Status and future of NNLO computations

Fabrizio Caola, CERN & IPPP Durham

CERN

IP3

University of Durham

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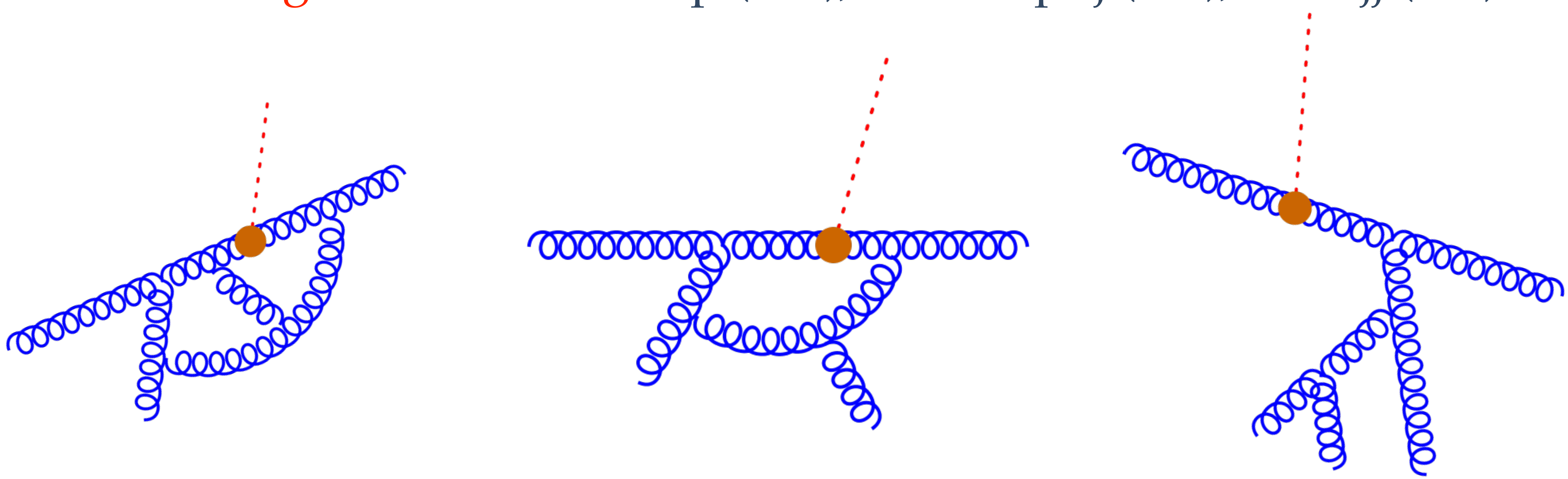
``Few percent accuracy'': NNLO

- $\alpha_s \sim 0.1 \rightarrow$ For TYPICAL PROCESSES, we need NLO for ~ 10% and NNLO for ~ 1% accuracy. Processes with large perturbative corrections (Higgs): N^{3}LO
- less dependence on unphysical variation ($\mu_{R,F}$) → dynamical scales and `art' of scale choice become less of an issue
- in several cases important test of perturbative stability (Higgs, VV…)
- F.O.: possible to accurately reproduce experimental fiducial volume

Different ingredients: two-loop (VV), one-loop+j (RV), tree+jj (RR)



TWO BIG PROBLEMS:

LOOP AMPLITUDES / IR STRUCTURE OF EXTRA EMISSION

Loop amplitudes: status

- Amplitude COMPLEXITY GROWS VERY FAST with the number of scales: invariants (~# legs) and particle masses

$$F^{\rm L}_{--++} = -(x^2+y^2)\Big[4{\rm Li}_4(-x) + \frac{1}{48}Z_+^4 + (\tilde{Y}-3\tilde{X}){\rm Li}_3(-x) + \Xi {\rm Li}_2(-x) + i\frac{\pi}{12}Z_+^3 + i\frac{\pi^3}{2}X - \frac{\pi^2}{12}X^2 - \frac{109}{720}\pi^4\Big] + \frac{1}{2}x(1-3y)\Big[{\rm Li}_3(-x/y) - Z_-{\rm Li}_2(-x/y) - \zeta_3 + \frac{1}{2}Y\tilde{Z}\Big] + \frac{1}{8}\left(14(x-y) - \frac{8}{y} + \frac{9}{y^2}\right)\Xi + \frac{1}{16}(38xy-13)\tilde{Z} - \frac{\pi^2}{12} - \frac{9}{4}\left(\frac{1}{y}+2x\right)\tilde{X} + \frac{1}{4}x^2\Big[Z_-^3 + 3\tilde{Y}\tilde{Z}\Big] + \frac{1}{4} + \Big\{t \leftrightarrow u\Big\},$$

→ gg→γγ [Bern, De Freitas, Dixon [2002]

gg→VV: ~ 10 MB expression

- Despite a lot of recent progress (some inspired by N=4 SYM ideas), still pretty limited knowledge. State of the art:
  - Analytically: 2 -> 2, external masses (pp->VV*) [FC, Henn, Melnikov, Smirnov, Smirnov (2014-15); Gehrmann, Manteuffel, Tancredi (2014-15)]
  - Numerically: 2->2, internal / external masses (pp-> tt, pp->HH) [Czakon; Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016)]

Loop amplitudes: general remarks

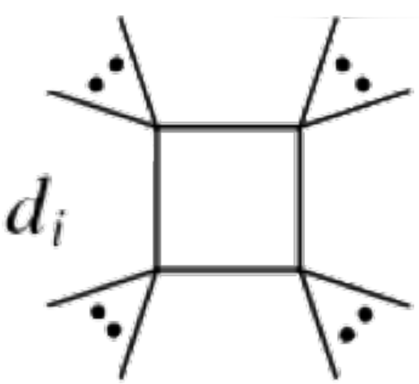
Computation of loop-amplitudes in two steps *(see Nicolas' talk)*:

1. reduce all the integrals of your amplitudes to a minimal set of independent `master' integrals
2. compute the independent integrals

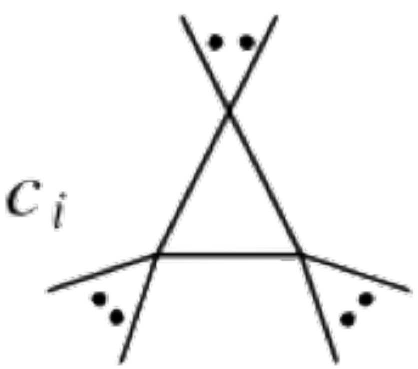
At one-loop:

- independent integrals are always the same (box, tri., bub., tadpoles)
- only (1) is an issue. Very well-understood (tensor reduction, unitarity…)

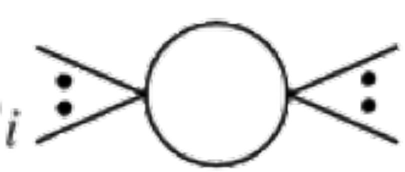
$$A_n^{1-\mathrm{loop}} = \sum_i d_i$$



$$+ \sum_i c_i$$



$$+ \sum_i b_i$$



$$+ R_n + O(\varepsilon)$$

Beyond one-loop: reduction not well understood, MI many and process-dependent (and difficult to compute…)

Two-loop: the integrand

- So far: based on traditional IBP-LI RELATIONS [Tkachov; Chetyrkin and Tkachov (1981); Gehrmann and Remiddi (2000)] / LAPORTA ALGORITHM [Laporta (2000)]

$$\int d^d k F(k;\{p_j\}) = \int d^d(k+\alpha q) F(k+\alpha q;\{p_j\})$$

- Going beyond: significant improvements of tools, NEW IDEAS
- Motivated by the one-loop success, many interesting attempts to generalize unitarity ideas / OPP approach to two-loop case
- A lot of recent progress → *see Ben's talk*
- Towards 2→3 processes: 5/6-gluon all-plus amplitudes at two-loops [Badger, Frellesvig, Zhang (2013); Badger, Mogull, Ochiruv, O'Connell (2015); Badger, Mogull, Peraro (2016)]
- Interesting numerical techniques (e.g. finite field reconstruction [von Manteuffel, Schabinger (2015); Peraro (2016)]) are being studied

Can these techniques be systematized and applied to GENERIC PROCESSES (many legs, massive particles…)?

