Recent news from the four-dimensional universe

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Outline

1. Intro on
   Four Dimensional Regularization / Renormalization
2. Recent news
3. Conclusions

Comparing two renormalization procedures

- "Canonical" renormalization:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_0 + \alpha \Delta \mathcal{L}^{(1)} + \alpha^2 \Delta \mathcal{L}^{(2)} + \cdots \\ &= \mathcal{L}_0 + \underbrace{\sum_{\ell=1}^{N} \alpha^\ell \Delta \mathcal{L}^{(\ell)}}_{\text{CTs up to the } N^{th} \text{ perturbative order}} + \mathcal{O}(\alpha^{N+1})\end{aligned}$$

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- FDR renormalization:

$$\mathcal{L} = \mathcal{L} \quad \text{in 4 dimensions}$$ **at all orders!**

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- FDR renormalization:

$$\mathcal{L} = \mathcal{L} \quad \text{in 4 dimensions} \quad \textbf{at all orders!}$$

FDR is conceptually simpler

(e.g. different diagrams can be computed by different students...)

What should be kept when renormalizing?

- $$\int_R d^4q_1 \cdots d^4q_\ell \frac{\not{D}_i}{D_0 \cdots \not{D}_i \cdots D_k} \stackrel{?}{=} \int_R d^4q_1 \cdots d^4q_\ell \frac{1}{D_0 \cdots D_k}$$

  $\Uparrow$

  Some UV regulator

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- Is this enough?

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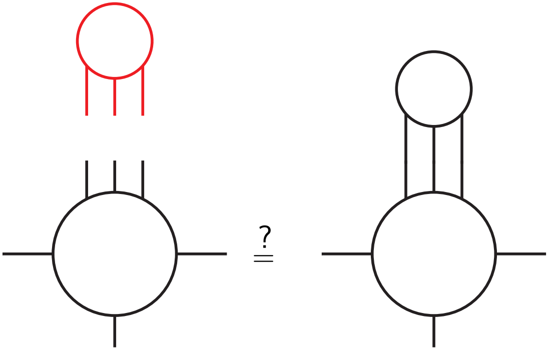
- I dub this **NUMDEN** cancellation, which is essential to ensure gauge cancellations $\Rightarrow$ **Gauge Invariance**
- Is this enough?

NO!

What should be kept when renormalizing?



- I dub this **SUBINTEGRATION** consistency, which is essential to ensure **Unitarity**

Dimensional regularization (DReg)

- DReg keeps **NUMDEN** cancellations and introduces the counterterms

$$\sum_{\ell=1}^{N} \alpha^{\ell} \Delta \mathcal{L}^{(\ell)}$$

to preserve **SUBINTEGRATION** consistency

(It is not enough to drop $\frac{1}{\epsilon^{\ell}}$ poles in the loop integrals to define a decent renormalization scheme beyond 1-loop!)

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- It is possible to simplify the whole picture by

encoding the UV subtraction directly in the definition of loop integration $\Rightarrow$ FDR

FDR

- Let a UV divergent integrand be $J(q) = \dfrac{1}{q^2(q+p)^2}$

$$\int [d^4q]\, J(q) \;\; \equiv \;\; \lim_{\mu \to 0} \int_R \left(J(q) - \frac{1}{\bar{q}^4} \right) \equiv \int [d^4q]\, \frac{1}{\bar{q}^2 \bar{D}_p}$$

$\Uparrow$ Subtraction term

$\Uparrow$ FDR integral

$$\begin{aligned} \bar{q}^2 &\equiv q^2 - \mu^2 \\ \bar{D}_p &\equiv (q+p)^2 - \mu^2 \end{aligned}$$

Extra Integrals

- Does *"naive"* **NUMDEN** cancellation work?

$$\int [d^4q] \frac{q^2}{\bar{q}^4 \bar{D}_p} \stackrel{?}{=} \int [d^4q] \frac{\not{q}^2}{\not{q}^2 \bar{q}^2 \bar{D}_p} = \int [d^4q] \frac{1}{\bar{q}^2 \bar{D}_p}$$

