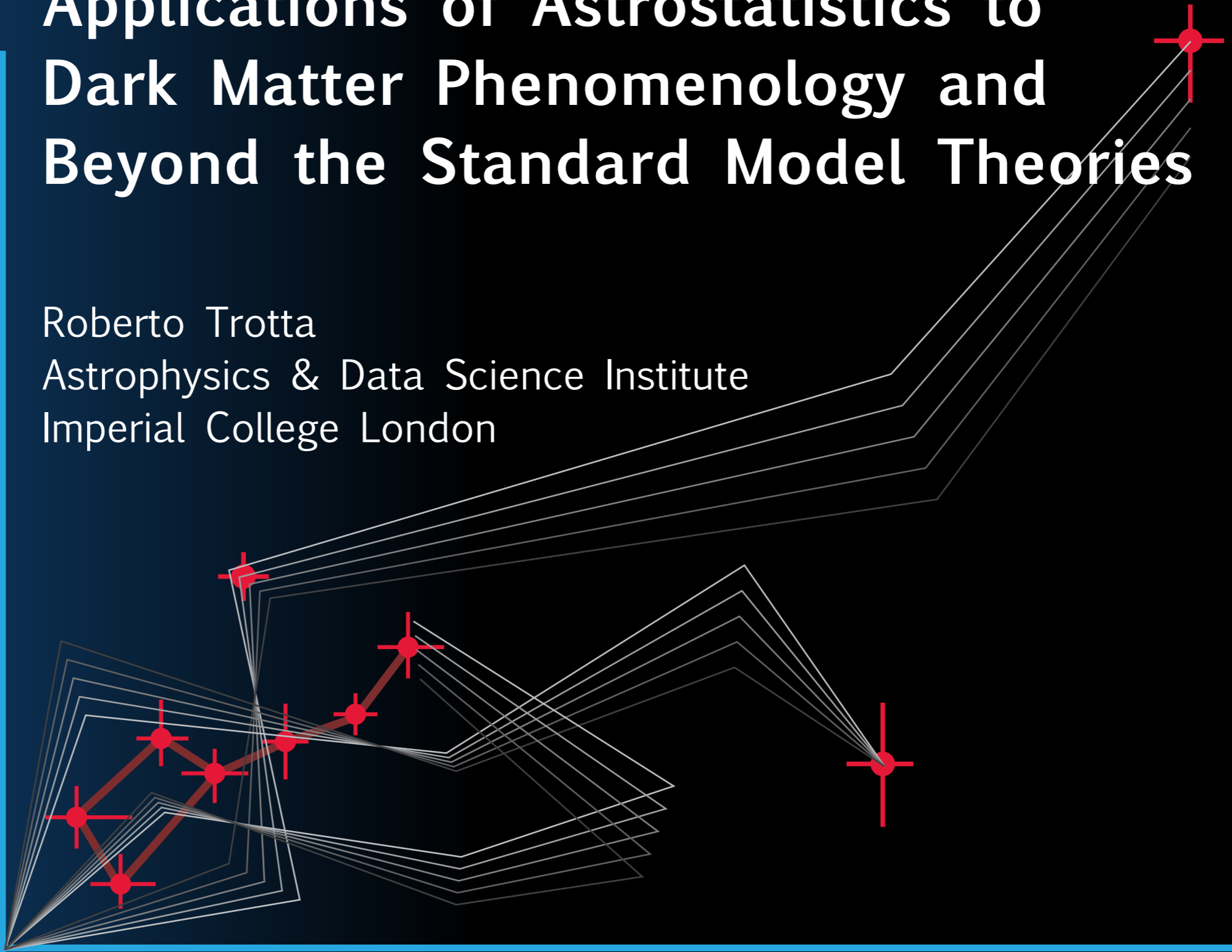


Applications of Astrostatistics to Dark Matter Phenomenology and Beyond the Standard Model Theories

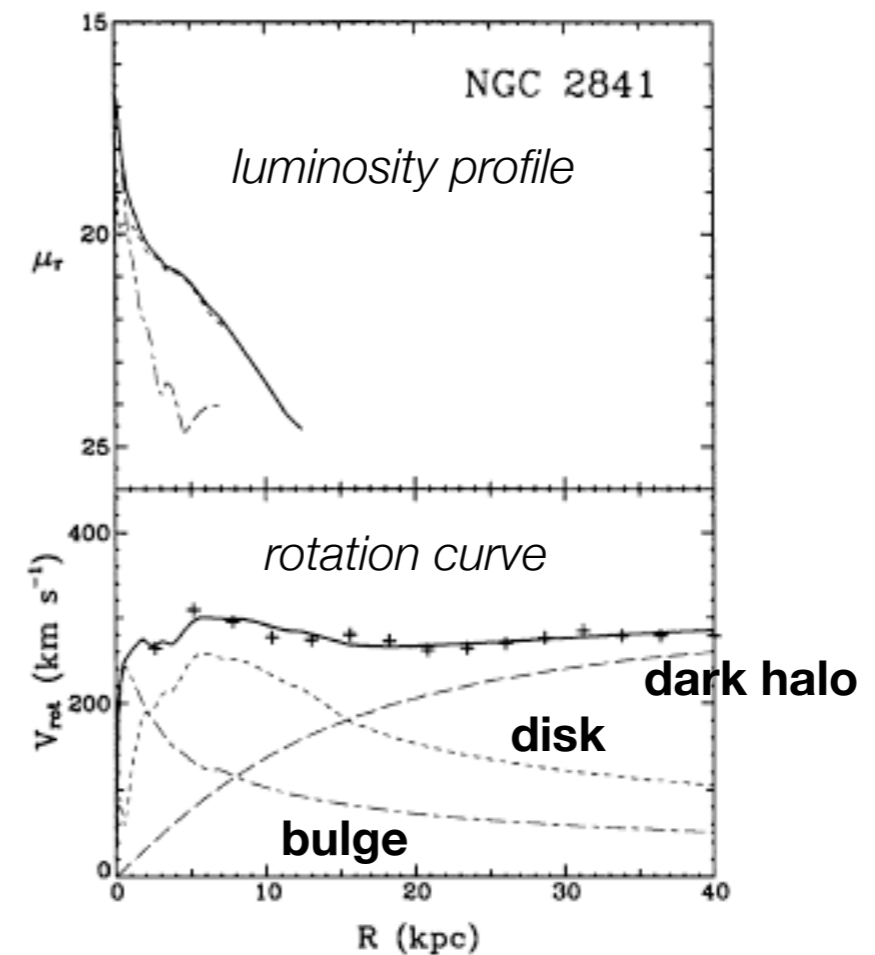
 @R_Trotta

Roberto Trotta
Astrophysics & Data Science Institute
Imperial College London



Evidence for dark matter

- Multiple astrophysical and cosmological probes strongly point to the existence of non-baryonic dark matter:



Kent 1987

Gravitational lensing

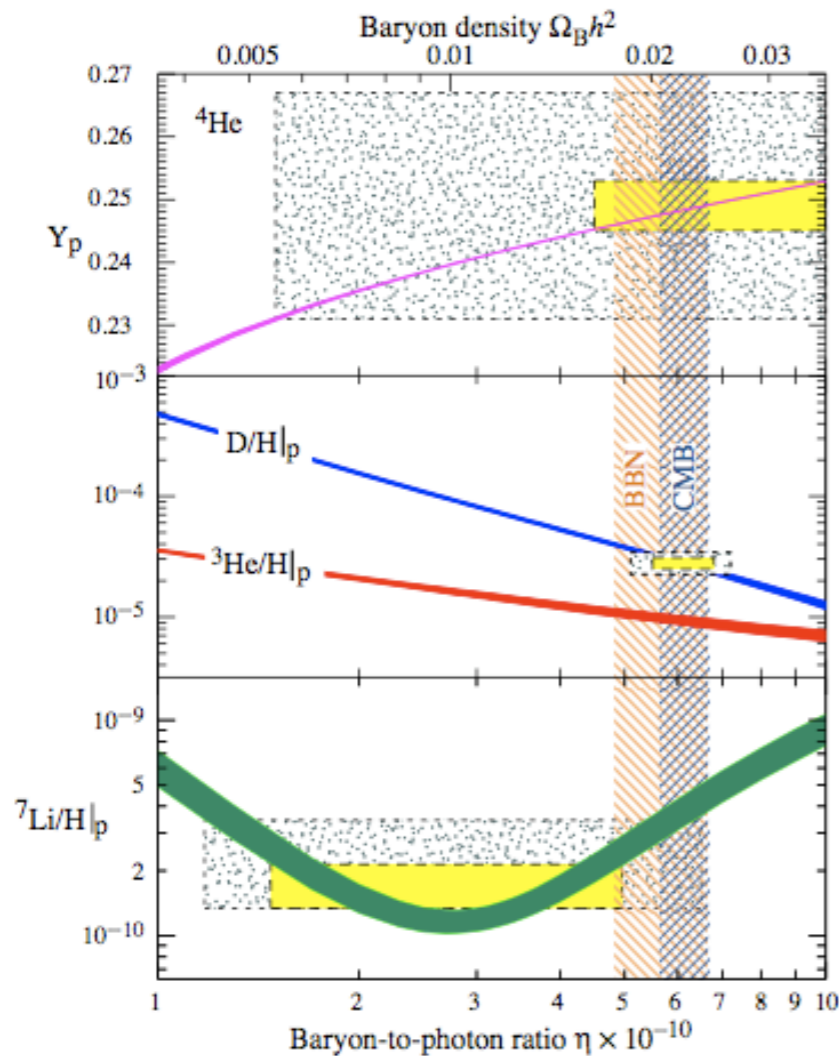
$$M/L \gg 1$$

Flat rotation curves

$$M/L \gg 1$$

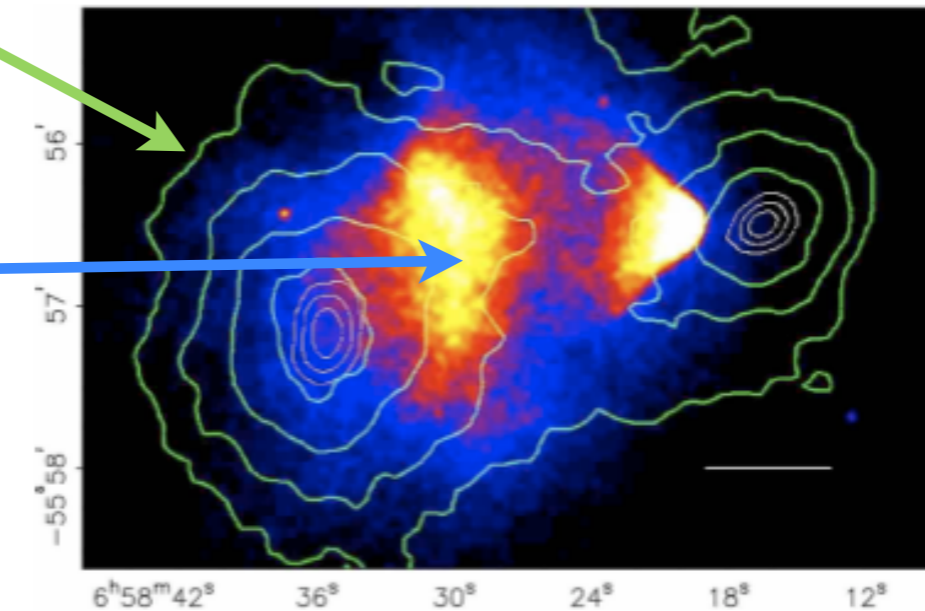
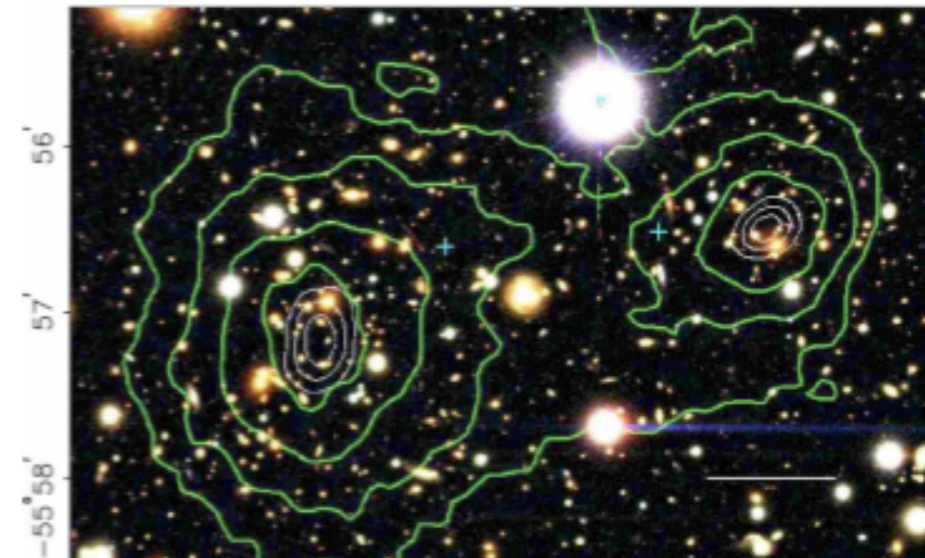
Evidence for dark matter (cont'ed)

Fields & Sarkar 2006



weak lensing

X-ray



Big Bang nucleosynthesis

$$4.7 < \eta_{10} < 6.5$$

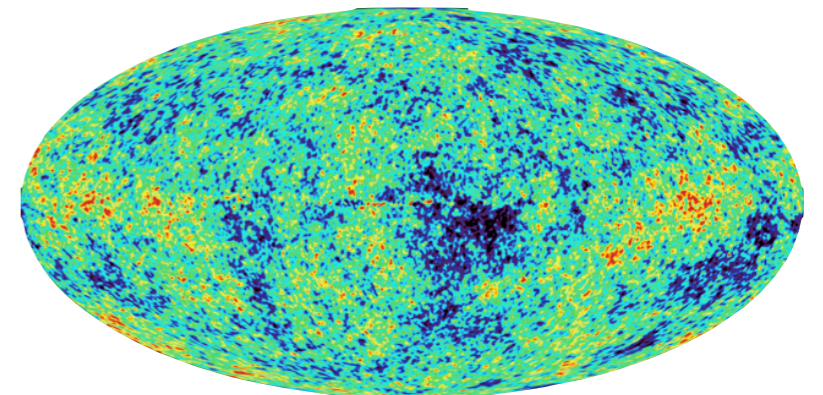
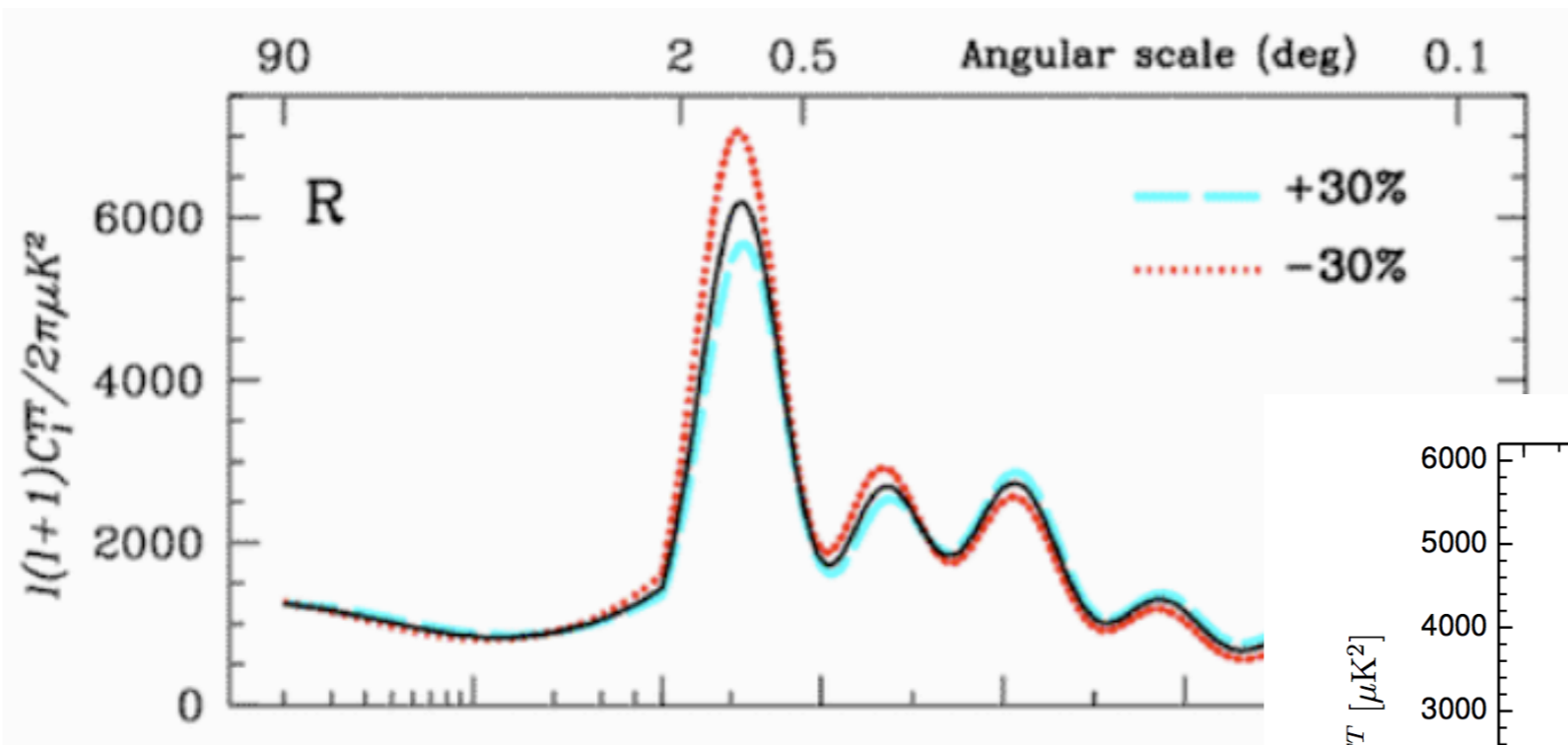
$$0.017 < \Omega_b h^2 < 0.024$$

Multi- λ data +
weak lensing

Clove et al 2006

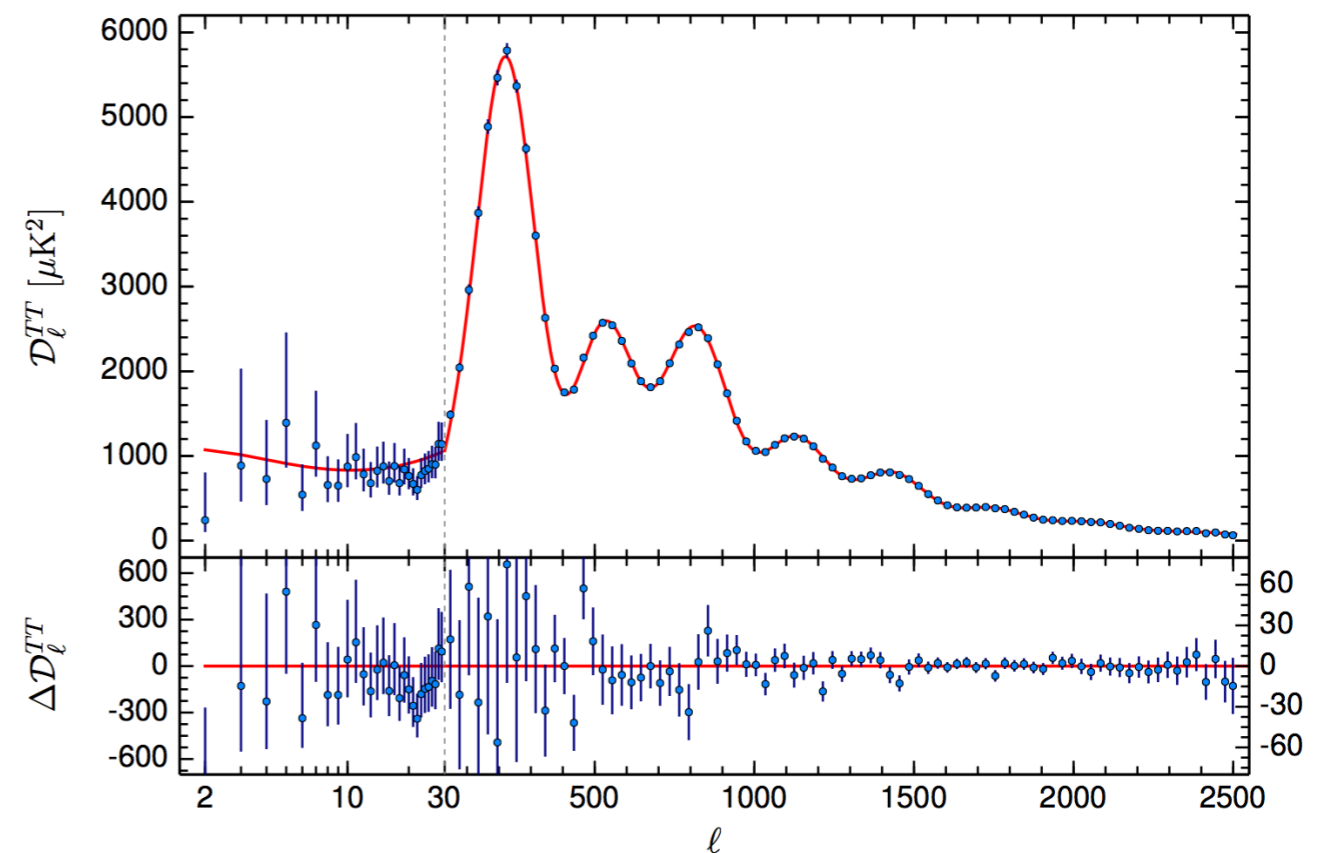
The cosmic microwave background

Change in the matter/radiation energy density ratio at decoupling:

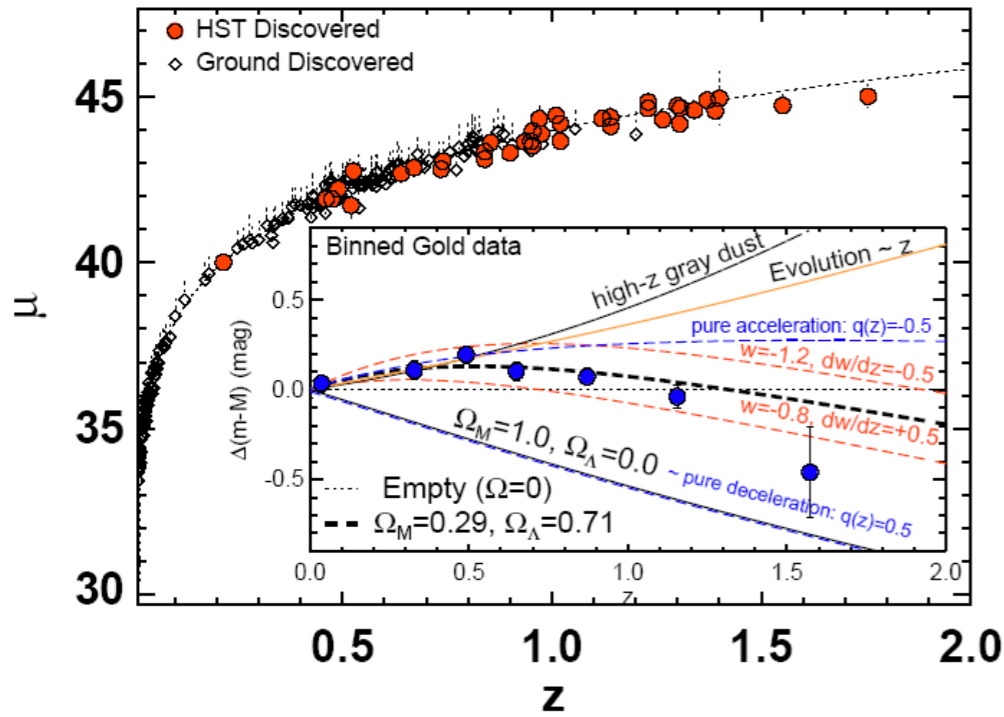


$$\Omega_{\text{cdm}}h^2 = 0.1099 \pm 0.0062$$

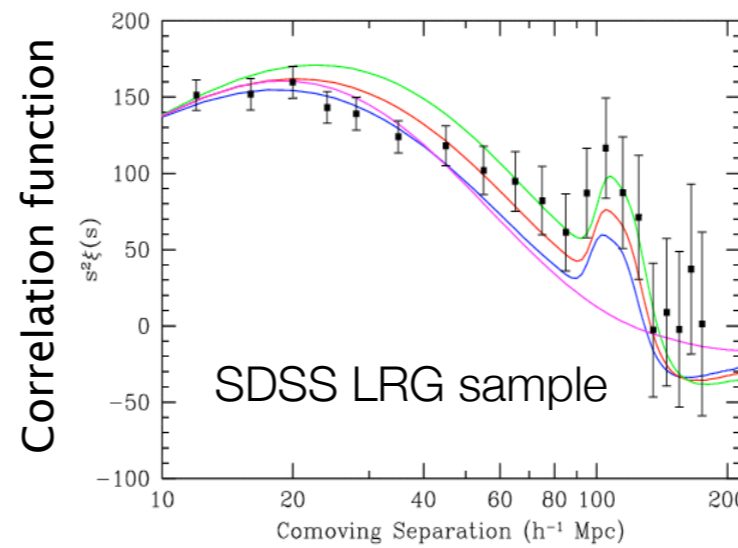
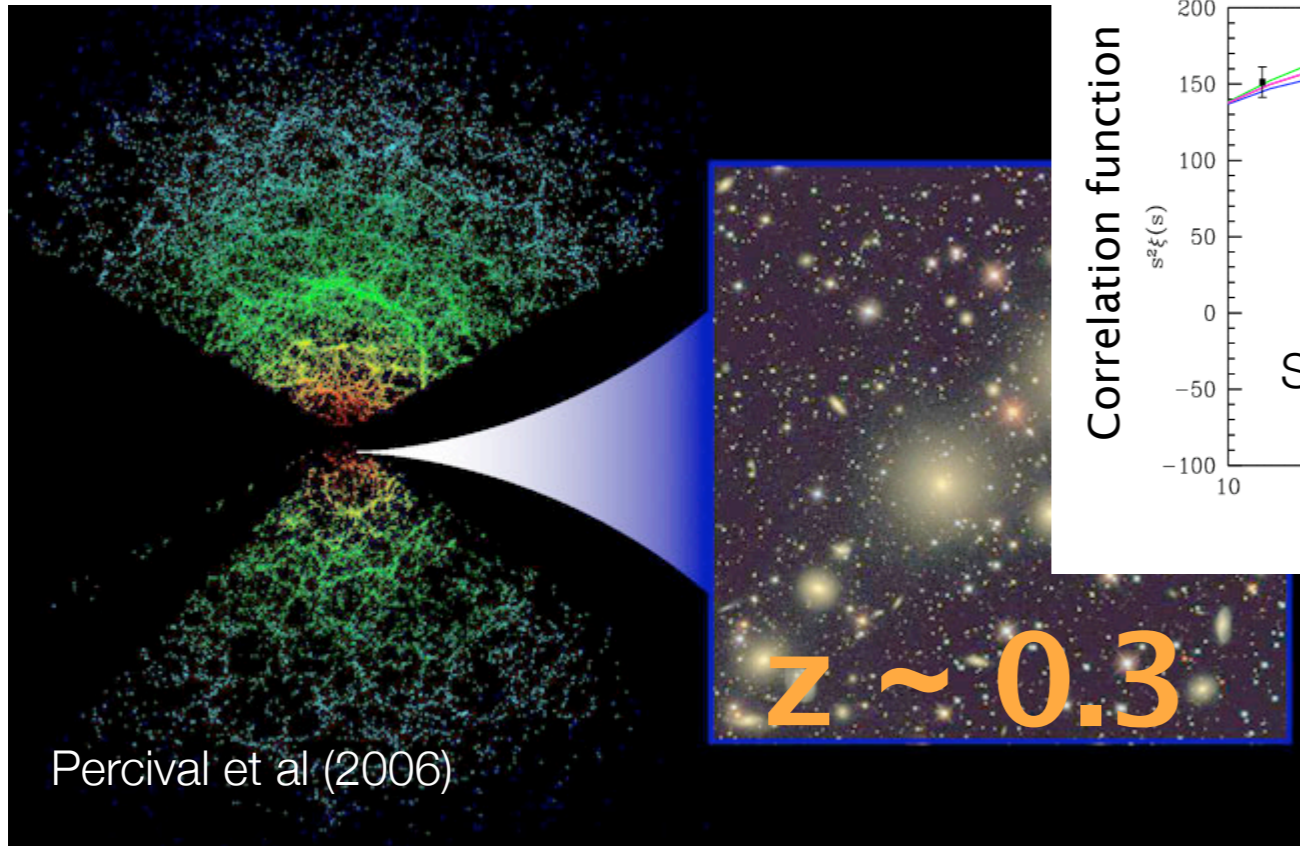
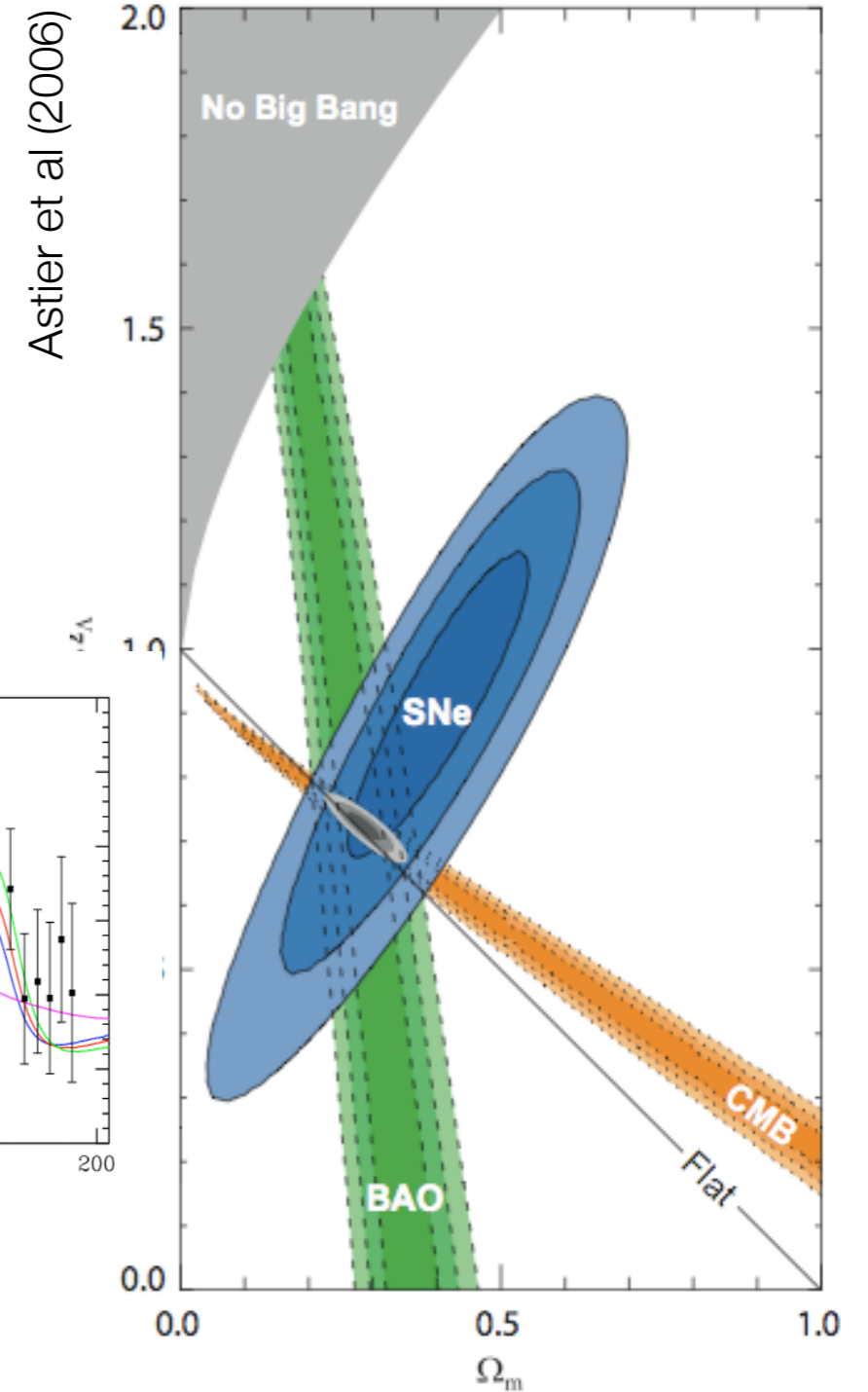
$$\Omega_{\text{m}} = 0.258 \pm 0.030$$



Combined cosmological probes



Riess et al (2006)



SUSY in a nutshell

- Every SM particle acquires a superpartner with the same quantum numbers but opposite spin statistics
- Unbroken SUSY: sparticles have the same mass as SM particles
- SUSY must be broken

Standard Model particles and fields		Supersymmetric partners			
Symbol	Name	Interaction eigenstates		Mass eigenstates	
Symbol	Name	Symbol	Name	Symbol	Name
$q = d, c, b, u, s, t$	quark	\tilde{q}_L, \tilde{q}_R	squark	\tilde{q}_1, \tilde{q}_2	squark
$l = e, \mu, \tau$	lepton	\tilde{l}_L, \tilde{l}_R	slepton	\tilde{l}_1, \tilde{l}_2	slepton
$\nu = \nu_e, \nu_\mu, \nu_\tau$	neutrino	$\tilde{\nu}$	sneutrino	$\tilde{\nu}$	sneutrino
g	gluon	\tilde{g}	gluino	\tilde{g}	gluino
W^\pm	W -boson	\tilde{W}^\pm	wino	$\tilde{\chi}_{1,2}^\pm$	chargino
H^-	Higgs boson	\tilde{H}_1^-	higgsino		
H^+	Higgs boson	\tilde{H}_2^+	higgsino	$\tilde{\chi}_{1,2,3,4}^0$	neutralino
B	B -field	\tilde{B}	bino		
W^3	W^3 -field	\tilde{W}^3	wino		
H_1^0	Higgs boson	\tilde{H}_1^0	higgsino		
H_2^0	Higgs boson	\tilde{H}_2^0	higgsino		
H_3^0	Higgs boson				

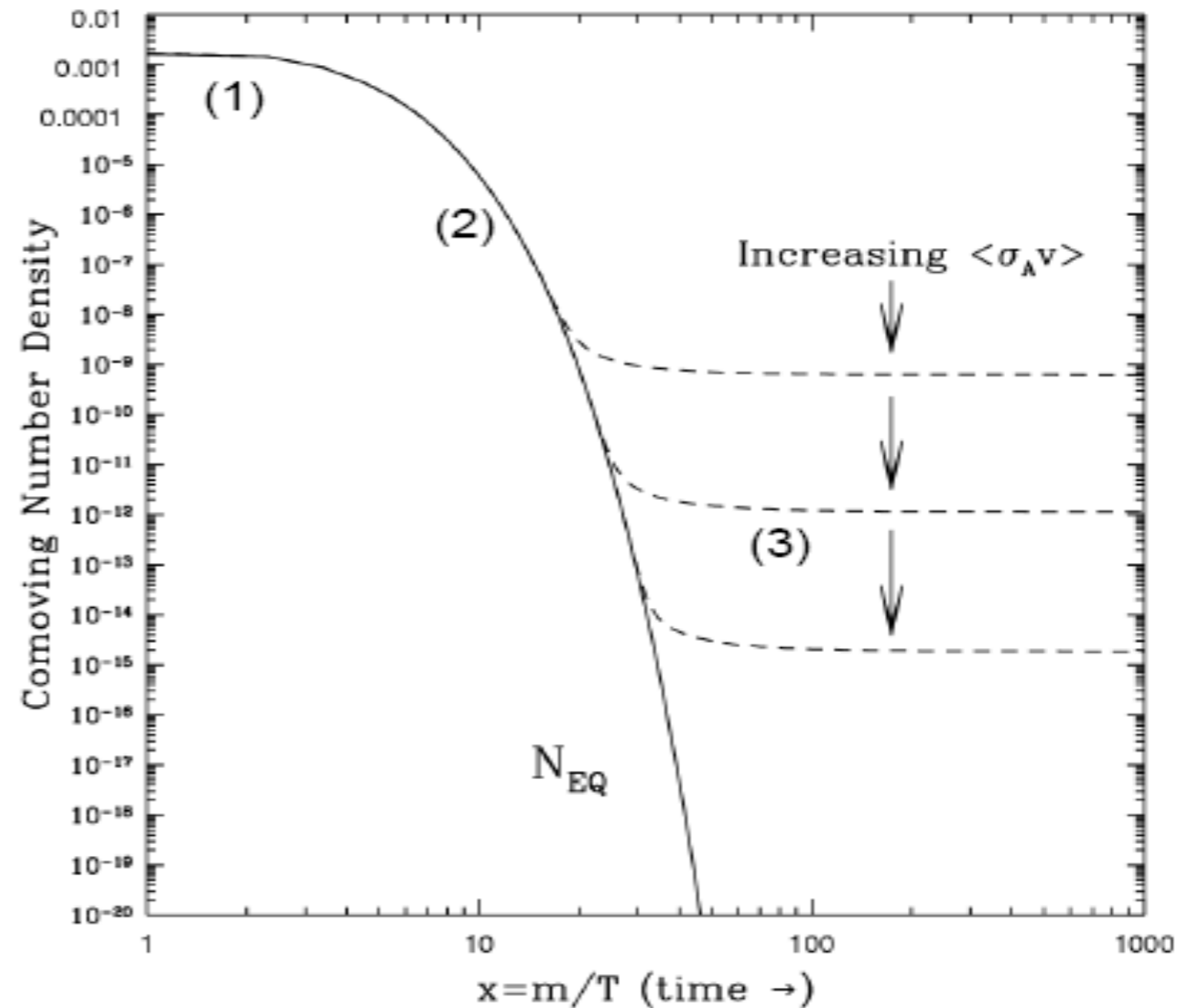
**A natural
DM
candidate**

Review article: Bertone, Hooper & Silk (2005)

A solution to the DM problem

- Under R-parity, the neutralino is stable
- Neutral, weakly interacting, mass in the range ~ 100 GeV to a few TeV
- Cosmological neutralino relic abundance:

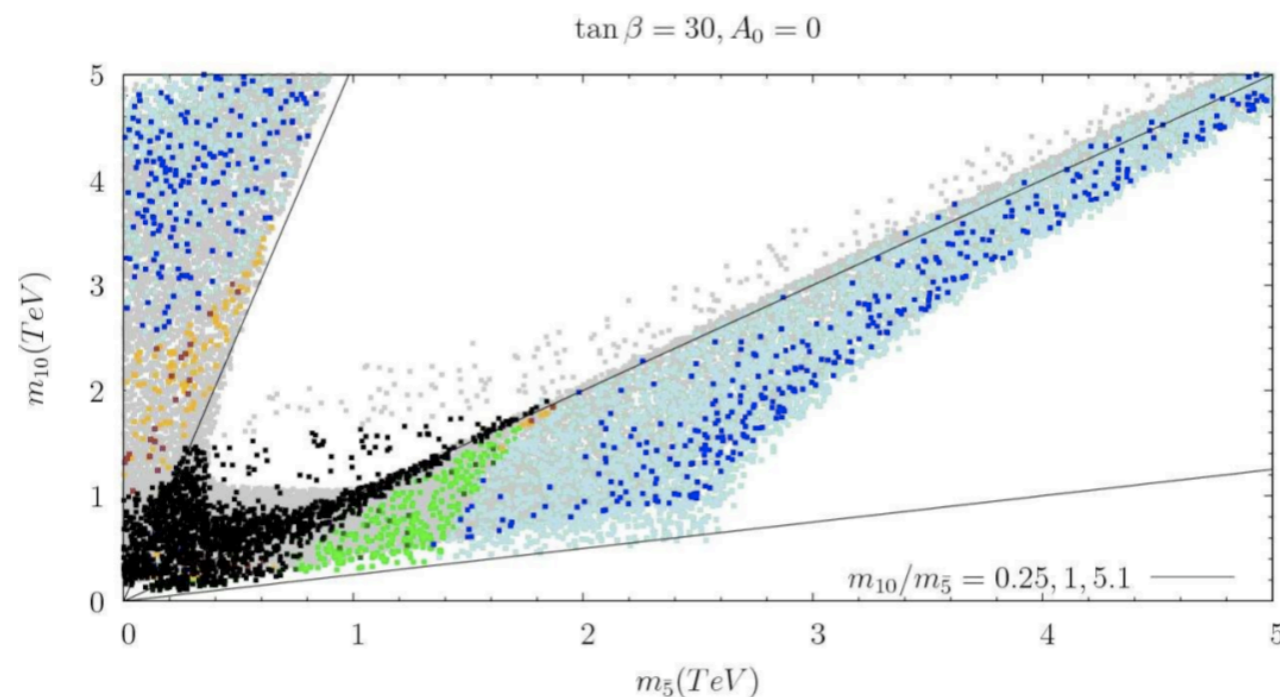
$$\Omega_{\chi} h^2 \sim \frac{3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma v \rangle}$$



Courtesy Sabine Kraml

- The general Minimal Supersymmetric Standard Model (MSSM):
105 free parameters!
- Need some (pretty strong) simplifying assumption:
the Constrained MSSM (CMSSM) reduces the free parameters to just **4 continuous variables plus a discrete one** ($\text{sign}(\mu)$).
- Clearly a highly constrained model (probably not the end of the story!)
- **Present-day data:** collider measurements of rare processes, CDM abundance (Planck), sparticle masses lower limits, EW precision measurements, Higgs mass and couplings.
- **Astrophysical direct and indirect detection techniques might also be competitive:** neutrino (IceCUBE), gamma-rays (Fermi), antimatter (PAMELA), direct detection (XENON1T, LUX, PandaX,...)