Two-loop: master integrals

- For a large class of processes (~ phenomenologically relevant scattering amplitudes with massless internal lines) we think we know (at least in principle) how to compute the (very complicated) MI. E.g.: differential equations [Kotikov (1991); Remiddi (1997); Henn (2013); Papadopoulos (2014)]. Recent results for very complicated processes: planar 3-jet [Gehrmann, Henn, Lo Presti (2015)], towards planar Vjj/Hjj [Papadopoulos, Tommasini, Wever (2016)]

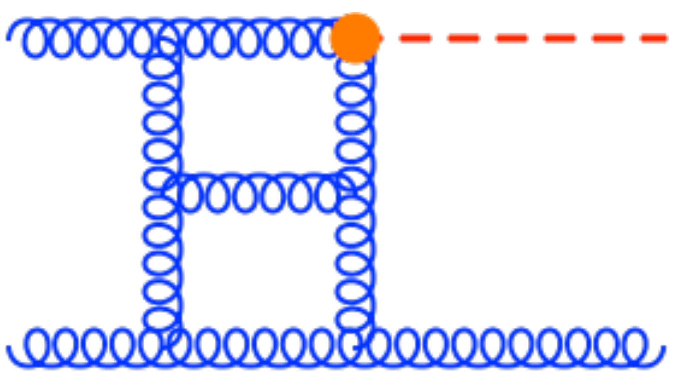
- In these cases, the basis function for the result is very well-known (Goncharov PolyLogs) and several techniques allow to efficiently handle the result (symbol, co-products…) and numerically evaluate it

- Unfortunately, we know that GPL are not the end of the story. Typical example: amplitudes with internal massive particles
- Progress in this cases as well (e.g. [Tancredi and Remiddi (2016); Adams, Bogner, Weinzierl (2015-16)]) but no satisfactory understanding yet. Last year: planar MI for Higgs p_t with exact mass dependence [Bonciani et al (2016)]

Can we find good ways to efficiently evaluate generic MIs (beyond GPLs)?

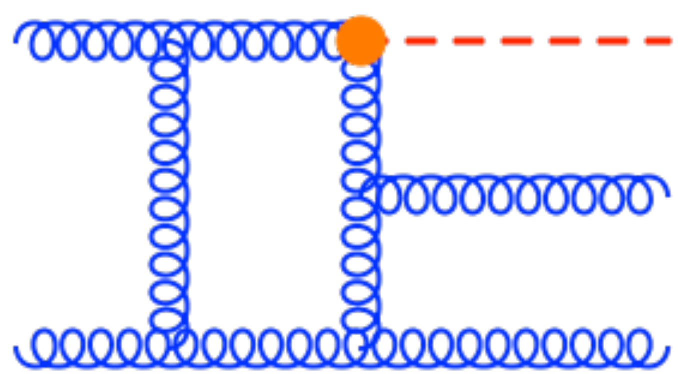
NNLO computations: IR subtraction

VV



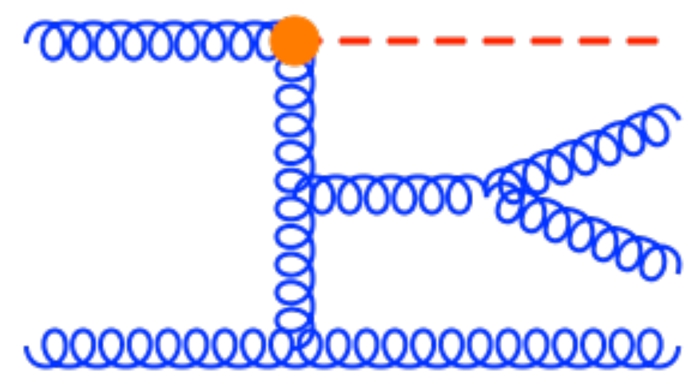
$$\int \left[\frac{\mathrm{vv}_4}{\epsilon^4} + \frac{\mathrm{vv}_3}{\epsilon^3} + \frac{\mathrm{vv}_2}{\epsilon^2} + \frac{\mathrm{vv}_1}{\epsilon} + \mathrm{vv}_0\right] d\phi_2$$

RV



$$\int \left[\frac{\mathrm{rv}_2}{\epsilon^2} + \frac{\mathrm{rv}_1}{\epsilon} + \mathrm{rv}_0\right] d\phi_3$$

RR



$$\int [\mathrm{rr}_0]\, d\phi_4$$

- IR divergences hidden in PS integrations
- After integrations, all singularities are manifest and cancel (KLN)
- We are interested in realistic setup (arbitrary cuts, arbitrary observables) $\rightarrow$ we need fully differential results, we are not allowed to integrate over the PS
- The challenge is to EXTRACT PS-INTEGRATION SINGULARITIES WITHOUT ACTUALLY PERFORMING THE PS-INTEGRATION

The solution: two philosophies

Same problem at NLO. Two different approaches have been developed

Phase space slicing

$$\int |M|^2 F_J d\phi_d = \int_0^\delta \left[|M|^2 F_J d\phi_d\right]_{s.c.} + \int_\delta^1 |M|^2 F_J \phi_4 + \mathcal{O}(\delta)$$

- conceptually simple, straightforward implementation
- must be very careful with residual δ dependence (esp. in diff. distr.)
- highly non-local → severe numerical cancellations

Subtraction

$$\int |M|^2 F_J d\phi_d = \int (|M|^2 F_J - \mathcal{S}) d\phi_4 + \int \mathcal{S} d\phi_d$$

- in principle can be made fully local → less severe numerical problems
- requires the knowledge of subtraction terms, and their integration

The solution: two philosophies

Both methods have proven useful for 2→ 2 computations

Phase space slicing

$$\int |M|^2 F_J d\phi_d = \int_0^\delta \left[|M|^2 F_J d\phi_d\right]_{s.c.} + \int_\delta^1 |M|^2 F_J \phi_4 + \mathcal{O}(\delta)$$

- q_t subtraction [Catani, Grazzini] → H, V, VH, VV, HH → *see Marius' talk*
- N-jettiness [Boughezal et al; Gaunt et al] → H, V, γγ, VH, Vj, Hj, single-top

Subtraction

$$\int |M|^2 F_J d\phi_d = \int (|M|^2 F_J - \mathcal{S}) d\phi_4 + \int \mathcal{S} d\phi_d$$

- antenna [Gehrmann-de Ridder, Gehrmann, Glover] → jj, Hj, Vj
- Sector-decomposition+FKS [Czakon; Boughezal, Melnikov, Petriello; Czakon, Heymes; FC, Melnikov, Röntsch] → ttbar, single-top, Hj
- P2B [Cacciari, Dreyer, Karlberg, Salam, Zanderighi] → VBF_H, single-top
- *Colorful NNLO [Del Duca, Somogyi, Tocsanyi, Duhr, Kardos]: only e^+e^- so far*

The solution: two philosophies

Some of these techniques are quite generic

IN PRINCIPLE, they allow for ARBITRARY COMPUTATIONS

IN PRACTICE: \`genuine' 2→2 REACTIONS, with big computer farms

TYPICAL RUNTIME: 100.000 CPU hours (typical setup)

Slicing: a closer look

Due to its highly non-local character, slicing leads to large numerical cancellations → abandoned at NLO

Why can we use it at NNLO?

- huge increase in computing power
- significant progress in NLO computations (speed / stability) → the CPU-intensive '+J' part is highly optimized for free (fully inherited by NLO)
- NNLO corrections smaller than NLO ones: can allow for larger uncertainty on them, without affecting the final result → δ_{cut} can be chosen not too prohibitively small (although careful if extreme precision is required, see m_W determinations)
- So far, relatively `simple' kinematics configurations tested. It would be interesting to stress-test slicing on e.g. 2→3 (impossible right now) or with intricate IR configurations (di-jet)
- Interesting theoretical development: towards leading power corrections in δ (would allow for larger δ_{cut}). Non trivial for generic processes

