Testing Naturalness at 100 TeV

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CP3 seminar

Naturalness



Hierarchy problem

The total Higgs mass-squared consistent of the tree-level mass-squared parameter and the sum of all loop contributions.

The dominant loop contributions are



A cut-off scale of $\Lambda \sim$ 10 TeV leads to

 $\sim -(2 \, {\rm TeV})^2$ $\sim (700 \, {\rm GeV})^2$ $\sim (500 \, {\rm GeV})^2$

For a fine-tuning of 10 %, the top, gauge, and Higgs loops must be cut off at

 $\Lambda_{top} \lesssim 2\,\text{TeV} \;, \qquad \quad \Lambda_{gauge} \lesssim 5\,\text{TeV} \;, \qquad \quad \Lambda_{Higgs} \lesssim 10\,\text{TeV} \;.$

Adding a new symmetry can protect the Higgs mass.

- Supersymmetry leads to scalar top partner
- Little Higgs mechanism leads to fermionic top partner

They couple in such a way to the Higgs that the contribution from the top quark is canceled.



Higgs as Pseudo Goldstone boson

Nambu Goldstone Theorem

For each spontaneously broken global symmetry generator there is a massless boson in the spectrum.

The Higgs can be seen as part of a Nambu Goldstone boson π with a VEV f.

$$\phi = \phi_0 \exp \frac{i\pi}{f} , \qquad \qquad \phi_0 = \begin{pmatrix} 0 \\ f \end{pmatrix}$$

The simplest little Higgs model breaks a global SU(3) to SU(2)_L

$$\pi = \begin{pmatrix} \Phi & H \\ H^{\dagger} & \eta \end{pmatrix} ,$$

H is the SU(2) Higgs doublet .

Expanded around its VEV the non-linear field ϕ reads

$$\phi = \begin{pmatrix} 0 \\ f \end{pmatrix} + i \begin{pmatrix} H \\ 0 \end{pmatrix} - \frac{1}{2f} \begin{pmatrix} 0 \\ H^{\dagger}H \end{pmatrix} + \dots ,$$

The Higgs kinetic term and interaction terms are given by

$$f^2 |\partial_\mu \phi|^2 = |\partial_\mu h|^2 + \frac{|\partial_\mu H|^2 H^{\dagger} H}{f^2} + \dots$$

This Higgs field is a massless NGB

$$f^2 \left| \partial_\mu \phi \right|^2 = \left| \partial_\mu h \right|^2 + \frac{\left| \partial_\mu H \right|^2 H^{\dagger} H}{f^2} + \dots \,.$$

- \cdot with non-renormalizable interactions suppressed by a symmetry breaking scale f
- \cdot This is an effective theory which becomes strongly coupled at $\Lambda \simeq 4\pi f$

Gauge interactions

Adding gauge interactions via $|{\it D}_{\mu}\phi|^2$ reintroduces the quadratic divergent terms

$$\left(\frac{g\Lambda}{4\pi}\right)^2 H^{\dagger}H \; .$$

Collective symmetry breaking

Arkani-Hamed et al. 2002]

If there are two different global symmetries, an exact Goldstone boson remains as long as one symmetry stays unbroken

Two copies of NGBs

$$\phi_{1/2} = \phi_0 \exp rac{i\pi_{1/2}}{f} \; .$$

The interaction with gauge bosons

$$\mathcal{L} = |D_{\mu}\phi_1|^2 + |D_{\mu}\phi_2|^2 ,$$

lead to quadratically divergent one-loop diagrams

$$\left(\frac{g}{4\pi}\right)^2 \left(\phi_1^{\dagger}\phi_1 + \phi_2^{\dagger}\phi_2\right) = \left(\frac{g\Lambda}{4\pi}\right)^2 \left(f^2 + f^2 + \dots\right) \ ,$$

which do not involve the Higgs doublet.