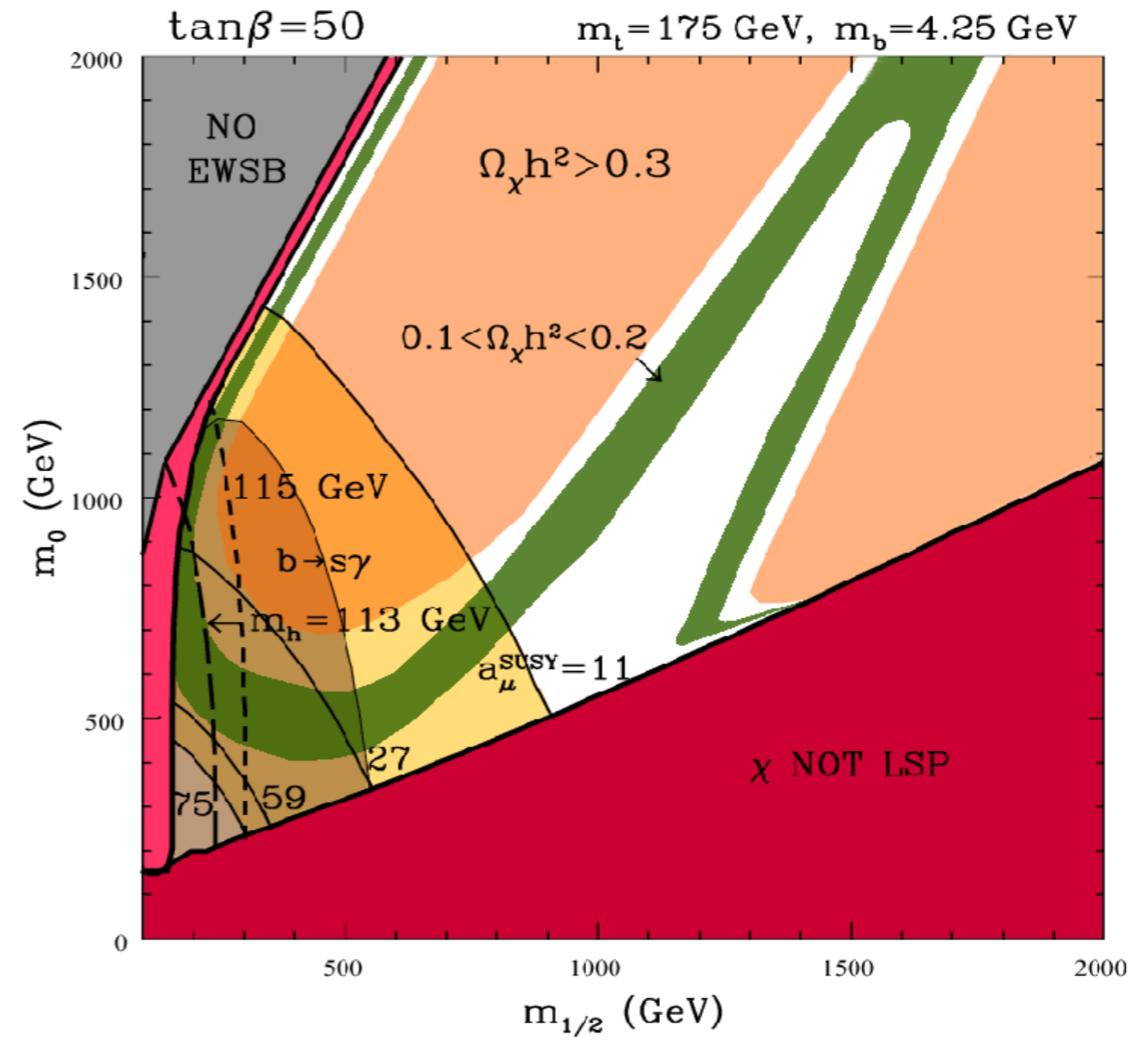
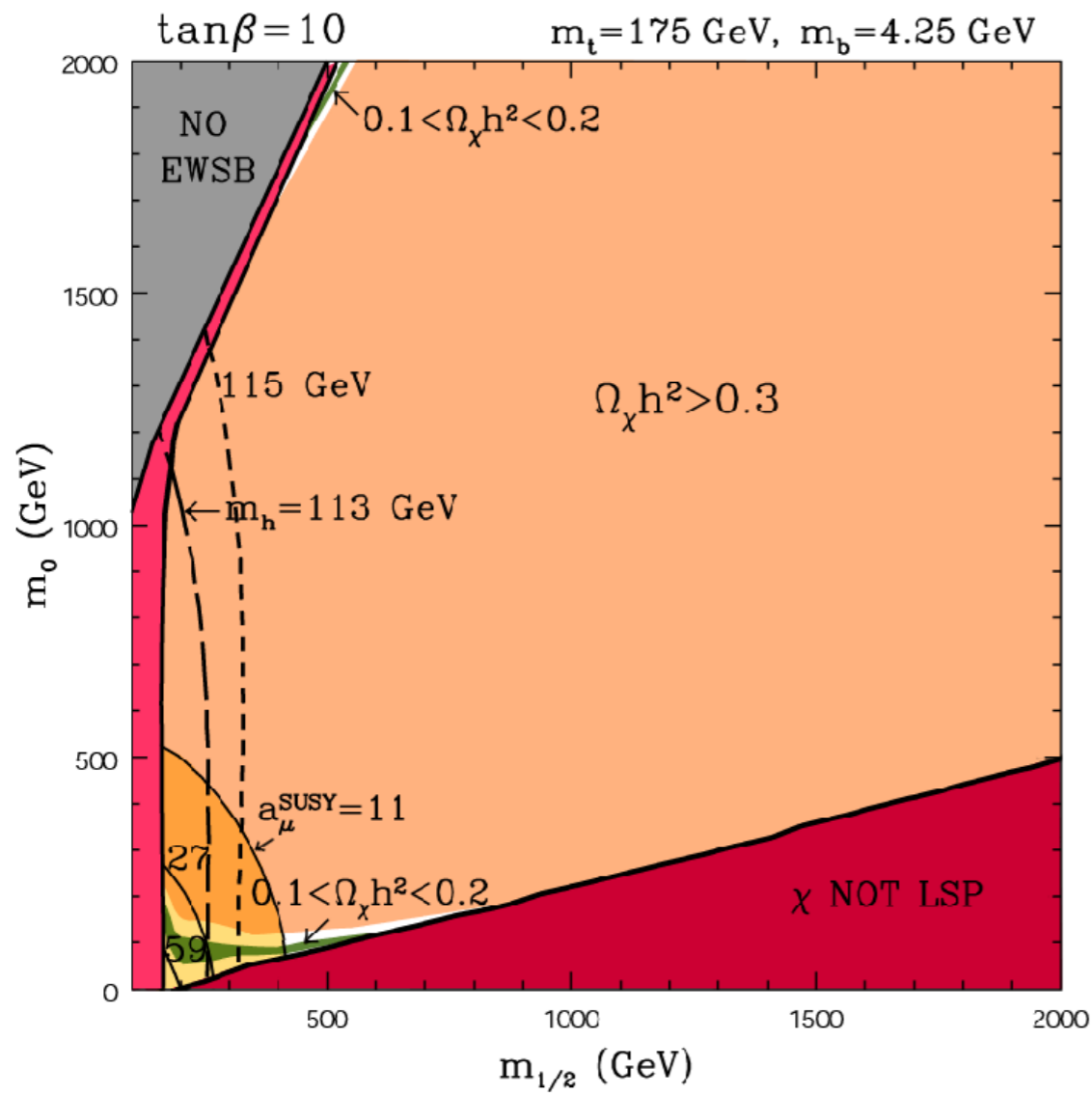
Exploration with “random scans”



Gogoladze et al (2008)

- Points accepted/rejected in a in/out fashion (e.g., 2-sigma cuts)
- No statistical measure attached to density of points: no probabilistic interpretation of results possible
- Inefficient/Unfeasible in high dimensional parameters spaces ($N > 3$)
- Explores only a very limited portion of the parameter space!

2 dimensional slices



Roszkowski et al (2001)

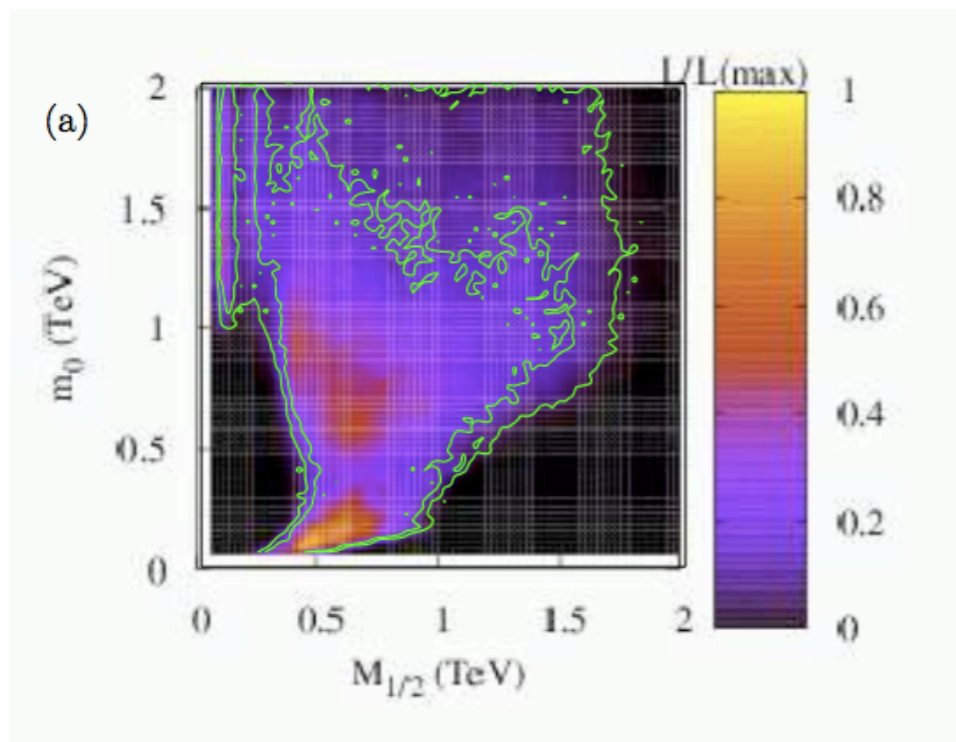
“Global fits”: multi-parameters, simultaneous likelihood-based fits to data in several observational channels (direct/indirect detection+colliders+cosmology), often including uncertainties from poorly-known nuisance parameters (e.g., astrophysical quantities)

Why global fits?

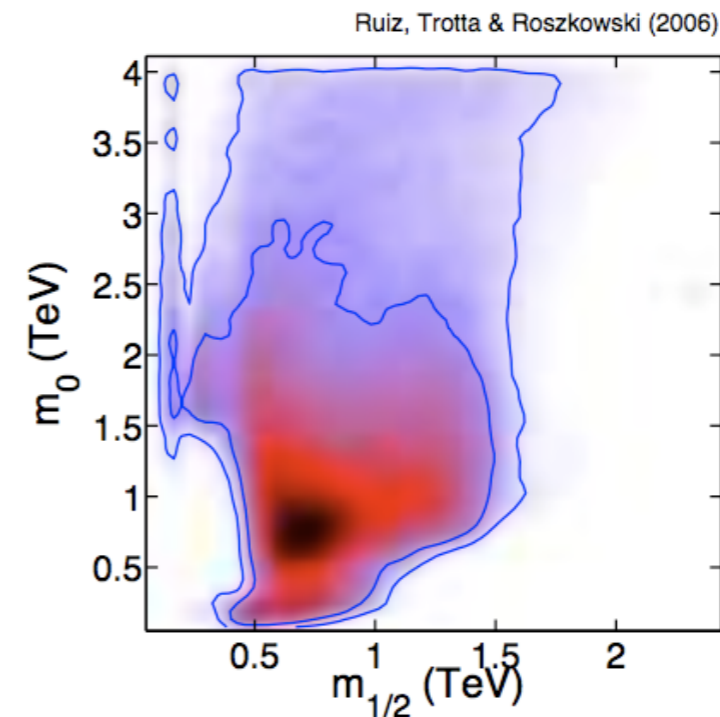
- The theoretical interpretation of a Dark Matter-like signal requires fitting an underlying model (e.g., SUSY, extra dimensions, etc) to the data.
- In case of a detection, a global multi-messenger approach will allow to check the consistency of the theory across observables and to obtain more stringent constraints on the Dark Matter properties.
- Robust and believable interpretation of direct and indirect detection data requires a careful modeling of astrophysical and experimental uncertainties.

Global CMSSM scans

- Bayesian approach introduced by two groups (early work by Baltz & Gondolo, 2004):
- Ben Allanach (DAMPT) and collaborators (Allanach & Lester, 2006 onwards)
- Ruiz de Austri, Roszkowski & RT (Ruiz de Austri et al, 2006 onwards)
+ Feroz & Hobson (MultiNest), + Silk (indirect detection), + Strigari (direct detection), + Martinez et al (dwarfs), + de los Heros (IceCube)

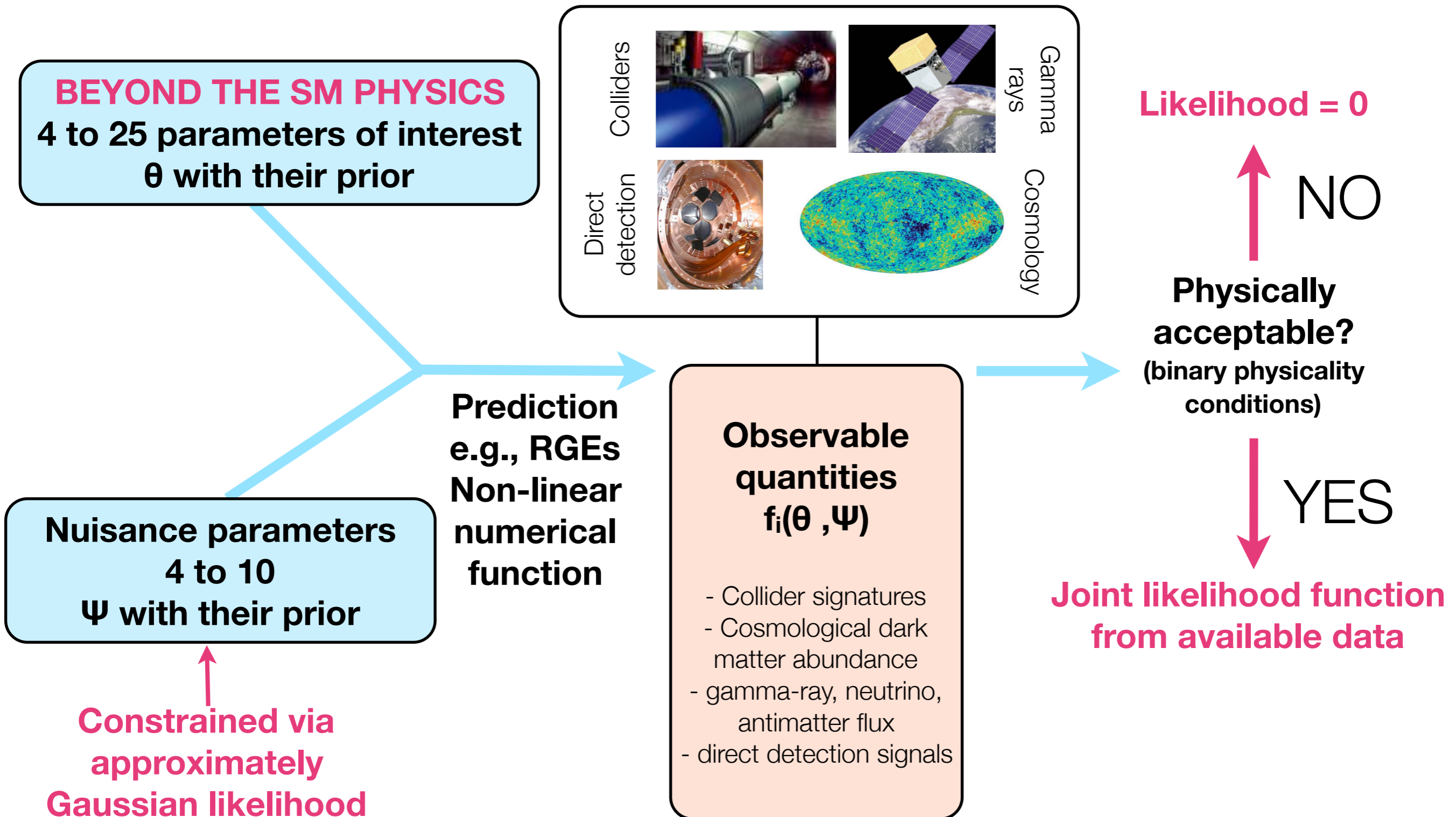


Allanach & Lester (2006)

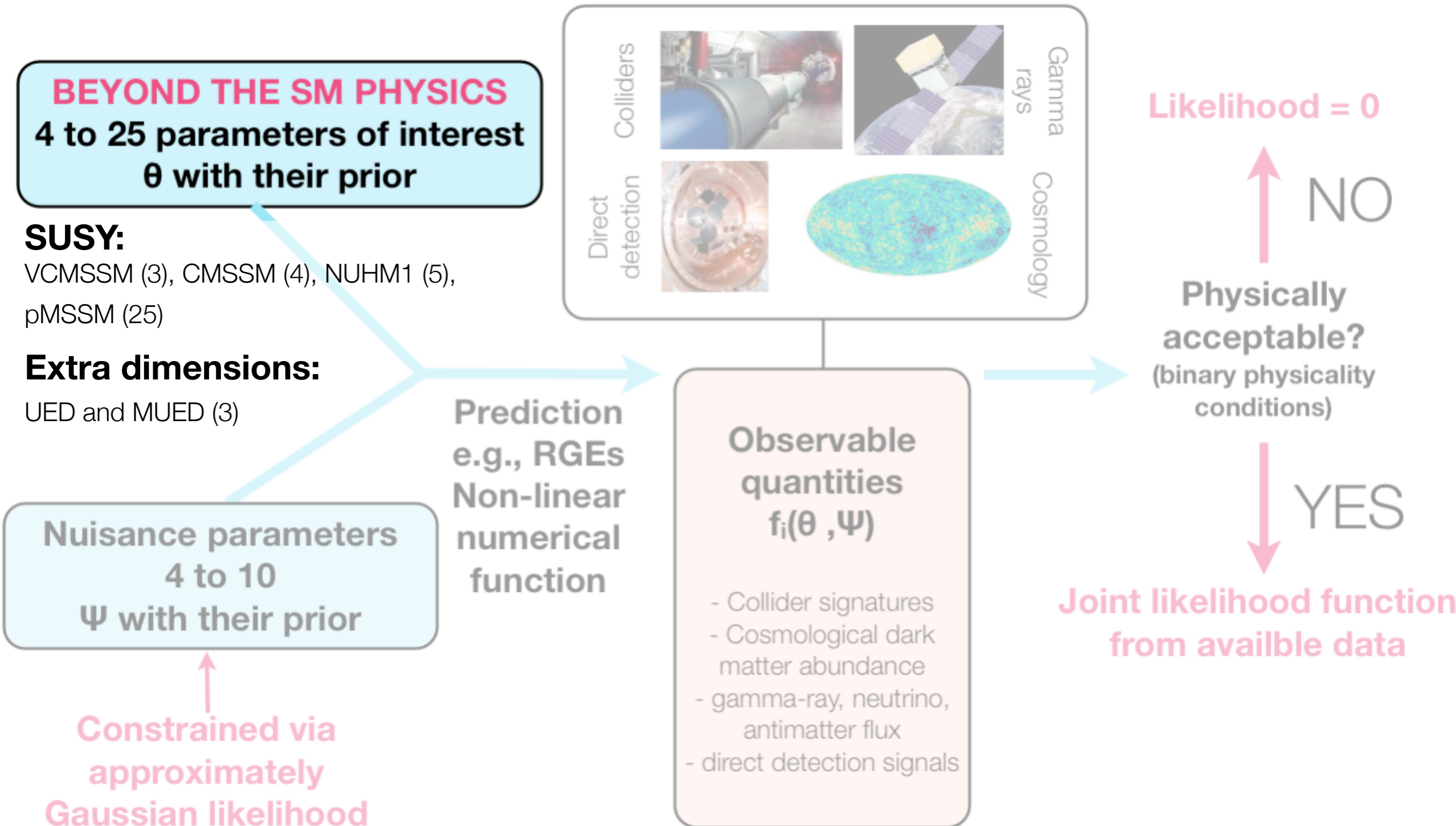


Ruiz de Austri, Roszkowski & RT (2006)

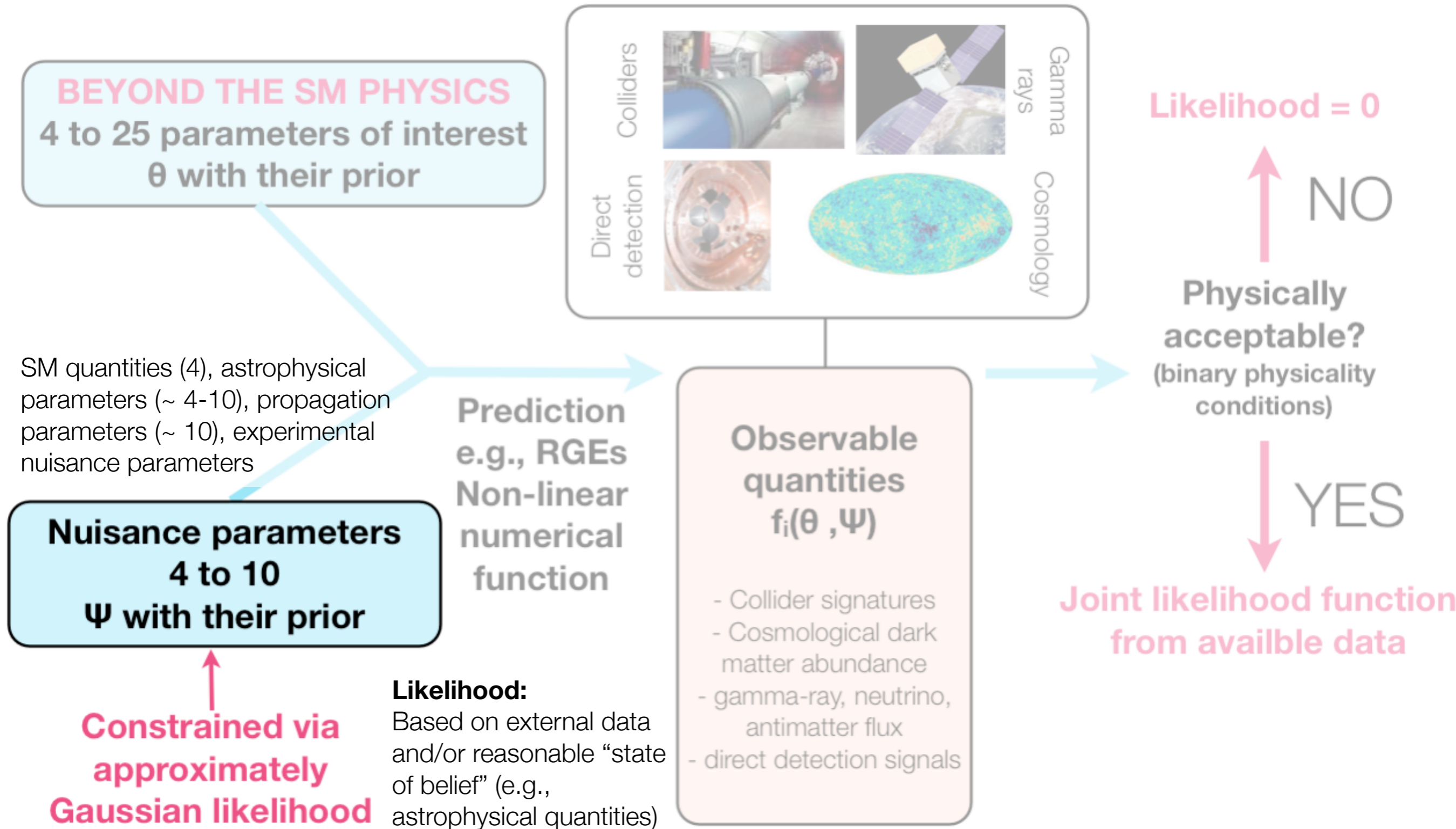
Generic analysis pipeline for BSM physics



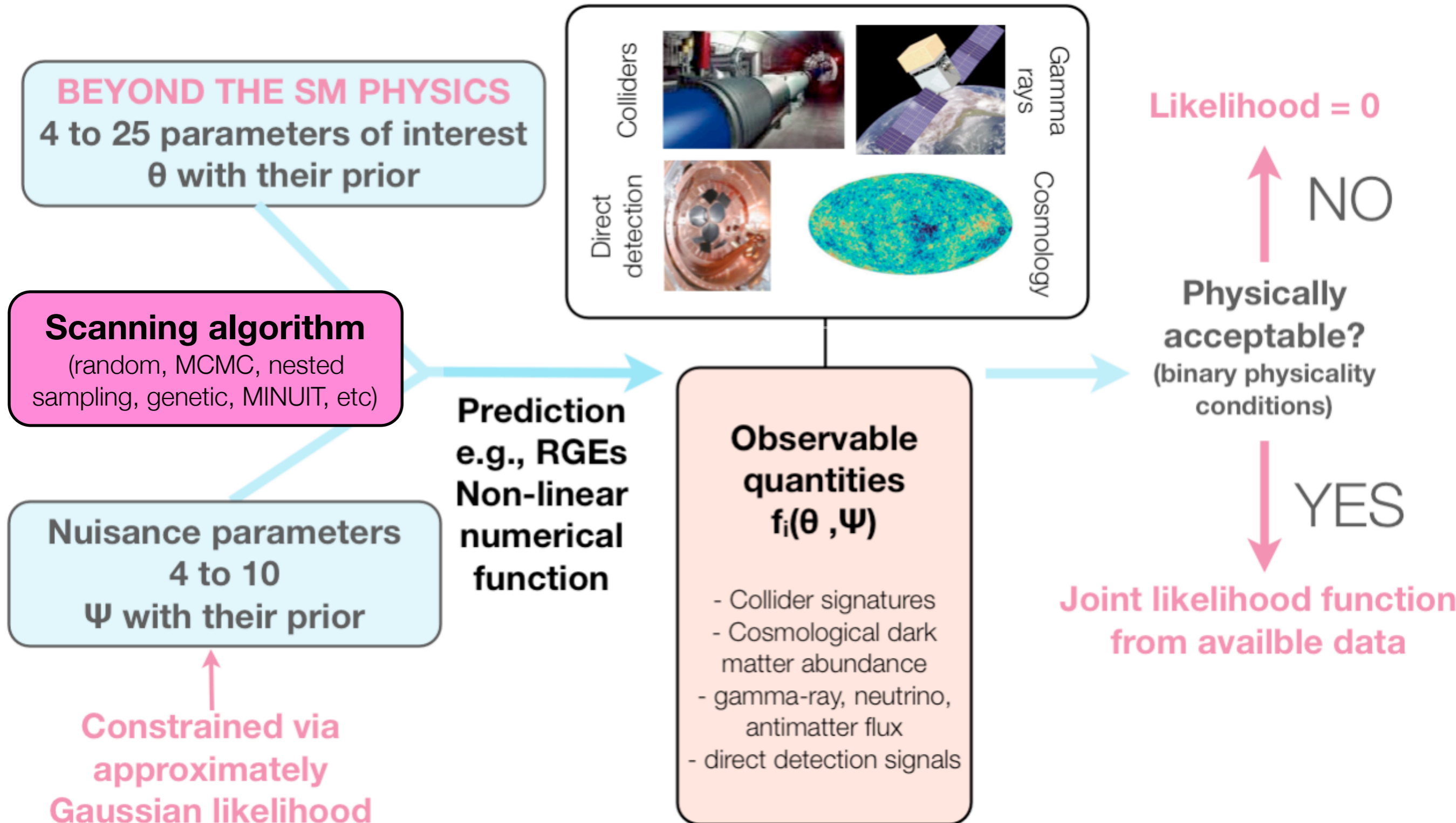
Analysis pipeline for BSM physics



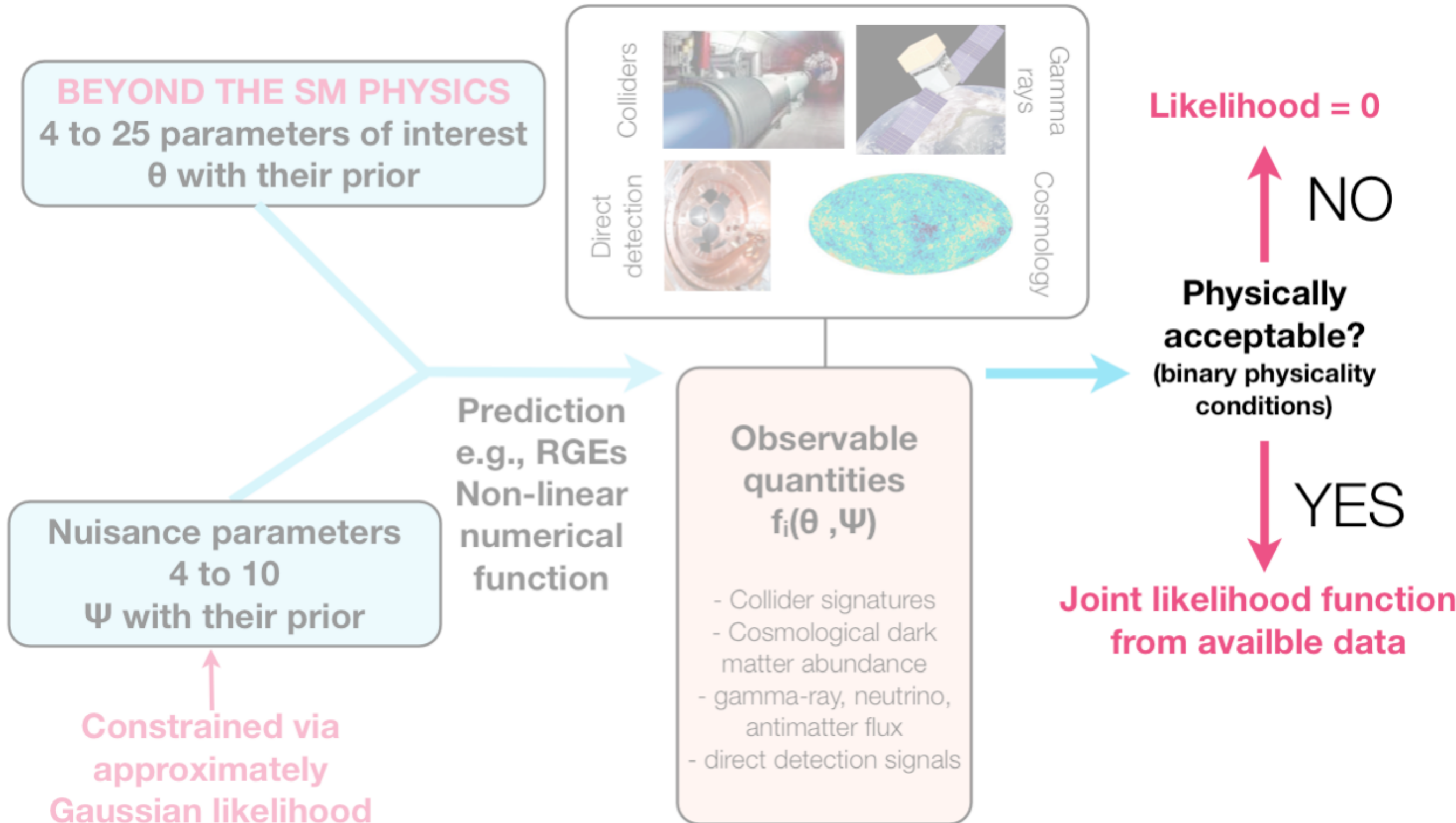
Analysis pipeline for BSM physics



Analysis pipeline for BSM physics



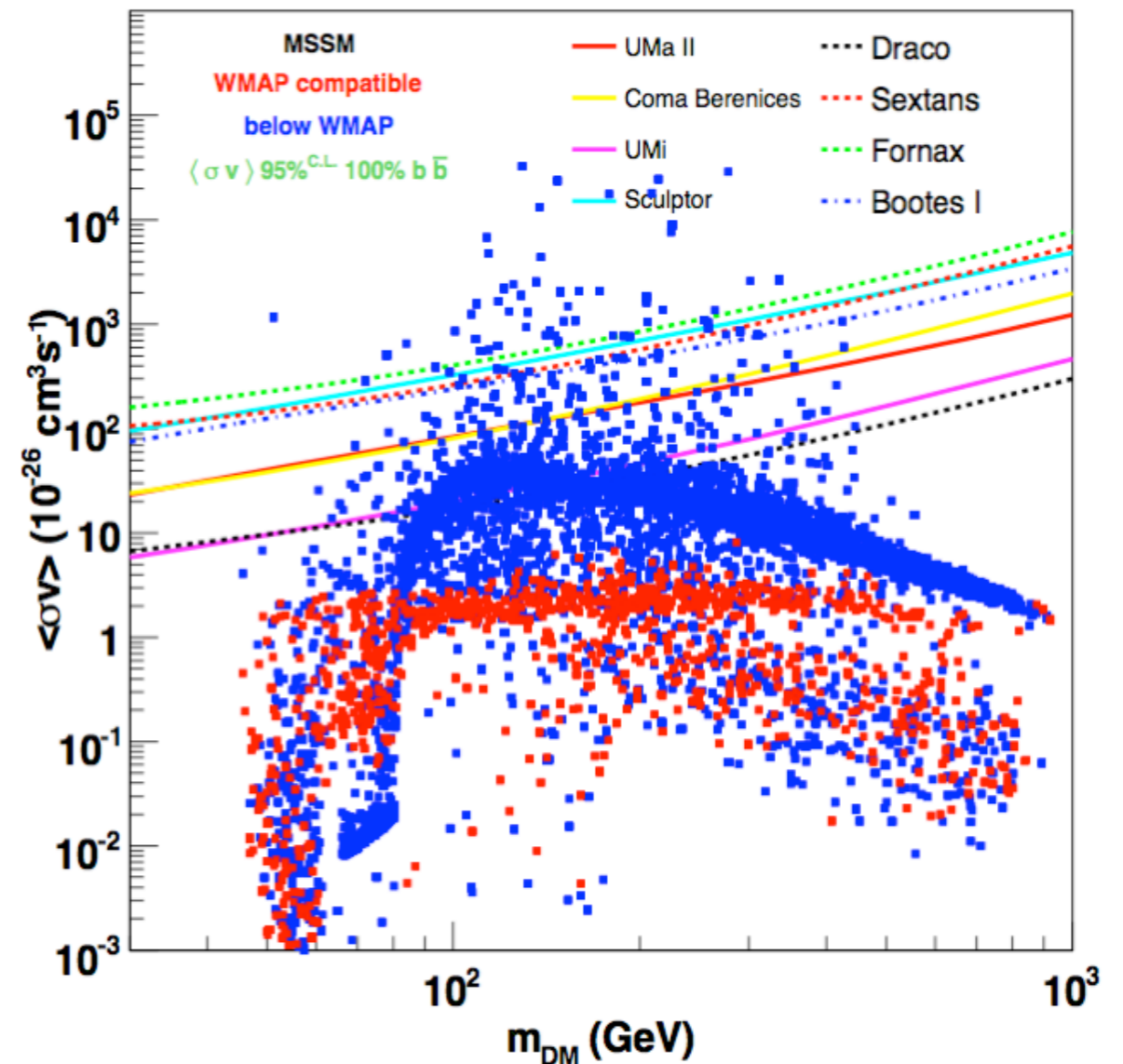
Analysis pipeline for BSM physics



Exploration with “random scans”