Subtraction: a closer look

Very different approaches, each with its own merits/problems

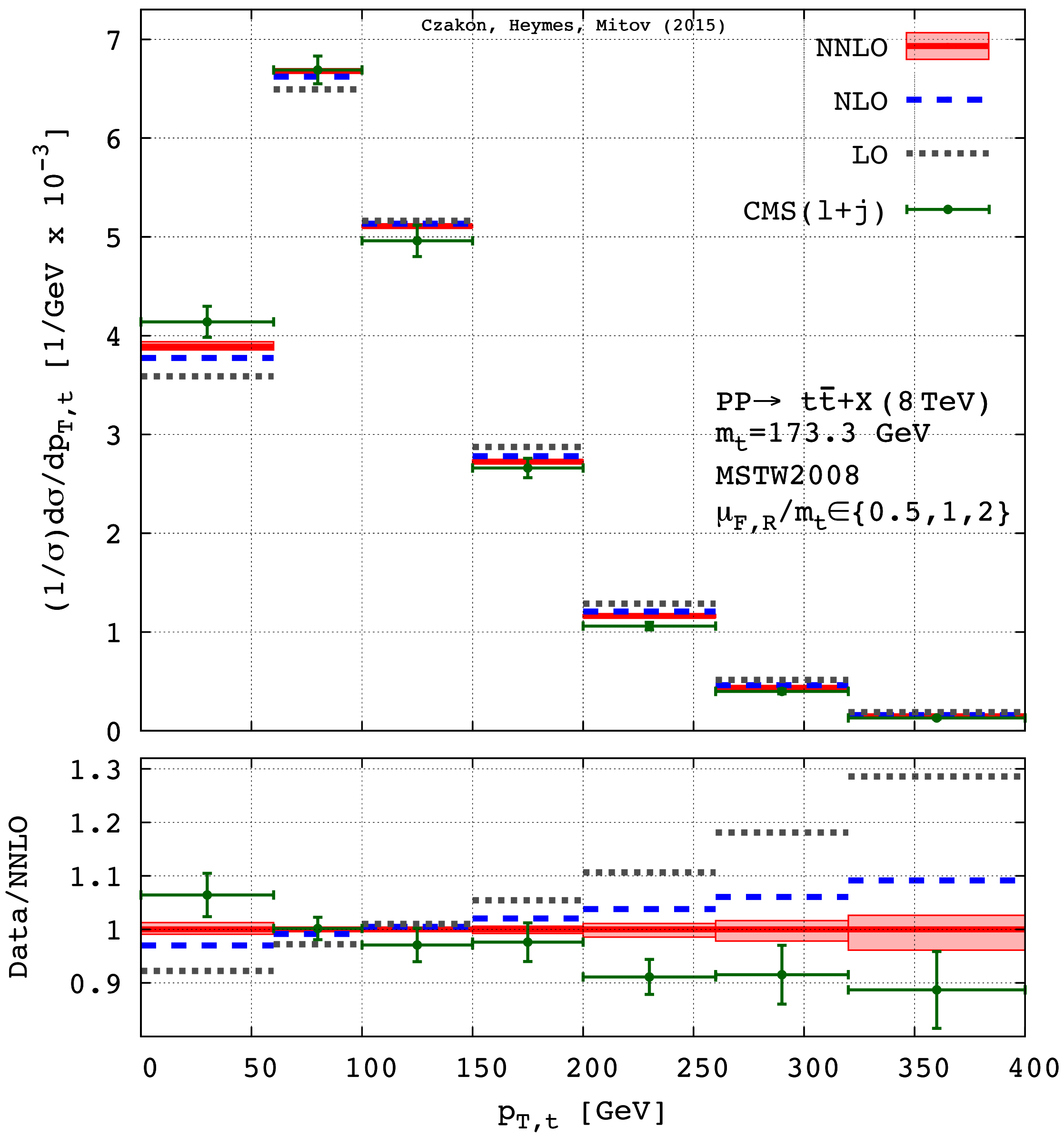
- antenna: almost fully local subtraction, fully analytic. Entirely worked out only for massless processes (technical problems, difficult integrated subtractions)
- sector-decomposition+FKS: fully local, numerical integration of integrated subtractions. As a consequence, massive processes are not a problem
- projection to Born: local, very nice trick to get integrated subtraction for free, but requires prior knowledge of $d\sigma^{NNLO}/d\Phi^{Born} \rightarrow$ limited applicability, small room for checks

Many results, but still in `proof-of-concept' phase

- an obviously optimal framework has not appeared yet
- despite flood of results, (a lot of) theoretical work still needed
- all the `latest technologies' in NLO not present here
- large room for improvement

Recent NNLO results: top

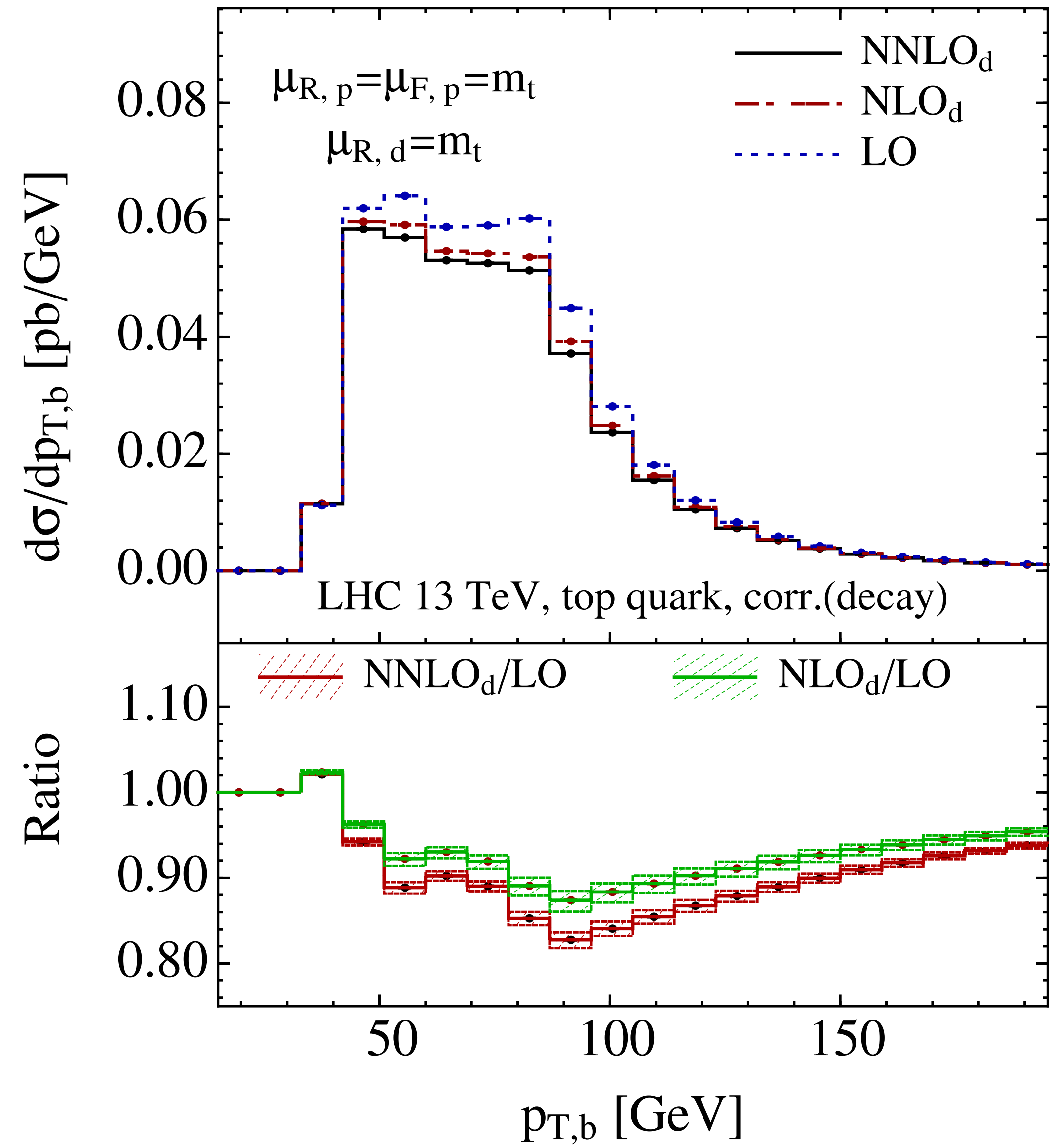
TTBAR DIFFERENTIAL DISTRIBUTIONS



[Czakon, Heymes, Mitov (2016)]

→ *see Davide's talk*

T-CHANNEL SINGLE-TOP PLUS TOP-DECAY (NWA)