$\Uparrow$ subtracts $q^2/\bar{q}^6$ $\Uparrow$ subtracts $1/\bar{q}^4$

- **NO**, because subtraction terms contribute despite $\mu \to 0$:

$$\left(\int [d^4q] \frac{\mu^2}{\bar{q}^4 \bar{D}_p} \equiv \right) \int_R d^4q \left(\frac{1}{\bar{q}^4} - \frac{q^2}{\bar{q}^6} \right) = -\mu^2 \int d^4q \frac{1}{\bar{q}^6} = \frac{i\pi^2}{2}$$

- The correct **NUMDEN** cancellation is:

$$\int [d^4q] \frac{\bar{q}^2 + \mu^2}{\bar{q}^4 \bar{D}_p} = \int [d^4q] \frac{\not{\bar{q}}^2}{\not{\bar{q}}^2 \bar{q}^2 \bar{D}_p} + \underbrace{\int [d^4q] \frac{\mu^2}{\bar{q}^4 \bar{D}_p}}_{\text{Extra Integral!}}$$

Global prescription (GP)

- To keep gauge cancellations

$$\boxed{q^2 \to \bar{q}^2 \text{ in denominators} \quad \Leftrightarrow \quad q^2 \to \bar{q}^2 \text{ in numerators}}$$

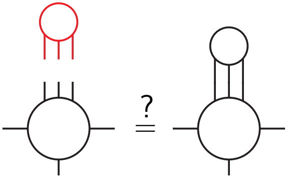
when q^2 originates from Feynman rules (not from reduction!)

- Apart from that

algebraic manipulations on FDR integrals are legal

such as tensor decomposition and IBP to reduce them to MI

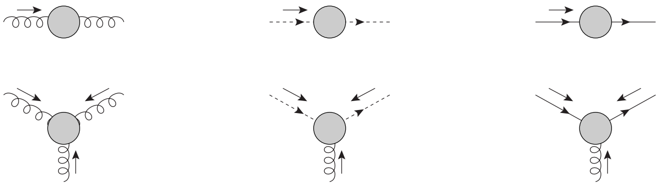
Extra Extra Integrals (EEIs)



- One needs **GP** at the level of the subamplitude  (Sub-Prescription, **SP**) and also **GP** at the level of the full amplitude on the right (**full GP**)
- **SP** and **full GP** clash with each other
- It is possible to correct for this mismatch and ensure **SUBINTEGRATION** consistency by adding **EEIs** derived by solely analyzing the loop diagrams on the right

Recent News

- FDR has been proven to renormalize consistently **off-shell** QCD up to 2 loops (B. Page, R.P., JHEP 1511 (2015) 183)



- The analysis of the **EEIs** led to a fix of 2-loop *"naive"* FDH:

$$\boxed{G^{(2-\text{loop})}_{\text{bare, DReg}}\Big|_{n_s=4} \to G^{(2-\text{loop})}_{\text{bare, DReg}}\Big|_{n_s=4} + \sum\nolimits_{\text{Diag}} EEI_b\Big|_{n_s=4}}$$

where $n_s = \gamma_\mu \gamma^\mu = g_{\mu\nu} g^{\mu\nu}$

$\mathbf{EEI_b s}$

- The **$\mathbf{EEI_b}$s** are obtained from the **EEIs** by using $R=$ **DReg** and *"dropping"* the subtraction term, e.g.:

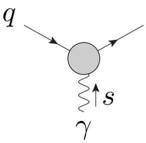
$$
\begin{aligned}
EEI &= \text{Const} \int [d^4 q] \frac{1}{\bar{q}^2 \bar{D}_p} = EEI_b + EEI_a \\
&= \text{Const} \left(\int d^n q \frac{1}{q^2 D_p} - \lim_{\mu \to 0} \int d^n q \frac{1}{\bar{q}^4} \right)
\end{aligned}
$$

- Note that **$\mathbf{EEI_b}$** contains logs which make it impossible to reabsorbe it in renormalization constants

