Top EFT at lepton colliders

Gauthier Durieux
(DESY Hamburg)

based on current work with Cen Zhang (Brookhaven),
Martín Perelló, Marcel Vos (Valencia, ATLAS-ILC)

LHCTheory ERC meeting
22-24 March 2017, CP3–UCLouvain

Introduction

The NP EFT

provides a systematic parametrization of the theory space
in direct vicinity of the SM

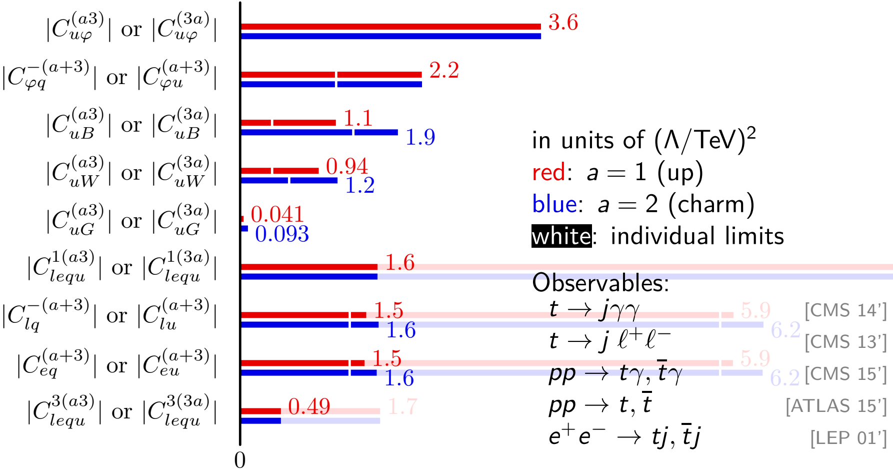
- in a low-energy limit
- through a proper QFT
- consistent when global

EFT analysis recipe:

1. Go global!
2. Combine observables!
3. Offer yourself NLO!
   - FCNCs [Degrande et al, 14']
   - top pair production [Franzosi et al. 15']
   - single top production [Zhang 16']
   - $t\bar{t}Z$, $t\bar{t}\gamma$ [Bylund et al. 16']
   - $t\bar{t}h$ [Maltoni et al. 16']
   - four-fermion operators [$\bar{l}l\bar{q}q$ OK, $\bar{q}q\bar{q}q$ ongoing]

Global top FCNC constraints at NLO in QCD

Showcase example: [GD et al. 14']



in units of $(\Lambda/\text{TeV})^2$

red: $a = 1$ (up)

blue: $a = 2$ (charm)

white: individual limits

Observables:

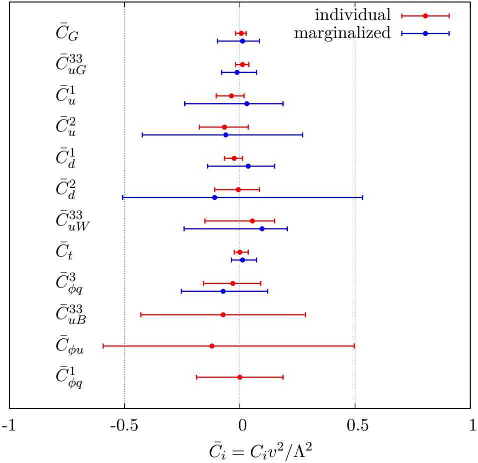
- $t \to j\gamma\gamma$ [CMS 14']
- $t \to j\,\ell^+\ell^-$ [CMS 13']
- $pp \to t\gamma, \bar{t}\gamma$ [CMS 15']
- $pp \to t, \bar{t}$ [ATLAS 15']
- $e^+e^- \to tj, \bar{t}j$ [LEP 01']

Global top EFT constraints from LHC

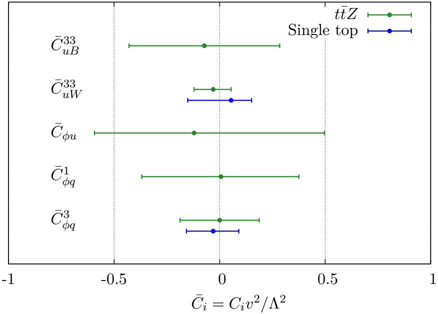
- 195 observables (174 from differential distributions), 12 operators
- mainly from $t\bar{t}$, then single top, charge asymmetries, associated production, W helicity fraction in decay
- standard-model only (N)NLO k-factors in each bin

[Buckley *et al.* Jun. 15]

[Buckley *et al.* Dec. 15]



individual 95% CL limits from $t\bar{t}Z$ and $t\bar{t}\gamma$:

