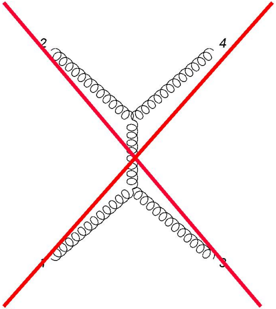

Colour Order in the Trace Basis: 1 $q\bar{q}$ Pair, $n\quad 2$ Gluons

1 Quark Pair QCD Amplitude Structure

$$M_{\ i}(q; g_1; \ldots; g_{n\ \ 2}; \bar{q}) = g_s^{n\ \ 2}(t^{a_1} \ldots t^{a_{n\ \ 2}})_{q\bar{q}} A_i(q^{h_q}; p_1^{h_1}; \ldots; p_{n\ \ 2}^{h_{n\ \ 2}}; \bar{q}^{h_{\bar{q}}})$$

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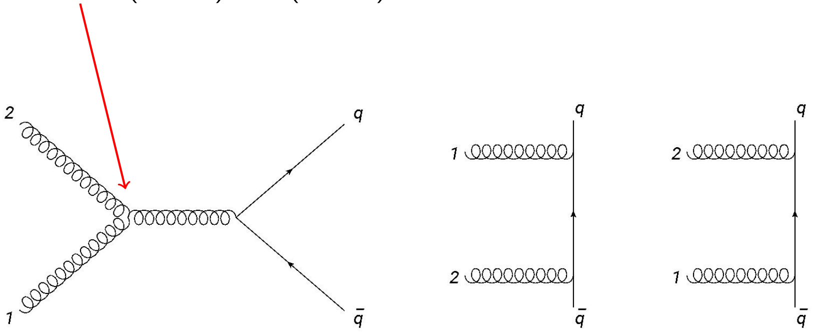

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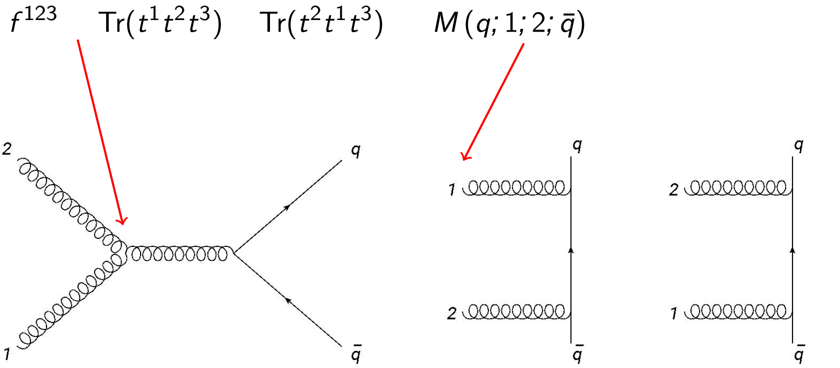

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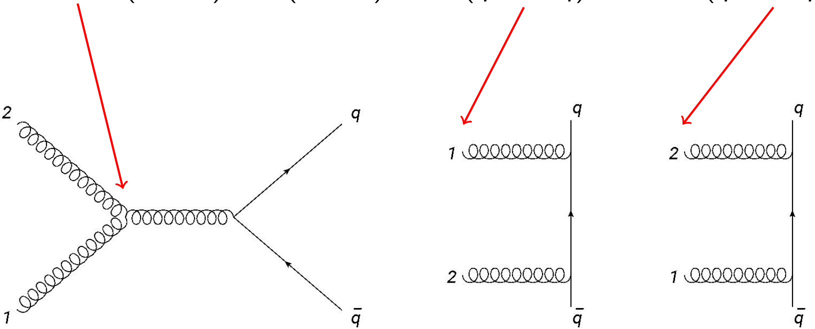

Colour Order in the Trace Basis: 2 $q\bar{q}$ Pairs, $n \quad 4$ Gluons

2 Quark Pair QCD Amplitude Structure 1

$$M_i(Q; 1; :::; k; q; q; k+1; :::; n \quad 4; Q) = g_s^{n \quad 2}$$
$$A_i(h_q; h_Q; h_g)(t^{a_1} ::: t^{a_k})_{Q\bar{q}}(t^{a_{k+1}} ::: t^{a_{n \quad 4}})_{q\bar{Q}}$$
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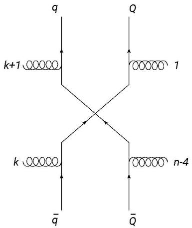

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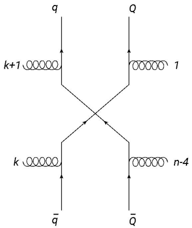