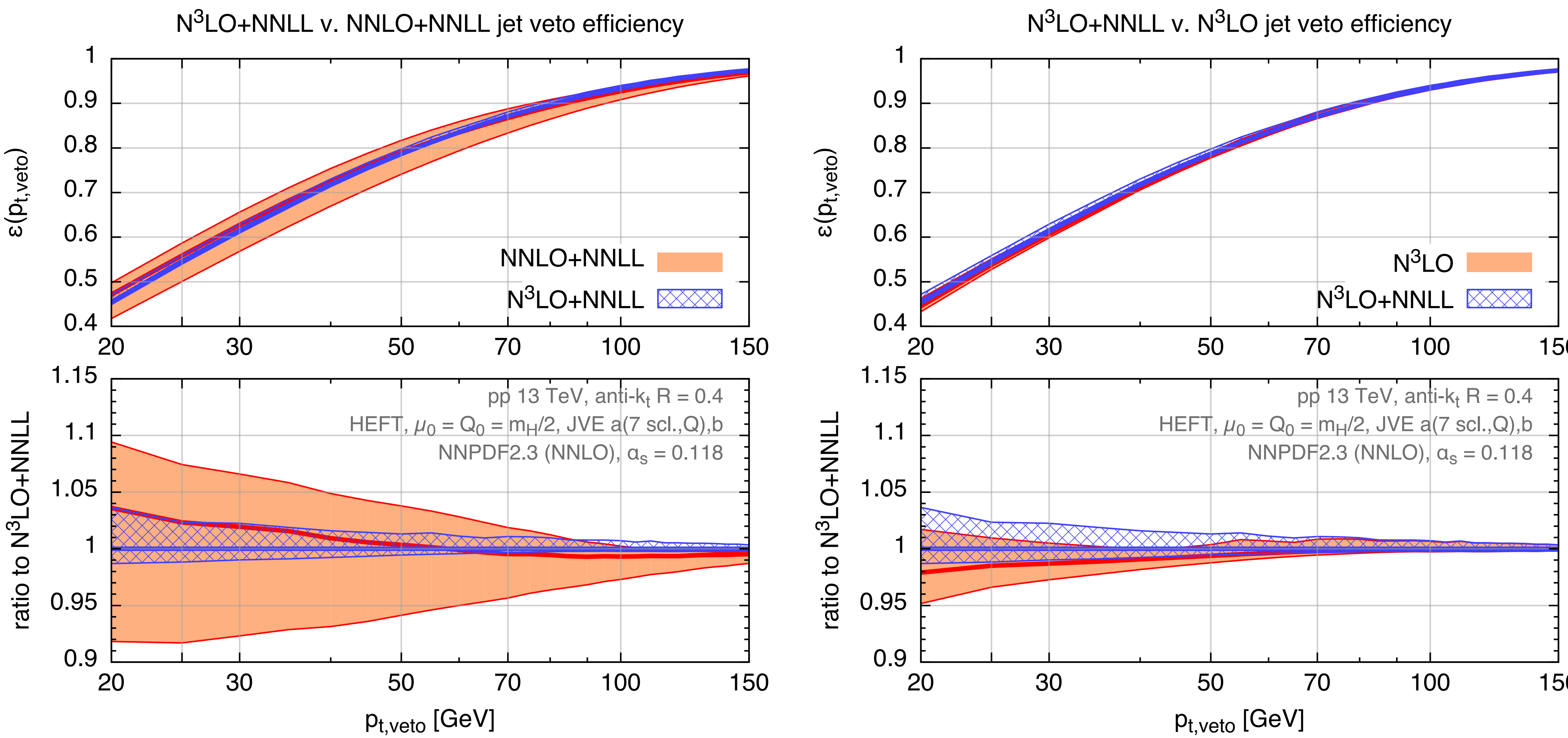


[Berger, Gao, Yuan, Zhu (2016)]

- Small inclusive corrections
- LARGE CORRECTIONS in exclusive region
- Similar behavior observed in Higgs in VBF [Cacciari et al (2015)]

Application of f.o. results: H and jet vetoes

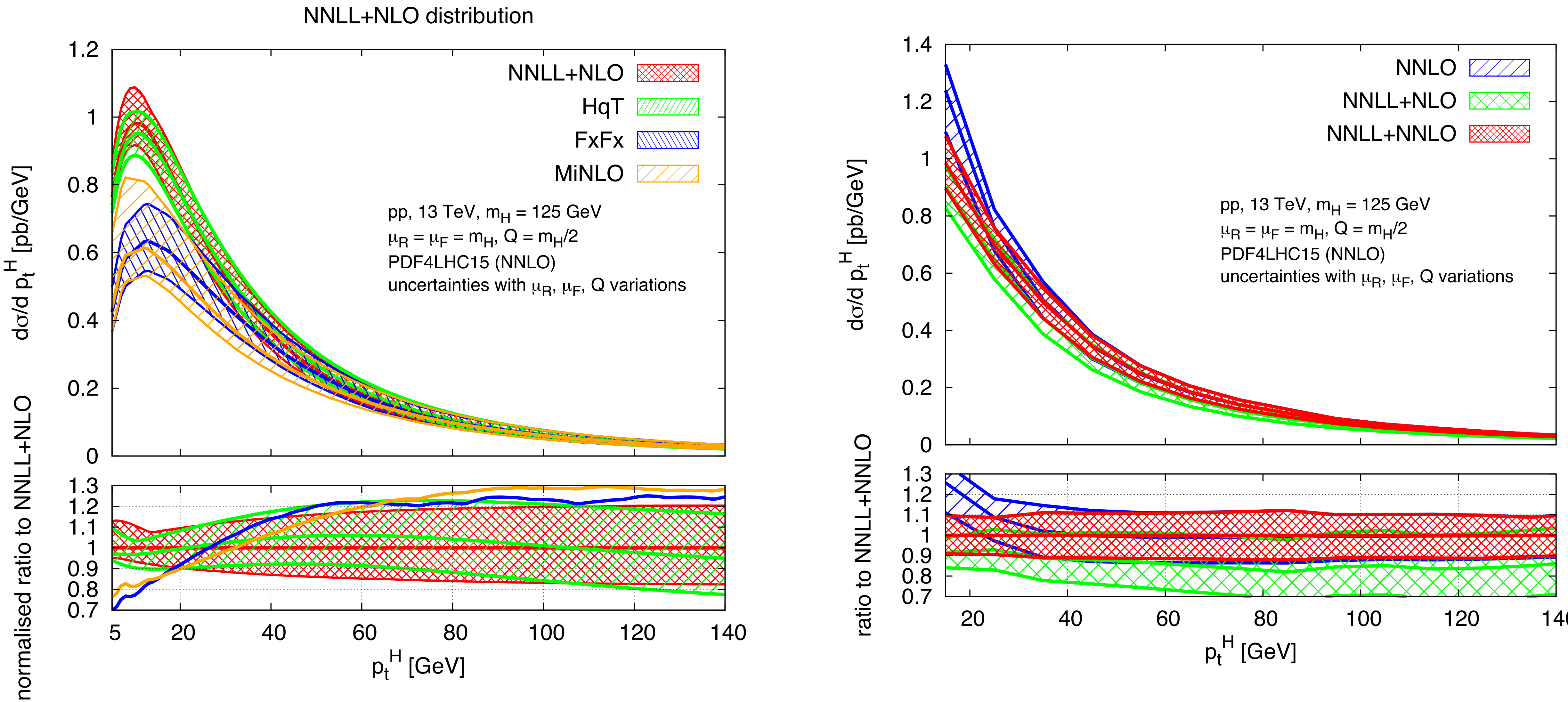
[Banfi, FC, Dreyer, Monni, Salam, Zanderighi, Dulat (2015)]



- Combination of f.o. N^3LO (Higgs inclusive) and NNLO (H+J exclusive) with NNLL resummation, LL_R resummation, mass effects…
- No breakdown of fixed (high) order till very low scales
- Even more so for Z+jet [Gerhmann-De Ridder et al (2016)]