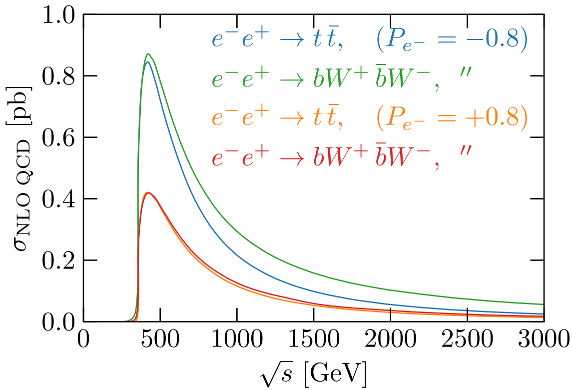


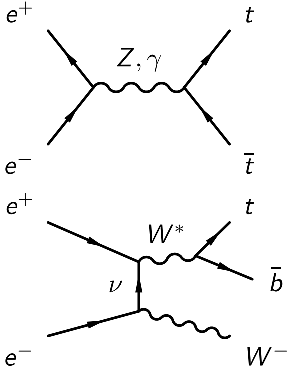
Top EFT at lepton colliders

Top physics at lepton colliders

Handles:

- $t\bar{t}$ threshold scan $\longrightarrow$ mass, width, Yukawa, etc.
- $t\bar{t}$ continuum:
  - · total rate
  - · production distributions
  - · decay distributions
- Polarized beams and $\sqrt{s}$ dependence (at linear colliders)
- $+\ \sigma(t\bar{t}h)$ above $\sqrt{s} \simeq 500$ GeV ($+\ t\bar{t}Z$, $t\bar{t}\gamma$, etc.)





Up-sector neutral current operators

[Grzadkowski et al 10']

Two-quark operators:

Scalar: $O_{u\varphi} \equiv \bar{q} u \, \tilde{\varphi} \quad \varphi^\dagger \varphi,$

Vector:

$$O^1_{\varphi q} \equiv \bar{q} \gamma^\mu q \quad \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi \quad \equiv [O^+_{\varphi q} + O^-_{\varphi q}]/2,$$

$$O^3_{\varphi q} \equiv \bar{q} \gamma^\mu \tau^I q \quad \varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi, \quad \equiv [O^+_{\varphi q} - O^-_{\varphi q}]/2 \qquad \text{(CC also)}$$

$$O_{\varphi u} \equiv \bar{u} \gamma^\mu u \quad \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi,$$

$$O_{\varphi ud} \equiv \bar{u} \gamma^\mu d \quad \tilde{\varphi}^\dagger i\overleftrightarrow{D}_\mu \varphi, \qquad \text{(CC only, } m_b \text{ int.)}$$

Tensor:

$$O_{uB} \equiv \bar{q} \sigma^{\mu\nu} u \, \tilde{\varphi} \quad B_{\mu\nu},$$

$$O_{uW} \equiv \bar{q} \sigma^{\mu\nu} \tau^I u \, \tilde{\varphi} \quad W^I_{\mu\nu}, \qquad \text{(CC also)}$$

$$O_{dW} \equiv \bar{q} \sigma^{\mu\nu} \tau^I d \, \tilde{\varphi} \quad W^I_{\mu\nu}, \qquad \text{(CC only, } m_b \text{ int.)}$$

$$O_{uG} \equiv \bar{q} \sigma^{\mu\nu} T^A u \, \tilde{\varphi} \quad G^A_{\mu\nu}.$$

Two-quark–two-lepton operators:

Scalar:

$$O^S_{lequ} \equiv \bar{l} e \;\; \varepsilon \;\; \bar{q} u, \qquad \text{(CC also, } m_e \text{ int.)}$$

$$O_{ledq} \equiv \bar{l} e \quad \bar{d} q, \qquad \text{(CC only, } m_e \text{ int.)}$$

Vector:

$$O^1_{lq} \equiv \bar{l} \gamma_\mu l \quad \bar{q} \gamma^\mu q \quad \equiv [O^+_{lq} + O^-_{lq}]/2,$$

$$O^3_{lq} \equiv \bar{l} \gamma_\mu \tau^I l \quad \bar{q} \gamma^\mu \tau^I q \quad \equiv [O^+_{lq} - O^-_{lq}]/2, \qquad \text{(CC also)}$$

$$O_{lu} \equiv \bar{l} \gamma_\mu l \quad \bar{u} \gamma^\mu u,$$

$$O_{eq} \equiv \bar{e} \gamma^\mu e \quad \bar{q} \gamma_\mu q,$$

$$O_{eu} \equiv \bar{e} \gamma_\mu e \quad \bar{u} \gamma^\mu u,$$

Tensor:

$$O^T_{lequ} \equiv \bar{l} \sigma_{\mu\nu} e \quad \varepsilon \quad \bar{q} \sigma^{\mu\nu} u. \qquad \text{(CC also, } m_e \text{ int.)}$$