2 Quark Pair QCD Amplitude Structure 2

$$M_i(q;1;\ldots;k;q;Q;k+1;\ldots;n\quad 4;Q) = \quad g_s^{n\quad 2}$$
$$\frac{1}{N_C}A_i(h_q;h_Q;h_g)(t^{a_1}\ldots t^{a_k})_{q\bar{q}}(t^{a_{k+1}}\ldots t^{a_{n\quad 4}})_{Q\bar{Q}}$$
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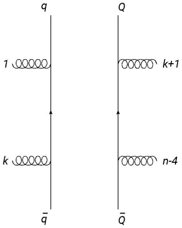

MHV Amplitudes: All-Gluon

Full Colour-Summed Amplitude (MHV = $M[(n \quad 2) \; ;2 \;])$

$$\mathrm{X}_{i} \; M_{i}(g_1; g_2; \ldots; g_n) = g_s^{n \quad 2} \; \mathrm{X}_{i \, 2 \, S_n = Z_n} \mathrm{Tr}(t^{a_{\; i(1)}} \ldots t^{a_{\; i(n)}}) A_{\; i}(\; _i(p_1^{h_1}); \ldots; \; _i(p_n^{h_n}))$$

Kinematic Amplitude

$$A_{\; i}(i \quad ; j \quad) = i \frac{hij i^4}{h12ih23i \ldots hn1i}$$

$$A_{\; i}(i^+; j^+) = i \frac{[ji]^4}{[1n][n(n \quad 1)] \ldots [21]}$$

- Flipping all helicities means $hij i \;\; ! \;\; [ji]$
- $j M j_h^2 = j M j^2_{\; h}$

MHV Amplitudes: All-Gluon

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$$\sum_{i} M_{i}(g_1; g_2; \ldots; g_n) = g_s^{n \quad 2} \sum_{i \in S_n / Z_n} \mathrm{Tr}(t^{a_{i(1)}} \ldots t^{a_{i(n)}}) A_{i}(\; {}_{i}(p_1^{h_1}); \ldots; \; {}_{i}(p_n^{h_n}))$$

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MHV Amplitudes: All-Gluon

Full Colour-Summed Amplitude (MHV $= M[(n \quad 2) \; ;2 \;])$

$$\mathop{\mathrm{X}}_{i} \; M_{\;i}(g_1; g_2; ::: ; g_n) = g_s^{n \quad 2} \mathop{\mathrm{X}}_{\;i \, 2 \, S_n = Z_n} \mathrm{Tr}(t^{a_{\;i(1)}} ::: t^{a_{\;i(n)}}) A_{\;i}(\;\;_i(p_1^{h_1}); ::: ; \;\;_i(p_n^{h_n}))$$

Kinematic Amplitude

$$A_{\;i}(i \quad ; j \quad) = i \frac{hij i^4}{h12ih23i ::: hn1i}$$

$$A_{\;i}(i^+; j^+) = i \frac{[ji]^4}{[1n][n(n \quad 1)] ::: [21]}$$

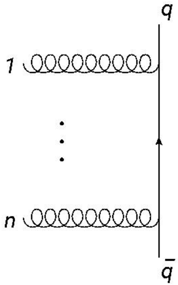
- Flipping all helicities means $hij i \quad ! \quad [ji]$
- $j M j_h^2 = j M j^2 \;\; _h$

MHV Amplitudes: 1 Quark Pair, $n \quad 2$ Gluons

Full Colour-Summed Amplitude

$$\underset{i}{X} M_{i}(q; g_1; :::; g_{n\ \ 2}; \bar{q}) = g_s^{n\ \ 2} \underset{i\,2\,S_{n\ \ 2}}{X} (t^{a\ \ i(1)}; :::; t^{a\ \ i(n\ \ 2)})_{q\bar{q}}$$

$$A_{\ i}\ \ q^{h_q};\ \ _i(p_1^{h_1}); :::;\ \ _i(p_{n\ \ 2}^{h_{n\ \ 2}}); \bar{q}^{h_{\bar{q}}}$$



Kinematic Amplitude

$$A_{\ i}(q\ \ ; i\ \ ; \bar{q}^{+}) = \frac{hqi\,i^3 h\bar{q}i\,i}{h\bar{q}qihq1ih12i:::h(n\quad 2)\bar{q}i}$$

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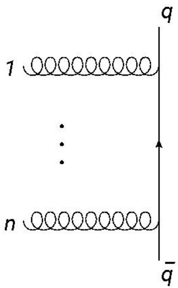
$$A_{\;i} \;\; q^{h_q}; \;\; {}_i(p_1^{h_1}); : : : ; \;\; {}_i(p_{n \;\; 2}^{h_n}); \bar{q}^{h_{\bar{q}}}$$