- Points accepted/rejected in a in/out fashion (e.g., 2-sigma cuts)
- No statistical measure attached to density of points: no probabilistic interpretation of results possible, although the temptation cannot be resisted...
- Inefficient in moderately large dimensional parameters spaces (even just $D > 5$)

E.g.: Fermi constraints on DM gamma-ray flux from dwarfs compared with theory



Abdo et al, arxiv: 1001.4531

- Given a model M , with parameters of interest θ , nuisance parameters Ψ , a prior pdf $P(\theta, \Psi | M)$, and available data d with likelihood $P(d | \theta, \Psi, M) = L(\theta, \Psi)$, we need an algorithm to compute the following statistical quantities:

1. Marginal posterior pdf:

$$P(\theta|d, M) = \int d\Psi P(\theta, \Psi|d, M) = \int d\Psi \frac{L(\theta, \Psi)P(\theta, \Psi|M)}{P(d|M)}$$

2. Profile likelihood ratio:

$$\lambda(\theta) = \frac{L(\theta, \hat{\psi})}{L(\hat{\theta}, \hat{\psi})}$$

where $\hat{\psi}$ is the conditional MLE and $\hat{\theta}$ the unconditional MLE

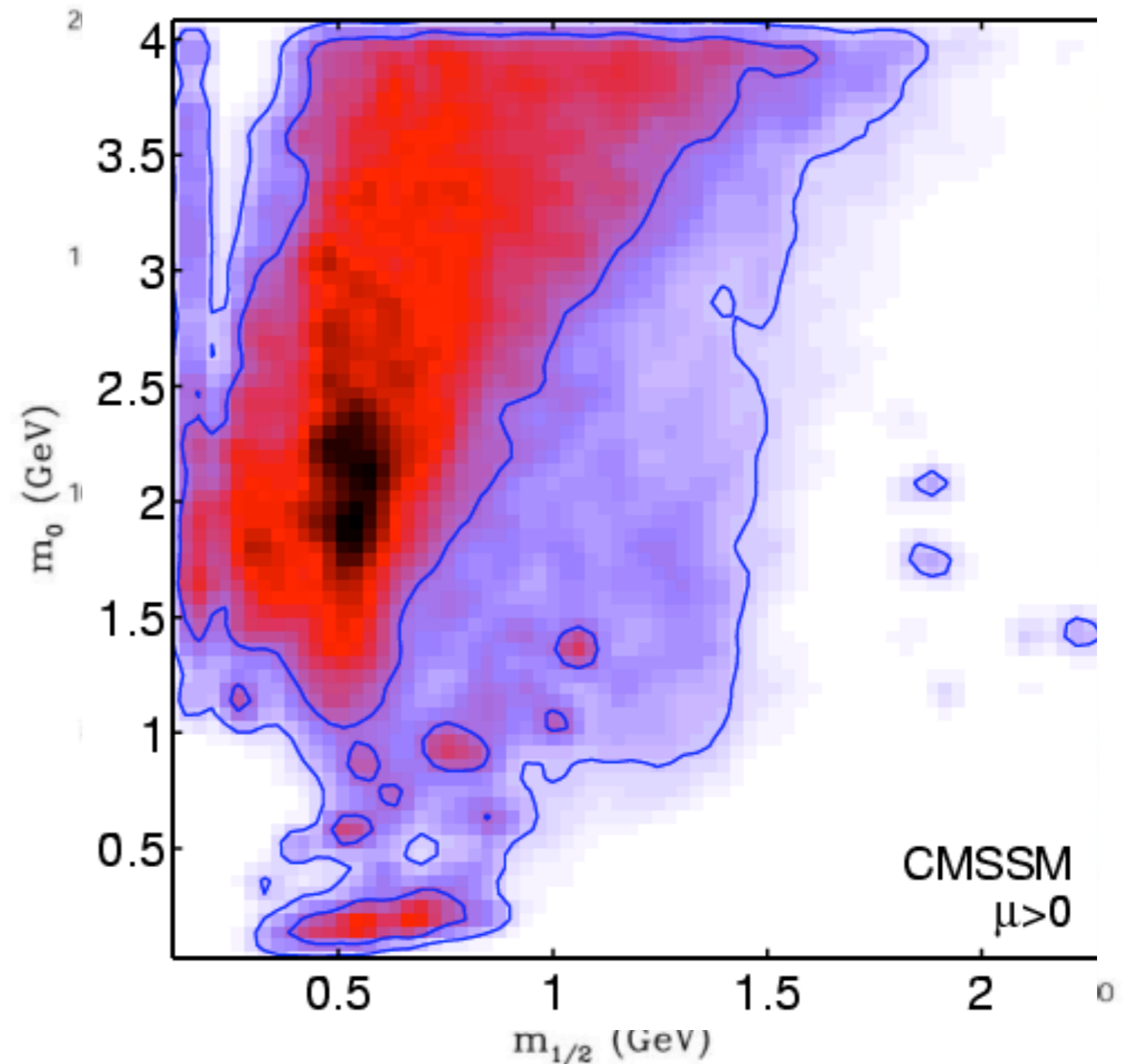
3. Bayesian evidence (model likelihood):

$$P(d|M) = \int d\theta d\Psi L(\theta, \Psi)P(\theta, \Psi|M)$$

Solution: global fits

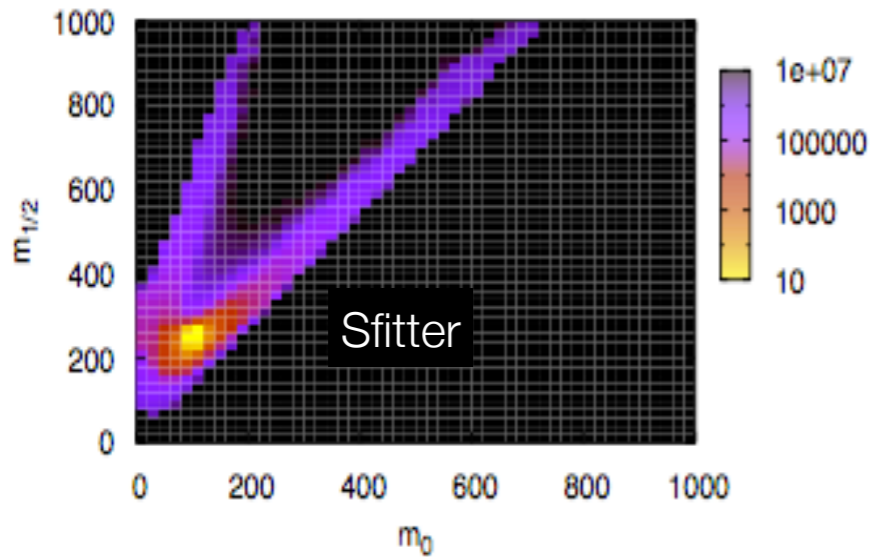
Carry out a **simultaneous fit** of all relevant SUSY and SM parameter to the experimental data/constraints.

Marginalize (= integrate) or maximise along the hidden dimensions to obtain results that account for the multi-dimensional nature of the problem.

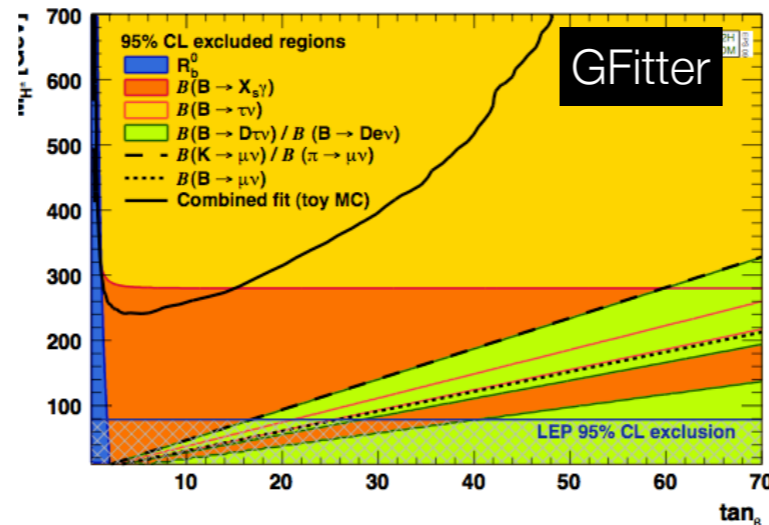


Global Fits: Some History

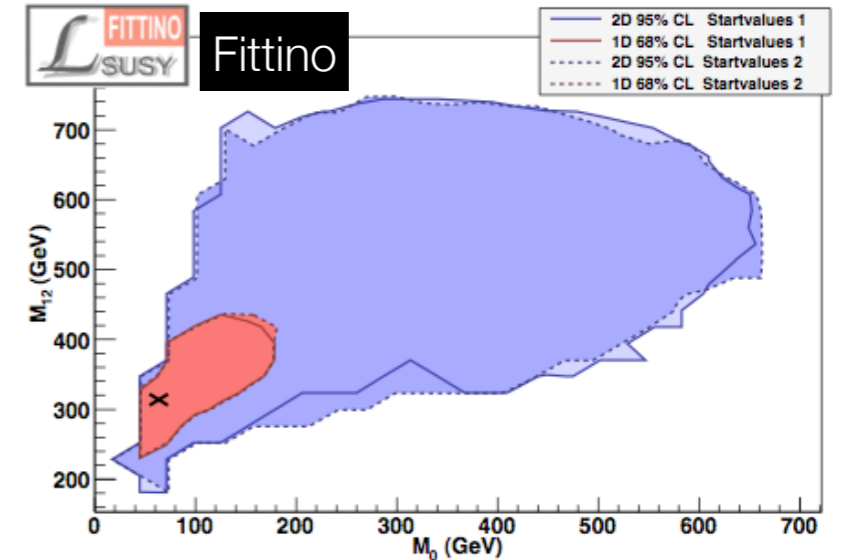
R. Lafaye, M. Rauch, T. Plehn, D. Zerwas



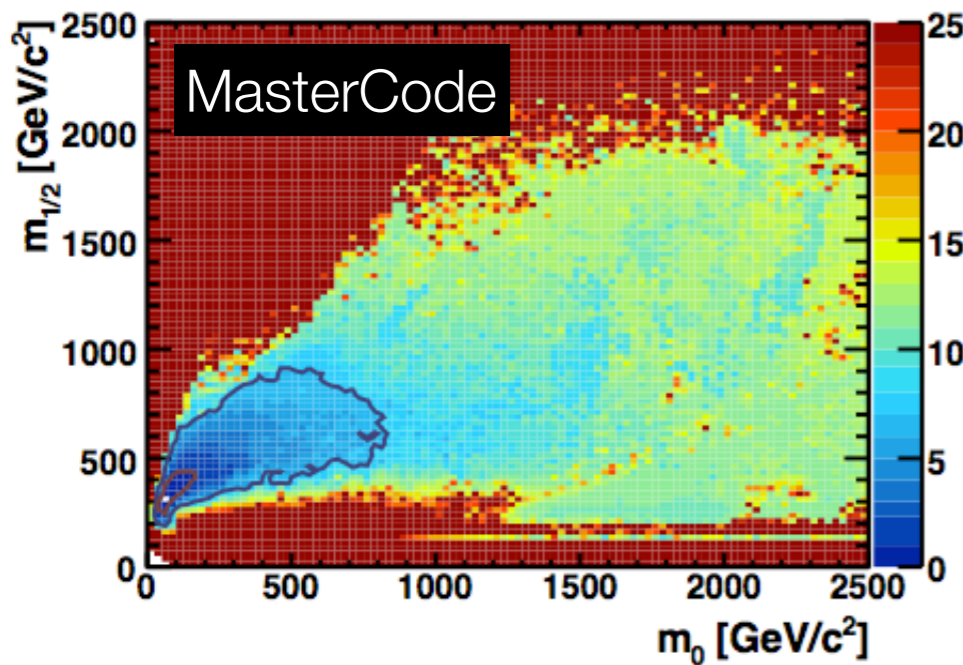
H. Flächer, M. Goebel, J. Haller, A. Höcker, K. Mönig, J. Stelzer



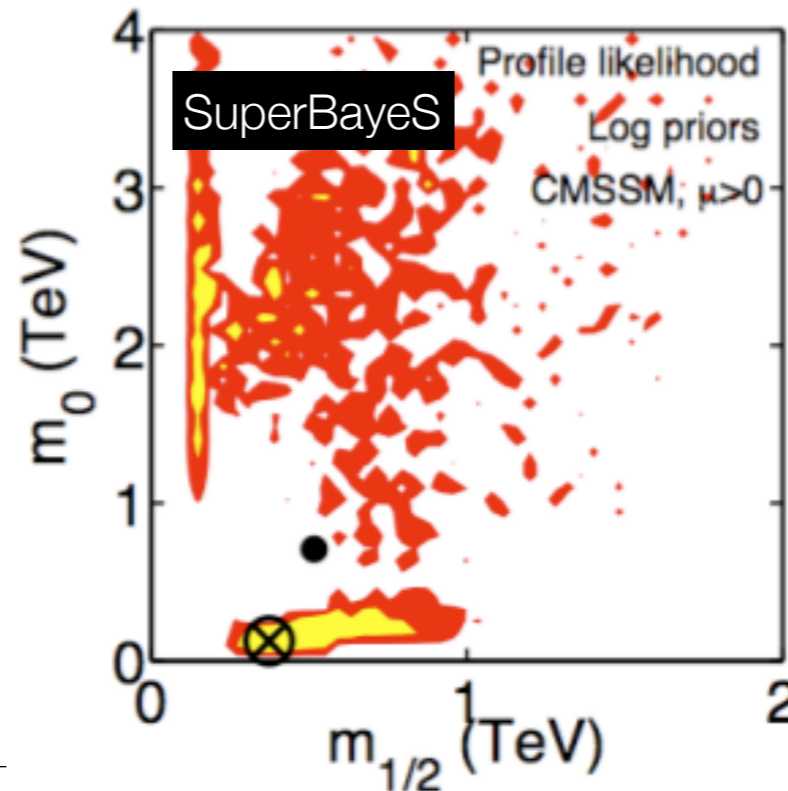
P. Bechtle, K. Desch, M. Uhlenbrock, P. Wienemann



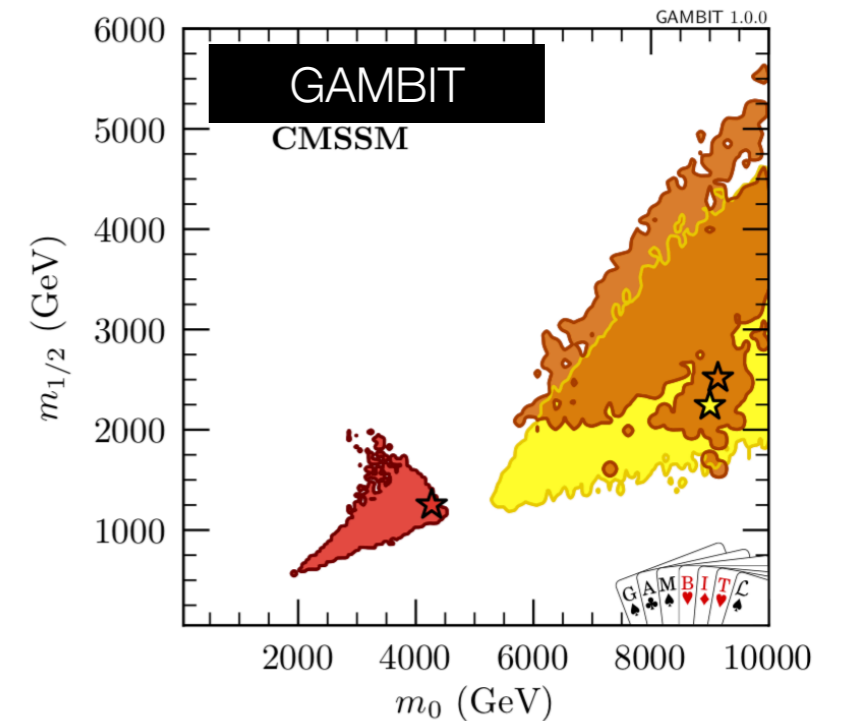
O. Buchmueller, R. Cavanaugh, A. De Roeck, J.R. Ellis, H. Flacher, S. Heinemeyer, G. Isidori, K.A. Olive, F.J. Ronga, G. Weiglein



F. Feroz, R. Ruiz de Austri, R. Trotta, M.P. Hobson



GAMBIT Collaboration (P. Scott, PI)



First Generation Global Fits: SuperBayeS

SuperBayeS

Supersymmetry Parameters Extraction Routines for Bayesian Statistics

- Implements the CMSSM, but can be easily extended to the general MSSM
- **Latest release (v 1.5.1) in April 2011:** linked to SoftSusy 2.0.18, DarkSusy 5.0, MICROMEAS 2.2, FeynHiggs 2.5.1, Hdecay 3.102. Uses MultiNest v 2.8.
- Includes up-to-date constraints from all observables, plotting routines, statistical analysis tools, posterior and profile likelihood plots. Fully parallelized, MPI-ready, user-friendly interface
- MCMC engine (Metropolis-Hastings, bank sampler), grid scan mode, multi-modal nested sampling MultiNest algorithm (Feroz & Hobson 2008)
A full 8D scan now takes less than 2 days on 8 CPUs.
- **Now discontinued**

Second Generation Global Fits: GAMBIT

GAMBIT: The Global And Modular BSM Inference Tool

gambit.hepforge.org

EPJC 77 (2017) 784

arXiv:1705.07908

- Extensive model database – not just SUSY
- Extensive observable/data libraries
- Many statistical and scanning options (Bayesian & frequentist)
- *Fast* LHC likelihood calculator
- Massively parallel
- Fully open-source
- Fast definition of new datasets and theories
- Plug and play scanning, physics and likelihood packages



Members of: ATLAS, Belle-II, CMS, CTA, *Fermi*-LAT, DARWIN, IceCube, LHCb, SHiP, XENON

Authors of: DarkSUSY, DDCalc, Diver, FlexibleSUSY, gamlike, GM2Calc, IsaJet, nulike, PolyChord, Rivet, SOFTSUSY, SuperIso, SUSY-AI, WIMPSim



Collaborators:

Peter Athron, Csaba Balázs, Ankit Beniwal, Florian Bernlochner, Sanjay Bloor, Torsten Bringmann, Andy Buckley, Eliel Camargo-Molina, Marcin Chrzęszc, Jan Conrad, Jonathan Cornell, Matthias Danninger, Tom Edwards, Joakim Edsjö, Ben Farmer, Andrew Fowlie, Tomás Gonzalo, Will Handley, Sebastian Hoof, Selim Hotinli, Felix Kahlhoefer, Suraj Krishnamurthy, Anders Kvellestad, Julia Harz, Paul Jackson, Tong Li, Greg Martinez, Nazilla Mahmoudi, James McKay, Are Raklev, Janina Renk, Chris Rogan, Roberto Ruiz de Austri, Patrick Stoecker, Roberto Trotta, Pat Scott, Nicola Serra, Daniel Steiner, Puwen Sun, Aaron Vincent, Christoph Weniger, Sebastian Wild, Martin White, Yang Zhang

40+ participants in 10 Experiments & 14 major theory codes

3

The **G**lobal **A**nd **M**odular **B**SM **I**nference **T**ool

- A new framework for BSM **global fits**
- Fully **open source**
- **Modular design:** easily extended with
 - new models
 - new likelihoods
 - new theory calculators
 - new scanning algorithms
- Use external codes (**backends**) as **runtime plugins**
 - Currently supported:
C, C++, Fortran, Mathematica
 - Coming soon: Python
- **Two-level parallelization** with MPI and OpenMP
- **Hierarchical** model database
- **Flexible output streams** (ASCII, HDF5, ...)
- Many **scanners** and **backends** already included



The screenshot shows the GAMBIT homepage layout. On the left is a navigation menu with a yellow background, listing: Home, Results & Publications, Talks, Collaboration, Download, Source Code, Support (with sub-items: FAQ, Compiler matrix, Known issues, Documentation, Configuration examples, Report issue), Mailing list, Contact, and Internal pages (with sub-items: Wiki, Git repos: gambit (dev fork), gambit_internal, gambit_results). On the right is a large graphic of a fan of playing cards. The top card is the Jack of Spades, with 'GAMBIT' written on it. The cards behind it are labeled G, A, M, B, I, T, C. Below the graphic, the text reads: 'GAMBIT The Global And Modular BSM Inference Tool'. A welcome message follows: 'Welcome to the GAMBIT homepage. GAMBIT is a global fitting code for generic Beyond the Standard Model theories, designed to allow fast and easy definition of new models, observables, likelihoods, scanners and backend physics codes.' Another message states: 'We have released GAMBIT to the public! Please check out the Source Code section and have fun with it!'. A final line says: 'You can read more about GAMBIT in this Physics World article.'

gambit.hepforge.org



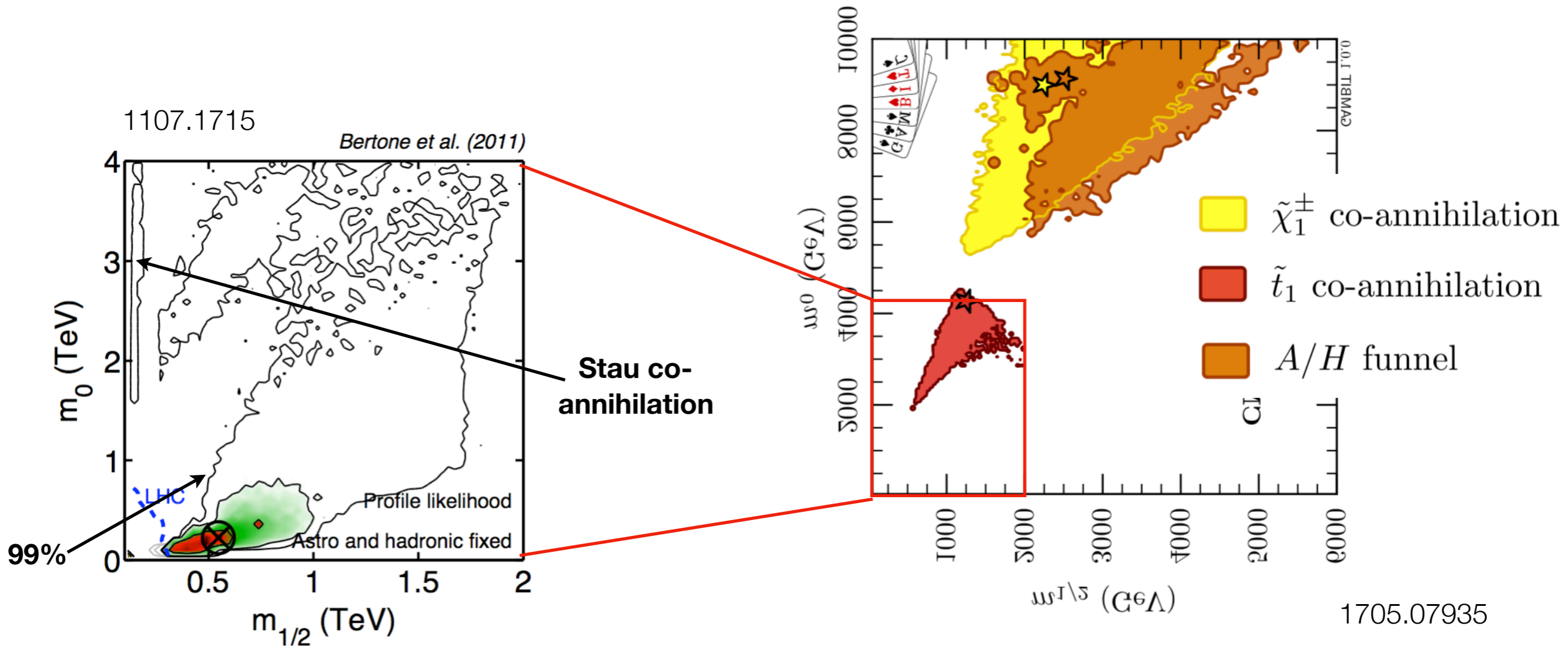
CMSSM: Frequentist Fits

SuperBayeS: profile likelihood

2011, 41M samples (~1 week on 8 CPUs)
incl. ATLAS 35 fb⁻¹ Run I and WMAP-7

GAMBIT: profile likelihood (95% CL)

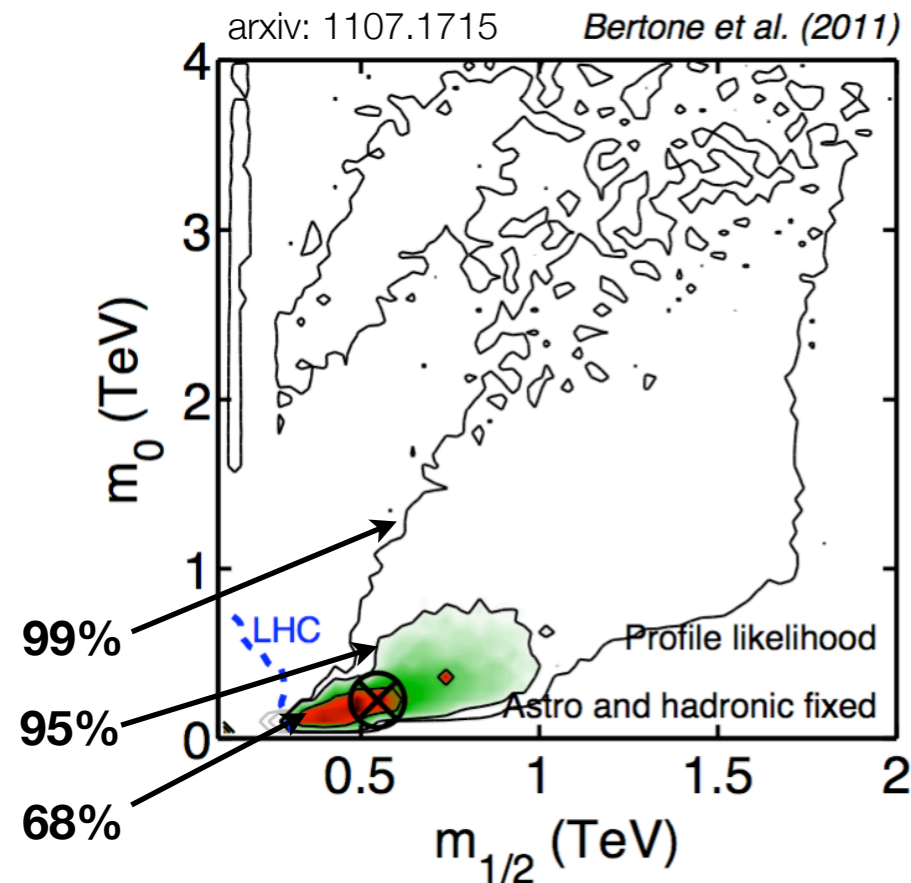
2018, 71M samples (3 days on 2400 CPUs)
incl. ATLAS/CMS Run I + II, Planck 2018



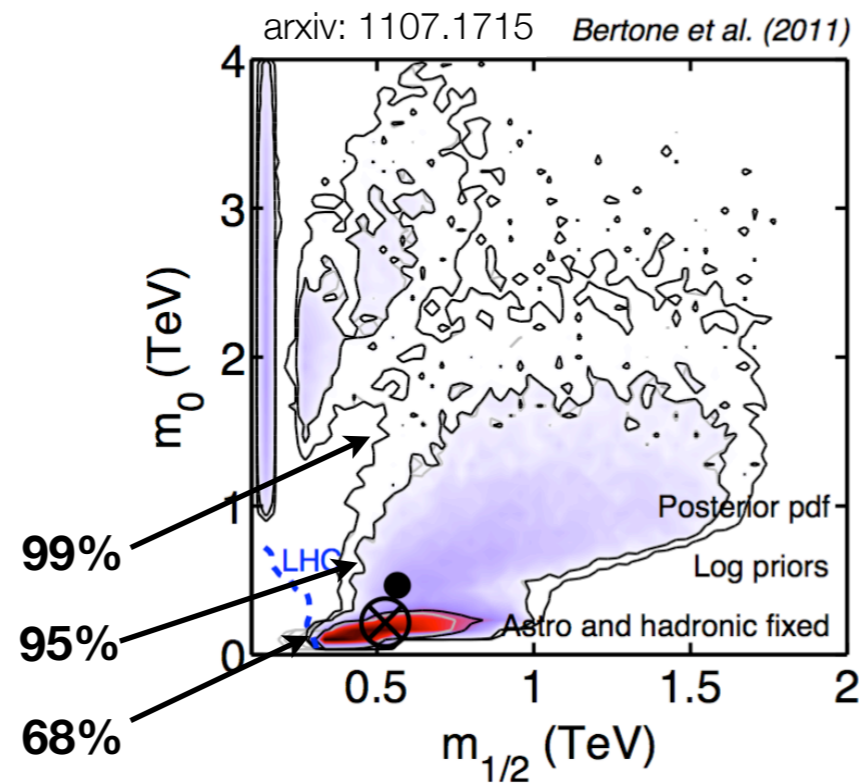
Stau co-annihilation region now ruled out at > 95% CL in the CMSSM.