Independent coefficients

Two-quark operators: 10 real degrees of freedom

Scalar: $C_{u\varphi}^{(33)}$,

Vector:

$$C_{\varphi q}^{+(33)} = C_{\varphi q}^{+(33)*}, \qquad \text{(down-}Z\text{, } tbW\text{)}$$

$$C_{\varphi q}^{V(33)} = C_{\varphi q}^{V(33)*} \equiv C_{\varphi u}^{(33)} + C_{\varphi q}^{-(33)}, \qquad \text{(up-}Z\text{, } tbW\text{)}$$

$$C_{\varphi q}^{A(33)} = C_{\varphi q}^{A(33)*} \equiv C_{\varphi u}^{(33)} - C_{\varphi q}^{-(33)}, \qquad \text{(up-}Z\text{, } tbW\text{)}$$

$$C_{\varphi ud}^{(33)}$$

Tensor:

$$C_{uA}^{(33)} \equiv C_{uW}^{(33)} + C_{uB}^{(33)},$$

$$C_{uZ}^{(33)} \equiv \text{cotan}\,\theta_W\, C_{uW}^{(33)} - \tan\theta_W\, C_{uB}^{(33)},$$

$$C_{uG}^{(33)}$$

Two-quark–two-lepton operators: 9×3^2 real degrees of freedom

Scalar: $C_{lequ}^{S(33)}$,

Vector:

$$C_{lq}^{+(33)} = C_{lq}^{+(33)*}, \qquad \text{(up–}\nu\text{, down–}\ell\text{)}$$

$$C_{lq}^{V(33)} = C_{lq}^{V(33)*} \equiv C_{lu}^{(33)} + C_{lq}^{-(33)}, \qquad \text{(up–}\ell\text{)}$$

$$C_{lq}^{A(33)} = C_{lq}^{A(33)*} \equiv C_{lu}^{(33)} - C_{lq}^{-(33)}, \qquad \text{(up–}\ell\text{)}$$

$$C_{eq}^{V(33)} = C_{eq}^{V(33)*} \equiv C_{eu}^{(33)} + C_{eq}^{(33)}, \qquad \text{(up–}\ell\text{, down–}\ell\text{)}$$

$$C_{eq}^{A(33)} = C_{eq}^{A(33)*} \equiv C_{eu}^{(33)} - C_{eq}^{(33)},$$

Tensor: $C_{lequ}^{T(33)}$.

Anomalous vertices

$$t\bar{t}\gamma: \quad \gamma^\mu \overbrace{\left(F^\gamma_{1V} + \gamma_5 F^\gamma_{1A}\right)}^{\sim\emptyset} + \frac{\sigma_{\mu\nu} i q^\nu}{2m_t} \overbrace{\left(F^\gamma_{2V} + i\gamma_5 F^\gamma_{2A}\right)}^{\sim \mathrm{Re,Im}\{C_{uA}\}}$$

$$t\bar{t}Z: \quad \gamma^\mu \overbrace{\left(F^Z_{1V} + \gamma_5 F^Z_{1A}\right)}^{\sim C^V_{\varphi u}, C^A_{\varphi u}} + \frac{\sigma_{\mu\nu} i q^\nu}{2m_t} \overbrace{\left(F^Z_{2V} + i\gamma_5 F^Z_{2A}\right)}^{\sim \mathrm{Re,Im}\{C_{uZ}\}}$$

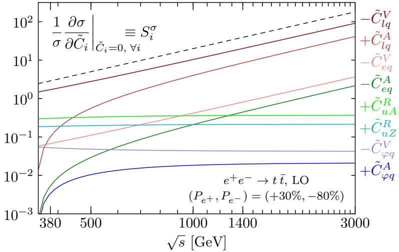
$$t\bar{b}W: \quad \gamma^\mu \overbrace{\left(F^W_{1V} + \gamma_5 F^W_{1A}\right)}^{\sim C_{\varphi ud}, C^+_{\varphi q} - \frac{1}{2}(C^V_{\varphi q} - C^A_{\varphi q})} + \frac{\sigma_{\mu\nu} i q^\nu}{2m_t} \overbrace{\left(F^W_{2V} + i\gamma_5 F^W_{2A}\right)}^{\sim \mathrm{Re,Im}\{s_W^2 C_{uA} + s_W c_W C_{uZ}\}}$$

Insufficiencies:

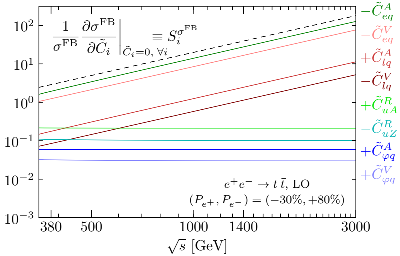
- Miss four-fermion operators
- Conflict with gauge invariance
  Do not allow for radiative corrections to be computed
- Complex couplings where the tree-level EFT prescribes real ones
- Hide correlations induced by gauge invariance
  Preclude the combination of measurements in various sectors