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MHV Amplitudes: 2 Quark Pairs, $n \geq 4$ Gluons

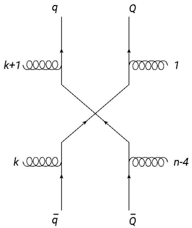
Full Colour-Summed Amplitude

$$\sum_{\sigma} M_{\sigma}(q, \bar{q}, Q, \bar{Q}, g_1, \dots, g_{n-4}) = g_s^{n-2} \frac{A_0(h_q, h_Q, h_g)}{\langle q\bar{q}g \rangle \langle Q\bar{Q}g \rangle} \left[\sum_{\sigma \in S_{n-4}} (t^{a_{\sigma(1)}} \dots t^{a_{\sigma(k)}})_{Q\bar{q}} \right.$$

$$\left. (t^{a_{\sigma(k+1)}} \dots t^{a_{\sigma(n-4)}})_{q\bar{Q}} \; A_{\sigma}(Q, 1, \dots, k, \bar{q}, q, k+1, \dots, n-4, \bar{Q}) - \frac{1}{N_C} \; \bar{q} \leftrightarrow \bar{Q} \right]$$

Kinematic Amplitude (part 1)

(h_q, h_Q, h_g)	$A_0(h_q, h_Q, h_g)$
$(+, +, +)$	$\langle qQ \rangle^2$
$(+, +, -)$	$[qQ]^2$
$(+, -, +)$	$\langle qQ \rangle^2$
$(+, -, -)$	$[qQ]^2$



MHV Amplitudes: 2 Quark Pairs, n 4 Gluons

Full Colour-Summed Amplitude

$$\mathrm{X}_{i} M_{i}(q; \bar{q}; Q; \bar{Q}; g_1; :::; g_{n\ \ 4}) = g_s^{n\ \ 2} \frac{A_0(h_q; h_Q; h_g)}{f q\bar{q}g f Q\bar{Q}g} \text{"} \mathrm{X}_{i\,2\,S_{n\ \ 4}} (t^{a\ \ i(1)} ::: t^{a\ \ i(k)})_{Q\bar{q}}$$

$$(t^{a\ \ i(k+1)} ::: t^{a\ \ i(n\ \ 4)})_{q\bar{Q}} \quad A_{\ i}(Q; 1; :::; k; \bar{q}; q; k+1; :::; n\ \ 4; \bar{Q}) \quad \frac{1}{N_C} \quad \bar{q} \ \$ \ \bar{Q} \ \#$$

Kinematic Amplitude (part 2)

$$A_{\ i} = \frac{f q\bar{Q}g}{f q1g f12g ::: f k\bar{Q}g} \frac{f Q\bar{q}g}{f Q(k+1)g f(k+1)(k+2)g ::: f(n\ \ 4)\bar{q}g}$$

- $f ijg = hiji$ if $h_g =$
- $f ijg = [ji]$ if $h_g = +$

MHV Amplitudes: 2 Quark Pairs, $n-4$ Gluons

Full Colour-Summed Amplitude

$$\sum_{\sigma} M_{\sigma}(q,\bar{q},Q,\bar{Q},g_1,\ldots,g_{n-4}) = g_s^{n-2}\frac{A_0(h_q,h_Q,h_g)}{\{q\bar{q}\}\{Q\bar{Q}\}}\sum_{\sigma\in S_{n-4}}\left[(t^{a_{\sigma(1)}}\ldots t^{a_{\sigma(k)}})_{Q\bar{q}}(t^{a_{\sigma(k+1)}}\ldots t^{a_{\sigma(n-4)}})_{q\bar{Q}}\,A_{\sigma}(Q,1,\ldots,k,\bar{q},q,k+1,\ldots,n-4,\bar{Q}) - \frac{1}{N_C}\,\bar{q}\leftrightarrow\bar{Q}\right]$$

Kinematic Amplitude (part 2)

$$A_{\sigma} = \frac{\{q\bar{Q}\}}{\{q1\}\{12\}\ldots\{k\bar{Q}\}}\,\frac{\{Q\bar{q}\}}{\{Q(k+1)\}\{(k+1)(k+2)\}\ldots\{(n-4)\bar{q}\}}$$

- $\{ij\} = \langle ij\rangle$ if $h_g = -$
- $\{ij\} = [ji]$ if $h_g = +$

MHV Amplitudes: Quark Pair, 1 Lepton Pair, $n-4$ Gluons

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$$\sum_{\sigma} M_{\sigma}(h_q; h_l; h_g) = i g_s^{n-4} \sum_{\sigma \in S_{n-4}} (t^{a_{\sigma(1)}}, \ldots, t^{a_{\sigma(n-4)}})_{q\bar{q}}$$

$$A_{\sigma}(q^{h_q}, \sigma(p_1^{h_1}), \ldots, \sigma(p_{n-4}^{h_{n-4}}), q^{h_{\bar{q}}}, l^{h_l}, l^{h_{\bar{l}}})$$