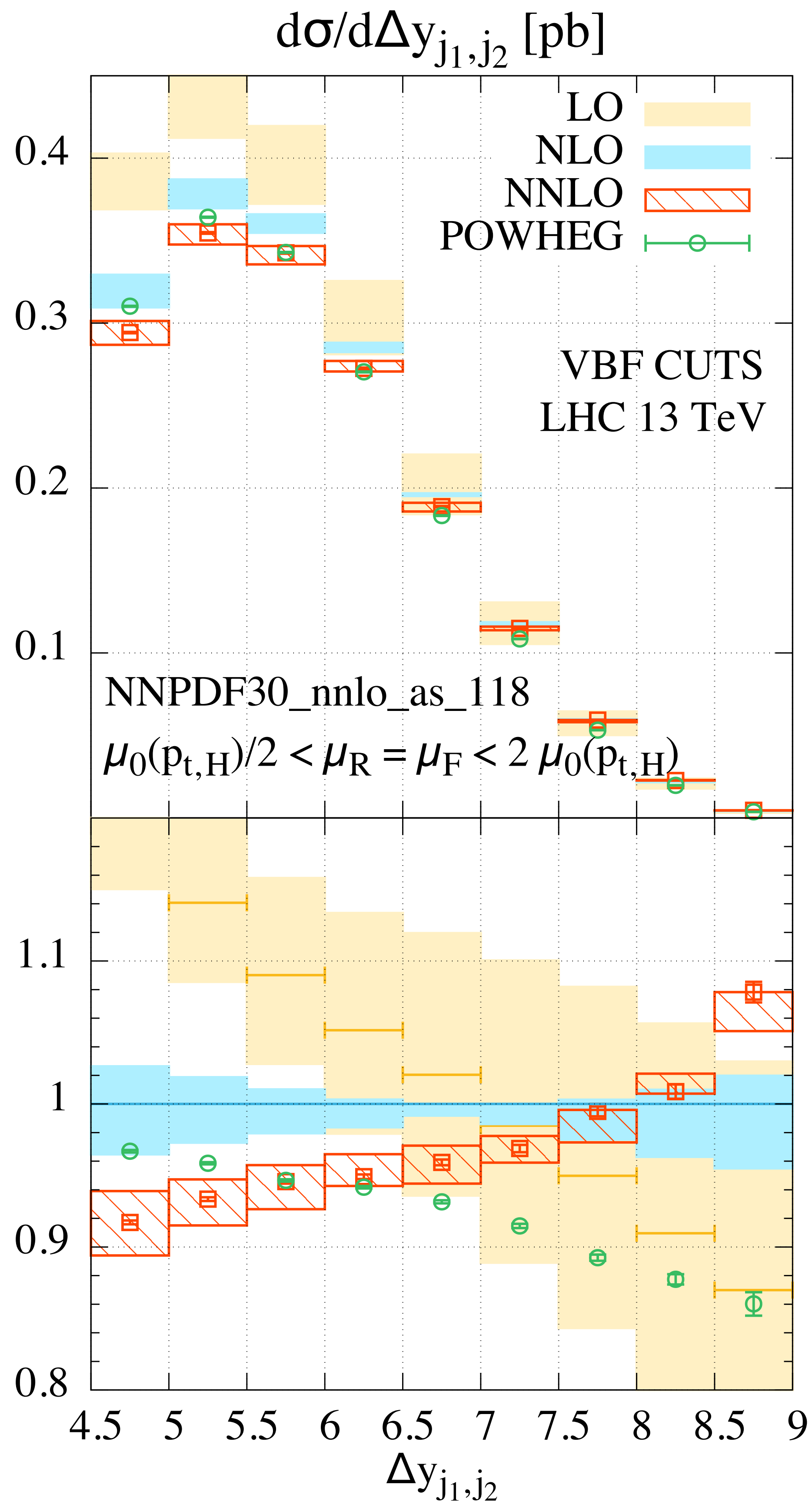
Application of NNLO results: H p_T

[Monni, Re, Torrielli (2016)]



- Matching of NNLO H+J with NNLL Higgs p_T resummation
- Significant reduction of perturbative uncertainties
- Again, no breakdown of perturbation theory (resummation effects: 25% at p_T = 15 GeV, ~0% at p_T = 40 GeV)

Inclusive vs exclusive: VBF@NNLO



[Cacciari et al (2015)]

VBF

- NNLO inclusive K-factor ~ 1%
- Actual correction ~ 10%
- Not captured by NLO or PS
- Non-trivial jet dynamics, modeled at NLO

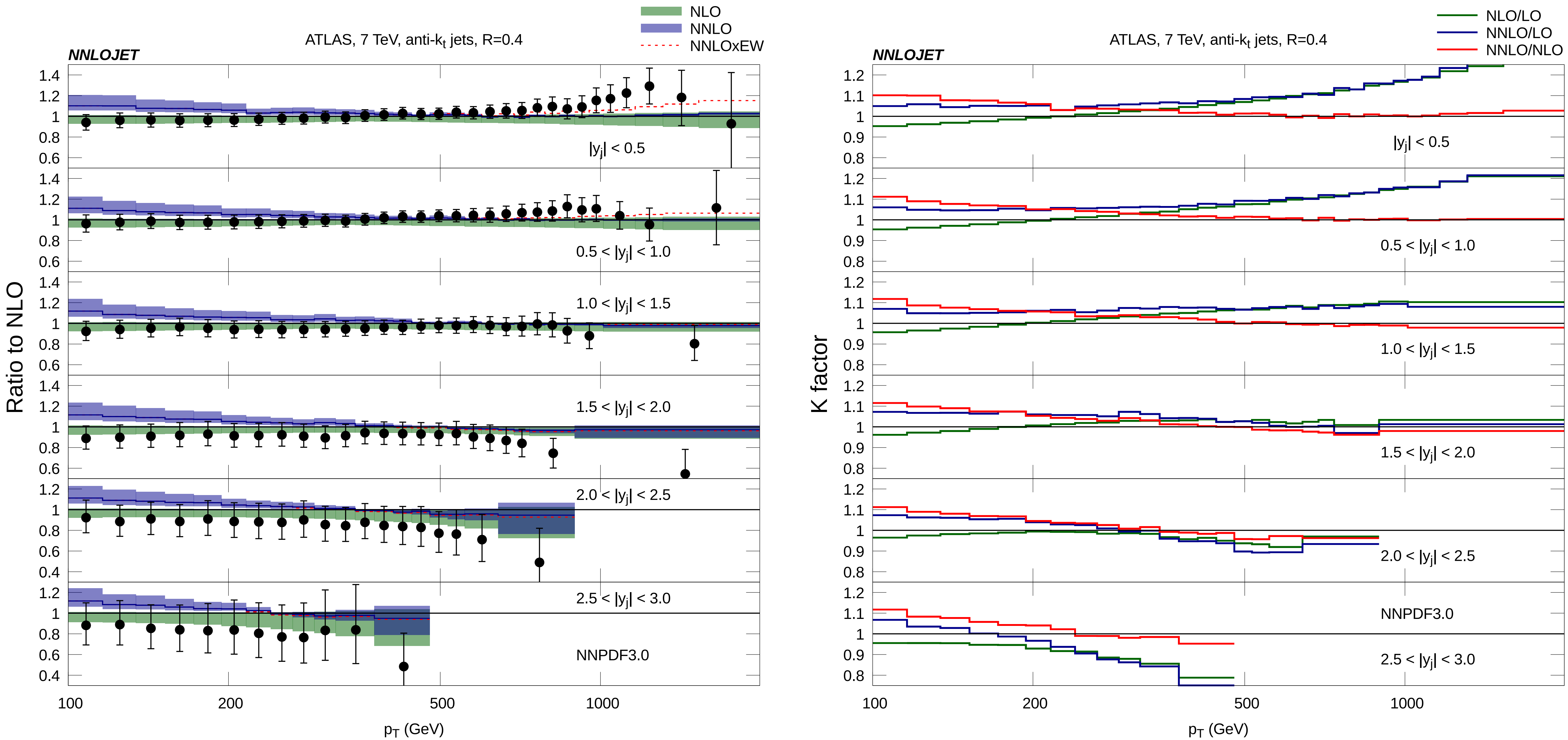
Similar considerations for pp→ VV (jet vetoes..) → *see Marius' talk*

Proper fiducial comparison very important, especially for processes involving jet or sophisticated event definition / reconstruction