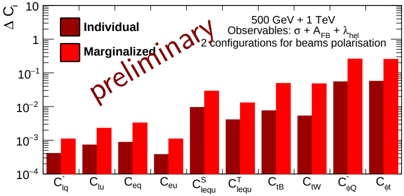
Operator sensitivities in $e^+e^- \to t\,\bar{t}$

Total cross section (left pol.):



FB-integrated cross section (right pol.):





Few features:

- quadratic energy growth for four-fermion operators
- no growth for two-fermion operators
  Some *azimuthal* $bW^+\bar{b}W^-$ observables have linearly growing sensitivity to C_{uZ}, C_{uA}.
- p-wave $\beta = \sqrt{1 - m_t^2/s}$ suppression of axial vectors at threshold
- enhanced sensitivity of axial vector operators in $\sigma^{\rm FB}$
- sensitivity sign flip for $C_{\varphi q}^V$ and C_{uZ}^R when polarization is reversed
- etc.

Helicity amplitude decomposition in $bW^+\bar{b}W^-$

[Jacob,Wick 59']

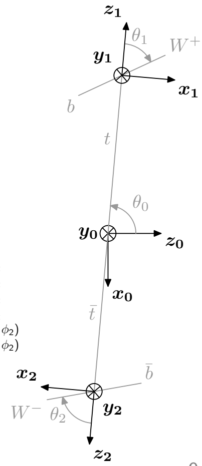
Production amplitudes: [Schmidt 95']

$$A_1(++) \sim \frac{2m_t}{\sqrt{s}}(V+A) + \sqrt{s}\,(T - \beta\tilde{T})$$
$$A_2(--) \sim \frac{2m_t}{\sqrt{s}}(V+A) + \sqrt{s}\,(T + \beta\tilde{T})$$
$$A_3(+-) \sim (V+\beta A) + 2m_t(T+\tilde{T})$$
$$A_4(-+) \sim (V-\beta A) + 2m_t(T+\tilde{T})$$

In terms of $\Omega = \{\theta_0, \theta_1, \phi_1, \theta_2, \phi_2\}$ helicity angles:

$$\frac{d\sigma}{d\Omega} =$$

$+3/4$		$(\lvert A_3\rvert^2+\lvert A_4\rvert^2)$	$\lvert a_2\rvert^2+\lvert a_4\rvert^2$	$\lvert b_1\rvert^2+\lvert b_3\rvert^2$	$(1+\cos^2\theta_0)$			
$+3/4$		$(\lvert A_3\rvert^2-\lvert A_4\rvert^2)$	$\lvert a_2\rvert^2+\lvert a_4\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$(1+\cos^2\theta_0)$		$\cos\theta_2$	
$+3/4$		$(\lvert A_3\rvert^2-\lvert A_4\rvert^2)$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_1\rvert^2+\lvert b_3\rvert^2$	$(1+\cos^2\theta_0)$	$\cos\theta_1$		
$+3/4$		$(\lvert A_3\rvert^2+\lvert A_4\rvert^2)$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$(1+\cos^2\theta_0)$	$\cos\theta_1$	$\cos\theta_2$	
$-3/2$		$(\lvert A_3\rvert^2-\lvert A_4\rvert^2)$	$\lvert a_2\rvert^2+\lvert a_4\rvert^2$	$\lvert b_1\rvert^2+\lvert b_3\rvert^2$	$\cos\theta_0$			
$-3/2$		$(\lvert A_3\rvert^2+\lvert A_4\rvert^2)$	$\lvert a_2\rvert^2+\lvert a_4\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$\cos\theta_0$		$\cos\theta_2$	
$-3/2$		$(\lvert A_3\rvert^2+\lvert A_4\rvert^2)$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_1\rvert^2+\lvert b_3\rvert^2$	$\cos\theta_0$	$\cos\theta_1$		
$-3/2$		$(\lvert A_3\rvert^2-\lvert A_4\rvert^2)$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$\cos\theta_0$	$\cos\theta_1$	$\cos\theta_2$	
$+3/2$		$(\lvert A_1\rvert^2+\lvert A_2\rvert^2)$	$\lvert a_2\rvert^2+\lvert a_4\rvert^2$	$\lvert b_1\rvert^2+\lvert b_3\rvert^2$	$\sin^2\theta_0$			
$-3/2$		$(\lvert A_1\rvert^2-\lvert A_2\rvert^2)$	$\lvert a_2\rvert^2+\lvert a_4\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$\sin^2\theta_0$		$\cos\theta_2$	
$+3/2$		$(\lvert A_1\rvert^2-\lvert A_2\rvert^2)$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_1\rvert^2+\lvert b_3\rvert^2$	$\sin^2\theta_0$	$\cos\theta_1$		
$-3/2$		$(\lvert A_1\rvert^2+\lvert A_2\rvert^2)$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$\sin^2\theta_0$	$\cos\theta_1$	$\cos\theta_2$	
$+3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_1^*A_4\}$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_3\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$\sin\theta_1$	$(1+\cos\theta_2)$	$\cos\phi_1$
$+3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_1^*A_4\}$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_1\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$\sin\theta_1$	$(1-\cos\theta_2)$	$\cos\phi_1$
$+3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_2^*A_3\}$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_1\rvert^2$	$\sin\theta_0(1-\cos\theta_0)$	$\sin\theta_1$	$(1+\cos\theta_2)$	$\cos\phi_1$
$+3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_2^*A_3\}$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_3\rvert^2$	$\sin\theta_0(1-\cos\theta_0)$	$\sin\theta_1$	$(1-\cos\theta_2)$	$\cos\phi_1$
$-3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_2^*A_4\}$	$\lvert a_4\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$(1+\cos\theta_1)$	$\sin\theta_2$	$\cos\phi_2$
$-3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_2^*A_4\}$	$\lvert a_2\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$(1-\cos\theta_1)$	$\sin\theta_2$	$\cos\phi_2$
$-3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_1^*A_3\}$	$\lvert a_2\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$\sin\theta_0(1-\cos\theta_0)$	$(1+\cos\theta_1)$	$\sin\theta_2$	$\cos\phi_2$
$-3/2$	$\sqrt{2}$	$\mathrm{Re}\{A_1^*A_3\}$	$\lvert a_4\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$\sin\theta_0(1-\cos\theta_0)$	$(1-\cos\theta_1)$	$\sin\theta_2$	$\cos\phi_2$
-3		$\mathrm{Re}\{A_1^*A_2\}$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$\sin^2\theta_0$	$\sin\theta_1$	$\sin\theta_2$	$\cos(\phi_1+\phi_2)$
$-3/2$		$\mathrm{Re}\{A_3^*A_4\}$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$\sin^2\theta_0$	$\sin\theta_1$	$\sin\theta_2$	$\cos(\phi_1-\phi_2)$
$+3/2$	$\sqrt{2}$	$\mathrm{Im}\{A_1^*A_4\}$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_3\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$\sin\theta_1$	$(1+\cos\theta_2)$	$\sin\phi_1$
$+3/2$	$\sqrt{2}$	$\mathrm{Im}\{A_1^*A_4\}$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_1\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$\sin\theta_1$	$(1-\cos\theta_2)$	$\sin\phi_1$
$-3/2$	$\sqrt{2}$	$\mathrm{Im}\{A_2^*A_3\}$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_1\rvert^2$	$\sin\theta_0(1-\cos\theta_0)$	$\sin\theta_1$	$(1+\cos\theta_2)$	$\sin\phi_1$
$-3/2$	$\sqrt{2}$	$\mathrm{Im}\{A_2^*A_3\}$	$\lvert a_2\rvert^2-\lvert a_4\rvert^2$	$\lvert b_3\rvert^2$	$\sin\theta_0(1-\cos\theta_0)$	$\sin\theta_1$	$(1-\cos\theta_2)$	$\sin\phi_1$
$+3/2$	$\sqrt{2}$	$\mathrm{Im}\{A_2^*A_4\}$	$\lvert a_4\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$(1+\cos\theta_1)$	$\sin\theta_2$	$\sin\phi_2$
$+3/2$	$\sqrt{2}$	$\mathrm{Im}\{A_2^*A_4\}$	$\lvert a_2\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$\sin\theta_0(1+\cos\theta_0)$	$(1-\cos\theta_1)$	$\sin\theta_2$	$\sin\phi_2$
$-3/2$	$\sqrt{2}$	$\mathrm{Im}\{A_1^*A_3\}$	$\lvert a_2\rvert^2$	$\lvert b_1\rvert^2-\lvert b_3\rvert^2$	$\sin\theta_0(1-\cos\theta_0)$	$(1+\cos\theta_1)$	$\sin\theta_2$	$\sin\phi_2$