CMSSM: Bayesian posteriors (2011)

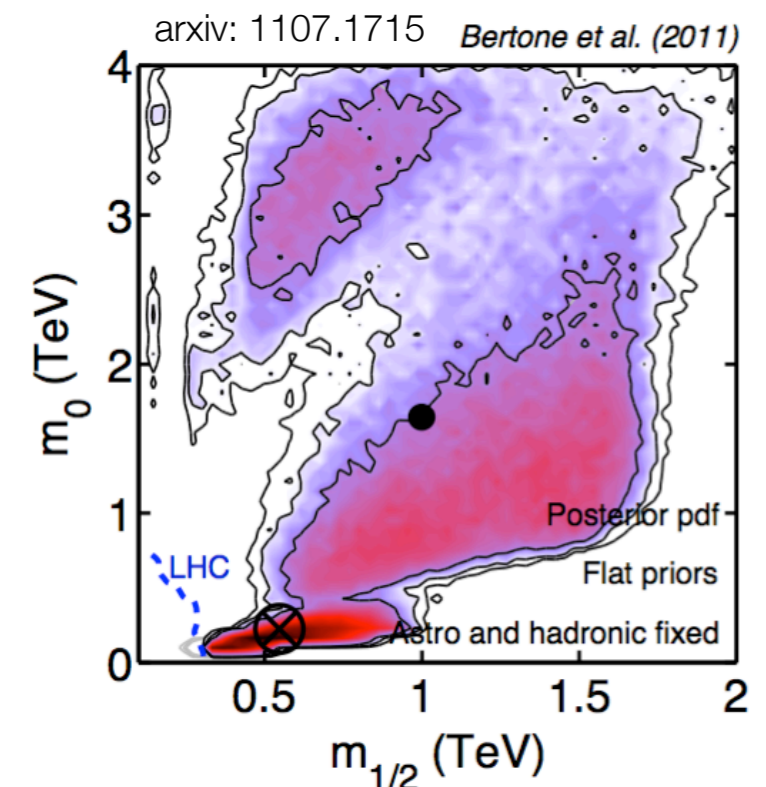
SuperBayeS: profile likelihood



SuperBayeS: Bayesian pdf "log" priors



"flat" priors

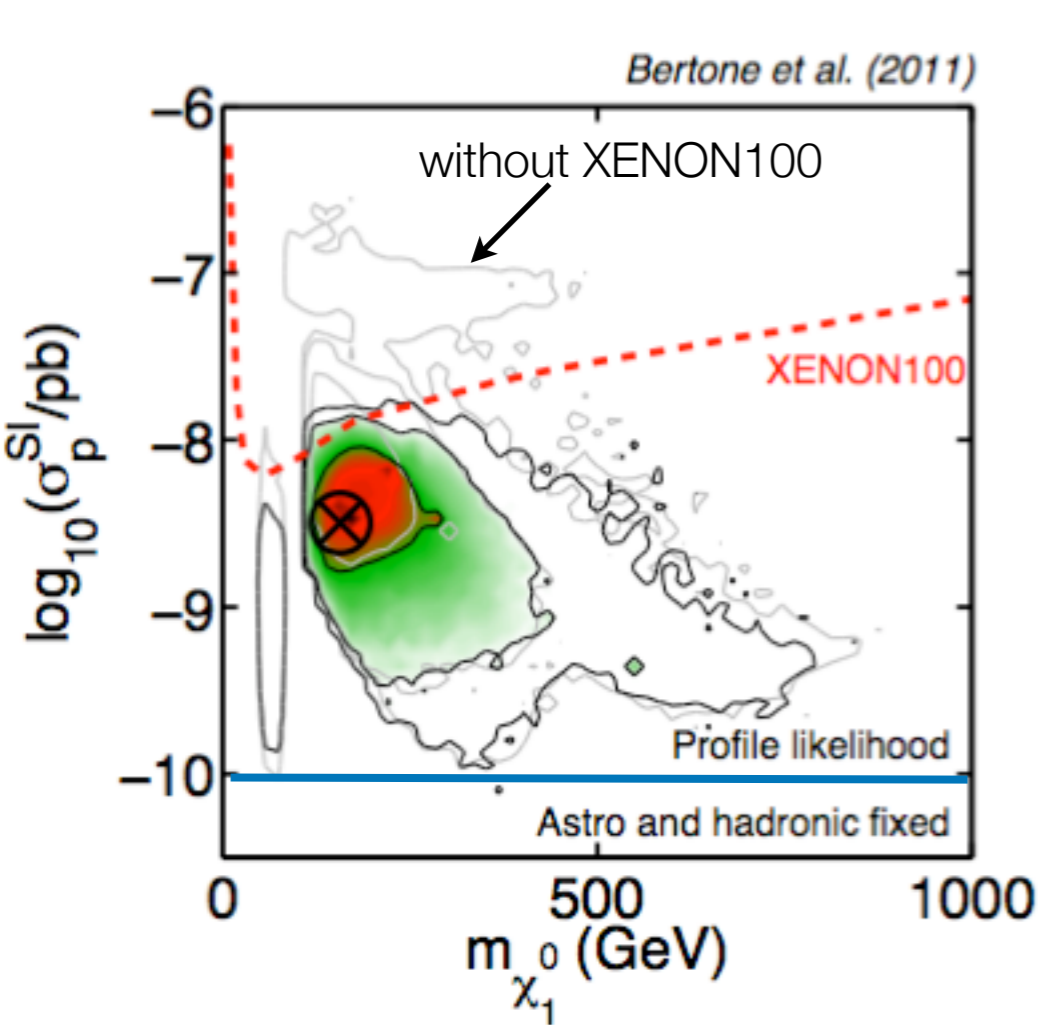


Tentative convergence between Frequentist and Bayesian scans has not improved despite 7 years of additional data (and stronger LHC lower limits).
The relative viability of the Focus Point region cannot be robustly established

Direct Detection Prospects

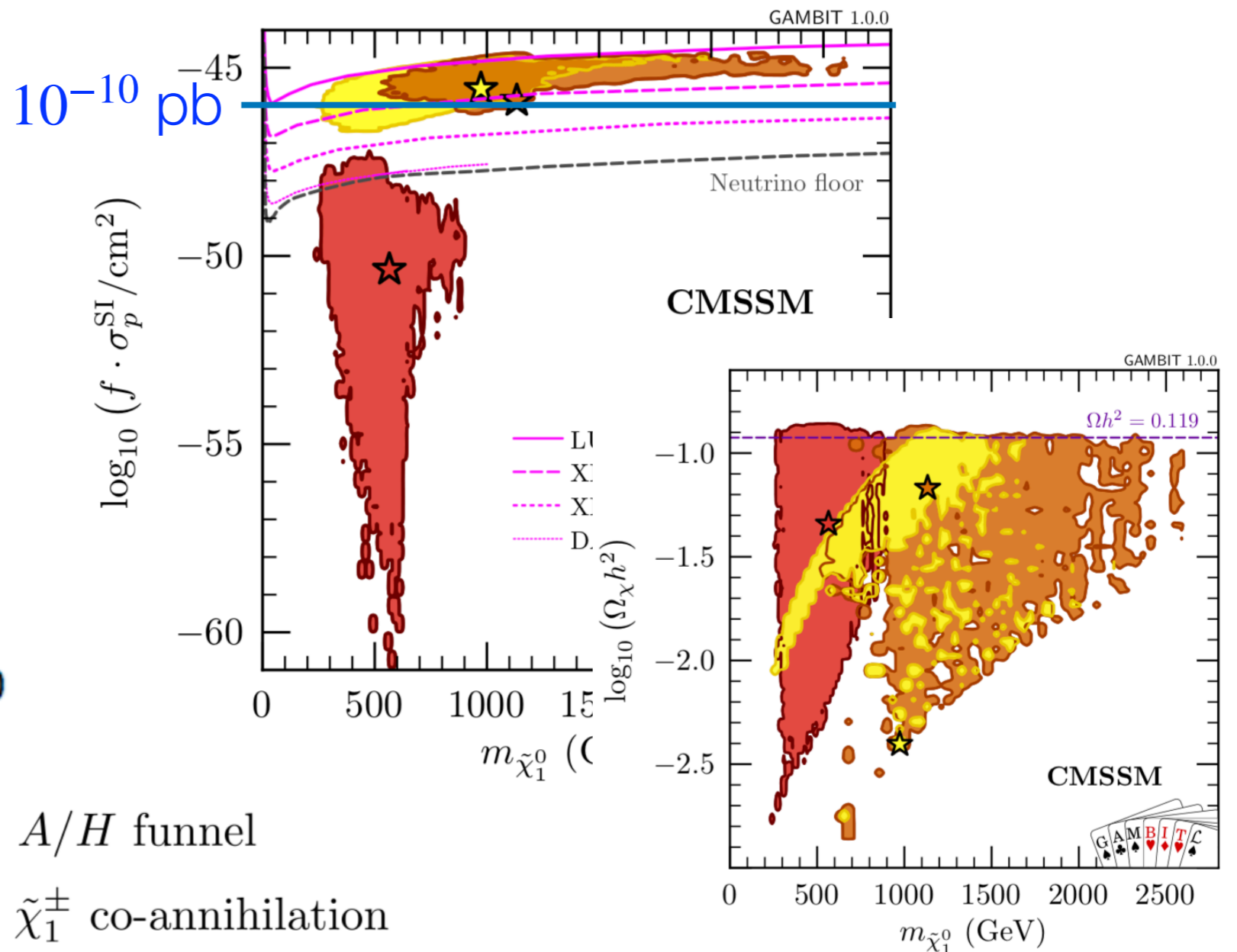
Profile likelihood, 2011

Astrophysical parameters fixed
Relic density constraint imposed



Profile likelihood, 2018

Astrophysical parameters varied
Relic density as upper limit only

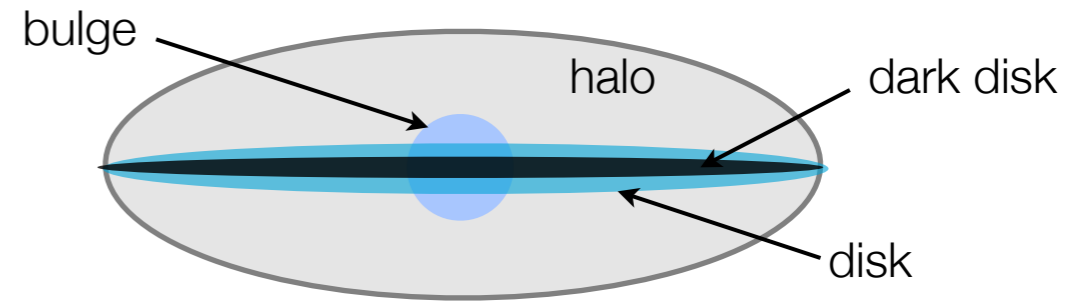


- A/H funnel
- $\tilde{\chi}_1^\pm$ co-annihilation
- \tilde{t}_1 co-annihilation

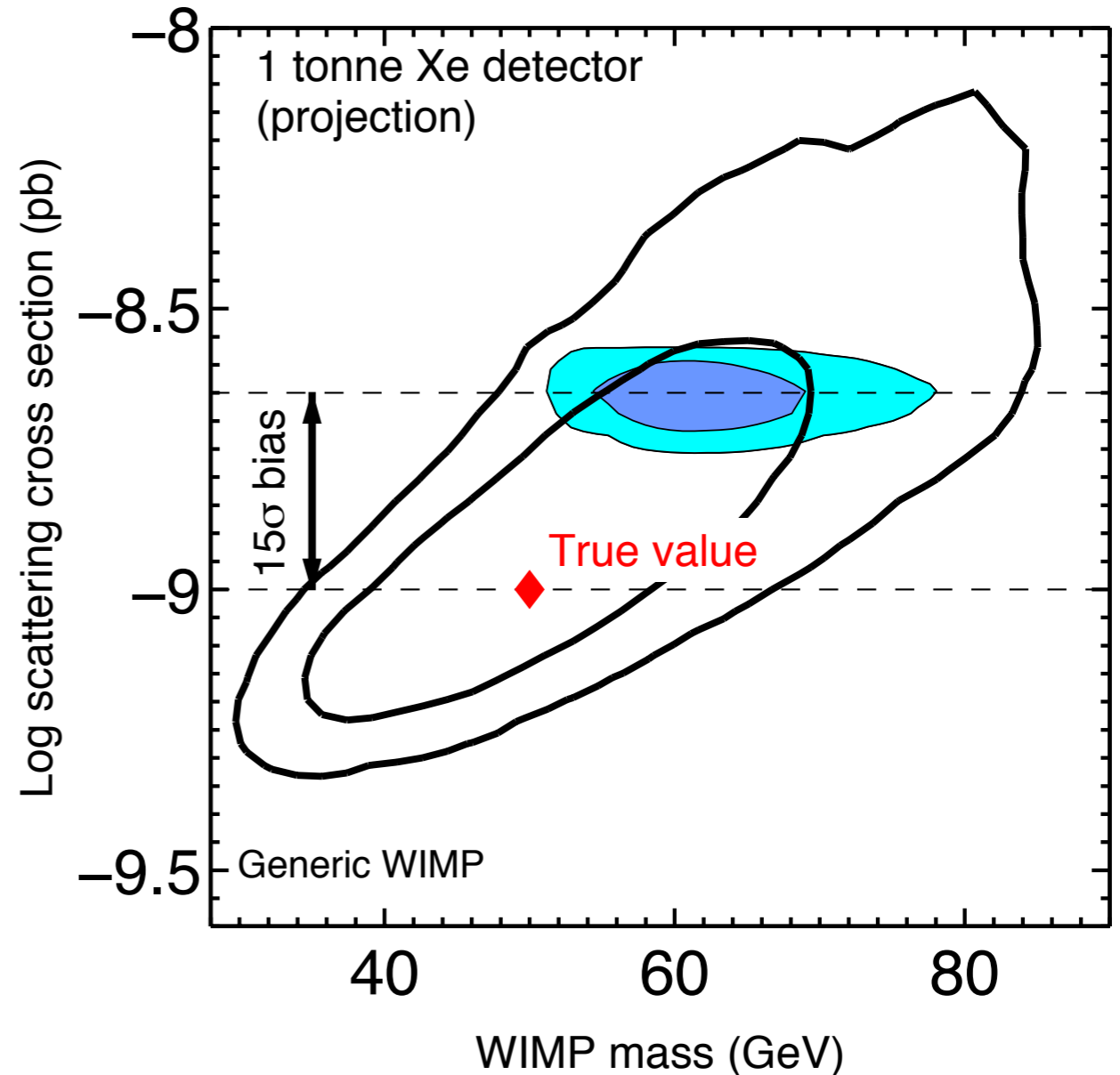
The importance of modeling the Milky Way

- Assuming an incorrect local density (by a factor of 2) can lead to **a 15 sigma bias** in the reconstructed cross section
- Milky Way modeled as a parameterized bulge+halo+disk spherical, isotropic superposition. Sloan-like data used to constrain the model's parameters
- **Marginalization over Milky Way halo model parameters converts catastrophic direct detection systematic errors into more manageable statistical errors**

Parameterized Milky Way model (7 parameters)



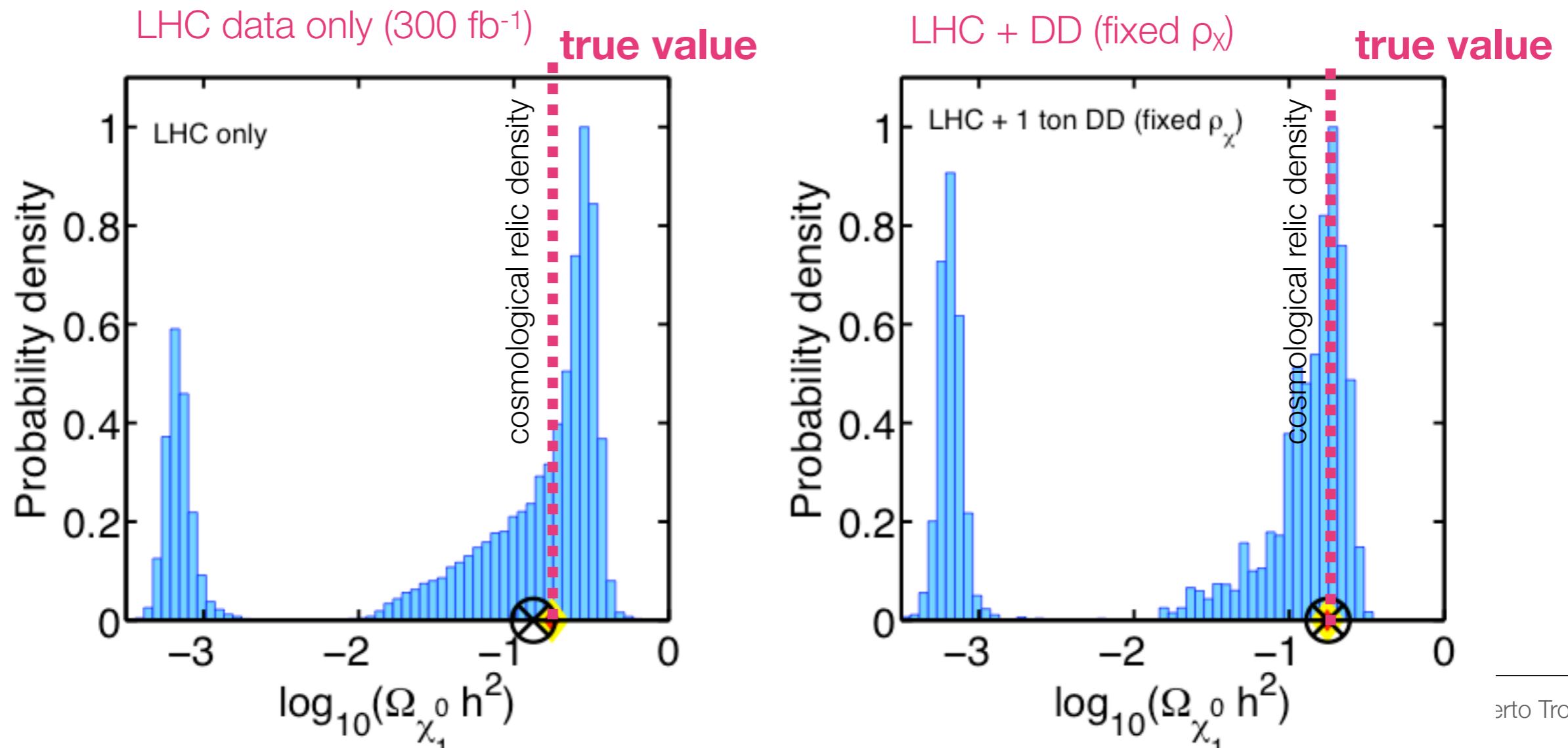
- Constraints including Milky Way modelling and Sloan
- Biased constraints without Milky Way modelling



Identification of Cosmological DM

If a signal is seen both at the LHC and in direct detection detectors, how can we check that this WIMP makes up the bulk of the cosmological relic density?

- Fit low-energy SUSY parameters and try to predict Ωh^2 from LHC data alone.
- **Problem:** LHC data alone are unable to constrain the relic abundance. Even DD data cannot break the degeneracy (if ρ_χ assumed fixed):

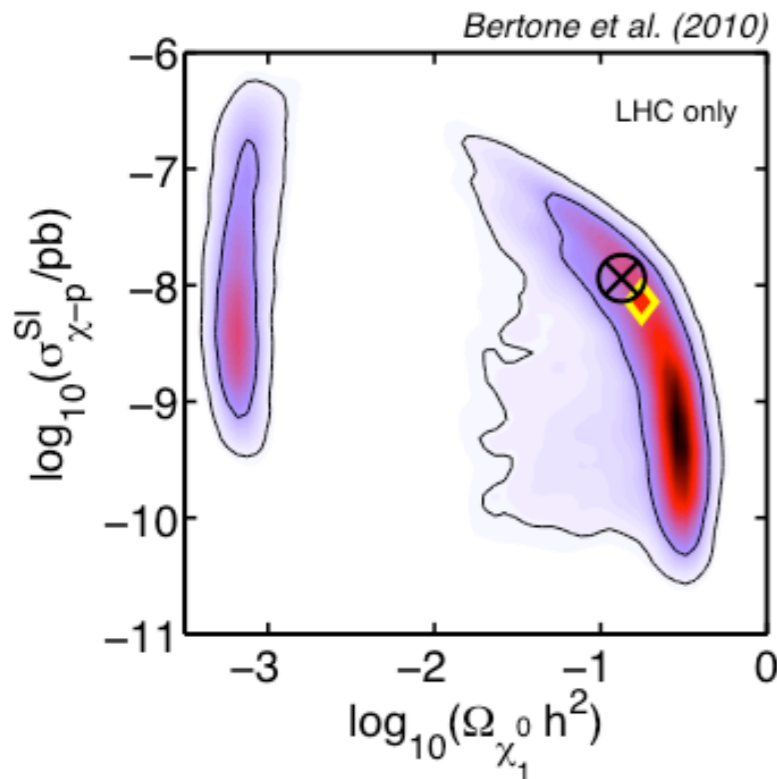


Complementarity of direct detection and LHC data

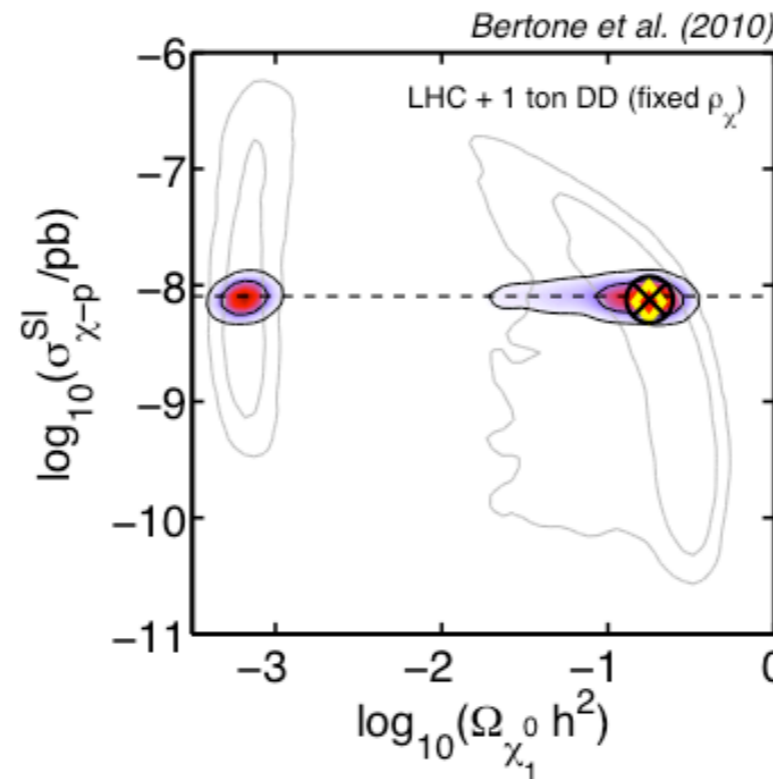
- **Strategy:** assume that the local density scales with the cosmological relic abundance (“*scaling Ansatz*”):

$$\rho_\chi \propto \Omega h^2 \chi$$

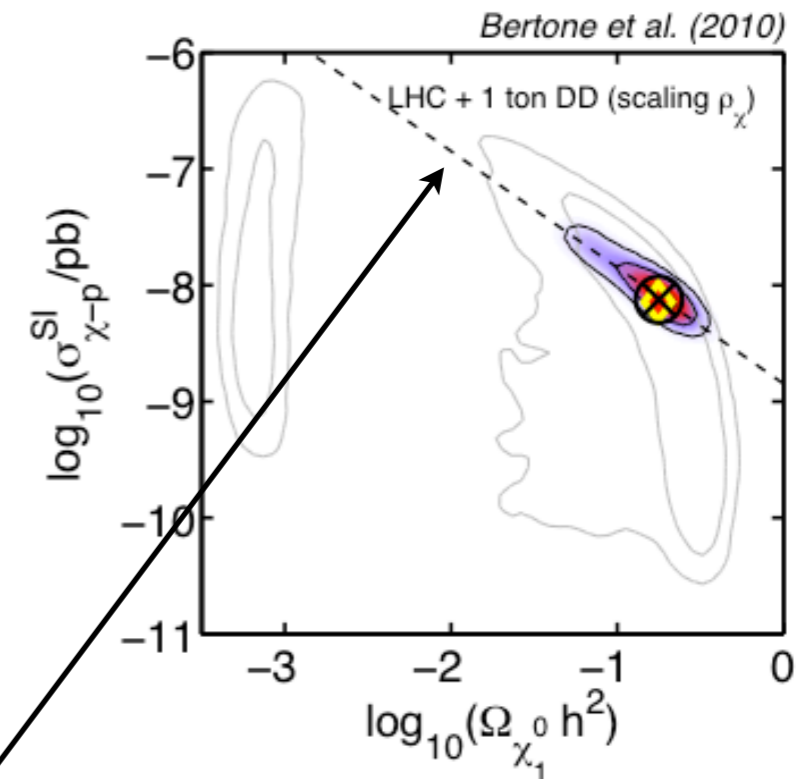
LHC data only (300 fb⁻¹)



LHC + DD (fixed ρ_χ)



LHC + DD (ρ_χ scales with Ωh^2)

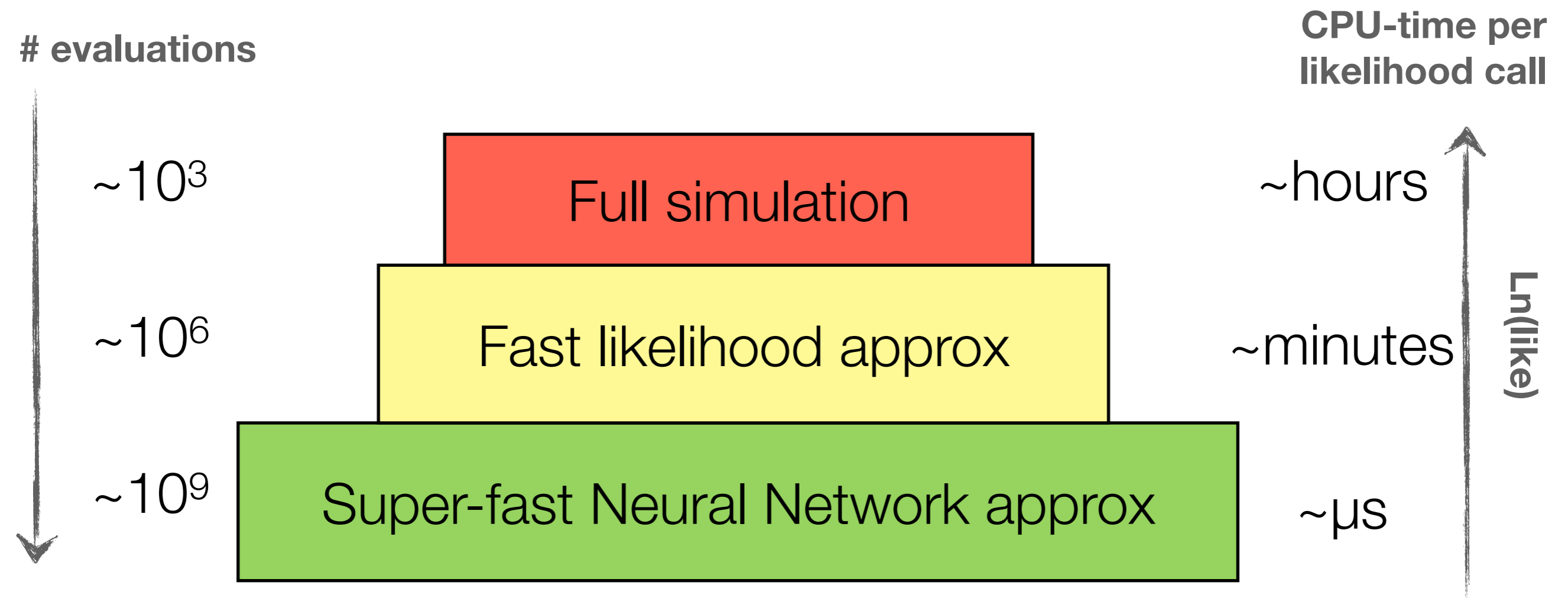


Bertone, RT et al, 1005.4280

**Our scaling Ansatz breaks
degeneracy in parameter space
Cosmological solution identified!**

Wedding-cake approach

- A full MSSM-15 profile likelihood scan might require $O(10^9)$ ATLAS likelihood evaluations
- Not feasible with full (expensive simulations). Wedding-cake strategy:



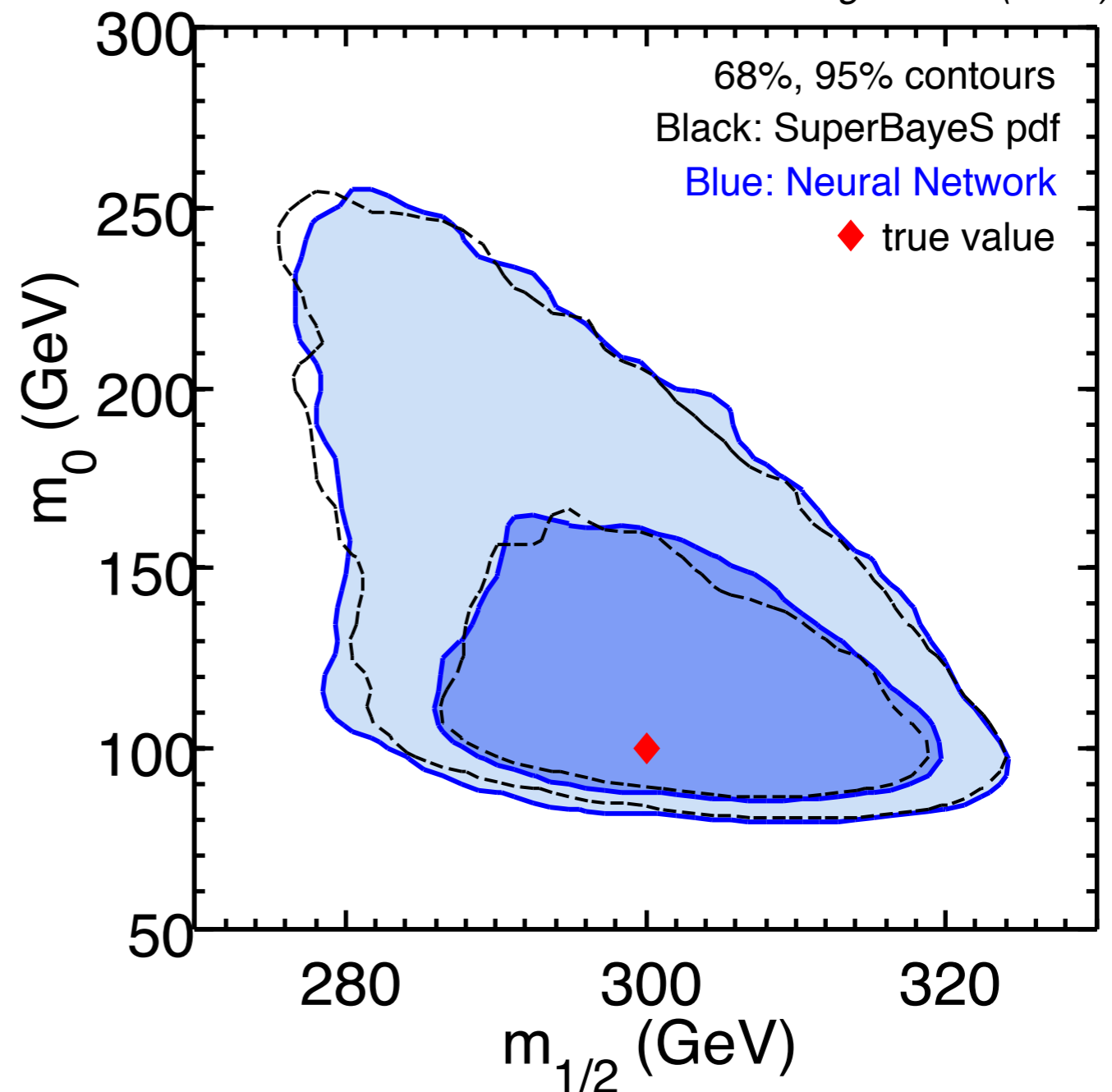
- Coverage of intervals is a frequentist property.
- Guaranteed when using e.g. Feldman-Cousins procedure to build intervals.
- Approximate confidence intervals are obtained via the Neyman construction with profile likelihood ratio as a test statistic.
- From a Bayesian perspective, coverage properties of credible intervals (if desired) can be used to calibrate priors.
- **Coverage studies are computationally expensive:**
 - (a) choose fiducial point in parameter space
 - (b) generate pseudo-data
 - (c) reconstruct credible/confidence interval
 - (d) check whether fiducial point within/without interval.

“Instantaneous” inference with neural networks

- **Standard MCMC**
(SuperBayeS v1.23, 2006 release)
720 CPU days
- **MultiNest**
(SuperBayeS v1.5, 2010 release)
16 CPU days
speed-up factor: ~ 50
- **SuperBayeS+Neural Networks**
(Bridges, Cranmer, Feroz, Hobson, Ruiz & RT, [1011.4306](#))
less than 1 CPU minute
speed-up factor: 30'000

Simulated ATLAS data

Bridges et al (2010)



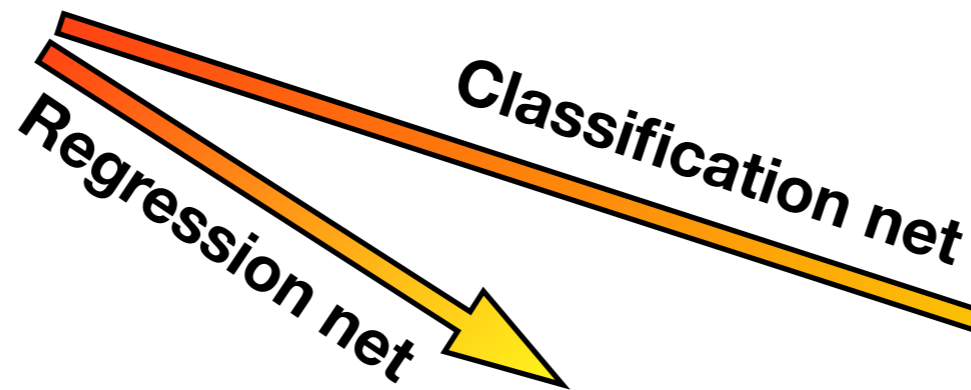
Neural nets shortcuts

SCANNING ALGORITHM

4 CMSSM parameters
 $\theta = \{m_0, m_{1/2}, A_0, \tan\beta\}$
(fixing $\text{sign}(\mu) > 0$)

4 SM “nuisance
parameters”
 $\Psi = \{m_t, m_b, \alpha_s, \alpha_{EM}\}$

↑
Data:
Gaussian likelihoods
for each of the Ψ_j
($j=1 \dots 4$)



RGE

**Non-linear
numerical
function**
via SoftSusy 2.0.18
DarkSusy 5.0
MICROMEAS 2.2
FeynHiggs 2.5.1
Hdecay 3.102

**Observable
quantities
 $f_i(\theta, \Psi)$**

CDM relic abundance
BR's
EW observables
g-2
Higgs mass
sparticle spectrum
(gamma-ray, neutrino,
antimatter flux, direct
detection x-section)

Likelihood = 0

↑ NO

Physically acceptable?
EWSB, no tachyons,
neutralino CDM

↓ YES

Joint likelihood function

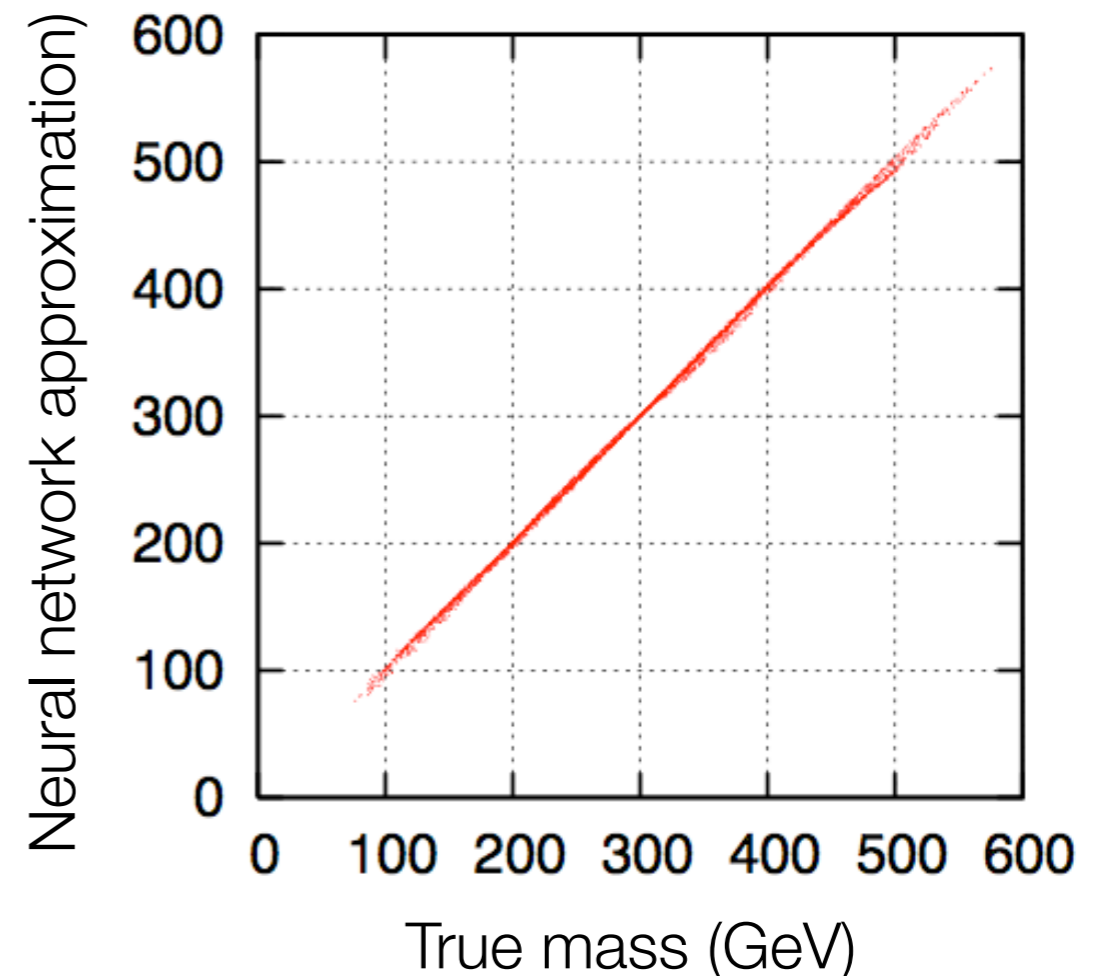
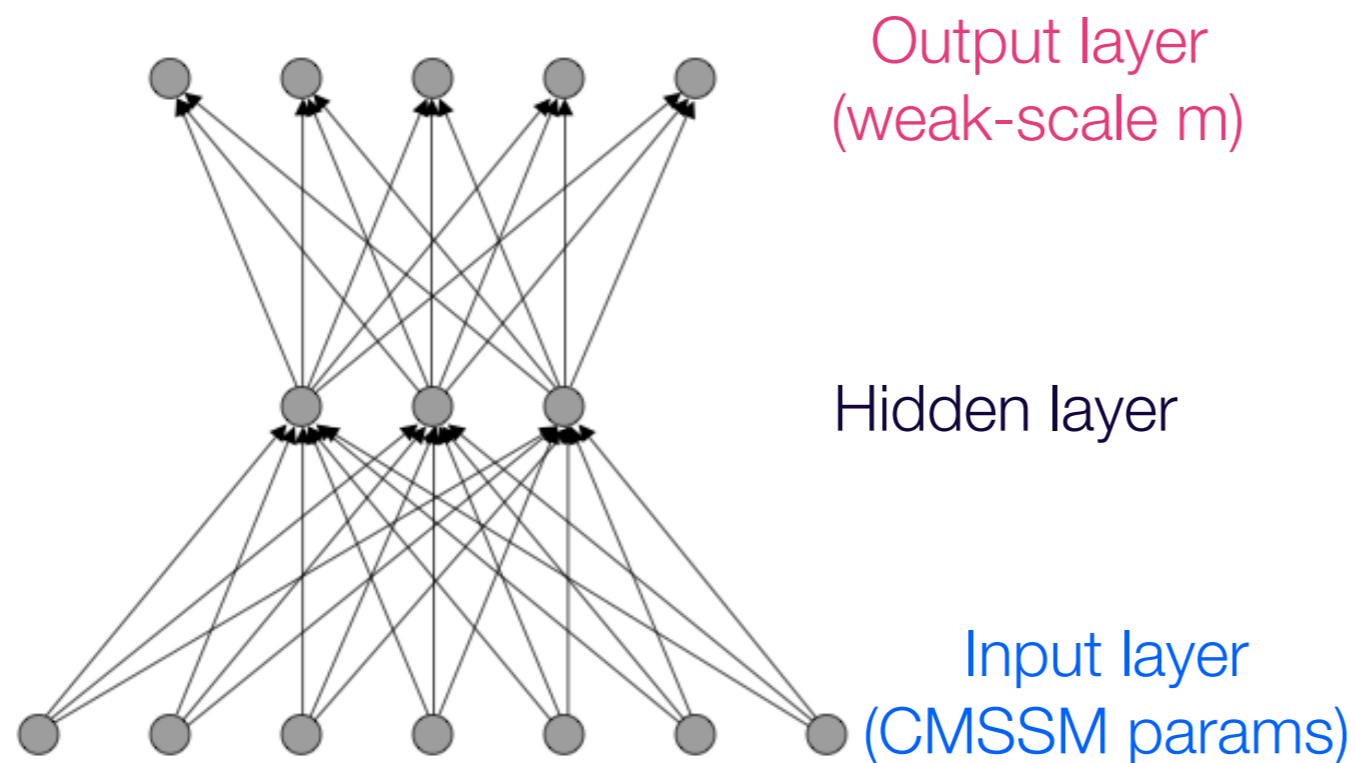
Data:
Gaussian likelihood
(CDM, EWO, g-2, $b \rightarrow s\gamma$, ΔM_{Bs})
other observables have
only lower/upper limits