The expansion of the product of both fields

$$\begin{split} \phi_1^{\dagger}\phi_2 &= \begin{pmatrix} 0 & f \end{pmatrix} \exp\left\{-\frac{2i}{f} \begin{pmatrix} 0 & h \\ h^{\dagger} & 0 \end{pmatrix}\right\} \begin{pmatrix} 0 \\ f \end{pmatrix} \\ &= f^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 2if \begin{pmatrix} 0 & h \\ h^{\dagger} & 0 \end{pmatrix} - 2 \begin{pmatrix} h^{\dagger}h & 0 \\ 0 & h^{\dagger}h \end{pmatrix} + \dots \\ &= f^2 - 2h^{\dagger}h + \dots, \end{split}$$

leads to a logarithmically divergent mass-squared for h equal to

$$\left(\frac{g^2}{4\pi}\right)^2 \log\left(\frac{\Lambda^2}{\mu^2}\right) \left|\phi_1^{\dagger}\phi_2\right|^2 \simeq \left(\frac{g^2}{4\pi}\right)^2 \log\left(\frac{\Lambda^2}{\mu^2}\right) f^2$$
 which is ~ $M_{\rm weak}^2$ for f ~ TeV.

Top sector

The coupling between Higgs fields and a third generation triplet $\boldsymbol{\Psi}$ is

$$\mathcal{L}_{yuk} = \lambda_1 \phi_1^{\dagger} \Psi t_1^c + \lambda_2 \phi_2^{\dagger} \Psi t_2^c , \qquad \Psi = (Q, T) ,$$

where T is a fermionic top-partner.

The expansion reads

$$\begin{split} \mathcal{L} &\sim \frac{\lambda}{\sqrt{2}} \left[fT(t_2^c + t_1^c) + iH^{\dagger}Q(t_2^c - t_1^c) - \frac{1}{2f}H^{\dagger}HT(t_2^c + t_1^c) + \dots \right] \\ &= \lambda H^{\dagger}Qt^c + \lambda f\left(1 - \frac{1}{2f^2}H^{\dagger}H\right)TT^c + \dots \,, \end{split}$$

where

$$T^c = (t_2^c + t_1^c)/\sqrt{2}$$
, $t^c = i(t_2^c - t_1^c)/\sqrt{2}$, $\lambda_1 \equiv \lambda_2 \equiv \lambda/\sqrt{2}$.

The divergent loop contribution cancel each other





Mirror Twin Higgs model

In mirror twin Higgs models the partner can be neutral under SM charges.

The Higgs H transforms as fundamental under SU(4) which is gauged into $SU(2)_{SM} \times SU(2)_{twin}$.

$$H_{SU(4)} = \begin{pmatrix} H_{SM} \\ H_{twin} \end{pmatrix}$$
,

The top sector

$$\begin{split} \mathcal{L} &= y H_{\text{SM}} Q_{\text{SM}} U_{\text{SM}}^{c} + y H_{\text{twin}} Q_{\text{twin}} U_{\text{twin}}^{c} \\ &\rightarrow y H Q_{\text{SM}} U_{\text{SM}}^{c} + y \left(f - \frac{|h|^2}{2f} \right) Q_{\text{twin}} U_{\text{twin}}^{c} \end{split}$$

Also in this case the divergent loop contribution cancel



but here the twin partner *T* is not charged under the SM gauge group.

Little Higgs models

- The SM Higgs mass is not stabilized against the sizeable loop corrections.
- Little Higgs models relieve the tension.
- The mirror twin-Higgs model provides the same mechanism and evades the strongest bounds from the LHC.

T-parity

T-parity forbids tree-level couplings between little Higgs particles and SM particles.

Simplified model

- We assume that a top partner of mass m_T has been discovered.
- We want to know which measurement is necessary in order to be sure that the top partner couplings cancel the top quark divergence.
- What will be the reach of a future collider for such a measurement?



We use a simplified Lagrangian describing the couplings between a Higgs doublet H and the third generation quarks q_3 and u_3^c with a vector-like pair (U^c , U) which are singlets under SU(2)_w.