NLO in QCD for $e^+e^- \to bW^+\bar{b}W^-$

For various beam polarizations and center-of-mass energies:

pol	$\sqrt{s}$ [GeV]	σ_{SM} [fb]	σ_i [fb] $|^{A}_{lq}$	$|^{A}_{eq}$	$|^{A}_{\phi q}$	$|^{V}_{lq}$	$|^{V}_{eq}$	$|^{V}_{\phi q}$	$|^{R}_{uZ}$	$|^{R}_{uA}$	$|^{I}_{uZ}$	$|^{I}_{uA}$	$|^{R}_{uG}$
00	300	2.92 ±0.3% (+1%/−1%) 1.15	0.353 ±0.2% (+1%/−1%) 1.15	−0.0856 ±0.4% (−2%/+2%) 1.27	0.14 ±0.1% (+2%/−1%) 1.2	−0.621 ±0.2% (−2%/+3%) 1.34	−0.303 ±0.4% (−3%/+4%) 1.51	−0.136 ±0.1% (−2%/+2%) 1.21	0.349 ±0.3% (+2%/−2%) 1.21	0.32 ±0.1% (+3%/−2%) 1.33	−0.000225 ±90% (−7%/+9%) —	−0.000125 ±90% (−9%/+10%) —	0.000214 ±6% (+10%/−9%) —
00	380	825 ±0.5% (+2%/−1%) 1.18	77.1 ±3% (+3%/−2%) 1.34	−55.6 ±20% (−2%/+3%) 1.32	53.1 ±0.1% (+2%/−1%) 1.2	−993 ±2% (−1%/+2%) 1.19	−635 ±1% (−1%/+1%) 1.16	−62.9 ±0.6% (−1%/+2%) 1.19	118 ±2% (+1%/−1%) 1.14	323 ±2% (+2%/−1%) 1.19	0.107 ±300% (+20%/−20%) −1.36	−0.434 ±80% (−10%/+10%) —	0.25 ±10% (+10%/−9%) —
00	500	669 ±0.5% (+0.4%/−0.5%) 0.952	258 ±7% (+0.6%/−0.7%) 0.936	−233 ±2% (−0.3%/+0.4%) 1.04	49.2 ±1% (+0.08%/−0.1%) 0.99	−1230 ±2% (−0.8%/+0.7%) 0.929	−750 ±4% (−2%/+1%) 0.872	−45.2 ±5% (−1%/+0.9%) 0.909	102 ±5% (+0.3%/−0.3%) 0.97	263 ±0.7% (+0.6%/−0.8%) 0.929	−2.08 ±50% (−6%/+8%) —	1.78 ±80% (+10%/−8%) —	0.715 ±30% (+10%/−9%) —
00	1000	221 ±0.5% (+1%/−1%) 0.897	756 ±9% (+2%/−1%) 1.21	−475 ±20% (−0.2%/+0.1%) 0.983	15.6 ±4% (+2%/−2%) 0.844	−1070 ±20% (−3%/+2%) 0.784	−940 ±6% (−0.6%/+0.5%) 0.95	−15.5 ±3% (−1%/+0.8%) 0.914	36.1 ±5% (+0.04%/−0.05%) 0.995	87.3 ±3% (+1%/−1%) 0.899	0.392 ±200% (+10%/−10%) —	−8.74 ±40% (−9%/+10%) —	0.907 ±5% (+10%/−9%) —
00	1400	132 ±3% (+0.6%/−0.7%) 0.936	391 ±30% (+7%/−9%) 0.555	−412 ±30% (−3%/+2%) 0.798	8.29 ±3% (+2%/−3%) 0.803	−1460 ±5% (−0.9%/+0.7%) 0.926	−816 ±10% (−3%/+2%) 0.794	−8 ±4% (−2%/+1%) 0.859	21.6 ±8% (+0.2%/−0.1%) 1.03	59.8 ±6% (+0.5%/−0.4%) 1.08	1.8 ±80% (+8%/−6%) —	0.257 ±200% (+20%/−20%) −0.906	0.414 ±100% (+10%/−10%) —
00	3000	40.2 ±10% (+1%/−1%) 1.13	1080 ±20% (+1%/−0.8%) 1.1	−70.4 ±200% (−70%/+60%) 0.128	1.28 ±10% (+8%/−9%) 0.533	−1270 ±20% (−0.3%/+0.2%) 0.981	−689 ±50% (−5%/+4%) 0.668	−0.717 ±30% (−20%/+10%) 0.406	10.[illegible] ±30% (+3%/−2%) 1.38	14.2 ±30% (+0.5%/−0.4%) 1.05	1.85 ±100% (+7%/−9%) 0.441	2.15 ±200% (+1%/−1%) 0.594	−0.261 ±200% (−7%/+8%) —
+−	300	2.73 ±0.3% (+1%/−1%) 1.14	0.351 ±0.2% (+1%/−1%) 1.14	—	0.126 ±0.2% (+2%/−1%) 1.19	−0.62 ±0.2% (−2%/+3%) 1.34	—	−0.14 ±[illegible] (−2%/+[illegible]2%) 1.22	0.376 ±[illegible]0.1% ([illegible]2%/−2%) 1.23	0.241 ±0.2% (+2%/−2%) 1.28	$6.25e-06$ ±2000% (+70%/−50%) −0.119	$2.7e-06$ ±2000% (+20%/−30%) 0.19	0.000197 ±4% (+10%/−9%) —
+−	380	579 ±2% (+2%/−1%) 1.19	73 ±1% (+3%/−2%) 1.35	—	36.1 ±0.5% (+2%/−1%) 1.18	−968 ±2% (−1%/+1%) 1.16	—	[illegible]4.5 ±[illegible]1% ([illegible]) [illegible]16	165 ±0.4% (+2%/−1%) 1.18	198 ±0.6% (+2%/−1%) 1.18	0.44 ±70% (+10%/−9%) —	−0.324 ±100% (−10%/+10%) —	0.185 ±10% (+10%/−9%) —