Neural networks technology

Bridges, KC,
RT et al ([1011.4306](#))

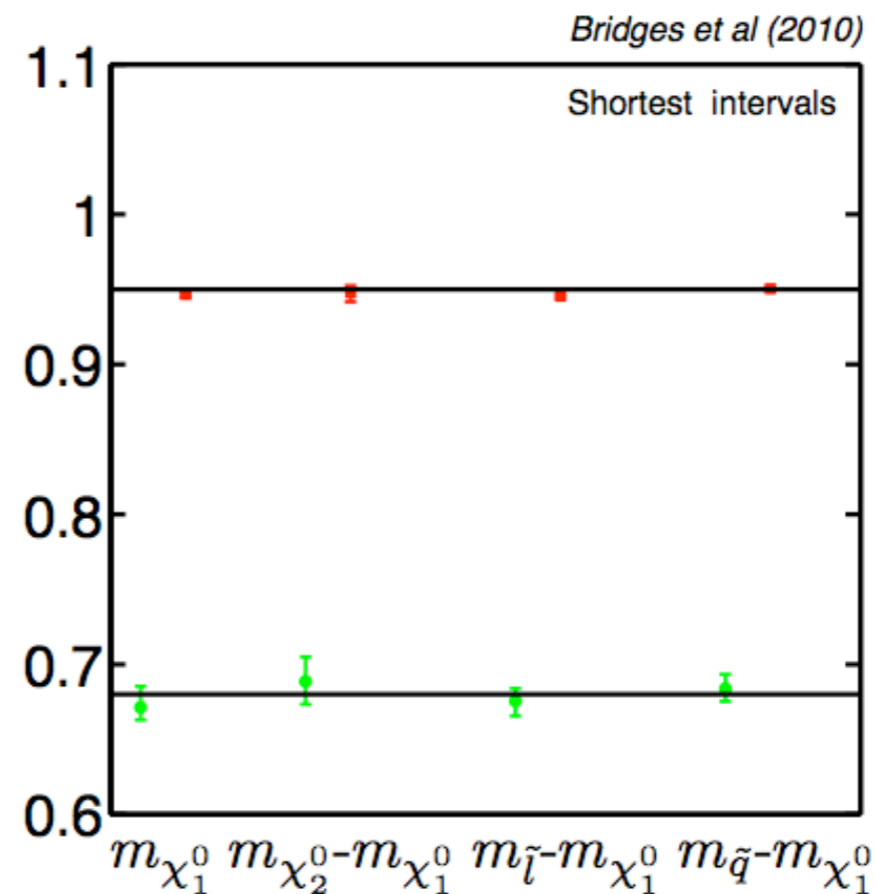
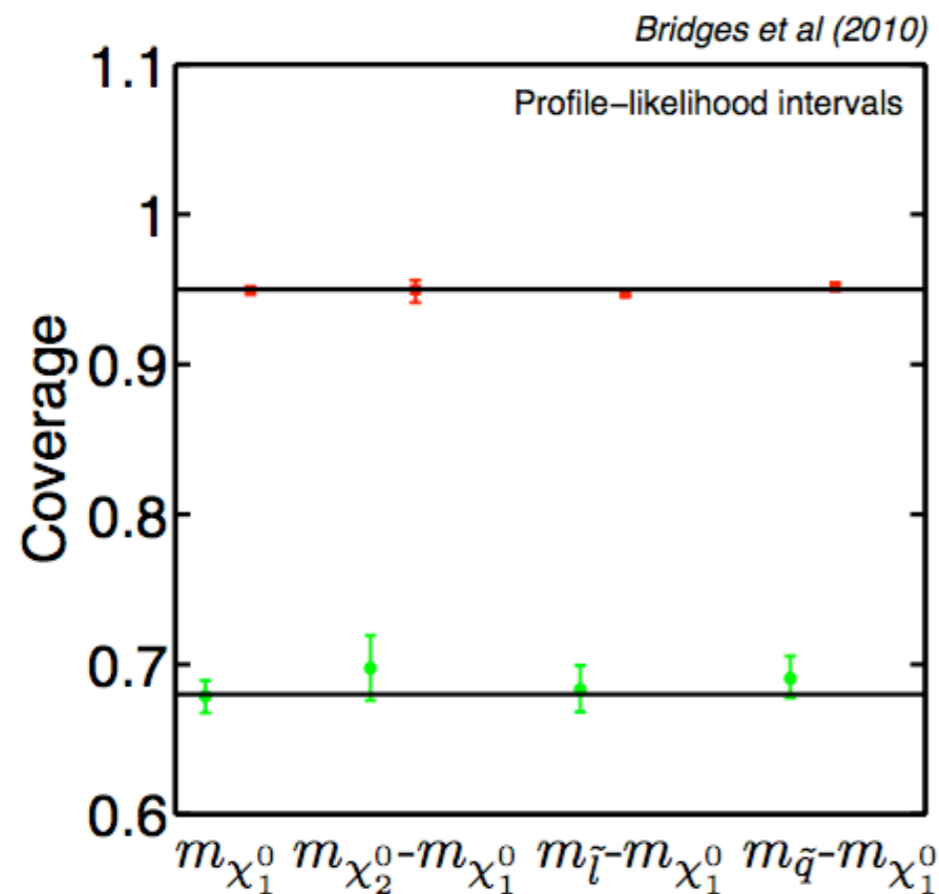
Imperial College
London

- We used a feed-forward multi-layer perceptron to “replace” SoftSusy in predicting the weak-scale masses from the CMSSM input parameters
- After training with a few 1000’s samples, the neural net achieved a correlation $> 99.99\%$



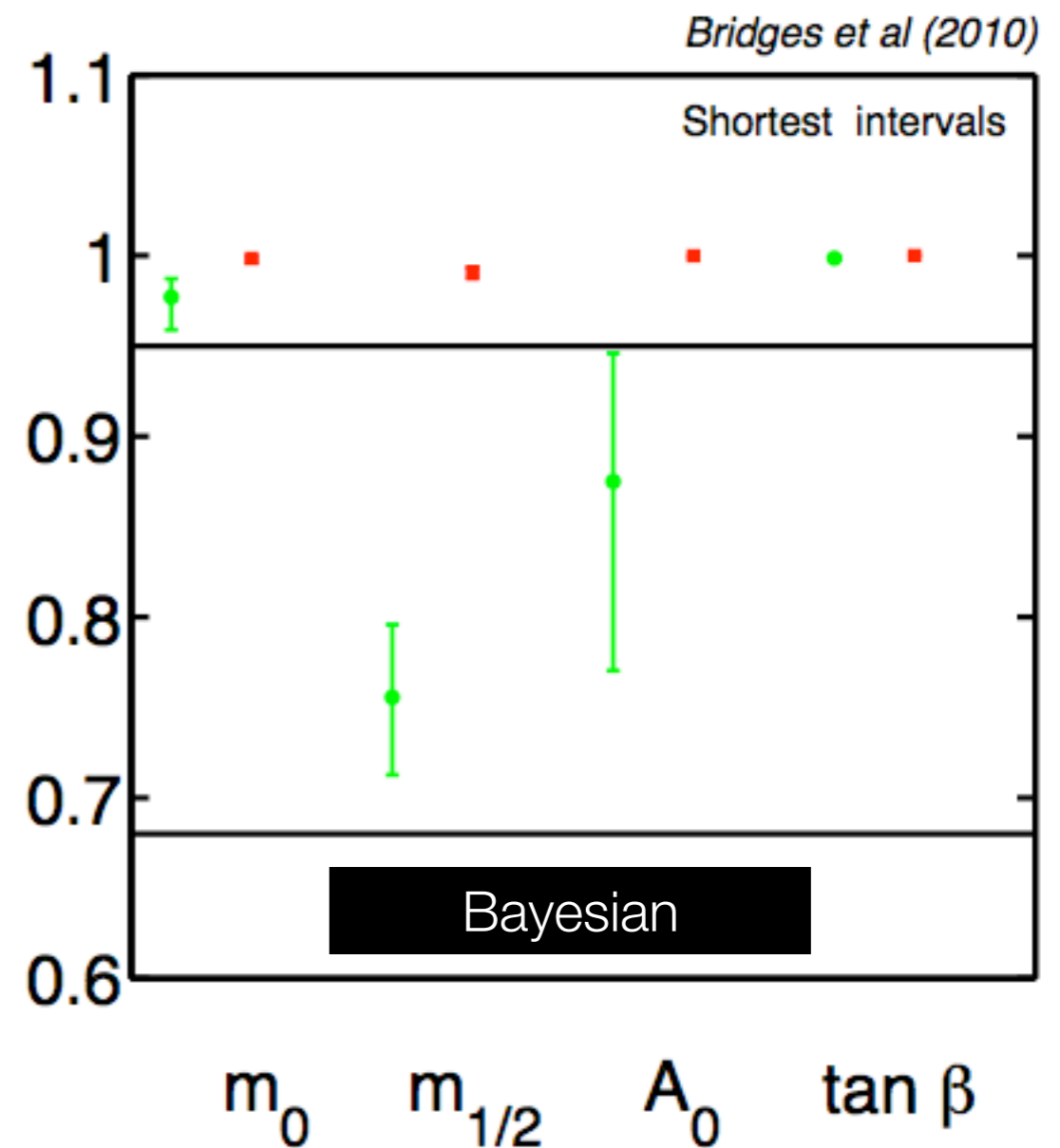
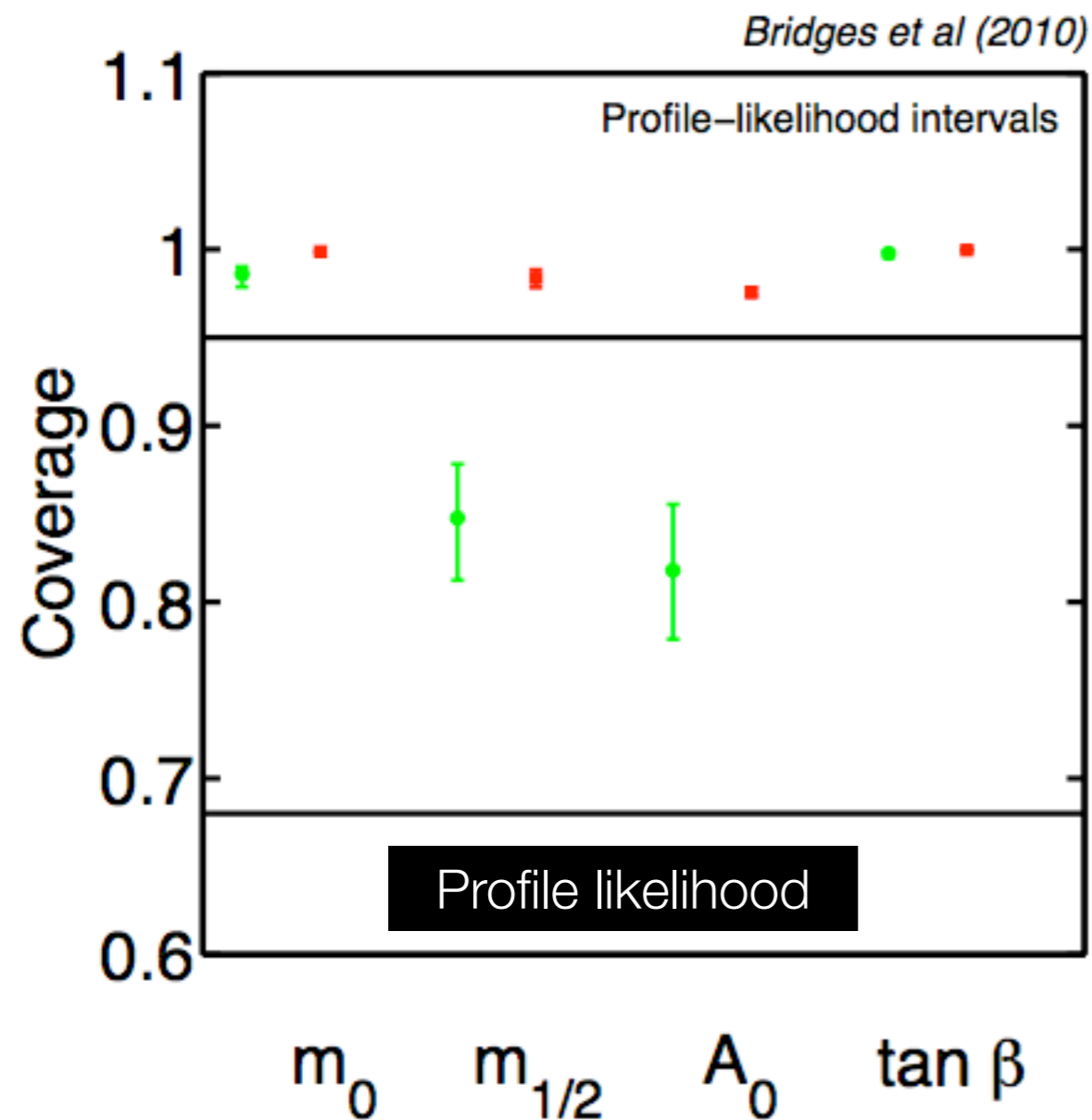
Coverage: are intervals what they say?

- We did **10,000 reconstructions, each with 1 million samples**. This would have taken 1,100 CPU yrs using standard methods. **Neural network speed-up is dramatic, of order 10^4** .
- **Test case:** use weak-scale masses as input, with Gaussian likelihood. Coverage is exact (within noise), as expected:



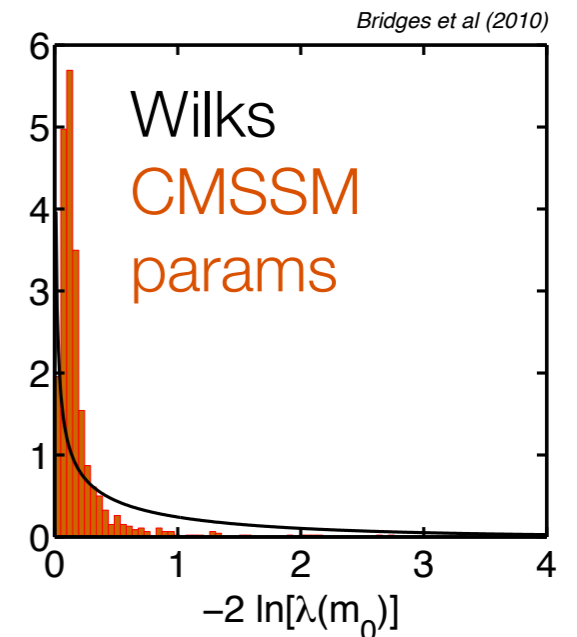
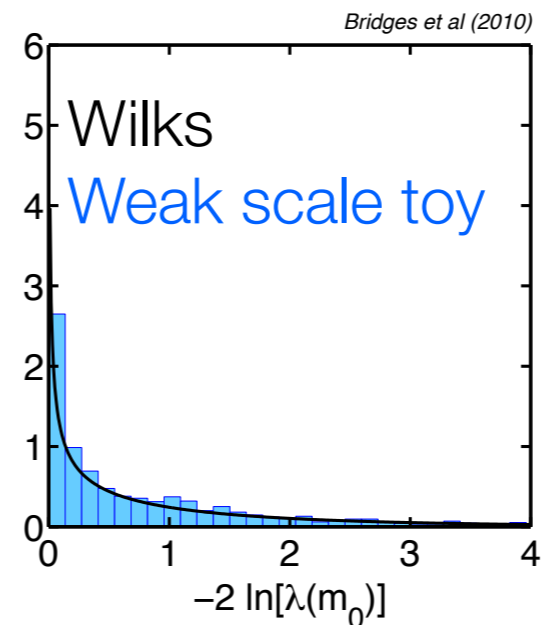
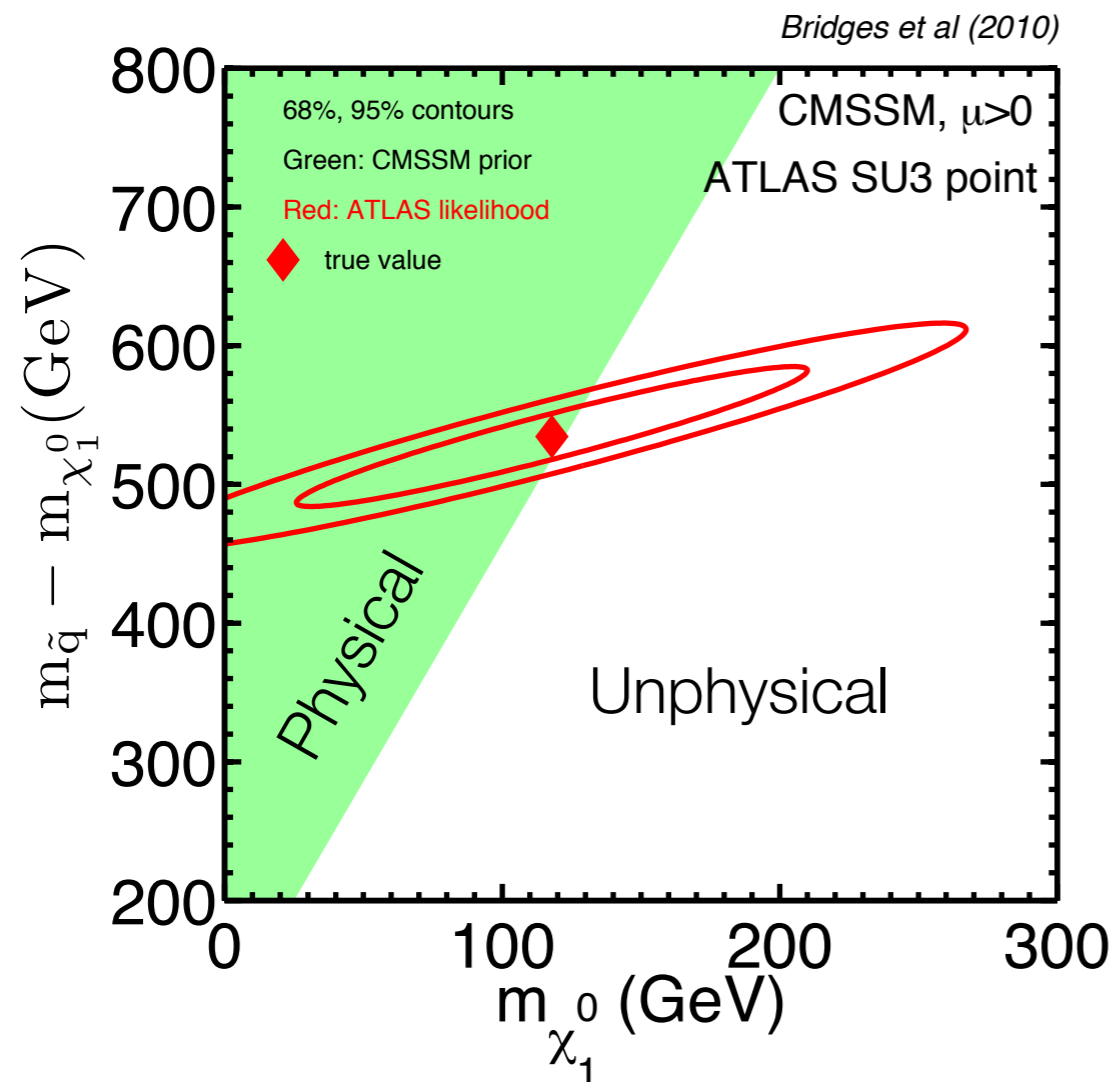
Coverage: are intervals what they say?

- Mapping back constraints to the CMSSM parameters, **we find substantial over-coverage** for both Bayesian and profile likelihood intervals:



Origin of over-coverage in the CMSSM

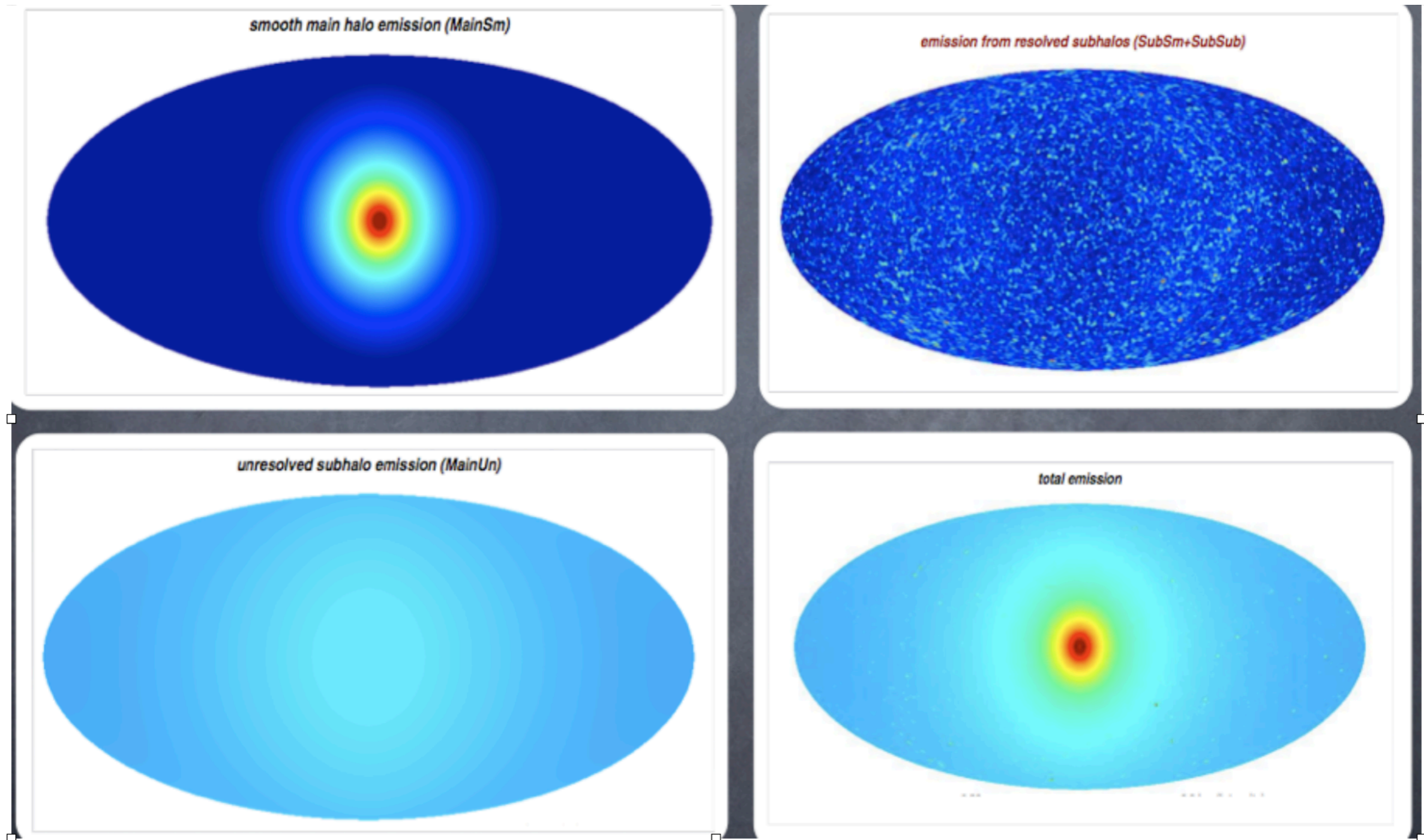
- The CMSSM prior introduces “physicality” boundaries in the weak-scale masses space. As a consequence, the distribution of $-2 \ln(\lambda)$ is not well approximated by χ^2 and Wilks’ theorem does not apply.



Bridges, KC, RT et al (1011.4306)

- Global scans of Beyond the Standard Model theories are now a mature field
- Ensuing predictions for direct/indirect detection experiments have to be interpreted with care due to weakness of current constraints
- BUT, quantitative, accurate inference tools will be required to pursue a robust and believable multi-messenger identification of DM
- Current tools have the ability to include statistical and systematic uncertainties in a statistically principled way
- Further work will focus on the careful assessment of their performance on simulated data sets under a variety of conditions

Gamma-ray from WIMP annihilation



Diemand et al (2008)

Predicting the gamma ray flux

Differential flux:

$$\frac{d\Phi_\gamma}{dE_\gamma} \propto \sum_i \left(\frac{\langle \sigma_i v \rangle}{m_\chi^2} \frac{dN_\gamma^i}{dE_\gamma} \right) \int \rho_\chi^2 dl$$

particle physics astrophysics

DM density profile:

$$\rho_\chi(r) = \rho_0 \frac{(r/r_0)^{-\gamma}}{[1 + (r/a)^\alpha]^{\frac{\beta-\gamma}{\alpha}}} [1 + (r_0/a)^\alpha]^{\frac{\beta-\gamma}{\alpha}}$$

Halo model	a (kpc)	α	β	γ
isothermal cored	3.5	2	2	0
NFW	20.0	1	3	1
NFW+ac	20.0	0.8	2.7	1.45
Moore	28.0	1.5	3	1.5
Moore+ac	28.0	0.8	2.7	1.65

Predicting the GC gamma ray signal

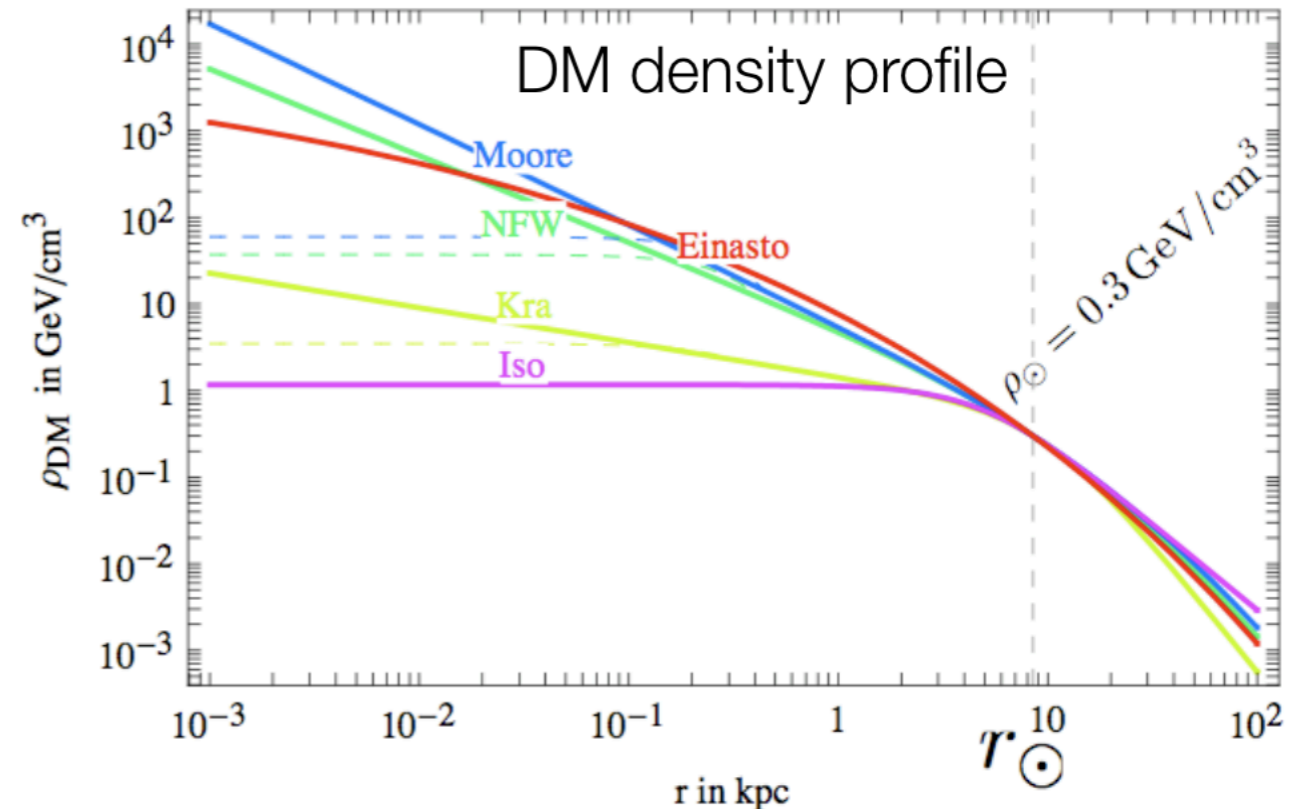
Einasto profile:

$$\rho_{\chi}(r) = \rho_0 \exp \left[-\frac{2}{\alpha} \left(\left(\frac{r}{r_s} \right)^{\alpha} - 1 \right) \right]$$

$$J(\Psi) = \int_{\text{los}} dl \rho_{\chi}^2(r(l, \Psi))$$

$$\bar{J} = \frac{1}{\Delta\Omega} \int_{\Delta\Omega} J(\Psi) d\Omega$$

DM density profile

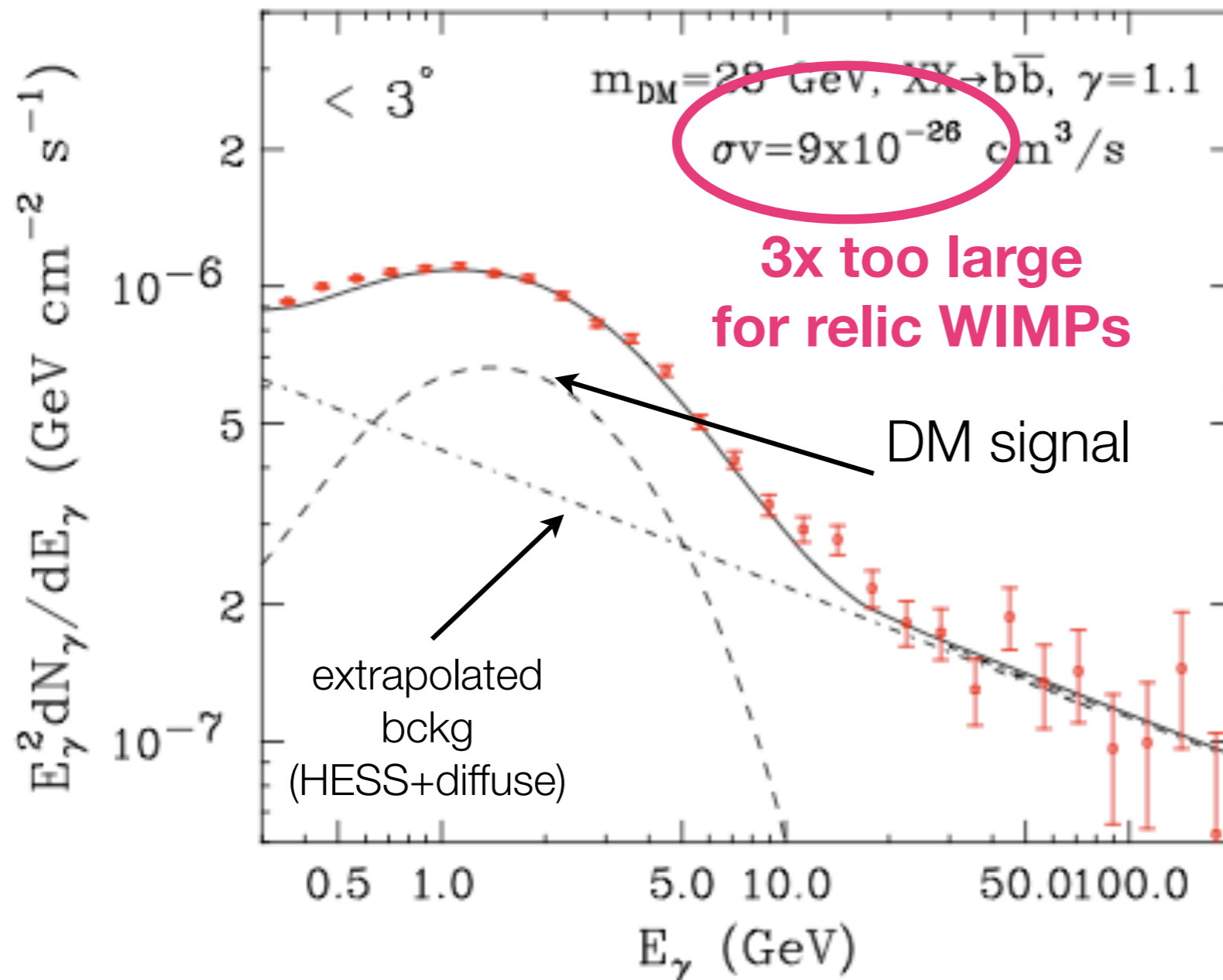


Cirelli (2009)

Halo model	a (kpc)	α	β	γ	$\bar{J}(10^{-3} \text{sr})$	$\bar{J}(10^{-5} \text{sr})$
isothermal cored	3.5	2	2	0	30.35	30.40
NFW	20.0	1	3	1	1.21×10^3	1.26×10^4
NFW+ac	20.0	0.8	2.7	1.45	1.25×10^5	1.02×10^7
Moore	28.0	1.5	3	1.5	1.05×10^5	9.68×10^6
Moore+ac	28.0	0.8	2.7	1.65	1.59×10^6	3.12×10^8

A GC Excess?