- **$\mathbf{EEI_b}$***s* reproduce the effect of the FDH/DRed evanescent operators in *"canonical"* renormalization, at least **off-shell**

On-shell QCD @2-loops

- Ben and I started considering the 2-loop IR divergent vertex



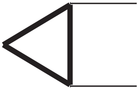
$$= \quad V^{(2)}$$

$$EEI_b(V^{(2)}) = \frac{\alpha_s^2}{16\pi^2} C_F \left(\frac{2N_c + n_f}{3} + \frac{1}{N_c} \right) \left(\frac{1}{\epsilon} + 1 + \ln \frac{\mu_{\mathrm{R}}^2}{-s - i\varepsilon} \right) V^{(0)}$$

- The *"dropping"* rule seems to survive IR divergences
- Work is ongoing with A. Signer and collaborators to establish the precise relation between $\mathbf{EEI_b}s$ and evanescent couplings
- This also paves the way for **mixed** FDR/DReg calculations

FDR treatment of IR infinities

- Adding μ^2 to propagators regulates **virtual** IR divergences



$$= \int [d^4q] \frac{1}{\bar{q}^2 \bar{D}_1 \bar{D}_2} \equiv \lim_{\mu \to 0} \int d^4q \frac{1}{\bar{q}^2 \bar{D}_1 \bar{D}_2}$$

giving rise to logs of μ

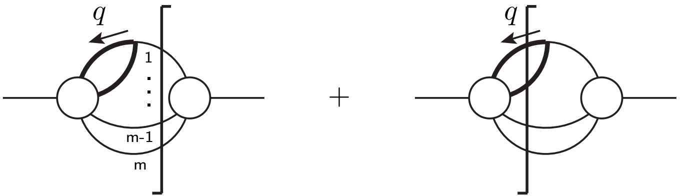
- **Real** matched via *cutting rules* $\boxed{\frac{i}{\bar{q}^2 + i\varepsilon} \to (2\pi)\, \delta_+(\bar{q}^2)}$ e.g.

$$\int_{\Phi_2} \Re \left(\int [d^4q] \frac{1}{\bar{q}^2 \bar{D}_1 \bar{D}_2} \right) = \int_{\bar{\Phi}_3} \frac{1}{\bar{s}_{13} \bar{s}_{23}} \qquad \begin{cases} \bar{s}_{ij} = (\bar{p}_i + \bar{p}_j)^2 \\ \bar{p}_{i,j}^2 = \mu^2 \end{cases}$$

- Logs of μ can be rewritten as counterterms integrated over a μ-**massive** phase-space $\bar{\Phi}_3$

Unobserved particles

- m-body virtual and $(m+1)$-body real IR divergences compensate each other



- In both cases the divergent splitting is regulated by **μ-massive unobserved particles:**

DReg vs FDR @NLO

- A one-to-one correspondence exists between DReg and FDR

$$\Gamma(1-\epsilon)\,\pi^{\epsilon}\int \frac{d^nq}{\mu_R^{-2\epsilon}}\,\Big(\cdots\Big)\Big|_{\mu_R=\mu \text{ and } \frac{1}{\epsilon^i}=0} = \int [d^4q]\,\Big(\cdots\Big)$$

for both UV and IR divergent loop integrals

- Analogously for the real contribution

$$\left(\frac{\mu_R^2}{s}\right)^{\epsilon}\int_{\phi_3} dx\,dy\,dz\Big(\cdots\Big)\delta(1-x-y-z)(xyz)^{-\epsilon}\Big|_{\mu_R=\mu \text{ and } \frac{1}{\epsilon^i}=0}$$

$$=\int_{\bar{\phi}_3} dx\,dy\,dz\Big(\cdots\Big)\delta(1-x-y-z+3\mu^2/s)$$

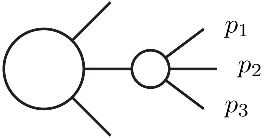
Local subtraction of IR divergences

- Disintegrating final-state virtual logs

$$\sigma_{\mathsf{NLO}} = \int_{\Phi_2} \left(|M|^2_{\mathsf{Born}} + \underbrace{|M|^2_{\mathsf{Virt}}}_{\text{devoid of logs of } \mu} \right) F_J^{(2)}(p_1, p_2)$$