+−	500	469 ±0.5% (+0.4%/−0.4%) 0.96	287 ±5% (+0.4%/−0.3%) 1.03	—	31.7 ±2% (+0.4%/−0.5%) 0.953	−1270 ±2% (−0.3%/+0.2%) 0.972	—	[illegible]1.1 ±3% (−0.2%/+0.2%) 0.982	130 ±1% (+0.8%/−0.9%) 0.918	164 ±0.5% (+0.5%/−0.6%) 0.943	1.35 ±50% (+10%/−9%) —	−0.442 ±200% (−6%/+8%) —	0.554 ±10% (+10%/−8%) —
+−	1000	160 ±1% (+0.9%/−1%) 0.902	470 ±8% (+3%/−4%) 0.742	—	11.8 ±4% (+0.6%/−0.8%) 0.931	−1450 ±4% (−0.8%/+0.7%) 0.926	—	−15.5 ±2% (−0.8%/+0.7%) 0.93	44.9 ±3% (+0.8%/−1%) 0.916	52.6 ±6% (+1%/−1%) 0.883	−0.663 ±200% (−7%/+9%) —	5.09 ±40% (+10%/−8%) —	0.587 ±4% (+10%/−9%) —
+−	1400	84.9 ±10% (+2%/−2%) 0.817	507 ±20% (+3%/−4%) 0.715	—	7.57 ±6% (+0.7%/−0.6%) 1.07	−1230 ±[illegible] (−3[illegible]/+3[illegible]) 0.772	—	−7.76 ±6% (−2%/+2%) 0.835	22.2 ±8% (+2%/−3%) 0.806	29.5 ±9% (+0.9%/−1%) 0.905	−1.22 ±300% (−8%/+10%) —	−2.38 ±90% (−10%/+10%) —	0.281 ±100% (+10%/−10%) —
+−	3000	23.8 ±7% (+2%/−3%) 0.787	356 ±30% (+10%/−10%) 0.414	—	0.574 ±30% (+20%/−20%) 0.338	−14[illegible]0 ±[illegible]0% (−2%/+1%) 0.853	—	−1.08 ±30% (−9%/+7%) 0.543	6.28 ±30% (+2%/−1%) 1.25	—	1.93 ±90% (+10%/−10%) —	8.36 ±30% (+7%/−5%) —	0.197 ±70% (+10%/−10%) —
−+	300	0.218 ±0.4% (−2%/+0%) 1.37	—	−0.0855 ±0.1% (−2%/+2%) 1.27	0.0147 ±0.1% (+3%/−3%) 1.42	—	−0.302 ±0.2% (−3%/+4%) 1.51	0.00343 ±0.1% (+4%/−3%) 1.59	−0.0259 ±0.1% (−3%/+4%) 1.5	0.0799 ±0.2% (+4%/−3%) 1.5	$3.38e-06$ ±300% (+6%/−5%) —	$-7.78e-06$ ±200% (−0.1%/+0.07%) 1.3	$1.84e-05$ ±4% (+10%/−9%) —
−+	380	249 ±0.9% (−1%/+0%) 1.19	—	−51.6 ±4% (−1%/+2%) 1.18	16.2 ±0.6% (+2%/−1%) 1.18	—	−649 ±0.2% (−1%/+2%) 1.18	3.31 ±1% (+1%/−1%) 1.16	−41.8 ±0.8% (−1%/+1%) 1.15	124 ±0.4% (+2%/−1%) 1.19	0.0946 ±200% (+10%/−9%) —	−0.0633 ±200% (−10%/+20%) —	0.000205 ±9000% (+200%/−200%) −0.0614
−+	500	203 ±1% (+0.5%/+0%) 0.948	—	−213 ±1% (−0.5%/+0.4%) 0.958	15.8 ±0.9% (+0.2%/−0.3%) 0.975	—	−810 ±0.7% (−0.8%/+0.6%) 0.933	0.767 ±4% (+2%/−3%) 0.783	−34.2 ±5% (−0.9%/+0.7%) 0.923	99.7 ±0.5% (+0.9%/−1%) 0.909	0.316 ±40% (+10%/−8%) —	0.187 ±200% (+10%/−8%) —	0.255 ±20% (+10%/−9%) —
−+	1000	63.4 ±5% (+0.9%/+0%) 0.911	—	−327 ±10% (−4%/+4%) 0.7	4.88 ±3% (+2%/−2%) 0.828	—	−810 ±5% (−2%/+2%) 0.82	−0.24 ±10% (−1%/+0.8%) 0.907	−10.4 ±6% (−2%/+2%) 0.812	34.1 ±5% (+0.9%/−1%) 0.906	−0.832 ±70% (−8%/+10%) —	0.255 ±300% (+3%/−2%) 1.26	0.39 ±10% (+10%/−8%) —
−+	1400	33.8 ±6% (+0.8%/+0%) 0.917	—	−493 ±10% (−0.4%/+0.3%) 0.93	2.86 ±4% (+1%/−1%) 0.897	—	−850 ±4% (−2%/+2%) 0.842	−0.208 ±10% (−0.4%/+0.5%) 1.06	−4.75 ±20% (−5%/+4%) 0.666	10.2 ±0% (+0%/+0%) 0.504	0.448 ±300% (+2%/−2%) 1.23	0.475 ±400% (+20%/−20%) −0.982	0.258 ±30% (+10%/−9%) —
−+	3000	—	—	−424 ±60% (−0.7%/+0.8%) 1.09	0.226 ±40% (+10%/−20%) 0.4	—	—	0.146 ±0% (+0%/+0%) —	−1.5 ±70% (−6%/+5%) 0.651	−497 ±0% (+0%/+0%) —	2.52 ±40% (+6%/−5%) —	2110 ±0% (+0%/+0%) —	−0.0453 ±200% (−0.04%/+0.04%) 1