Recent NNLO results: dijet

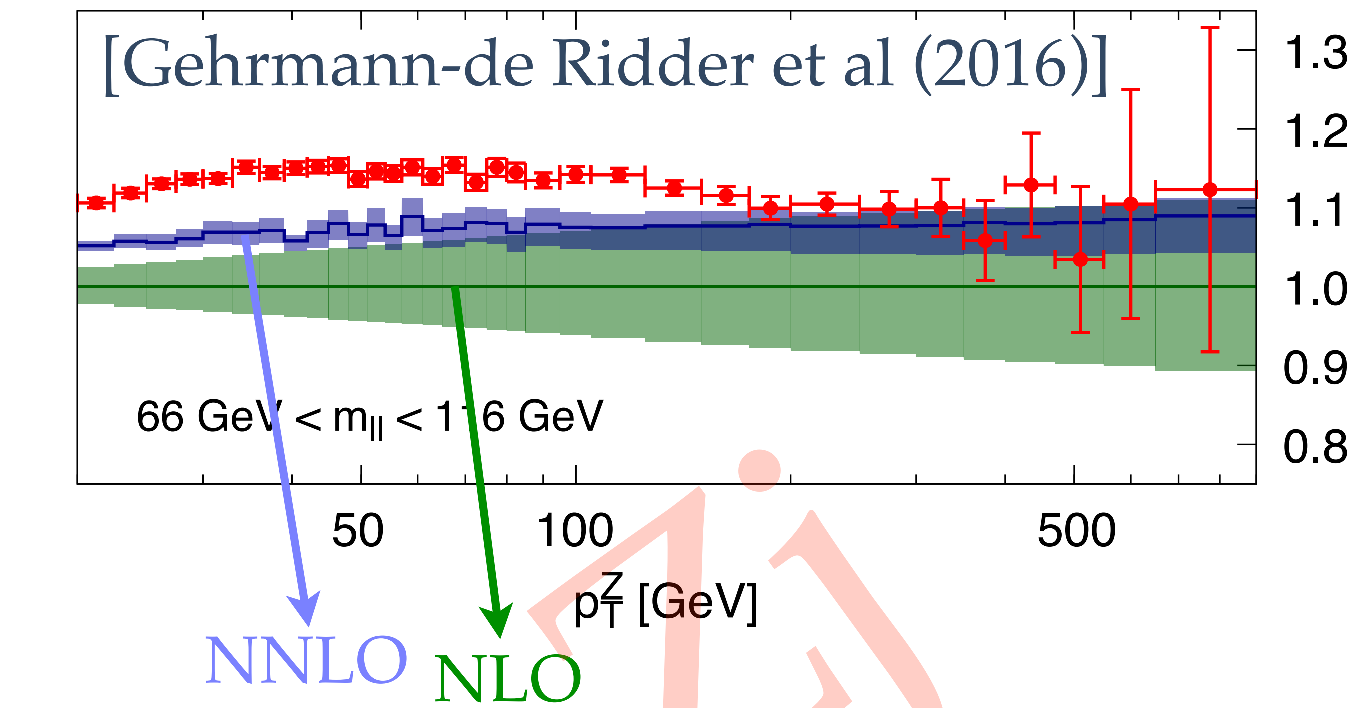
[Currie, Glover, Pires (2016)]

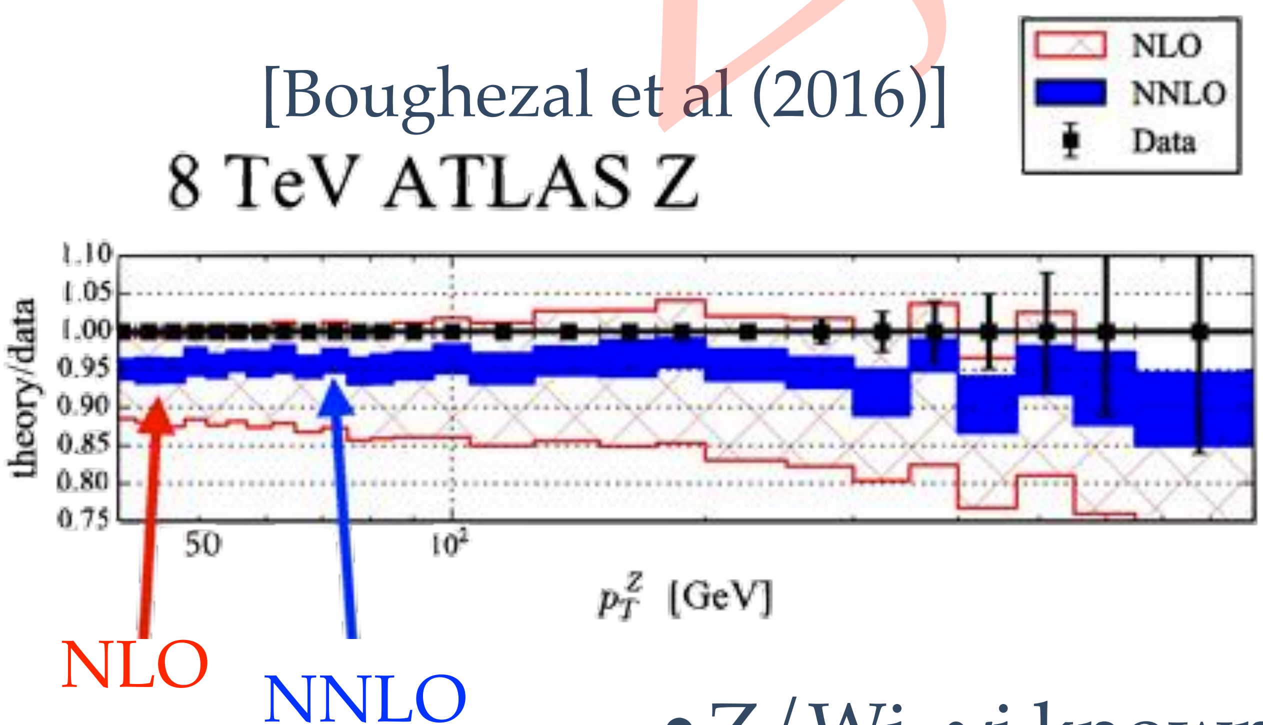
~40 partonic channels, highly non-trivial color flow. Realistic jet

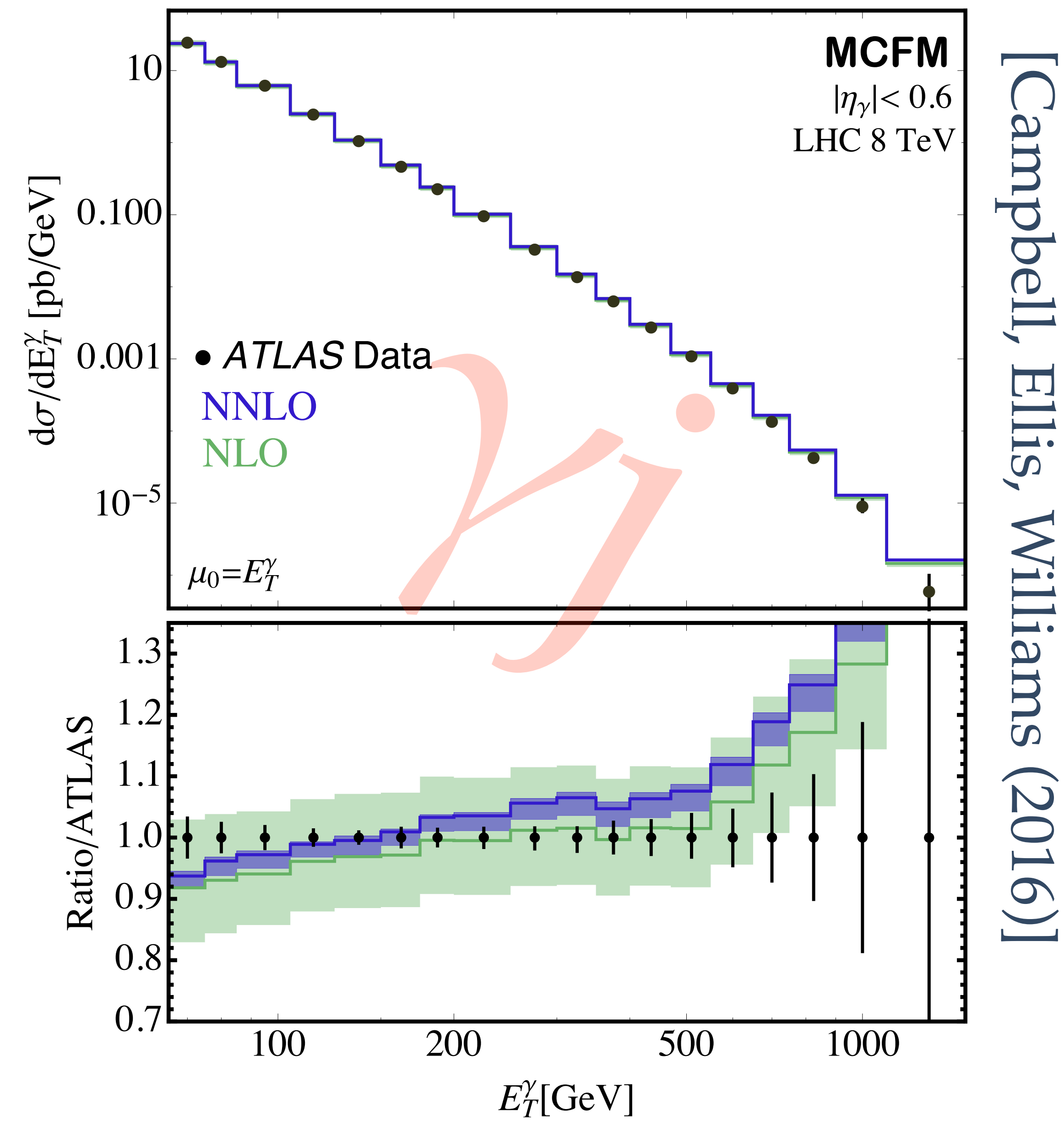


- Non trivial shape correction (scale choice?), sizable effect (jet dynamics?)
- Large effect on PDF? (see also jj in DIS [Niehues, Currie, Gehrmann (2016)])

Recent NNLO results: VJ







- Z/Wj, γj known. Zj: independent computations
- Highly improved theoretical accuracy (~exp error)
- Small deviations evident (PDFs? NP? Isolation?)