The couplings up to second order in H are

$$\begin{aligned} \mathcal{L}_U &= u_3^c \left(c_0 f U + c_1 H^{\dagger} q_3 + \frac{c_2}{2f} H^{\dagger} H U + \dots \right) \\ &+ U^c \left(\widehat{c}_0 f U + \widehat{c}_1 H^{\dagger} q_3 + \frac{\widehat{c}_2}{2f} H^{\dagger} H U + \dots \right) + \text{h.c.} \,. \end{aligned}$$

- No additional symmetry is assumed
- The $H^{\dagger}H$ term can cancel the loop contribution from the linear term
- The parameter $c_{0,1,2}$ describe the couplings of the right handed quark u_3^c
- The parameter $\widehat{c}_{0,1,2}$ describe the couplings of the right handed partner $\mathit{U^c}$

Mass eigenstates

The rotation into mass eigenstates											
	top quarks	$t'^c = \frac{\widehat{c}_0 u_3^c - c_0 U^c}{c} \; , \qquad$	$t'=q_3\;,$								
t	op partner	$T'^c = \frac{\widehat{c}_0 U^c + c_0 u_3^c}{c} ,$	T'=U,								
and definiti	on of the cou	olings									
	Mass	$m_{T'}=fc$,	$c=\sqrt{c_0^2+\widehat{c}_0^2}\;,$								
Yu	kawa	$\lambda_{t'} = \frac{\widehat{c}_0 c_1 - c_0 \widehat{c}_1}{c} ,$	$\lambda_{T'} = \frac{c_0 c_1 + \widehat{c}_0 \widehat{c}_1}{c} ,$								
H	[†] Hff ^c	$\alpha_{t'} = \widehat{c}_0 c_2 - c_0 \widehat{c}_2 \; , \qquad$	$\alpha_{T'} = c_0 c_2 + \widehat{c}_0 \widehat{c}_2 \;,$								

leads to the mass eigenstate Lagrangian

$$\begin{split} \mathcal{L}_{T'} &= m_{T'}T'^{c}T' + \lambda_{t'}H^{\dagger}t'^{c}t' + \lambda_{T'}H^{\dagger}T'^{c}t' \\ &+ \frac{\alpha_{t'}}{2m_{T'}}H^{\dagger}Ht'^{c}T' + \frac{\alpha_{T'}}{2m_{T'}}H^{\dagger}HT'^{c}T' + \mathcal{O}\left(H^{3}\right) + \text{h.c.} \;, \end{split}$$

Mass eigenstate Lagrangian

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If we want to forbid couplings between top quarks and top partner we have to ensure

$$\lambda_{T'} = \frac{c_0 c_1 + \widehat{c}_0 \widehat{c}_1}{c} \equiv 0 , \qquad \qquad \alpha_{t'} = \widehat{c}_0 c_2 - c_0 \widehat{c}_2 \equiv 0$$

Examples of models where this is given are

Models

Using a vector-like pair (Q^c, Q) of $SU(2)_W$ doublets (instead of singlets)

$$\begin{split} \mathcal{L}_{Q} &= \left(Q^{c}c_{0}f + u_{3}^{c}c_{1}H^{\dagger} + Q^{c}\frac{c_{2}}{2f}H^{\dagger}H + \dots \right)q_{3} \\ &+ \left(Q^{c}\widehat{c}_{0}f + u_{3}^{c}\widehat{c}_{1}H^{\dagger} + Q^{c}\frac{\widehat{c}_{2}}{2f}H^{\dagger}H + \dots \right)Q + \text{h.c.} \, . \end{split}$$
 leads to the an identical mass eigenstate Lagrangian with the replacement of $T^{\prime c} \leftrightarrow T^{\prime} \, , \qquad t^{\prime c} \leftrightarrow t^{\prime} \, . \end{split}$