- Same as 2 quark pairs with 1 pair not radiating
- Correct for coupling

Kinematic Amplitude

$$A_n(q, 1, \ldots, n-4, \bar{q}, l, \bar{l}) = \sum_{V=\gamma, Z, W} M_V^l(h_l; h_q; h_g) \frac{1}{\langle q1 \rangle \langle 12 \rangle \ldots \langle (n-4)\bar{q} \rangle}$$

$$M_V^l(h_l; h_q; h_g) = \frac{A_0(h_l; h_q; h_g)[\bar{l}l](g^l_{h_l})_V (g^q_{h_q})_V}{\langle l\bar{l} \rangle [\bar{l}l] - M_V^2 + i\Gamma_V M_V}$$

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Full Colour-Summed Amplitude

$$\sum_{\lambda_i} M_{\lambda_i}(h_q; h_l; h_g) = i g_s^{n-4} \sum_{\sigma_i \in S_{n-4}} (t^{a_{\sigma_i(1)}}; \ldots; t^{a_{\sigma_i(n-4)}})_{q\bar{q}}$$
$$A_{\lambda_i}(q^{h_q}; \sigma_i(p_1^{h_1}); \ldots; \sigma_i(p_{n-4}^{h_{n-4}}); \bar{q}^{h_{\bar{q}}}; l^{h_l}; \bar{l}^{h_{\bar{l}}})$$

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Polarising with MHV Amplitudes

$$P(h|i) = \frac{\mathrm{VC}_i^h}{\sum_{h'} \mathrm{VC}_i^{h'}} = \frac{\mathrm{FC}^h \mathrm{LC}_i^h}{\sum_j \mathrm{LC}_j^h} \left(\sum_{h'} \frac{\mathrm{FC}^{h'} \mathrm{LC}_i^{h'}}{\sum_k \mathrm{LC}_k^{h'}} \right)^{-1} = \frac{|A_n^h|^2}{\sum_{h'} |A_n^{h'}|^2}$$

What is A_n^h??

Process	Negative-helicity particles	$A_n^h(1, \ldots, n)$
All-gluon	i, j	$\langle ij \rangle^4$
Single Quark Pair	q, i	$\langle qi \rangle^3 \langle \bar{q}i \rangle$
Single Quark Pair	$\bar{q}, i$	$\langle qi \rangle \langle \bar{q}i \rangle^3$
Quark and Lepton Pairs		$A_0(h_l, h_q, +)(g^l_{h_l})_V (g^q_{h_q})_V$

- If 2 same-flavour quark pairs no factorisation (since $(h_q = h_Q)$ has a different colour structure to $(h_q \neq h_Q)$)

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Calculating the Antenna Functions

- All antennae have to be positive-definite
- Sum of all antennae must equal the unpolarised antenna
- In collinear and soft limits must reproduce DGLAP splitting
- Quarks cannot change helicity
- The hard branch cannot change helicity
- An initial gluon **can** change helicity
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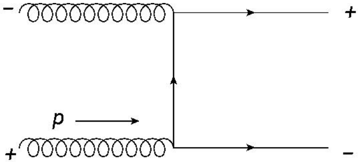

Splitting!

Speed Test of Shower

- Helicity shower better for > 2 ME-corrected legs
- At 3 ME-corrected legs, MHV saves 15%

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E ects on Late Branchings gg ! gg

- Corrected rst 2 emissions
- Large d_{56} (large $\log_{10}(d_{56}=d_{12})$ 0) expect MECs important
- Pythia has no MECs
- Vincia and Pythia showers intrinsically di erent

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- MHV kinematics can be factorised into helicity and colour parts

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$$FC^h = |A_n^h(1, \ldots, n)|^2 \sum \left| \frac{1}{\langle (1) \; (2)\rangle \ldots \langle (n) \; (1)\rangle} C(\; (t^1), \ldots, \; (t^n)) \right|^2$$

$$= |A_n^h|^2 \sum \left| F(\;) \right|^2$$

$$LC_i^h = |A_n^h|^2 |F(\;_i)|^2$$

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$$= |A_n^h|^2 \left| \sum F(\;) \right|^2$$

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Match to Full (L0) Matrix Element

- Requires ME of all possible histories
- Many of these historical MEs are MHV ((n 2) , 2)
- If nal-state is MHV, all historical states are either:
  - (n 2) , 1 , i.e. unphysical
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- So MHV multi-parton states are recursively faster
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