$$+ \underbrace{\int_{\Phi_3}}_{\mu \to 0 \text{ in here!}} \left(|M|^2_{\mathsf{Real}}\, F_J^{(3)}(p_1, p_2, p_3) - |M|^2_{\mathsf{CT}}\, F_J^{(2)}(\underbrace{\hat{p}_1, \hat{p}_2}_{\text{mapped kinematics}}) \right)$$

- *"Tripole"* arrangements when more particles



$e^+e^- \to \gamma^* \to q\bar{q}$ @NLO

- The local counterterm reads

$$|M|^2_{\mathsf{CT}} = \frac{16\pi\alpha_s}{s} C_F |M|^2_{\mathsf{Born}}(\hat{p}_1, \hat{p}_2)\left(\frac{s^2}{s_{13}s_{23}} - \frac{s}{s_{13}} - \frac{s}{s_{23}} + \frac{s_{13}}{2s_{23}} + \frac{s_{23}}{2s_{13}} - \frac{17}{2}\right)$$

- The mapping is: $\hat{p}_1^{\alpha} = \kappa \Lambda^{\alpha}_{\ \beta}\, p_1^{\beta} \left(1 + \frac{s_{23}}{s_{12}}\right)$, $\hat{p}_2^{\alpha} = \kappa \Lambda^{\alpha}_{\ \beta}\, p_2^{\beta} \left(1 + \frac{s_{13}}{s_{12}}\right)$

  where $\kappa = \sqrt{\frac{s s_{12}}{(s_{12}+s_{13})(s_{12}+s_{23})}}$ and $\Lambda^{\alpha}_{\ \beta}$ brings $\hat{p}_1 + \hat{p}_2 = (\sqrt{s}, 0, 0, 0)$

- The correct limiting behavior is obtained for both $s_{13} \to 0$ and $s_{23} \to 0$ $\Rightarrow$ *"tripole"*

Results

- Sanity checks:
  - Inclusive $\sigma_{\text{NLO}} = \sigma_0 \left(1 + C_{\text{F}} \frac{3}{4} \frac{\alpha_s}{\pi}\right)$ reproduced by montecarlo
  - $\sigma_{\text{NLO}}^{\text{cut}}$ (when available analytically) reproduced by montecarlo

- $e^+e^- \to \gamma^* \to \text{ 2 jets}$ @FCC-ee

($\sqrt{s} = 300$ GeV, interfaced with `FastJet`)

R	$\sigma_{2\text{–jets}}^{\texttt{FastJet}}/\sigma_{\text{NLO}}$	$\sigma_{2\text{–jets}}^{p_T > 20 \text{ GeV}}/\sigma_{\text{NLO}}$
2.1	0.580(3)	0.9360(2)
2.0	0.490(4)	0.9264(2)
1.9	0.379(4)	0.9152(3)
1.8	0.242(4)	0.9025(3)
1.0	–	0.7809(3)
0.6	–	0.6933(4)

Conclusions

1. **FDR** is turning to a convenient tool to compute RC
   - $\mathcal{L} = \mathcal{L}$ avoids the introduction of UV CTs
   - Better control of UV/IR infinities (well localized in integrals)
2. Going on-shell @NNLO seems feasible
   - A fix to "*naive*" FDH avoiding evanescent couplings is now available for realistic observables
   - Possibility envisaged of mixed FDR(UV)/DReg(IR) schemes
3. IR regularization *à la* FDR well understood @NLO for FSR
   - 2-jet cross section with a local IR subtraction worked out (more to come)
4. To do list:
   - ISR @NLO (should be trivial)
   - IR @NNLO (does FDR $\Leftrightarrow$ DReg persist?)

Thanks!