Goodenough & Hooper (2009)



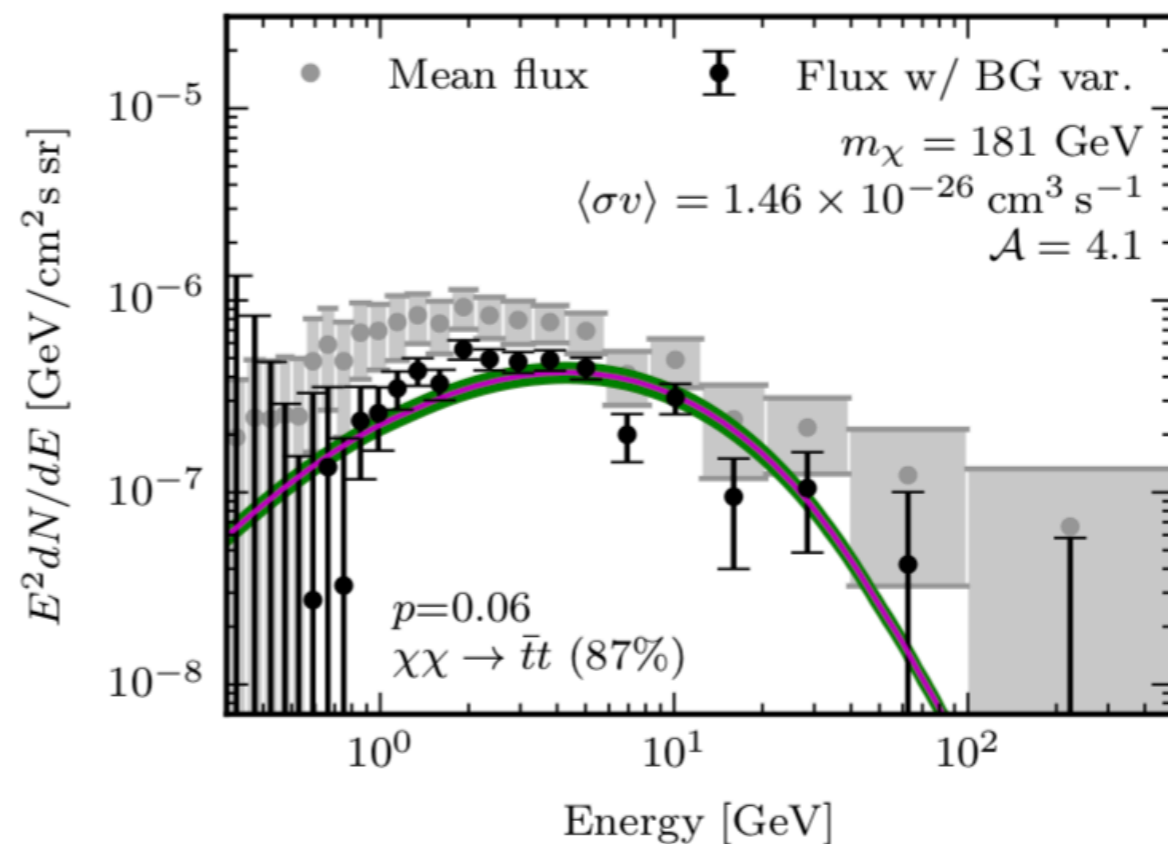
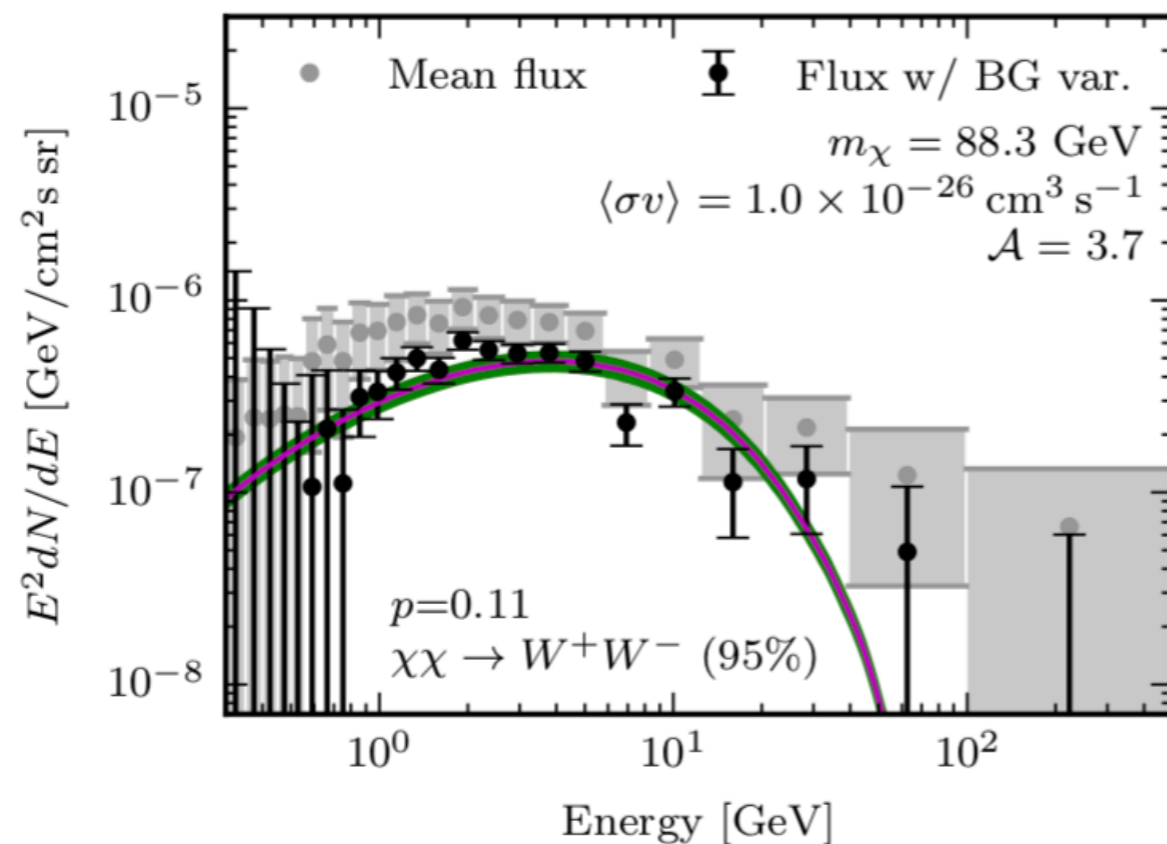
- Initial hints of a DM signal from the galactic center (Goodenough & Hooper 2009)

- Caveats:

- GC very complex region: point sources, diffuse emission, etc
- Strong bounds from PAMELA antiproton data and radio data constraining synchrotron emission
- The Hooper & Goodenough model is effectively ruled out.

GC Excess from the pMSSM

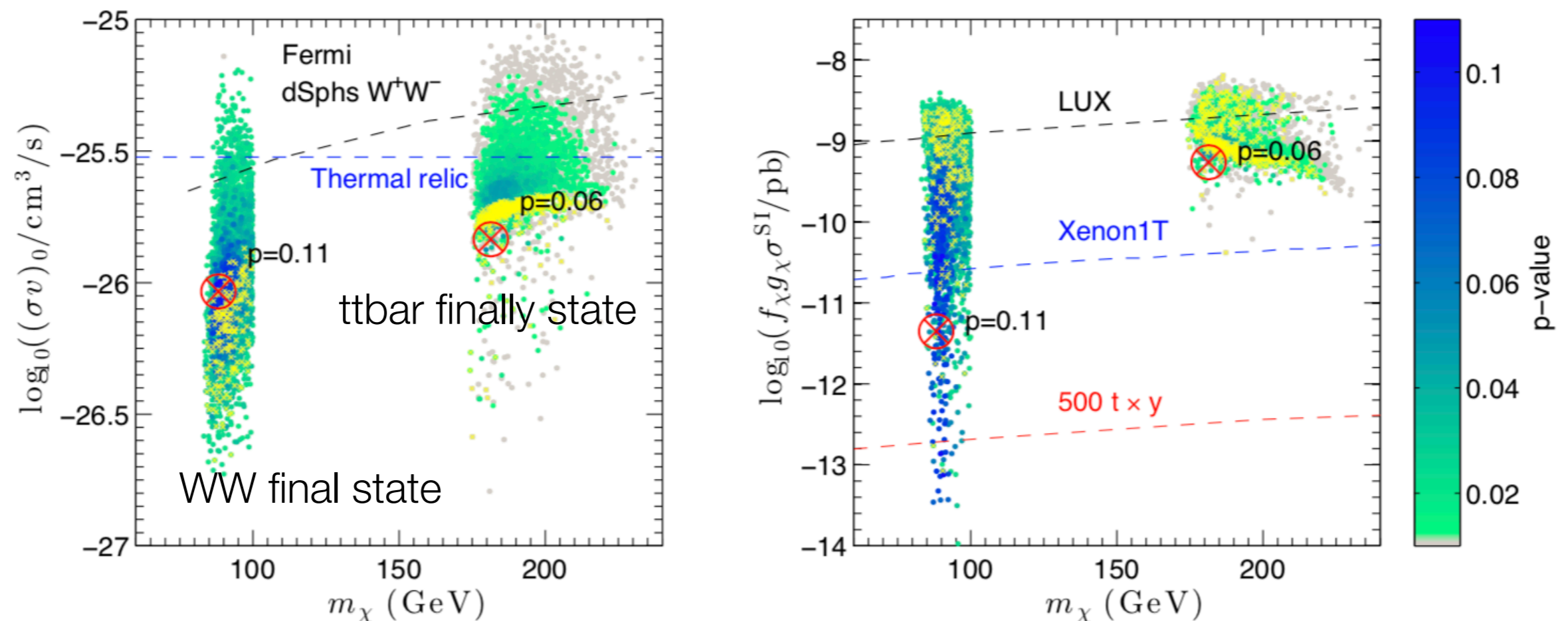
- Astrophysical explanations most plausible for the “excess” (unresolved population of millisecond pulsars, inverse Compton from cosmic rays).
- Can the putative excess be explained with a SUSY model? YES



Bertone, RT et al 2016

GC Excess from the pMSSM

- There exist points in the 19-dimensional pMSSM that can explain the GC excess while satisfying all other constraints
- Since this study was done, direct detection constraints have further improved, largely ruling out the $t\bar{t}$ island.

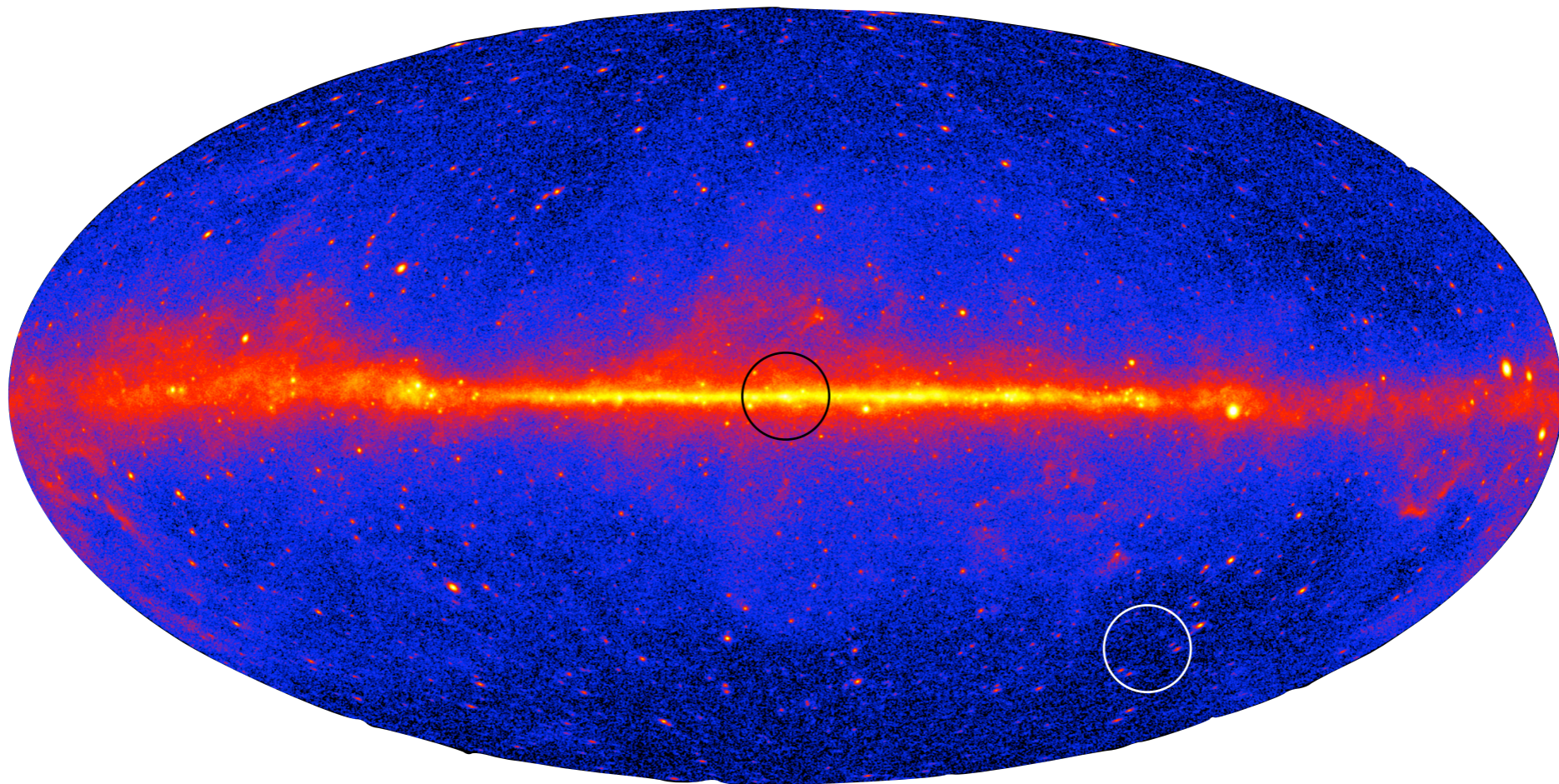


Bertone, RT et al 2016

Milky Way dwarf galaxies

Nearby, lots of dark matter ($\log_{10} J \sim 18 - 20$)

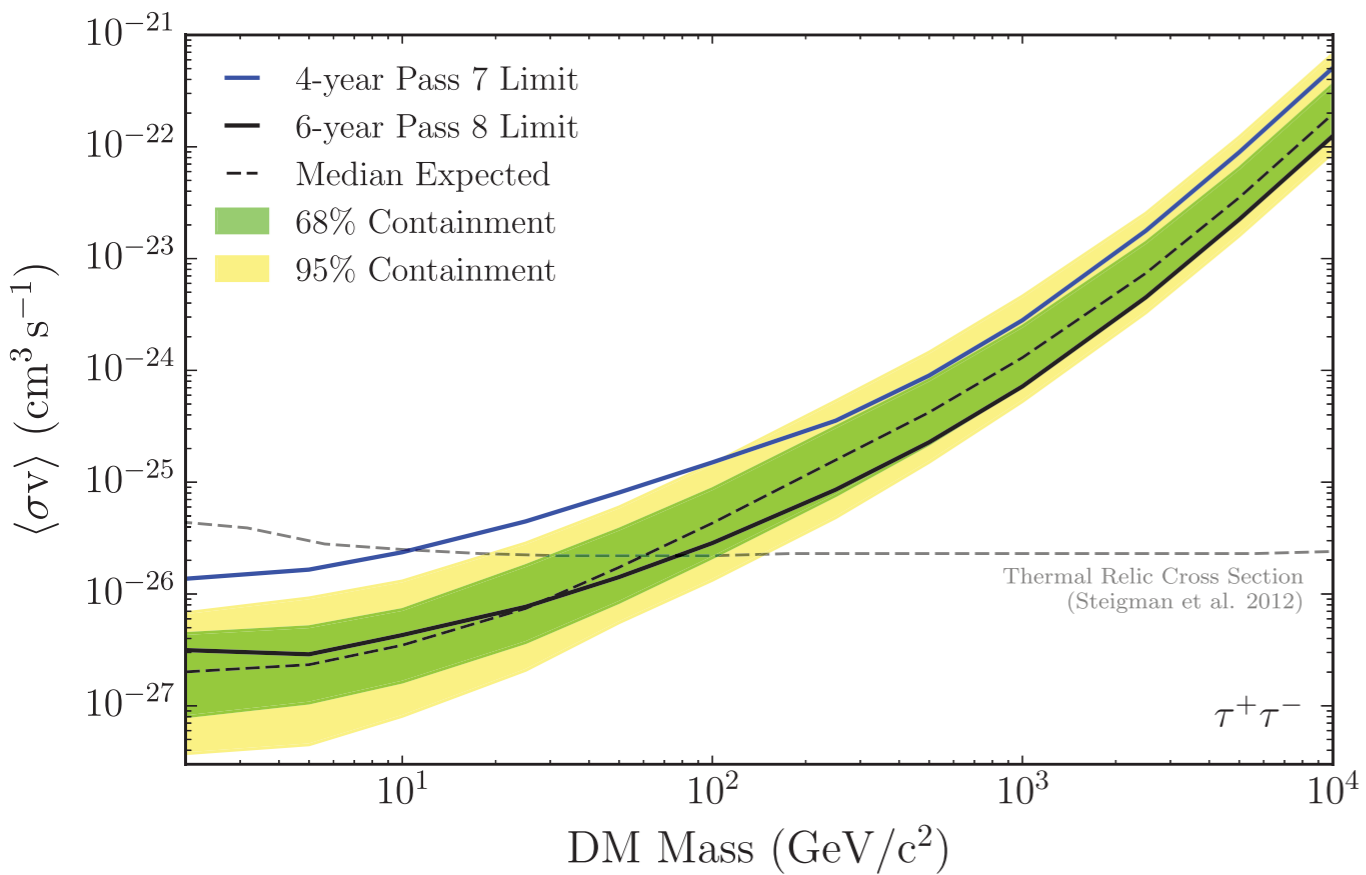
Not much else: no astrophysical background*
compare to Galactic center



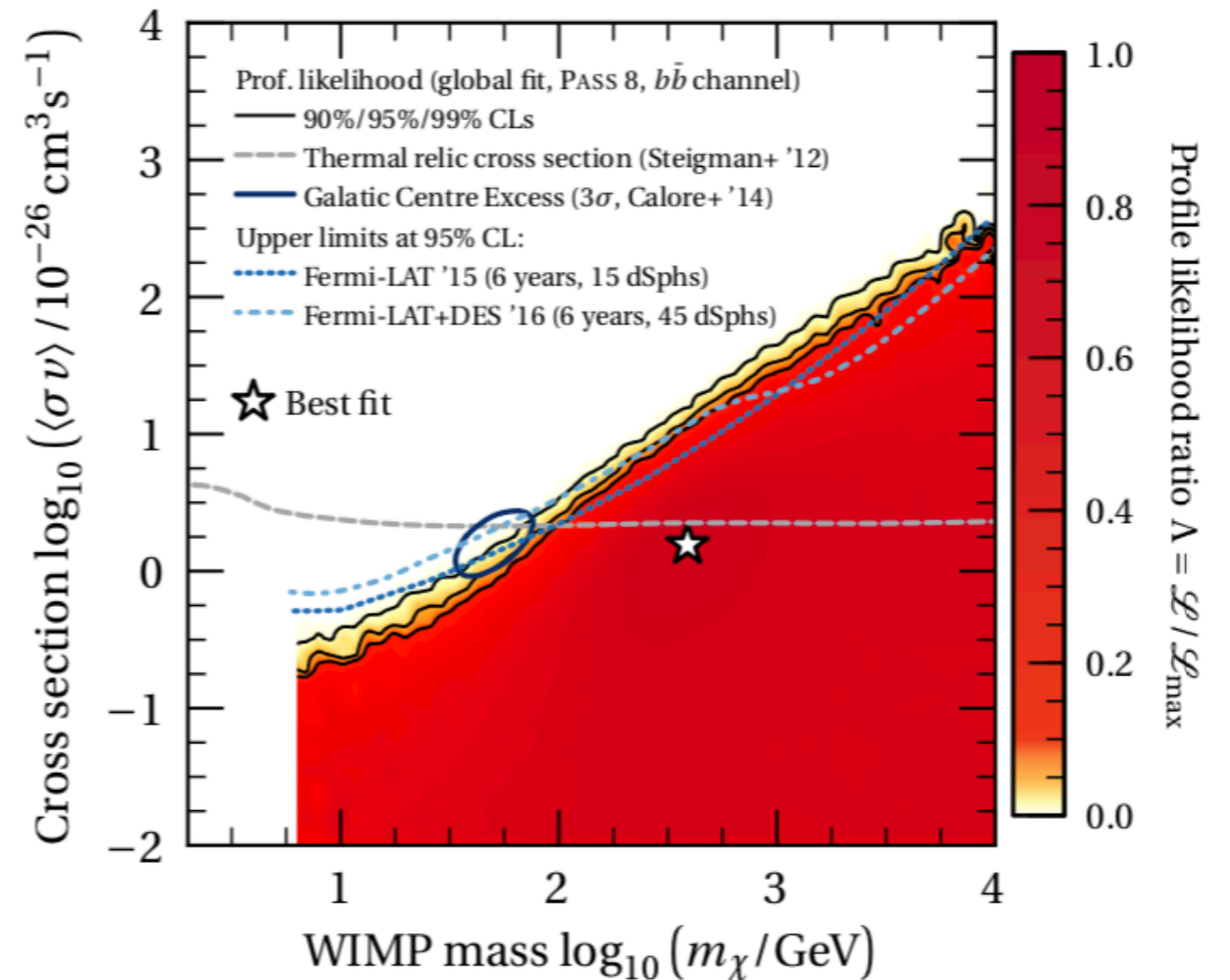
Dwarf searches reach the relic cross section

27 dwarfs, 10 yrs of data

Cross section limits



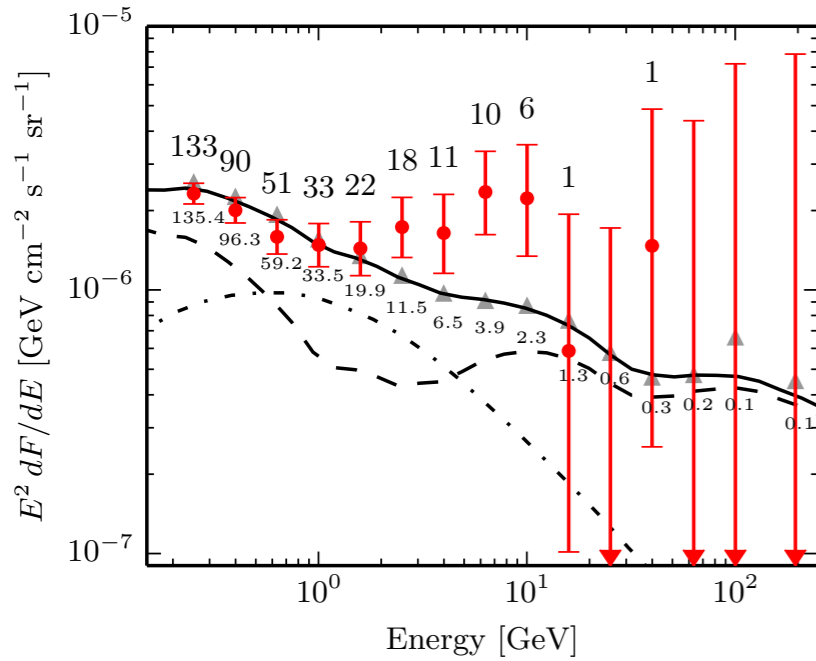
Fermi collaboration 1503.02641 (PRL)



Hoof, Geringer-Sameth & RT (in prep)

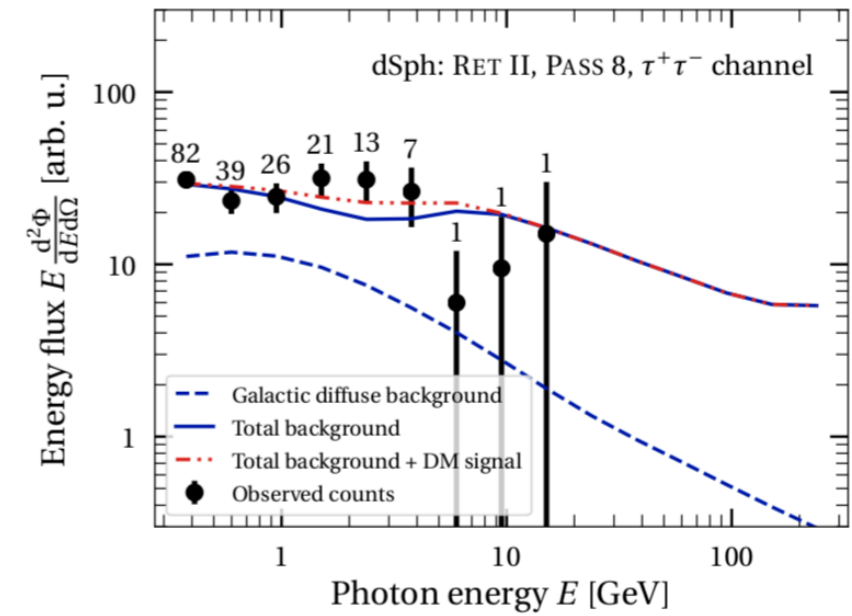
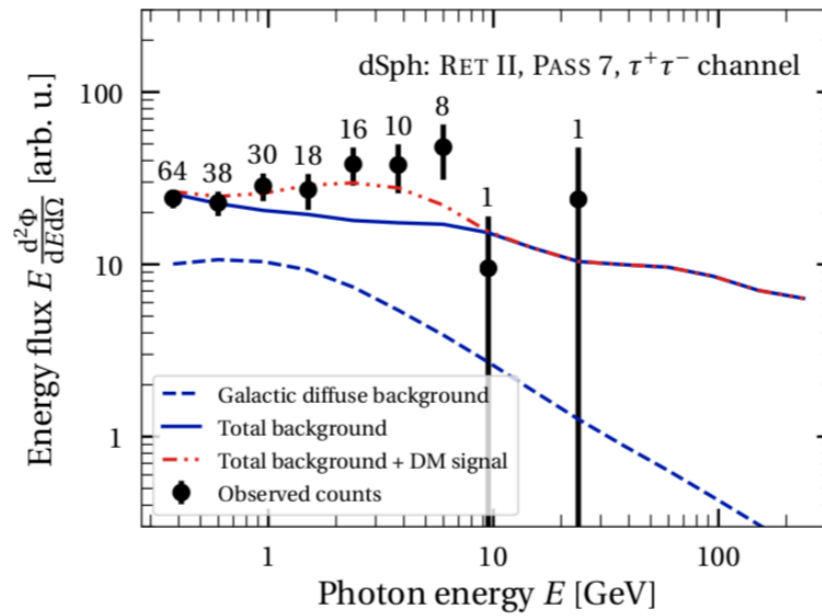
Case study: Reticulum II

6.5 yr Pass 7 Fermi LAT data

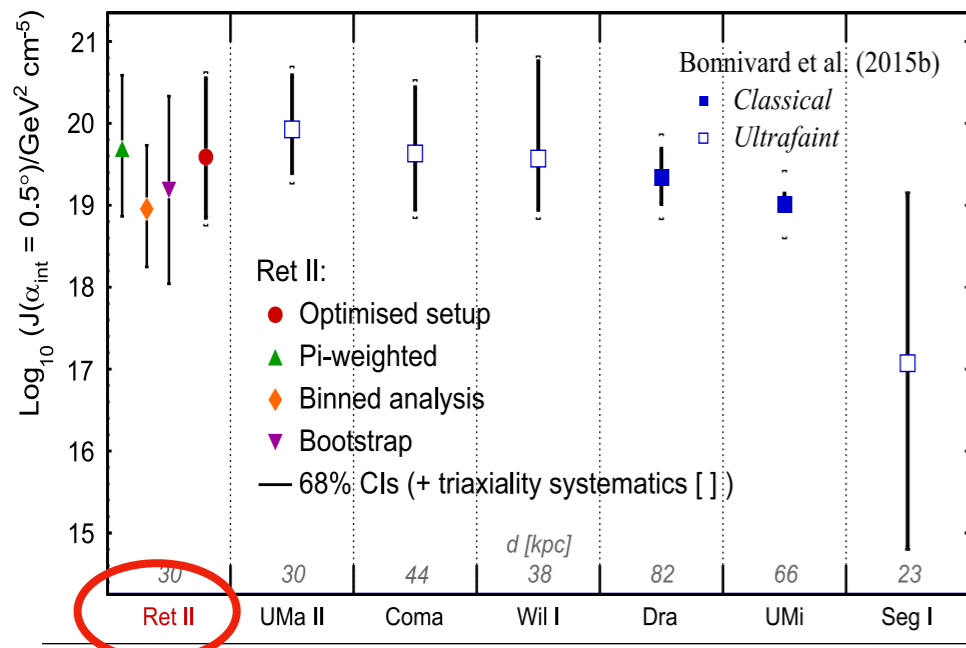


Geringer-Sameth+1503.02320 (PRL)

6.5 yr Pass 8 Fermi LAT data

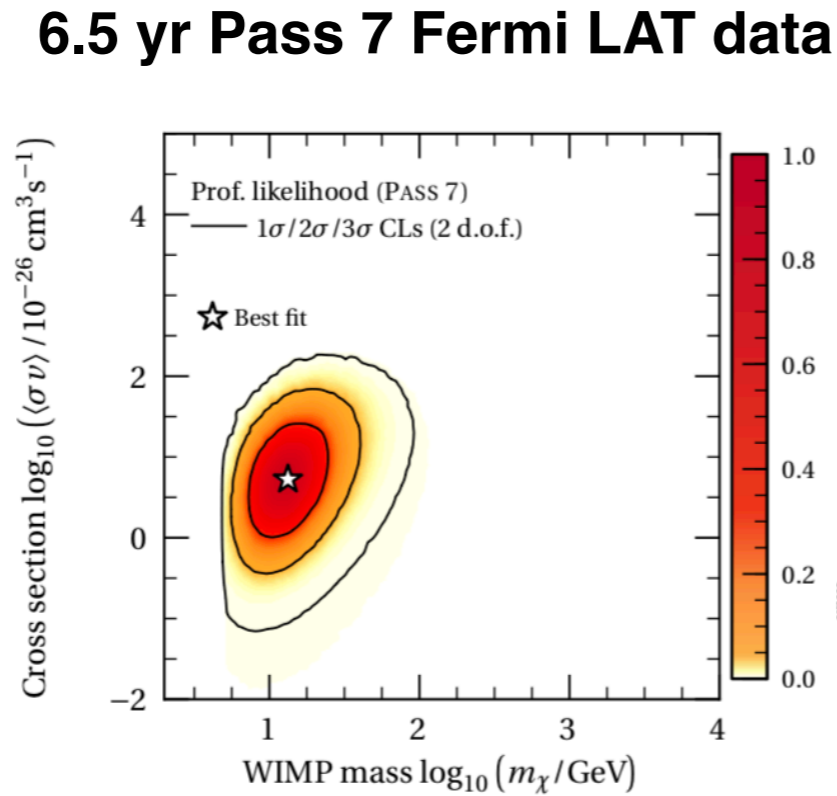


Hoof, Geringer-Sameth & RT (in prep)

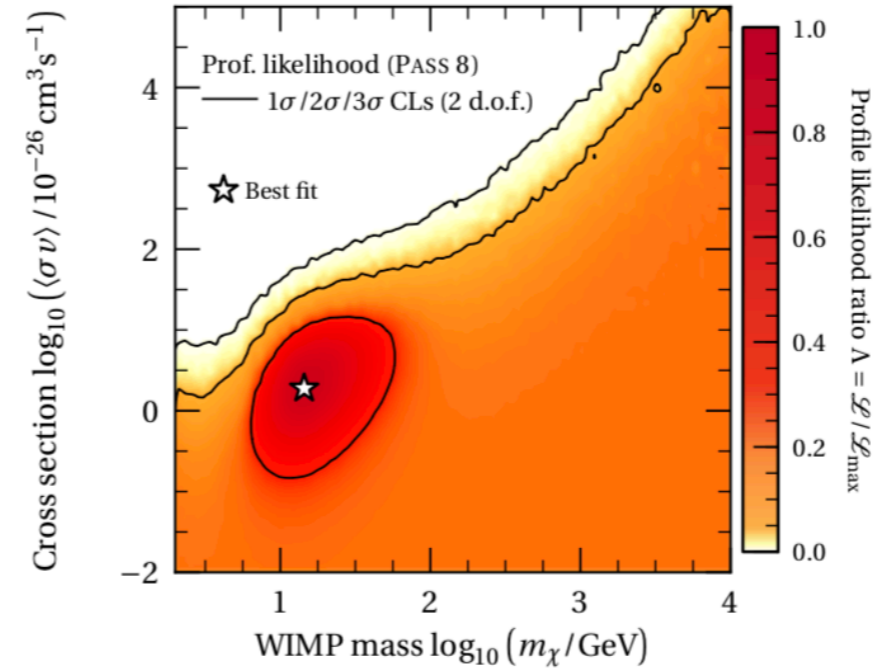


Bonnivard+ 1504.03309 (ApJL)

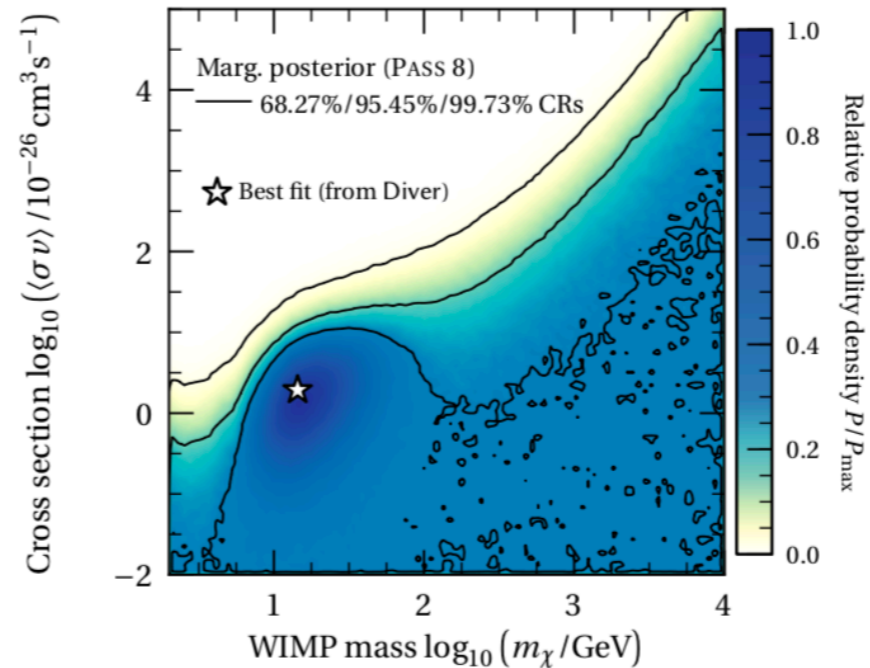
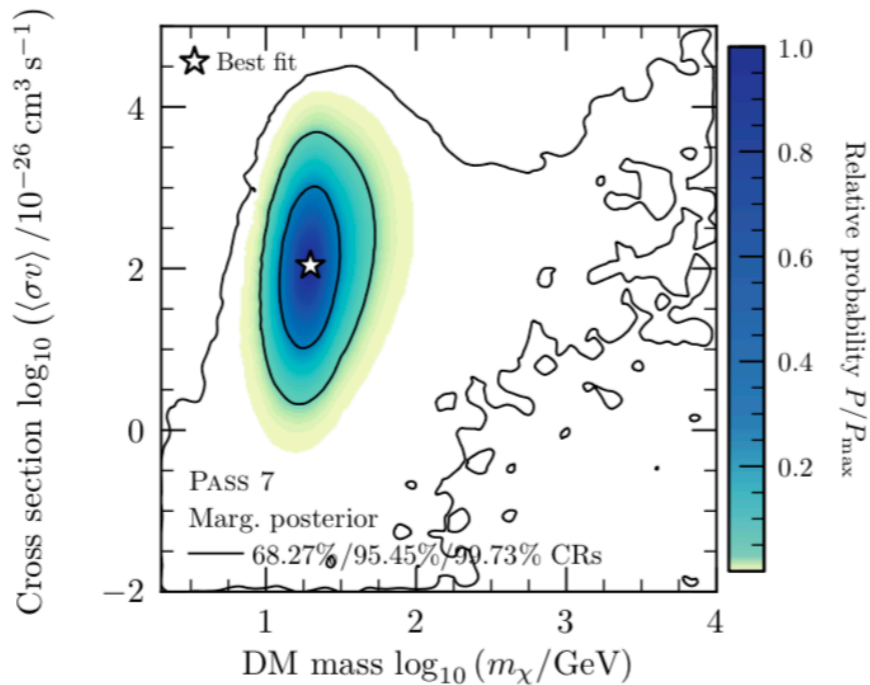
Profile likelihood



6.5 yr Pass 8 Fermi LAT data



Posterior pdf



Hoof, Geringer-Sameth & RT (in prep)

To discover dark matter annihilation using dwarfs:

1. Gamma-ray data is inconsistent with background
2. Inconsistent with any other possible source
(e.g. non-DM astrophysics, incorrect diffuse bg models)
3. Consistent with dark matter annihilation
(compare with other dwarfs, other experiments)

Two ways to model background give two different significances.