Various models can be described by these Lagrangians

Model	Coset		SU(2) C ₀	<i>C</i> ₁	C2	\widehat{C}_0	\widehat{C}_1	\widehat{C}_2
Toy model	SU(3) SU(2) Pe	relstein, Peskin, Pierce 2004	1	λ_1	$-\lambda_1$	$-\lambda_1$	λ_2	0	0
Simplest	$\left(\frac{SU(3)}{SU(2)}\right)^2$	Kaplan, Schmaltz 2003	1	λ	$-\lambda$	$-\lambda$	λ	λ	$-\lambda$
Littlest Higgs	SU(5) SO(5)	Arkani-Hamed et al. 2002	1	λ_1	$-\sqrt{2}i\lambda_1$	$-2\lambda_1$	λ_2	0	0
Custodial	SO(9) SO(5) SO(4)	Chang 2003	2	<i>Y</i> ₁	$\frac{i}{\sqrt{2}}y_1$	$-\frac{1}{2}y_1$	<i>y</i> ₂	0	0
T-parity invaria	ant <u>SU(3)</u> SU(2)	Cheng, Low, Wang 2006	1	λ	$-\lambda$	$-\lambda$	$-\lambda$	$-\lambda$	λ
T-parity invaria	ant <u>su(s)</u> so(s)	Cheng, Low, Wang 2006	1	λ	$-\sqrt{2}i\lambda$	-2λ	$-\lambda$	$-\sqrt{2}i\lambda$	2λ
Mirror twin Hig	gs SU(4) U(1) SU(3) U(1)	Burdman et al. 2015	1	0	$i\lambda_t$	0	λ_t	0	$-\lambda_t$

Quadratic divergence

Mass eigenstate Lagrangian

$$\begin{split} \mathcal{L}_{T'} &= m_{T'}T'^cT' + \lambda_{t'}H^{\dagger}t'^ct' + \lambda_{T'}H^{\dagger}T'^ct' \\ &+ \frac{\alpha_{t'}}{2m_{T'}}H^{\dagger}Ht'^cT' + \frac{\alpha_{T'}}{2m_{T'}}H^{\dagger}HT'^cT' + \mathcal{O}\left(H^3\right) + \text{h.c.} \;, \end{split}$$

The fermion mass matrix in the basis (t'^c, T'^c) and (t', T') is

$$\mathcal{M}(H) = \begin{pmatrix} 0 & 0 \\ 0 & m_{T'} \end{pmatrix} + \begin{pmatrix} \lambda_{t'} & 0 \\ \lambda_{T'} & 0 \end{pmatrix} H + \begin{pmatrix} 0 & \alpha_{t'} \\ 0 & \alpha_{T'} \end{pmatrix} \frac{H^{\dagger}H}{2m_{T'}} + \mathcal{O}\left(H^3\right) \ .$$

Coleman-Weinberg potential

$$V_{\text{quad}}^{T} = \frac{\Lambda^2}{16\pi^2} \operatorname{tr} \mathcal{M}^2 , \qquad \qquad \mathcal{M}^2 = \mathcal{M}(H)^{\dagger} \mathcal{M}(H)$$

Absence of quadratically divergent contributions to the Higgs mass for

$$\Delta m_{\rm quad}^2 = \frac{1}{2} \left. \frac{\partial^2 V_{\rm quad}^I}{\partial H^2} \right|_{H=0} \equiv 0 ,$$

Naturalness condition

Requiring the coefficients of the $H^{\dagger}H$ term to vanish leads to the condition

$$\alpha_{\mathcal{T}'} = -|\lambda_{\mathcal{T}'}|^2 - |\lambda_{t'}|^2$$

which corresponds to a cancellation between



 $lpha_{t'}$ turns out to be unconstrained by this consideration

In terms of the original parameter this condition reads

$$|c_1|^2 + c_0 c_2 = - \left|\widehat{c}_1\right|^2 - \widehat{c}_0 \widehat{c}_2 \; .$$

this holds for example in little and twin Higgs models

Little Higgs $|c_1|^2 = -c_0c_2$, $|\widehat{c}_1|^2 = -\widehat{c}_0\widehat{c}_2$,Mirror twin Higgs $|c_1|^2 = -\widehat{c}_0\widehat{c}_2$, $c_0 = \widehat{c}_1 = c_2 = 0$.