preliminary

(MG, complex mass scheme, $m_b \to 0$, total width EFT dependence not included)

A couple of additional ideas

Statistically optimal observables

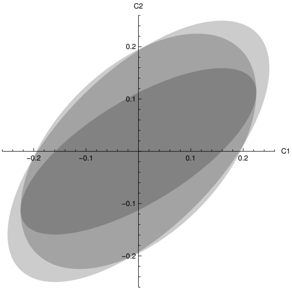
[Atwood,Soni 92']

[Diehl,Nachtmann 94']

minimize the one-sigma ellipsoid in EFT parameter space.

(*joint efficient* set of estimator, saturating the Rao-Cramér-Fréchet bound: $V^{-1} = I$)

For small C_i, with a phase-space distribution $\sigma(\phi) = \sigma_0(\phi) + \sum_i C_i \, \sigma_i(\phi)$,
the statistically optimal set of observables is: $O_i(\phi) = \sigma_i(\phi)/\sigma_0(\phi)$.



e.g. $\sigma(\phi) = 1 + \cos(\phi) + C_1 \sin(\phi) + C_2 \sin(2\phi)$

1. asymmetries: $O_i \sim \text{sign}\{\sin(i\phi)\}$
2. moments: $O_i \sim \sin(i\phi)$
3. statistically optimal: $O_i \sim \dfrac{\sin(i\phi)}{1 + \cos\phi}$

$\Longrightarrow$ area ratios 1.9 : 1.7 : 1
(total rate not used)

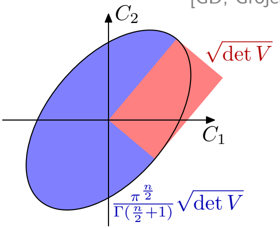
Global determinant parameter (GDP)

[GD, Grojean, Gu, Wang]

In a *n*-dimensional Gaussian fit

$$\mathrm{GDP} \equiv \sqrt[2n]{\det V}$$

provides a geometric average of the constraints strengths.

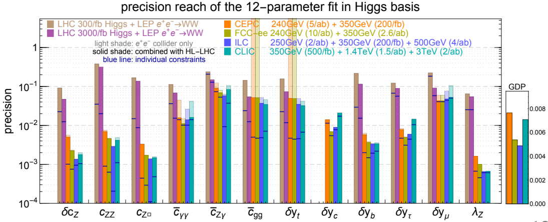


Interestingly, GDP ratios are operator-basis independent!

- as the volume scales linearly with coupling normalization
- as the volume is invariant under rotations

$\Longrightarrow$ conveniently assess constraint strengthening.

e.g. Higgs fit at lepton colliders:



Summary

The EFT parametrizes systematically the parameter space in direct vicinity of the standard model.

A global analysis of future-lepton-collider constraints on the top EFT is ongoing.

Statistically optimal observables are surprisingly unexploited.

Global determinant parameter ratios assess the strengthening of global constraints, basis independently.