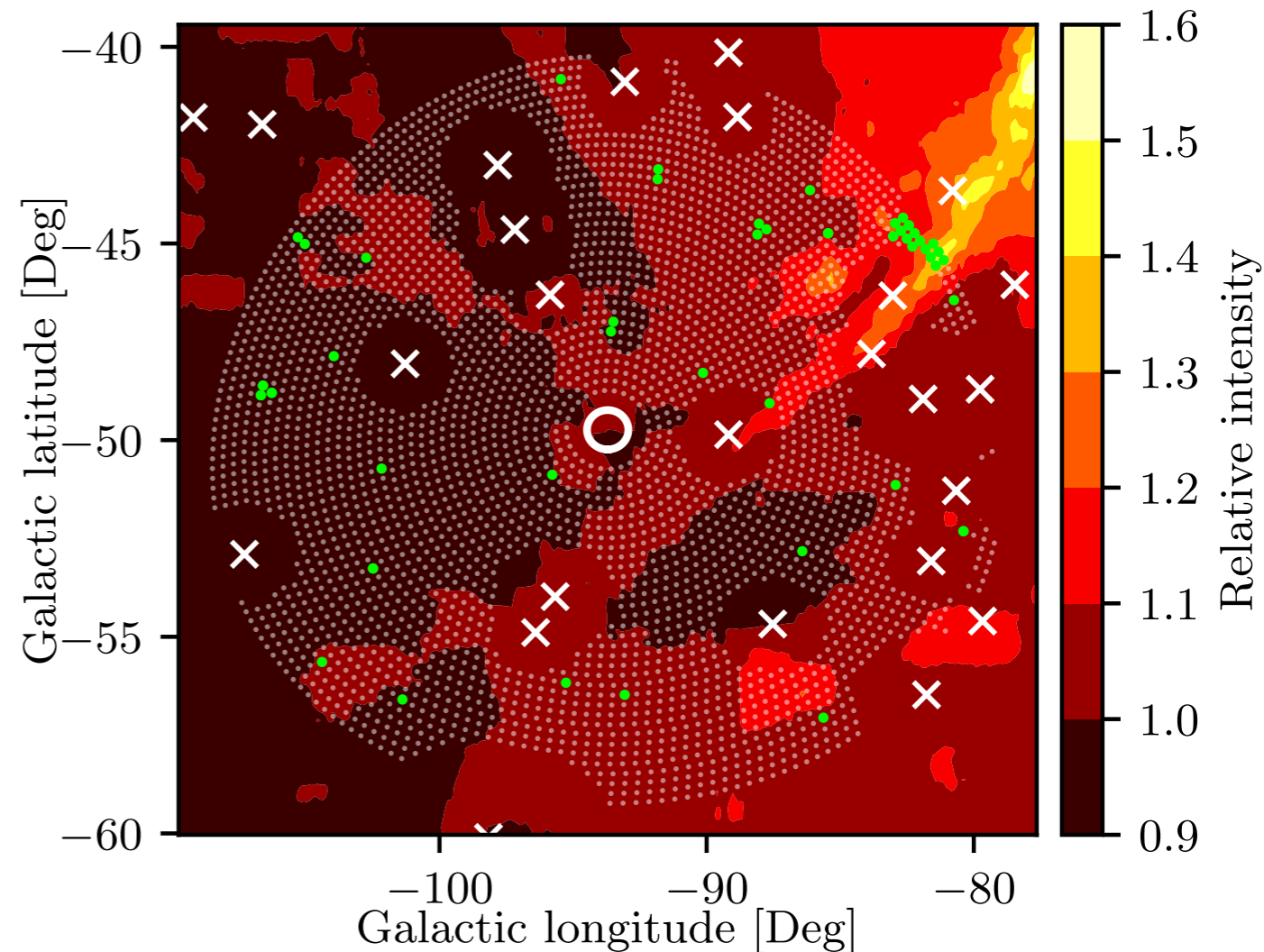
Beware of the Look Elsewhere Effect, too!

Diffuse background model

- Poisson with given spectrum:
 - “physical” model — cosmic ray interactions in Milky Way, extragalactic isotropic emission, charged particle misidentification
 - No additional non-DM sources along line of sight towards dwarf

H_0 : No additional source
 $p\text{-value} = 0.0001$

Empirical background from sampling



H_0 : No dark matter annihilation
 $p\text{-value} = 0.01$

To discover dark matter annihilation using dwarfs:

1. Gamma-ray data is inconsistent with background

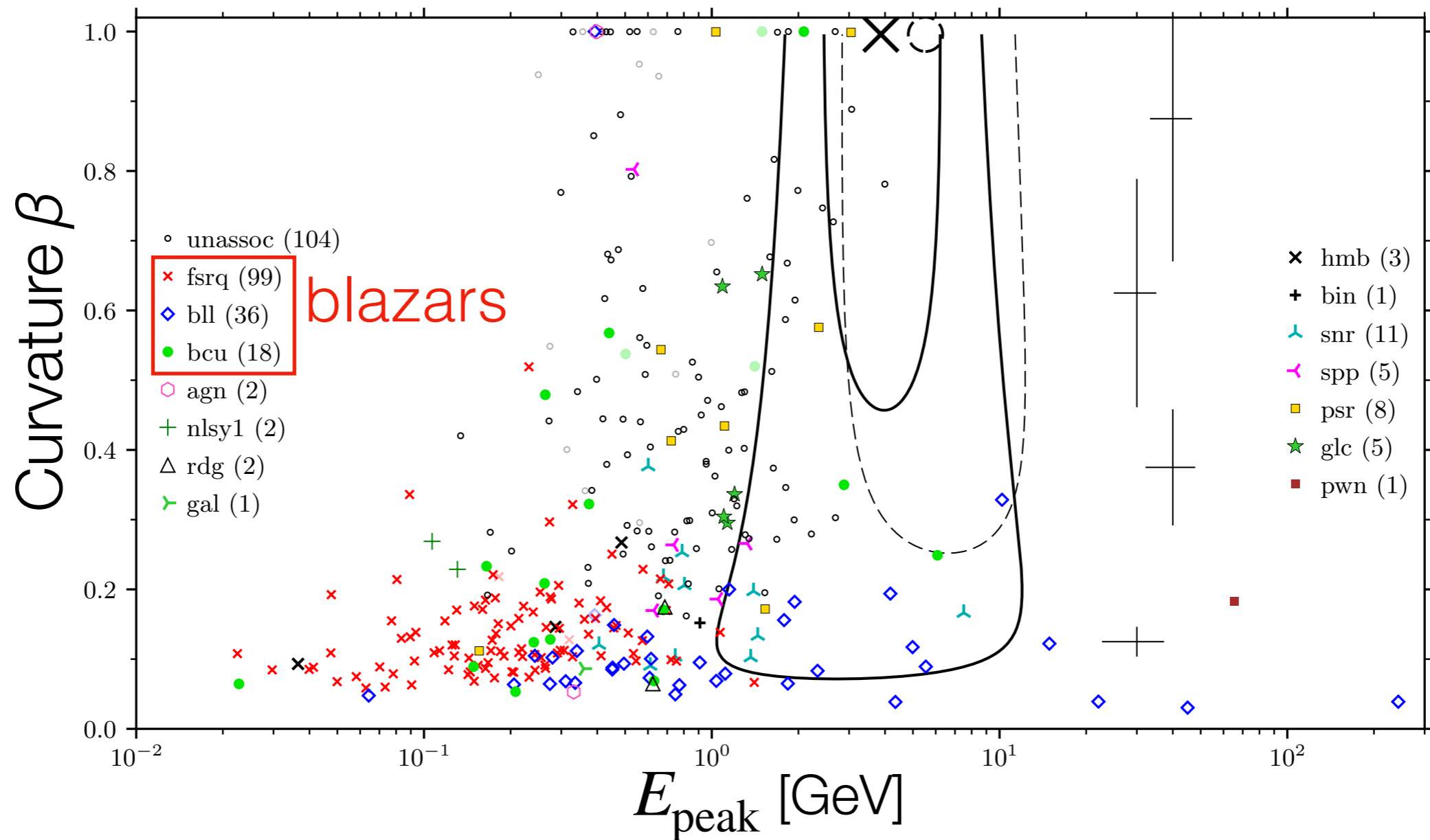
2. Inconsistent with any other possible source
(e.g. non-DM astrophysics, incorrect diffuse bg models)

3. Consistent with dark matter annihilation
(compare with other dwarfs, other experiments)

Compare with known classes of gamma-ray sources

$$\frac{dF(E | \theta)}{dE} = F_0 \left(\frac{E}{E_0} \right)^{-\alpha - \beta \log(E/E_0)}$$

E_{peak} = energy at peak of SED $E^2 dF/dE$



Combine all three tests into a single search using a Bayesian approach

Gamma-ray data is inconsistent with background

+

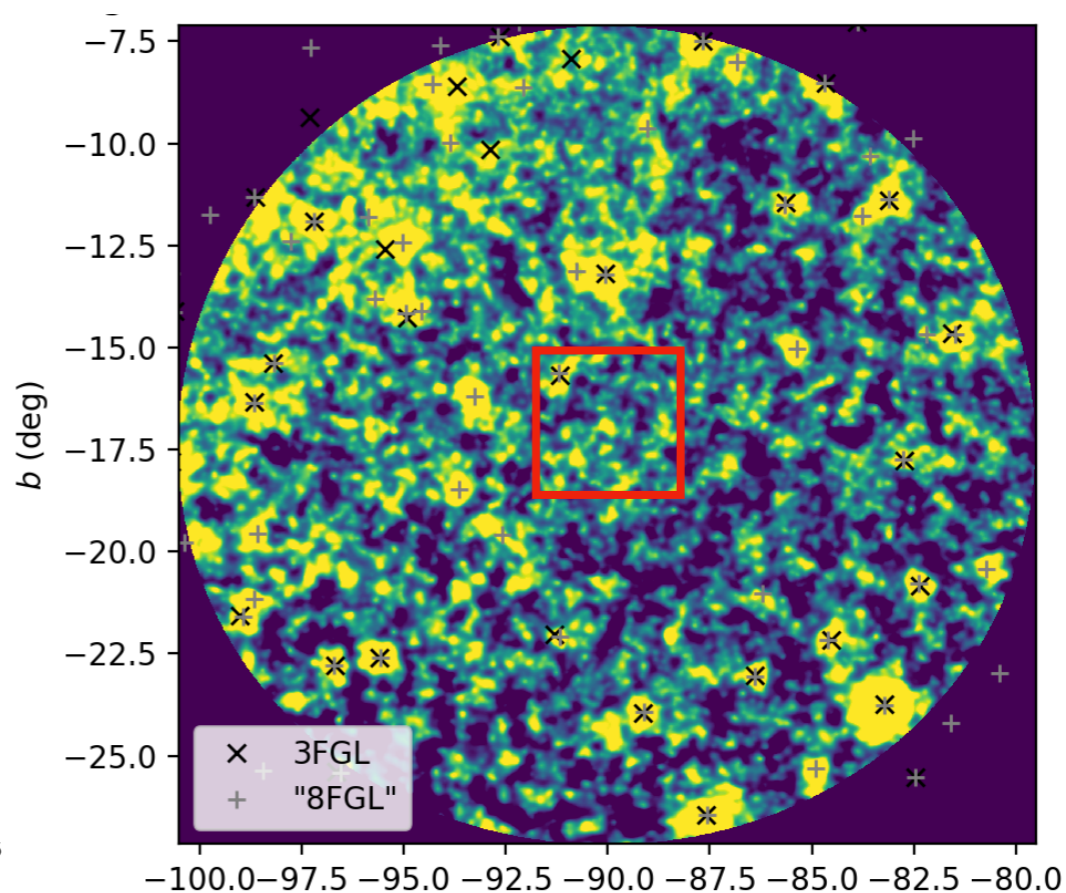
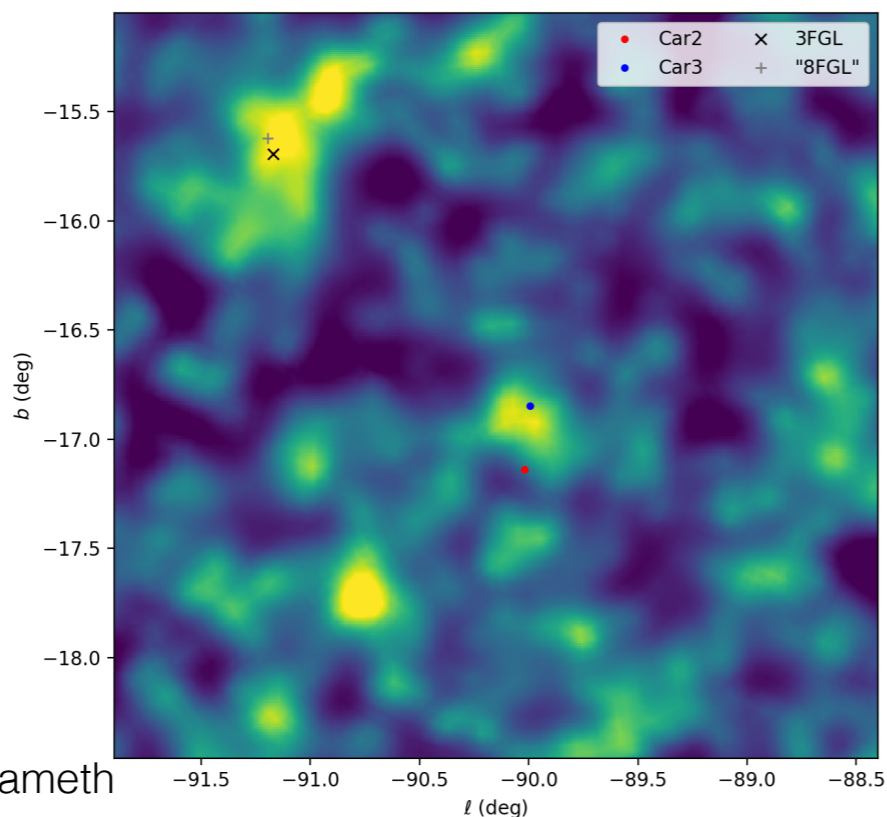
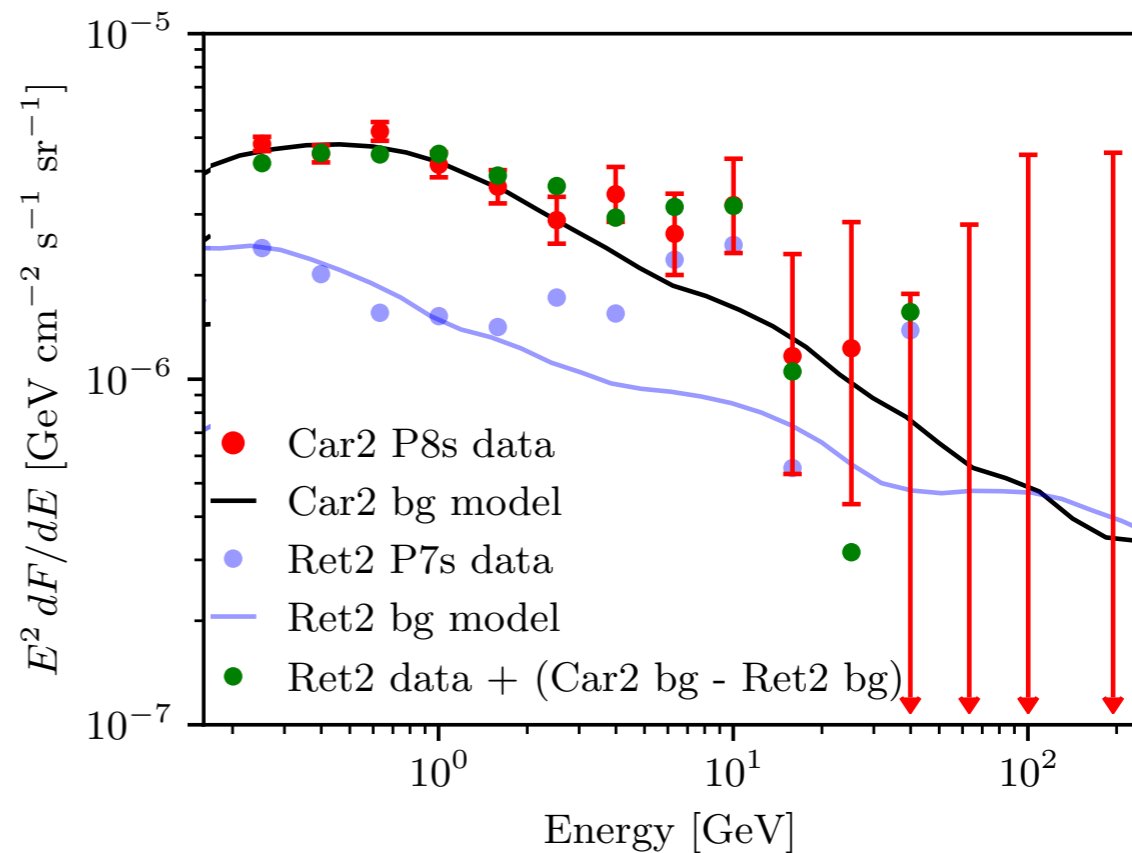
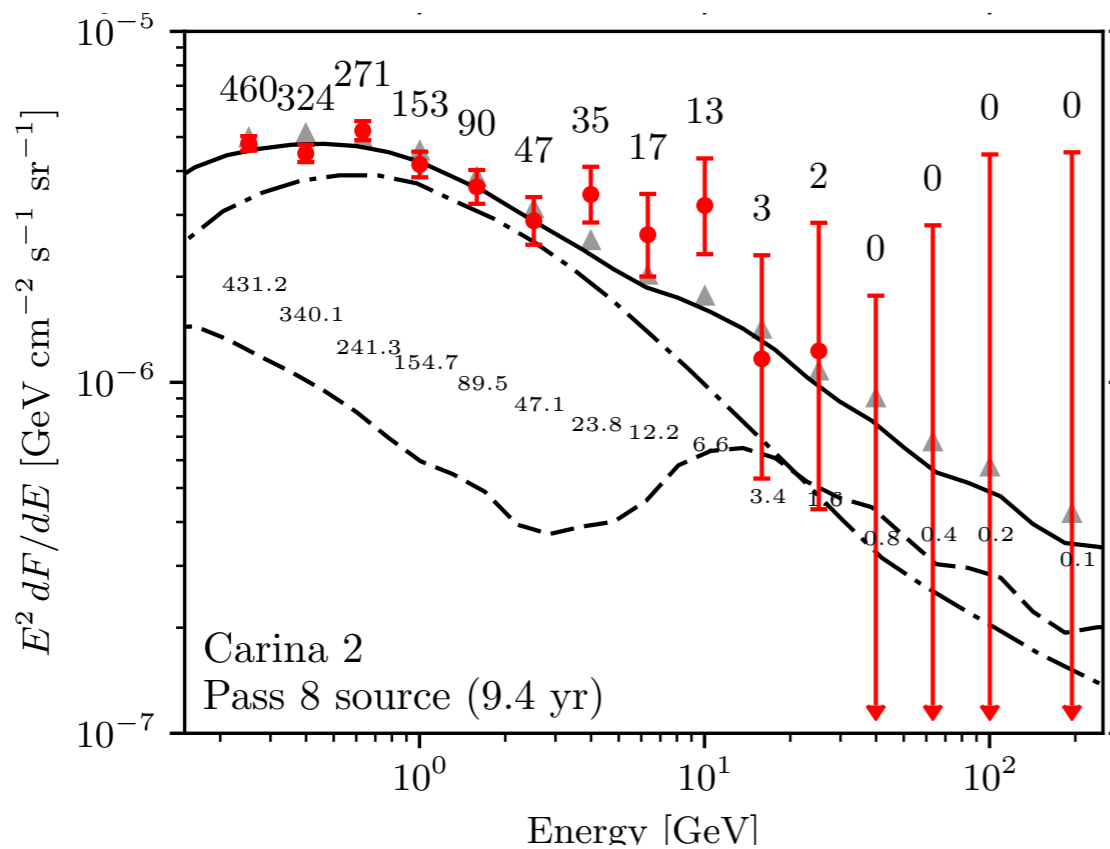
Inconsistent with any other possible source
(e.g. non-DM astrophysics, incorrect diffuse bg models)

+

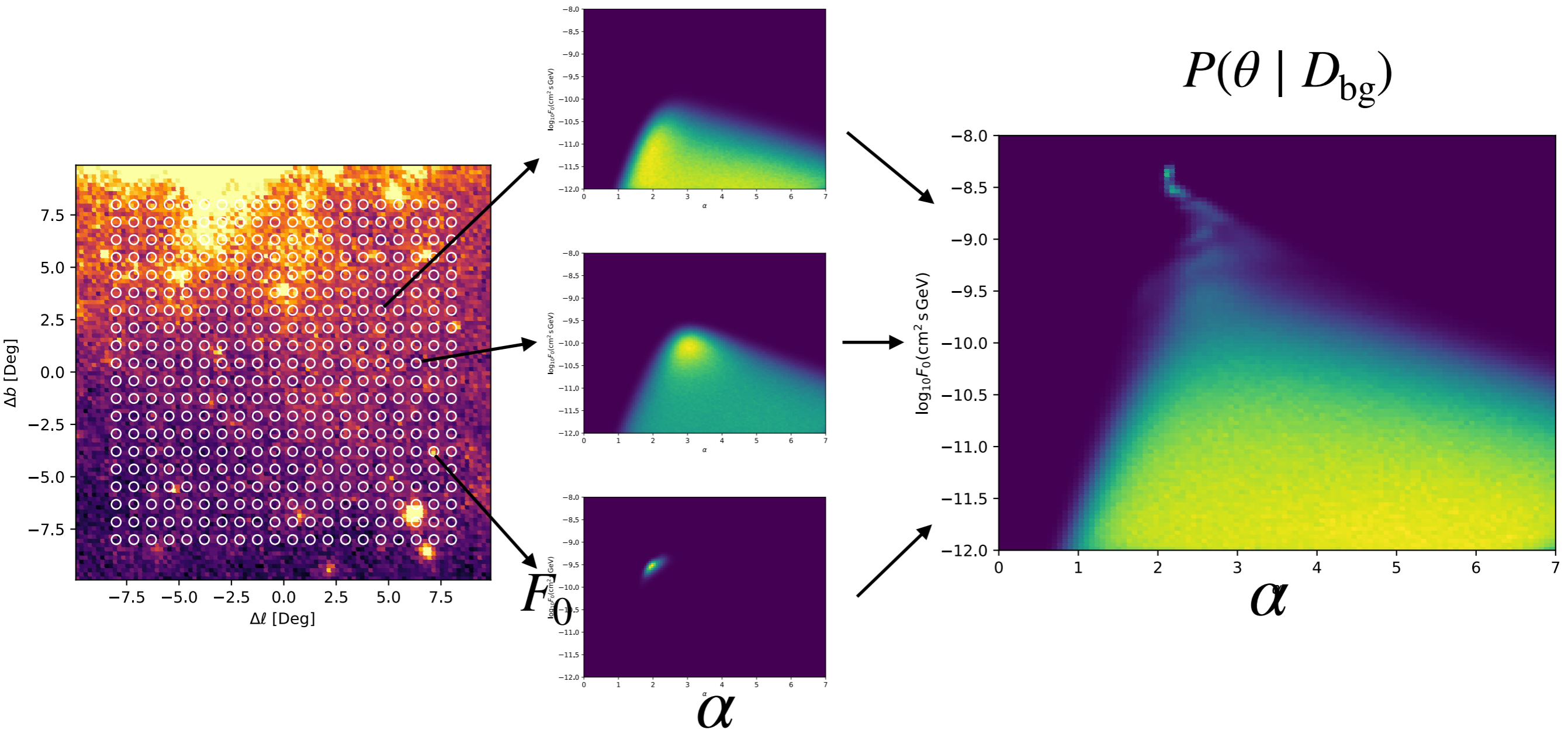
Consistent with dark matter annihilation

Carina 2 and 3

Discovered by Torrealba+ 1801.07279 (MNRAS)



Idea: empirical prior distribution of background sources leads to empirical prior on non-DM point-like sources around the target



fit each bg region with diffuse + point source $\frac{dF(E)}{dE} = F_0 \left(\frac{E}{E_0} \right)^{-\alpha}$

Conclusions - Constraints on DM from dwarfs

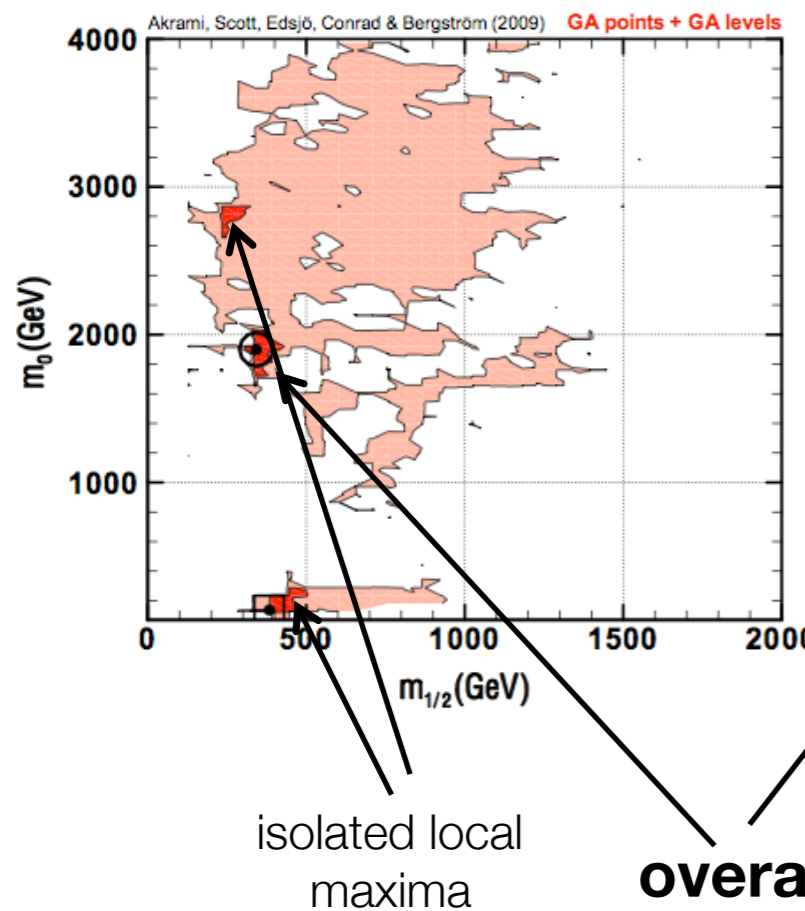
- Ruling out diffuse bg model is not enough
- Want to distinguish DM annihilation from non-DM source populations without sacrificing sensitivity
- Methods apply to any dwarf which is a promising DM target and shows evidence for gamma-ray emission along line of sight
- Should be simple to extend to any dark matter target where you expect localized emission (e.g. galaxy clusters, groups, dark subhalos)

SUPPLEMENTARY MATERIAL

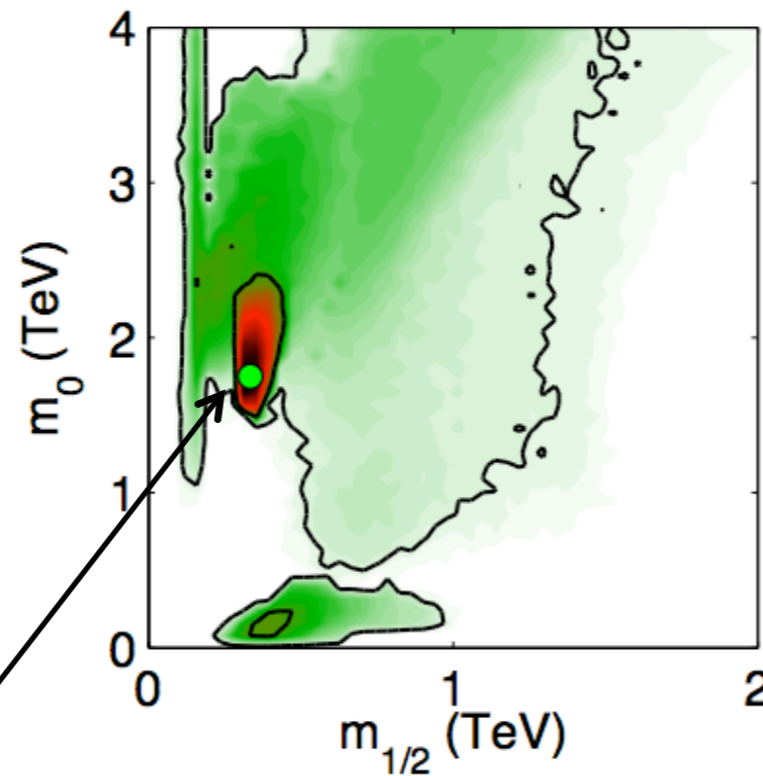
Profile likelihood results: comparison

- Akrami et al (0910.3950) adopted a genetic algorithm (GA) to map out the profile likelihood.
- This allows to find isolated spikes in the likelihood in the focus point region:
is this something other frequentist fits might have missed?

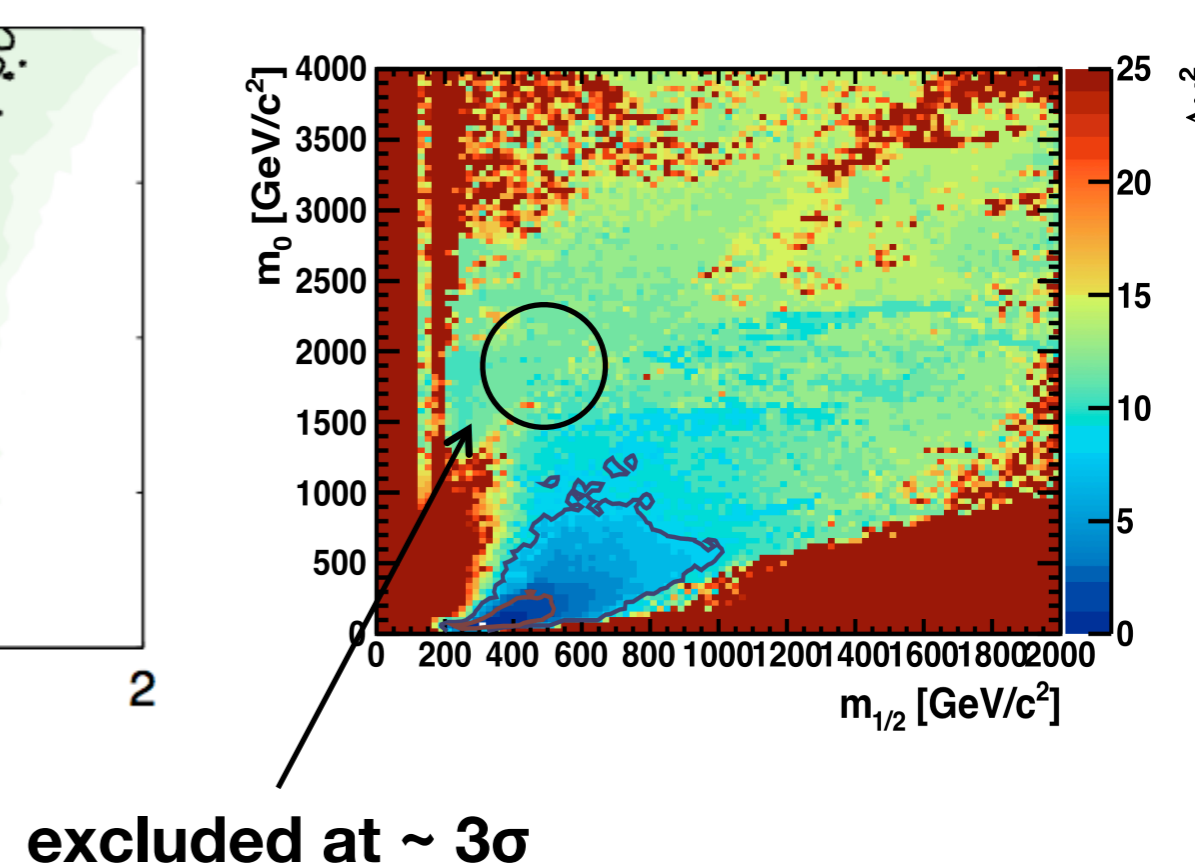
Genetic Algorithm
profile likelihood



MultiNest
profile likelihood

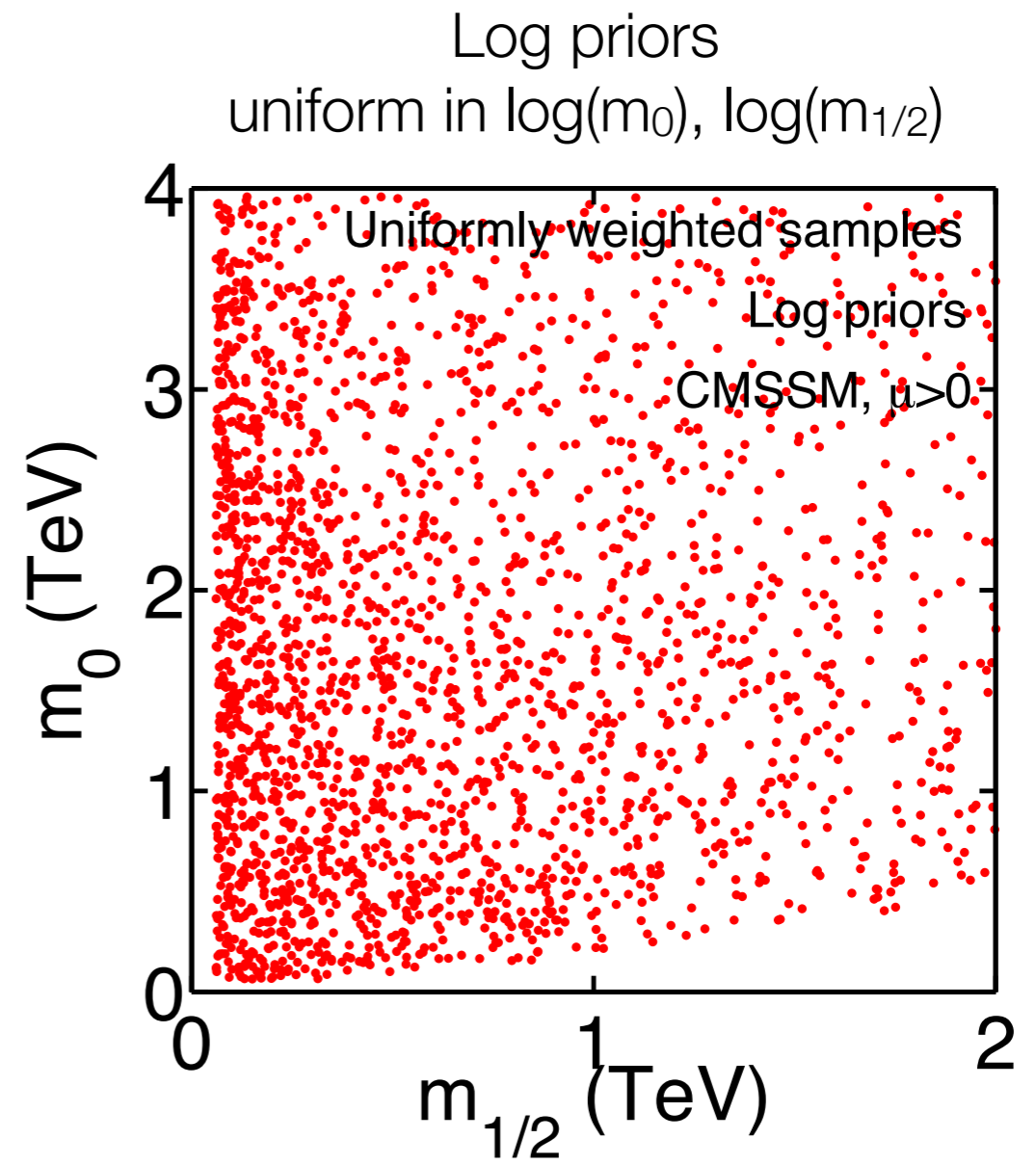
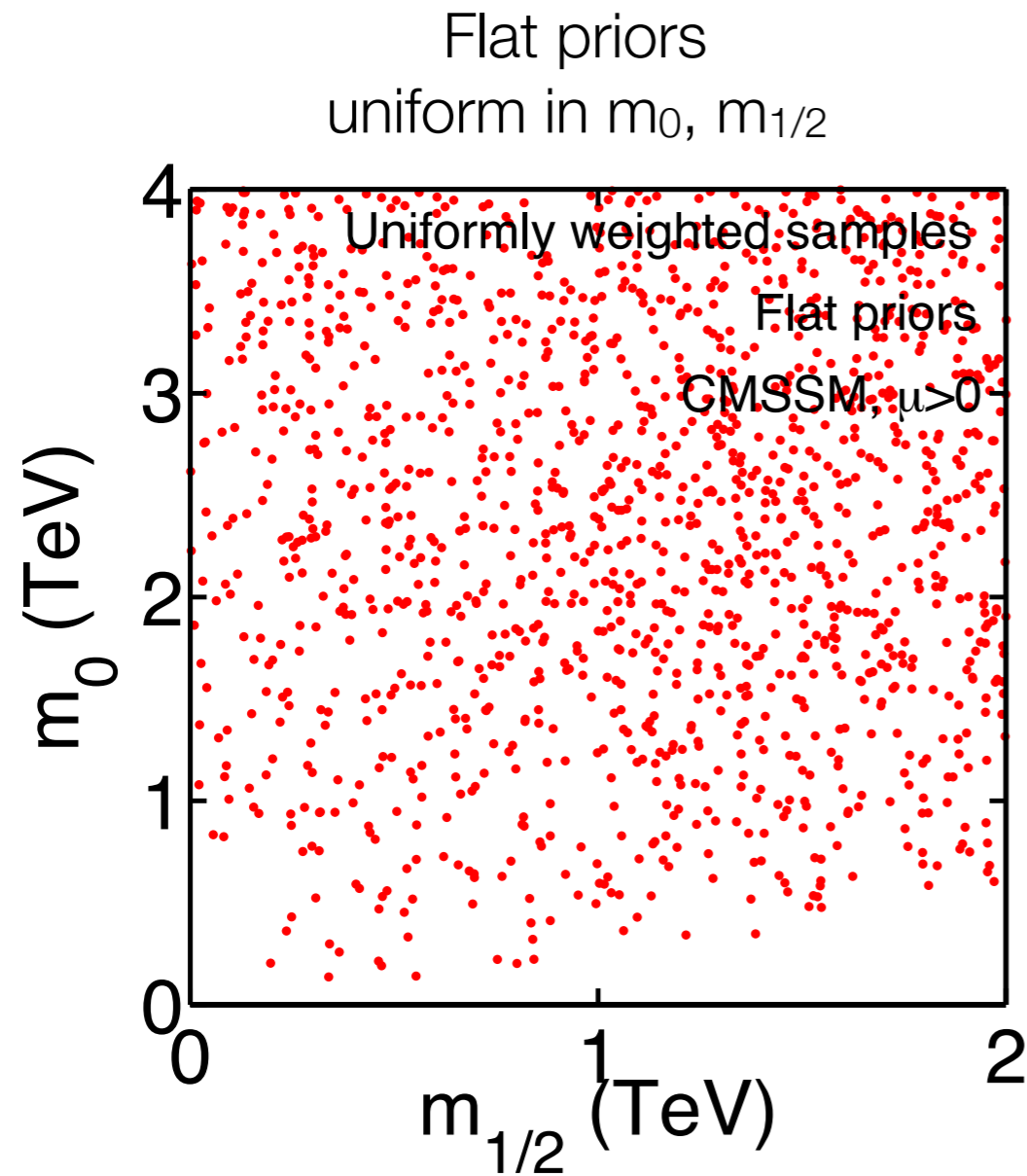