NNLO: status and future

- A lot of theoretical progress in the recent past
- This lead to realistic $2\to 2$ *PHENOMENOLOGY AT NNLO*
- Many interesting features
  - Greatly reduced th. uncertainties (expected)
  - Stability w.r.t. logarithmic corrections (not so obvious) → fiducial region
- And a few surprises
  - Non trivial jet dynamics (larger than naively expected corrections)
  - Curious data/theory discrepancies (PDFs? NP?)
- A lot more to explore
  - More pheno: e.g. jet dynamics @ NNLO vs mergedPS, $\text{NNLO}_{\text{prod}\otimes\text{decay}}$…
  - $2\to 2$ in ``extreme'' kinematics (boosted/off-shell H+j and $pp\to VV$)
  - better understanding of jet dynamics: $pp \to 3j$. Also: α_s, maybe some extra handle to understand NP effects?
  - Important backgrounds / precision tests: Hjj (VBF contamination, jet-bin correlations…), Vjj, ttj

NNLO: status and future

- Almost all these direction would require significant theoretical progress. For example
  - extreme kinematics: quark-mass effects in the loops, mixed QCD-EW → new ways of computing / evaluating MI?
- Breaking the 2 → 2 barrier highly non trivial
  - 2-loop: better integrands methods, efficient evaluation of MI
  - 1-loop: stable / fast 2→4 loop amplitudes in the soft / collinear region
  - more efficient IR subtraction (2→1 ~ 100 CPU hours, 2→2 ~ 100.000 CPU hours). *E.g.: NLP soft/collinear terms in slicing, different slicing variables, ``import" NLO technology in subtraction schemes…*
  - even if the goal is ≠ from NLO, at least some degree of automation
- Beyond NNLO?
  - N^3LO beyond the Higgs and ``simple" processes?
  - NNLO and beyond: $\lesssim 1\% \sim \Lambda_{QCD}/100$ GeV. NP effects (already now for m_t, m_W, $p_{t,Z}$?)

EXCITING TIMES AHEAD!

Thank you
very much for
your attention!

``Few percent'': the theory side

$$\mathrm{d}\sigma = \int \mathrm{d}x_1 \mathrm{d}x_2 f(x_1) f(x_2) \mathrm{d}\sigma_{\mathrm{part}}(x_1, x_2) F_J (1 + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q))$$

Input parameters: ~few percent.
In principle improvable

NP effects: ~ few percent
No good control/understanding of them at this level. LIMITING FACTOR FOR FUTURE DEVELOPMENT

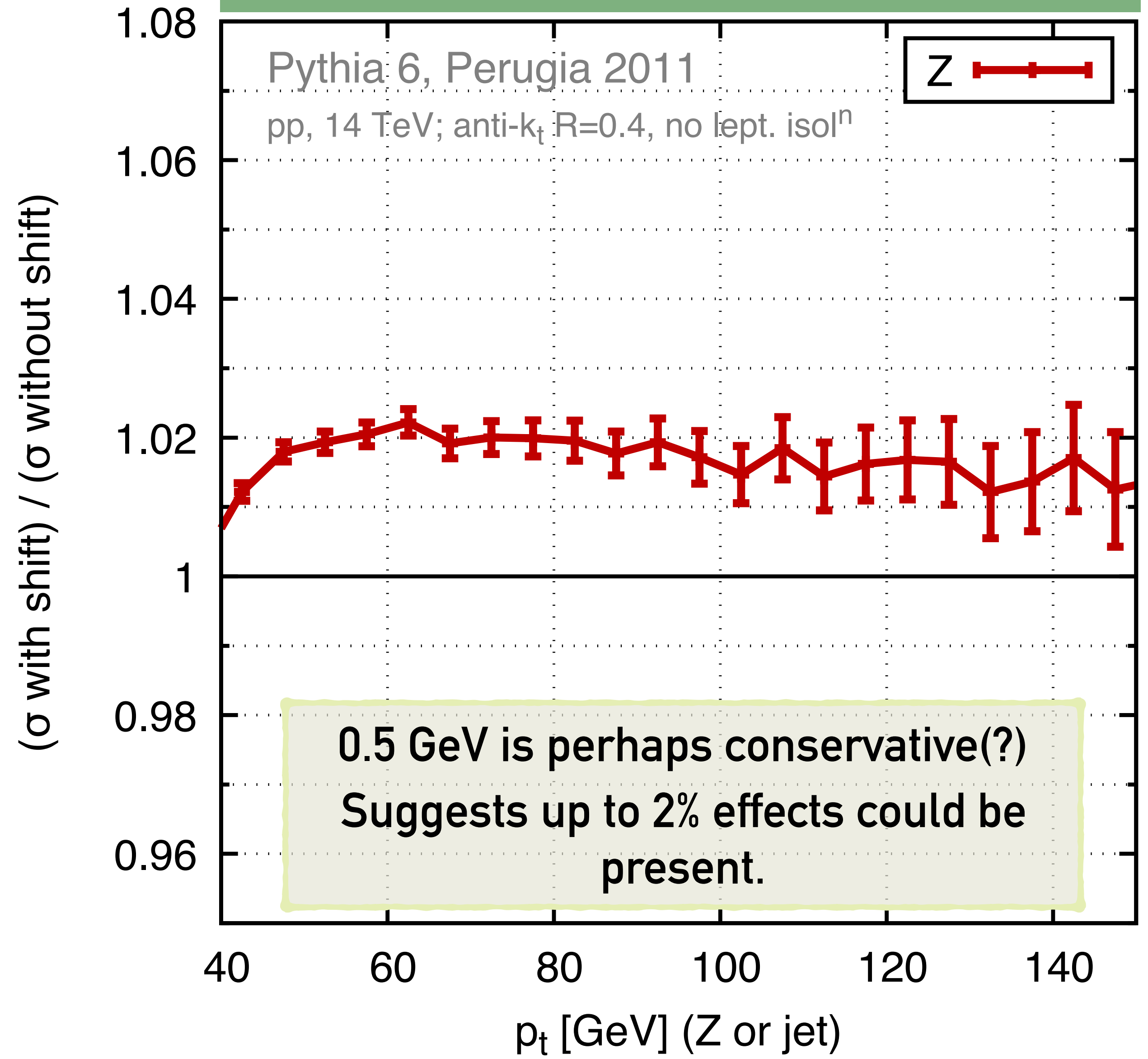
HARD SCATTERING MATRIX ELEMENT

- $\alpha_s \sim 0.1 \rightarrow$ For TYPICAL PROCESSES, we need NLO for ~ 10% and NNLO for ~ 1 % accuracy. Processes with large perturbative corrections (Higgs): N^3LO
- Going beyond that is neither particularly useful (exp. precision) NOR POSSIBLE GIVEN OUR CURRENT UNDERSTANDING OF QCD, even if we knew how to compute multi-loop amplitudes and had N^KLO subtraction schemes (NP effects)

Non-perturbative effects in Z p_T

- Inclusive Z cross section should have $\sim\Lambda^2/M^2$ corrections ($\sim 10^{-4}$?)
- Z p_T is **not inclusive** so corrections can be $\sim\Lambda/M$.
- Size of effect can't be probed by turning MC hadronisation on/off
  [maybe by modifying underlying MC parameters?]
- Shifting Z p_T by a finite amount illustrates what could happen