Testing the naturalness relation

Naturalness relation

$$\alpha_{\mathcal{T}'} = - |\lambda_{\mathcal{T}'}|^2 - |\lambda_{t'}|^2$$

This can be tested with the knowledge of

- Top Yukawa coupling
- Higgs coupling to a top quark and its partner (can be equal to 0)
- \cdot Coupling of a top partner pair to a Higgs pair (large phase space suppression)

It has previously been suggested to test this equation via

[Perelstein, Peskin, Pierce 2004]

$$\lambda_{T'} \frac{m_{T'}}{f} = |\lambda_{T'}|^2 + |\lambda_{t'}|^2 ,$$

- Measure λ_T with the total decay width: $\Gamma_T = rac{m_T \lambda_T^2}{16\pi}$
- Measure f in the gauge sector: $M_{Z_H} = \sqrt{\frac{g_L^2 + g_R^2}{2}} f = \frac{\sqrt{2g}}{\sin 2\psi} f$

Downsides

- · These measurements are non-trivial
- This naturalness relation is model dependent (e.g. vanishes for $\lambda_{T'} = 0$)

Electroweak symmetry breaking

Expanding the Higgs field around its VEV

$$H = \frac{1}{\sqrt{2}} (h^+, v + h)^T$$
, $v = 246 \,\text{GeV}$

and rotating the fermion fields into mass eigenstates

top quarks
$$t^c = t'^c$$
, $t = t' - T' \frac{v}{m_{T'}} \lambda_{T'}^*$,top partner $T^c = T'^c$, $T = T' + t' \frac{v}{m_{T'}} \lambda_{T'}$,

leads to the Lagrangian

$$\begin{split} \mathcal{L}_{T} &= m_{T}T^{c}T + \lambda_{t}vt^{c}t + \frac{\lambda_{t}}{\sqrt{2}}ht^{c}t + \frac{\lambda_{T}}{\sqrt{2}}hT^{c}t + \frac{\alpha_{t}}{4m_{T}}h^{2}t^{c}T + \frac{\alpha_{T}}{4m_{T}}h^{2}T^{c}T \\ &+ \frac{a_{t}v}{\sqrt{2}m_{T}}ht^{c}T + \frac{a_{T}v}{\sqrt{2}m_{T}}hT^{c}T + \mathcal{O}\left(h^{3}, \frac{v^{2}}{m_{T}^{2}}\right) + \text{h.c.} \;. \end{split}$$

The couplings m_T , $\lambda_{t,T}$ and $\alpha_{t,T}$ are up to linear order in v/m_T identical to their primed counterparts.

The new VEV induced couplings are

$$a_t = \alpha_{t'} + \lambda_{T'}^* \lambda_{t'}$$
, $a_T = \alpha_{T'} + |\lambda_{T'}|^2$,

Naturalness condition after electroweak symmetry breaking

Lagrangian after electroweak symmetry breaking

$$\begin{split} \mathcal{L}_{T} &= m_{T}T^{c}T + \lambda_{t}vt^{c}t + \frac{\lambda_{t}}{\sqrt{2}}ht^{c}t + \frac{\lambda_{T}}{\sqrt{2}}hT^{c}t + \frac{\alpha_{t}}{4m_{T}}h^{2}t^{c}T + \frac{\alpha_{T}}{4m_{T}}h^{2}T^{c}T \\ &+ \frac{a_{t}v}{\sqrt{2}m_{T}}ht^{c}T + \frac{a_{T}v}{\sqrt{2}m_{T}}hT^{c}T + \mathcal{O}\left(h^{3}, \frac{v^{2}}{m_{T}^{2}}\right) + \text{h.c.} \;. \end{split}$$

The *T^cTh* coupling together with the naturalness condition

$$a_T = \alpha_{T'} + |\lambda_{T'}|^2 , \qquad \qquad \alpha_{T'} = -|\lambda_{T'}|^2 - |\lambda_{t'}|^2 ,$$

leads to the relation

$$a_{T} = - \left|\lambda_{t}\right|^{2} + \mathcal{O}\left(\frac{v^{2}}{m_{T}^{2}}\right) ,$$

In order to verify the cancellation between the loop contribution of the top quarks and their partners only the measurement of the $T^{c}Th$ coupling is necessary.