MasterCode
profile likelihood



Samples from priors only

- No data in the likelihood, non-physical points discarded

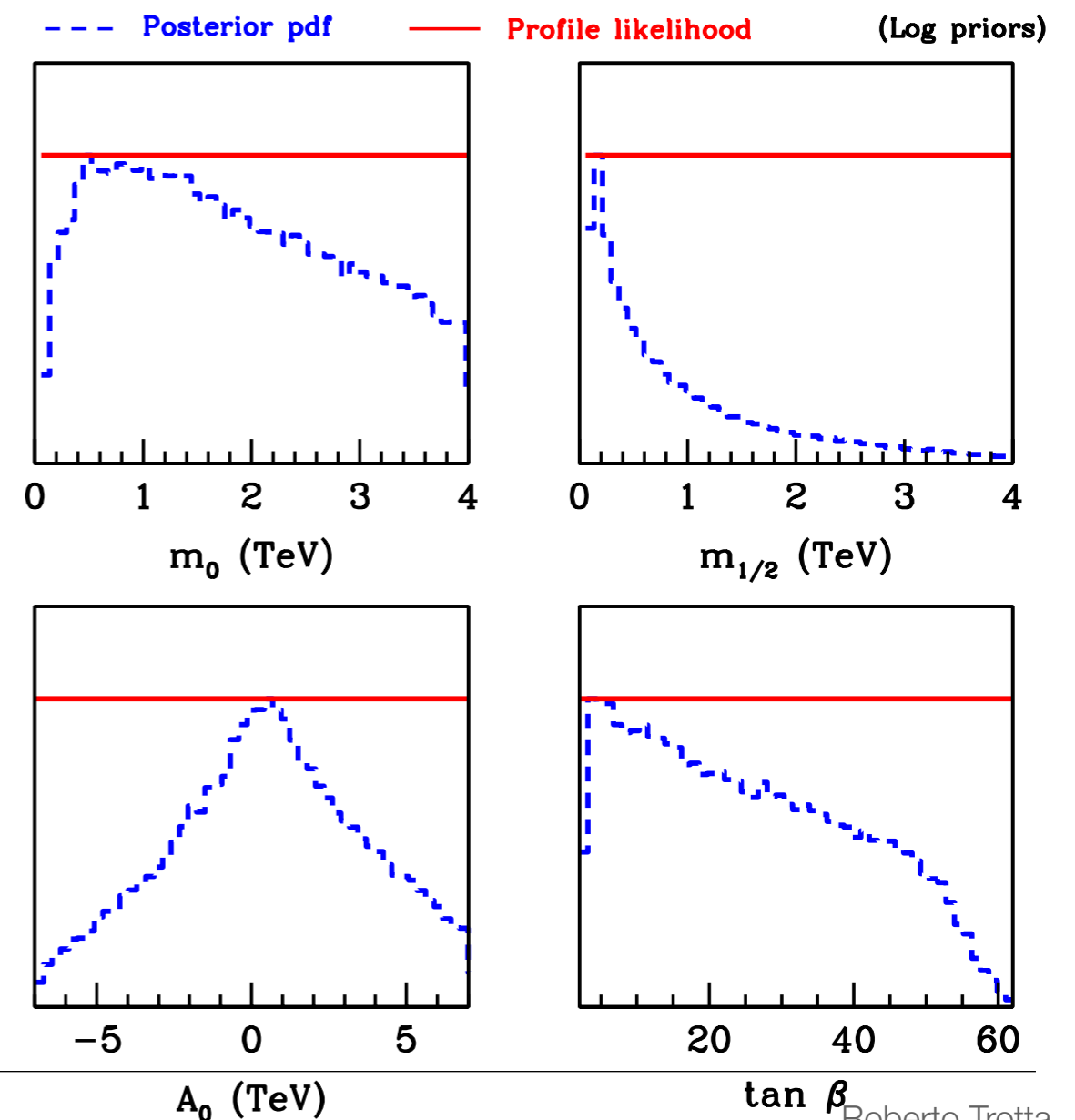
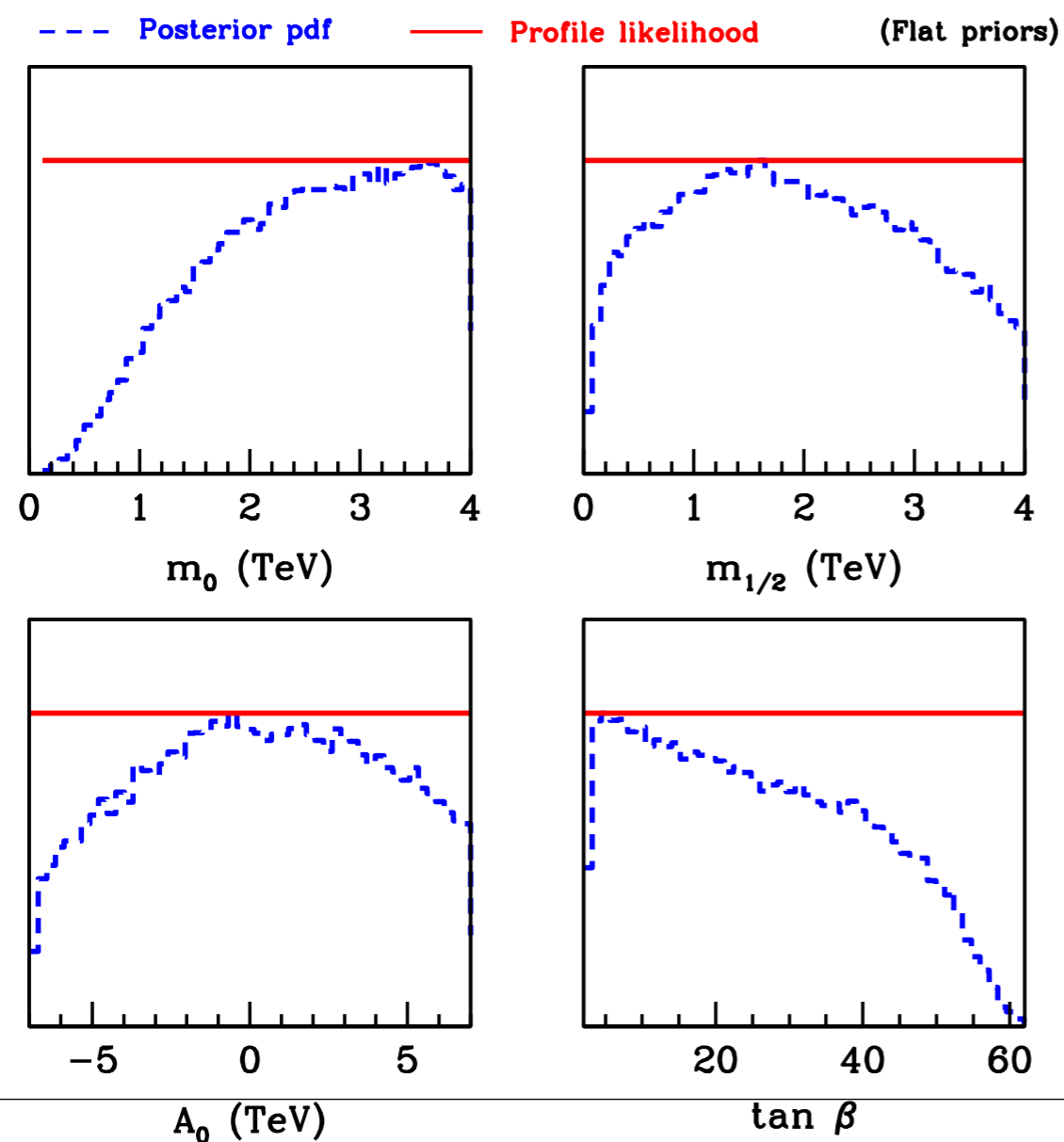


Samples from priors only

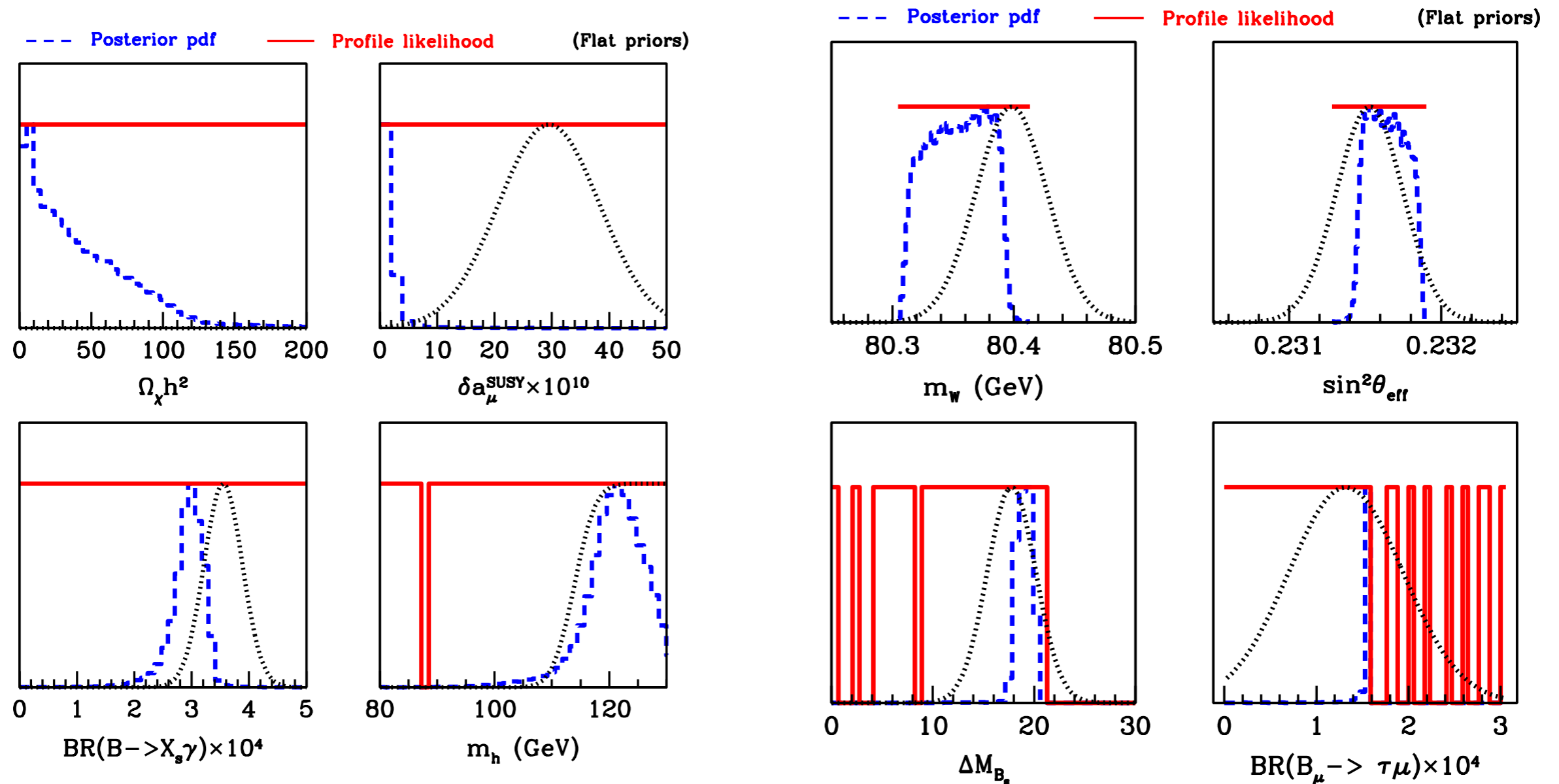
- Volume effect from the non-physical regions

Flat priors

Log priors



Prior distribution for observables



Priors are highly informative regarding the quantities being constrained!

Indirect observables

Observable	Mean value	Uncertainties		ref.
	μ	σ (exper.)	τ (theor.)	
M_W	80.398 GeV	25 MeV	15 MeV	[30]
$\sin^2 \theta_{\text{eff}}$	0.23153	16×10^{-5}	15×10^{-5}	[30]
$\delta a_\mu^{\text{SUSY}} \times 10^{10}$	29.5	8.8	1.0	[31]
$BR(\bar{B} \rightarrow X_s \gamma) \times 10^4$	3.55	0.26	0.21	[32]
ΔM_{B_s}	17.77 ps^{-1}	0.12 ps^{-1}	2.4 ps^{-1}	[33]
$BR(\bar{B}_u \rightarrow \tau \nu) \times 10^4$	1.32	0.49	0.38	[32]
$\Omega_\chi h^2$	0.1099	0.0062	$0.1 \Omega_\chi h^2$	[34]
	Limit (95% CL)		τ (theor.)	ref.
$BR(\bar{B}_s \rightarrow \mu^+ \mu^-)$	$< 5.8 \times 10^{-8}$		14%	[35]
m_h	$> 114.4 \text{ GeV}$ (SM-like Higgs)		3 GeV	[36]
ζ_h^2	$f(m_h)$ (see text)		negligible	[36]
$m_{\bar{q}}$	$> 375 \text{ GeV}$		5%	[25]
$m_{\bar{g}}$	$> 289 \text{ GeV}$		5%	[25]
other sparticle masses	As in table 4 of ref. [6].			

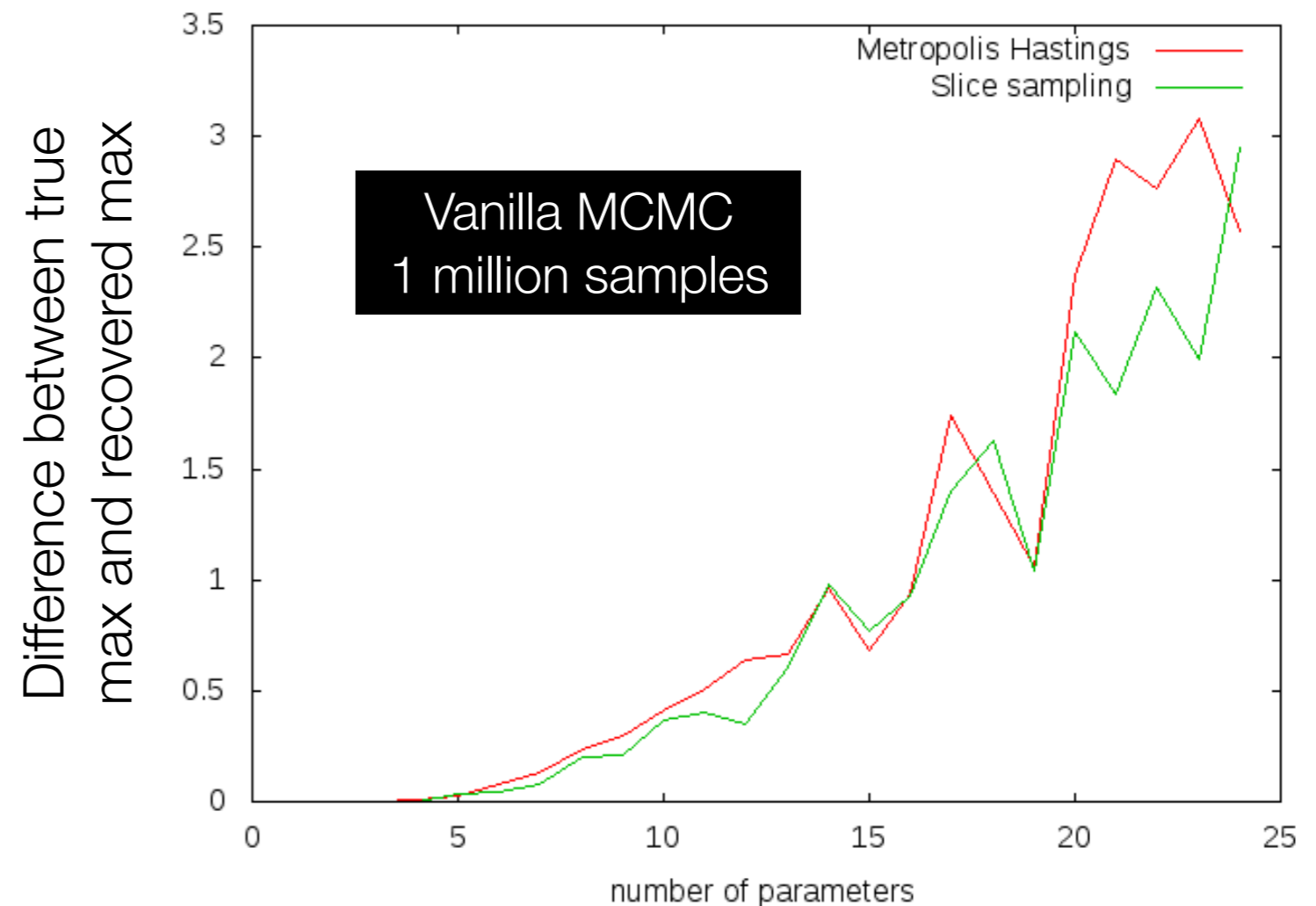
SM parameters

SM (nuisance) parameter	Mean value μ	Uncertainty σ (exper.)	Ref.
M_t	172.6 GeV	1.4 GeV	[24]
$m_b(m_b)^{\overline{MS}}$	4.20 GeV	0.07 GeV	[25]
$\alpha_s(M_Z)^{\overline{MS}}$	0.1176	0.002	[25]
$1/\alpha_{\text{em}}(M_Z)^{\overline{MS}}$	127.955	0.03	[26]

Plus consistency with astrophysical probes

- MCMC/MultiNest are not designed to find the best-fit point. Bayesian algorithms are designed to map out regions of significant posterior probability mass
- Even for a simple Gaussian toy model, this becomes difficult to do as the number of dimensions of the parameter space increases
- **Profiling with vanilla MCMC or MultiNest scans has to be done with caution!**

Toy multinormal likelihood



Posterior pdf from MultiNest scans

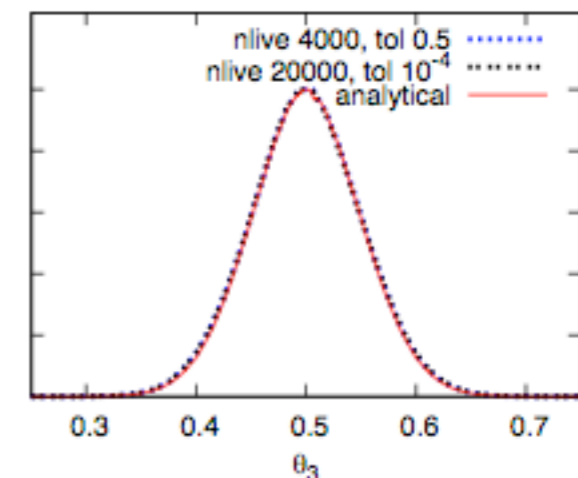
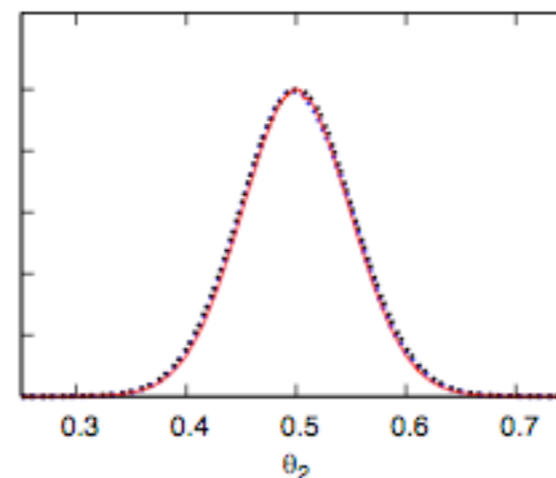
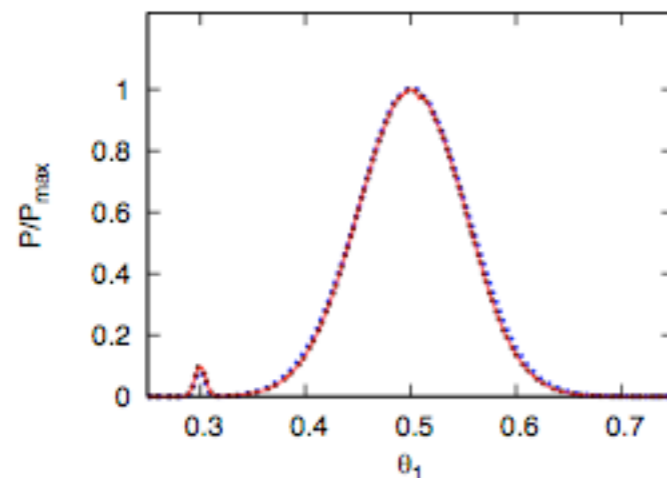
- MultiNest is primarily aimed at evaluation of the posterior pdf. It does an excellent job even for multi-modal problems. 8D toy case (Feroz, KC, RT et al, in prep)
- The tolerance parameter (**tol**) determines the stopping criterium (based on the incremental change of the value of the local evidence). Lower tol gives a finer exploration around the peak, important for profile likelihood evaluation

red: analytical

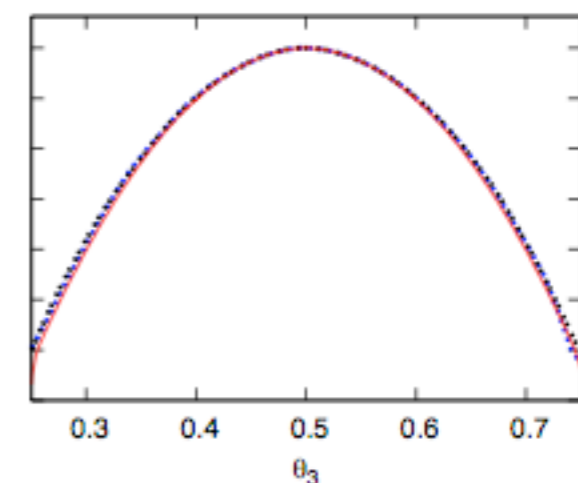
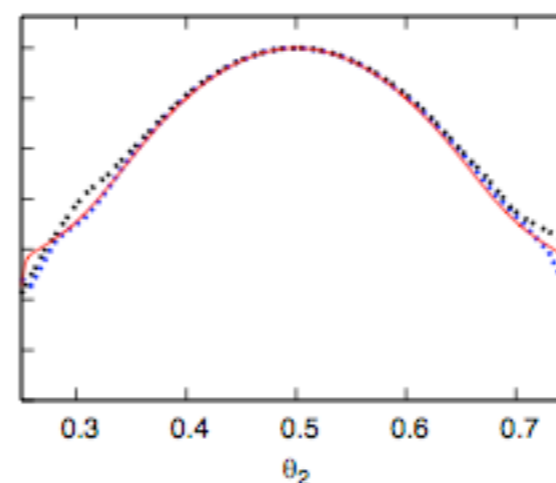
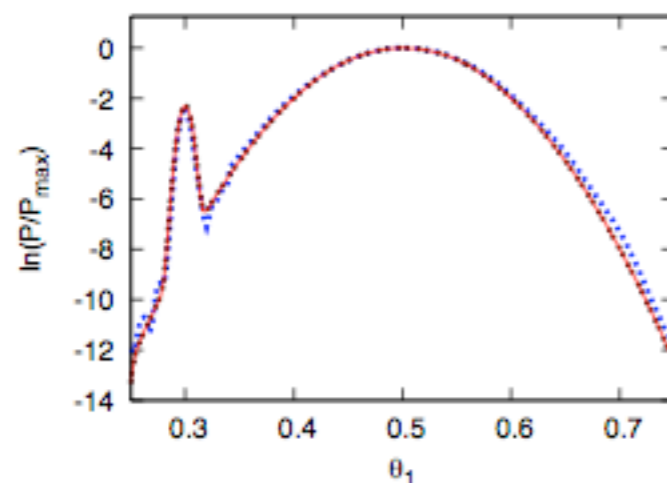
blue: MN, tol=0.5

black: MN, tol=0.0001

Posterior



Log(posterior)



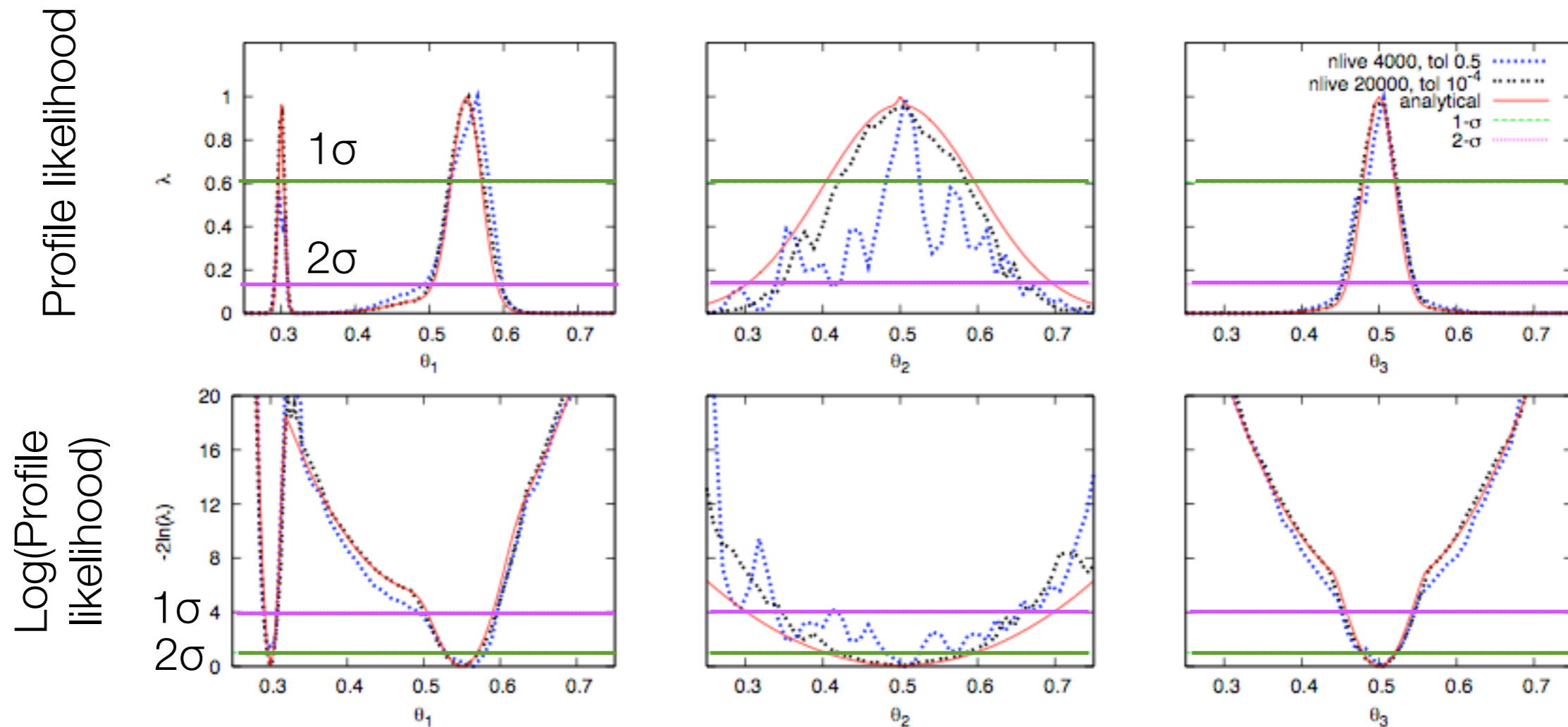
Profile likelihood from MultiNest scans

- A fairly accurate the profile likelihood can be obtained with MultiNest by tuning the tolerance (lower, **tol=0.0001**) and the number of live points (higher, **n_{live}=20,000**) (Feroz, KC, RT et al, in prep), even for highly multi-modal distributions. 8D toy:

red: analytical

blue: MN, tol=0.5

black: MN, tol=0.0001

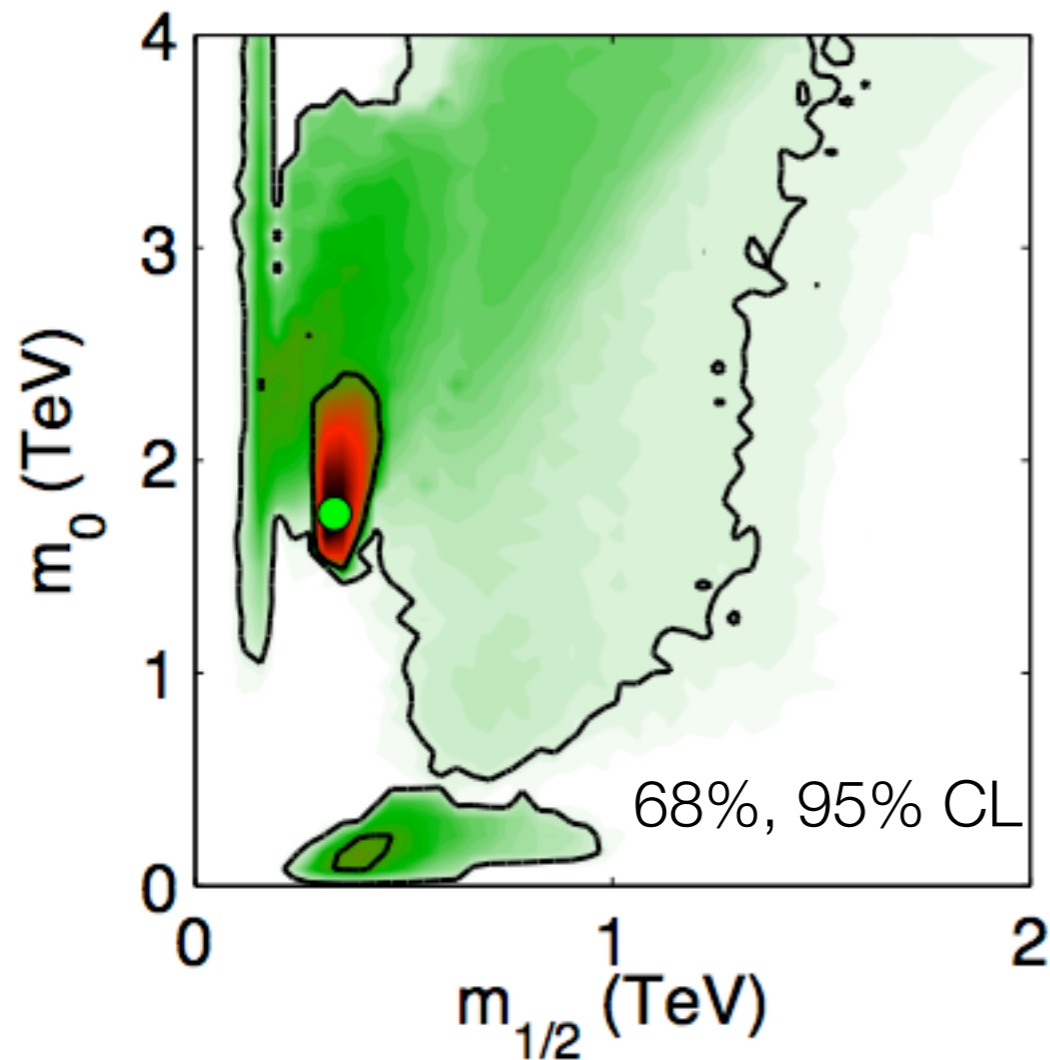


Profile likelihood from MultiNest

- MultiNest scan with 20,000 live points (usually: 4,000) and tolerance 0.0001 (usually: 0.5) results in 5.5 million likelihood evaluations (Akrami et al, GA: 3 million), and best-fit chi-square = 9.26 (Akrami et al, GA: 9.35).

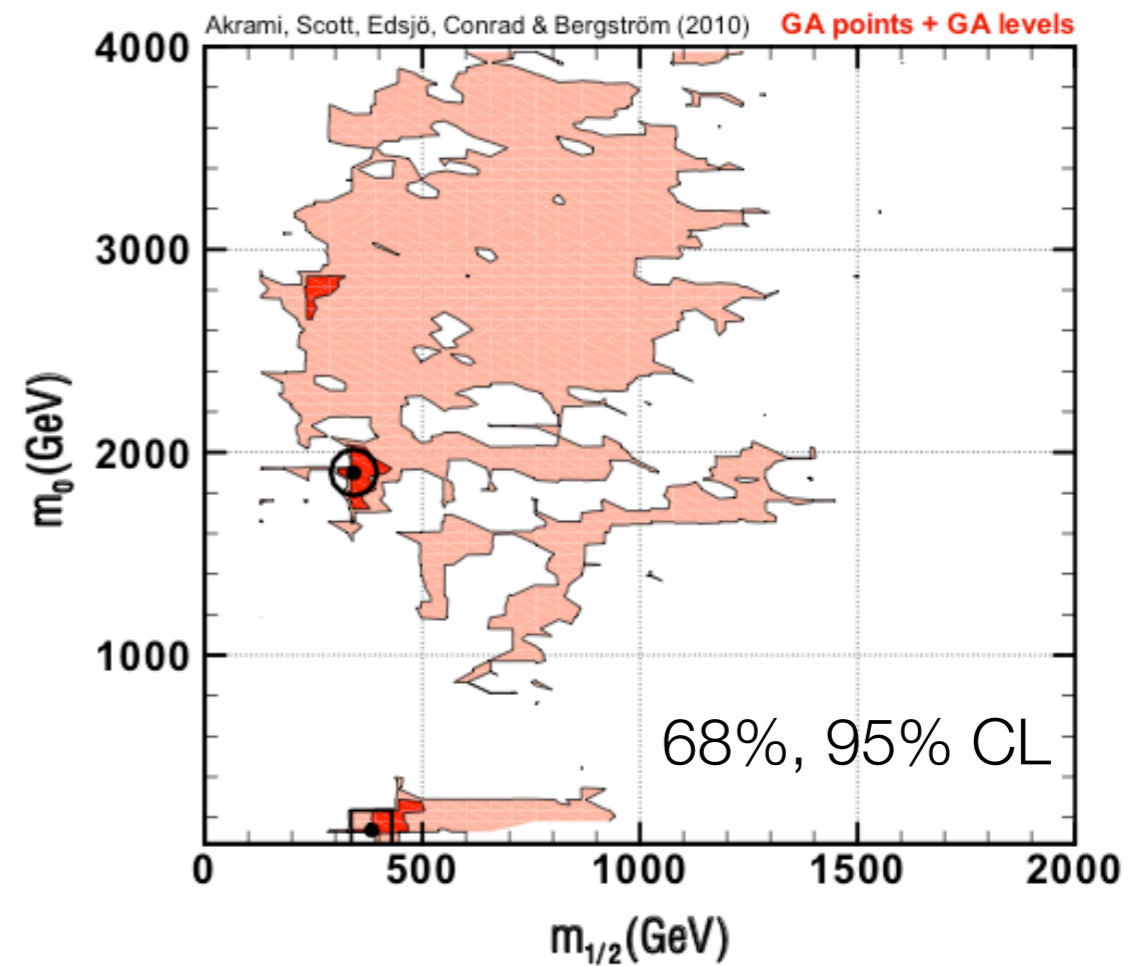
MultiNest finds a better best-fit + smoother contours than GA.

Profile likelihood MultiNest, $\text{tol}=10^{-4}$
Merged log and flat priors scans



Feroz, KC, RT et al (2011)

Profile likelihood
Genetic algorithm



Akrami et al (2010)