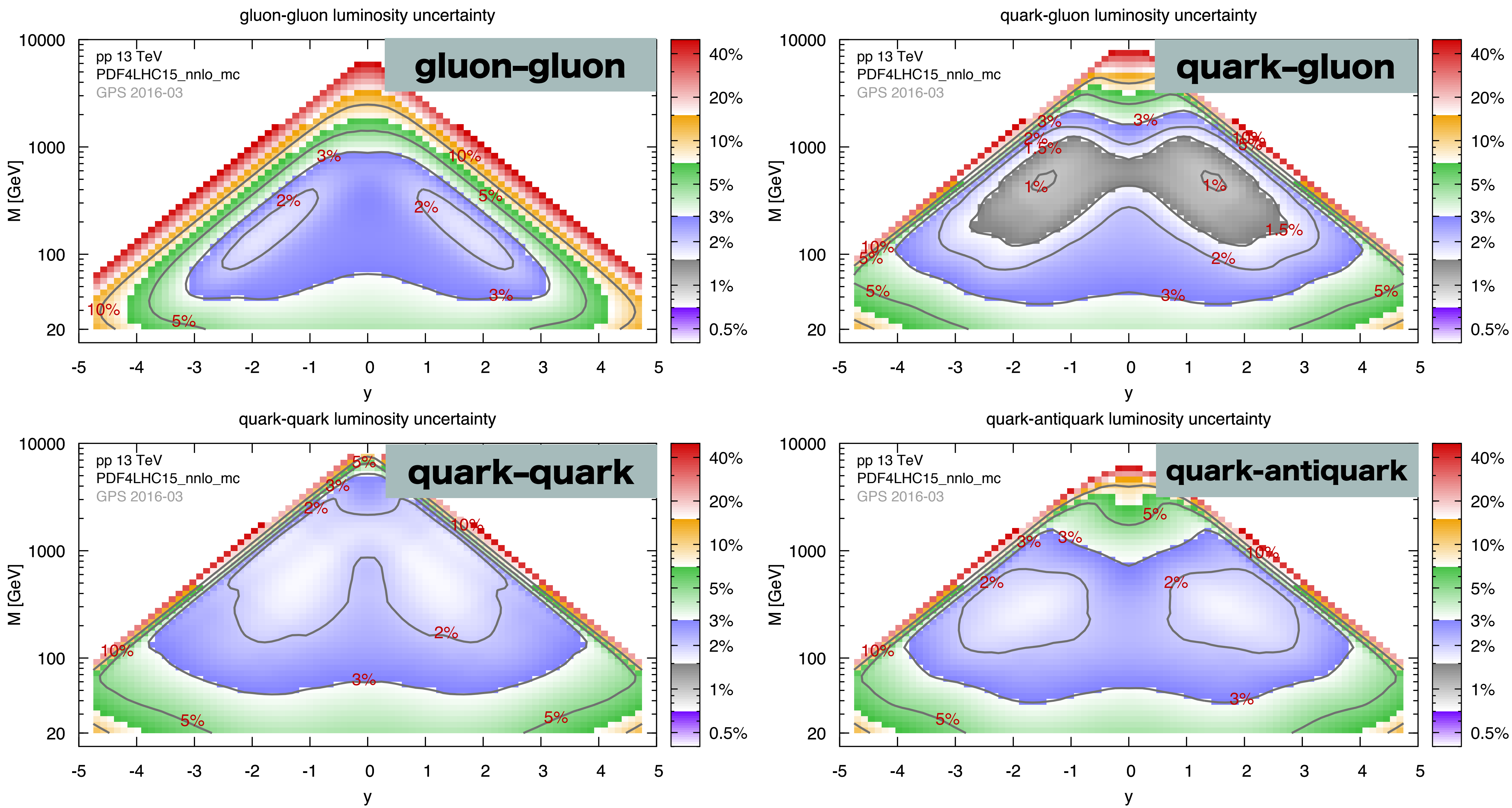
impact of 0.5 GeV shift of Z p_T



A conceptually similar problem is present for the W momentum in top decays

[G. Salam, ``Future challenges for precision QCD'']

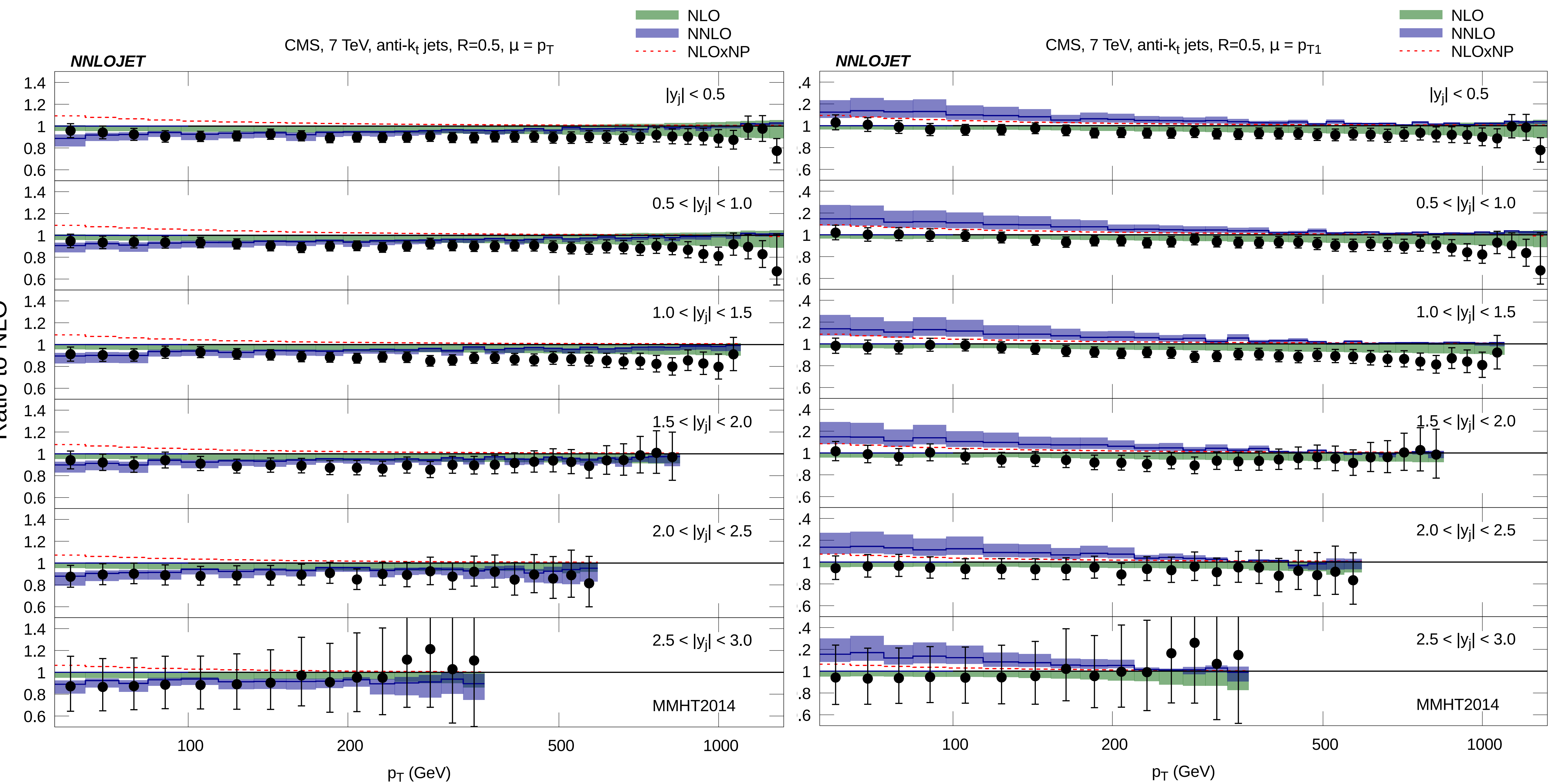
Parton distribution functions circa 2016



[plots by G. Salam]

- Big improvement w.r.t. few years ago [better handling on fit, larger data coverage (LHC)]. Reasonable consensus among different groups
- For Central EW Production: 2/3% precision
- *Going below may require some rethinking of PDF uncertainty*

dijet: p_t vs $p_{t,1}$



[J. Currie, CMS workshop Jan. 2017]