Naturalness parameter

$$\mu = -\frac{\Delta m_{H}^{2}|_{\rm NP}}{\Delta m_{H}^{2}|_{\rm SM}} = -\frac{a_{\rm T}}{\lambda_{t}^{2}} + \mathcal{O}\left(\frac{v^{2}}{m_{\rm T}^{2}}\right) , \qquad \mu|_{\rm natural ness} = 1 .$$

Collider analysis

Signal and Background



Branching Ratio

$$BR(T \to th) \simeq BR(T \to tZ)$$
$$\simeq \frac{1}{2} BR(T \to Wb) \simeq 25\%.$$

Crosssection of TTh in fb at 100 TeV



The horizontal line at $|\mu_t| = 1$ indicates theories with loop cancellation.

Background

We require the Higgs to be unboosted and the decay products of the top partners to be boosted, this ensures that *ttjjjj* is the dominant background. The two leading jets must have $p_T > 400$ GeV.

Analysis

Collider analysis at 100 TeV

- \cdot Madgraph
- Delphes
- · Boosted Collider Analysis (BoCA)

BoCA

https://github.com/BoostedColliderAnalysis/BoCA

- C++ Analysis software
- \cdot uses boosted decision trees for particle reconstruction and background rejection

Hadronic top BDT response





For boosted objects with $1 \text{ TeV} < p_T < 1.5 \text{ TeV}$ at a 100 TeV collider.



For boosted objects with $1 \text{ TeV} < p_T < 1.5 \text{ TeV}$ at a 100 TeV collider.

Result

Discovery reach and S/B

Top partner pair production in association with one Higgs boson



Signal over background



for a fixed luminosity of 3 ab^{-1}

Discovery reach and S/B

Top partner pair production in association with one Higgs boson



Signal over background

for luminosities of 0.3, 3, 30 ab^{-1} and a fixed significance of 5

The significance of *n* observed events is defined via the log-likelihood ratio

$$Z(x|n) = \sqrt{-2 \ln \frac{L(x|n)}{L(n|n)}}, \qquad L(x|n) = \frac{x^n e^{-x}}{n!}$$

where x is the predicted number for the hypothesis which is tested.

2500

3000

Exclusion limit of unnatural theories with $\mu \neq 1$, against the assumption that the observation at the collider is consistent with the prediction of a natural theory with $\mu = 1$.



Precision of measuring μ

Precision of measuring the naturalness parameter, defined as the uncertainty

$$\delta\mu = \sqrt{\left(-\frac{1}{\lambda_t^2}\delta a_T\right)^2}$$

$$\delta a_T = \frac{a_T}{2Z(b|n)}$$



Precision of measuring μ

Precision of measuring the naturalness parameter, defined as the uncertainty

$$\delta \mu = \sqrt{\left(-\frac{1}{\lambda_t^2}\delta a_T\right)^2 + \left(2\frac{a_T}{\lambda_t^3}\delta\lambda_t\right)^2}, \qquad \qquad \delta a_T = \frac{a_T}{2Z(b|n)}.$$



In order to reach a large precision in the naturalness parameter it is crucial to also improve the precision of the top Yukawa coupling.

Conclusion

- We want to distinguish between models with and without cancellation of the quadratic loop divergence in the top sector.
- We have found that after electroweak symmetry breaking generically one simple requirement ensures the cancellation between top quarks and their partner:

$$a_{T} = -\left|\lambda_{t}\right|^{2} + \mathcal{O}\left(\frac{v^{2}}{m_{T}^{2}}\right)$$

- This relation can be tested at a 100 TeV collider up to some TeV.
- This result is easily generalizable to an arbitrary number of top-partners in generic representations.

Thank you