

# HIGGS PRODUCTION AT NLO IN THE STANDARD MODEL EFT

Nicolas Deutschmann

Work in progress with

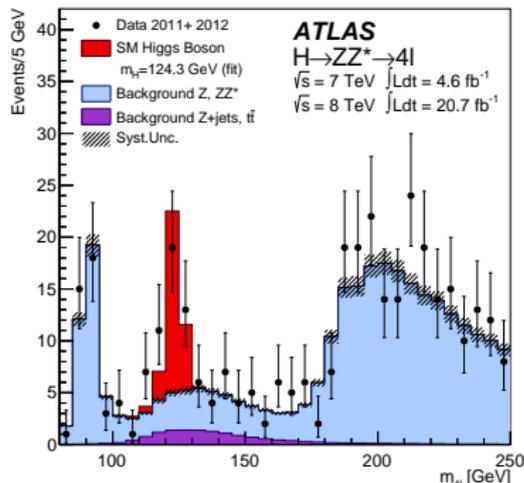
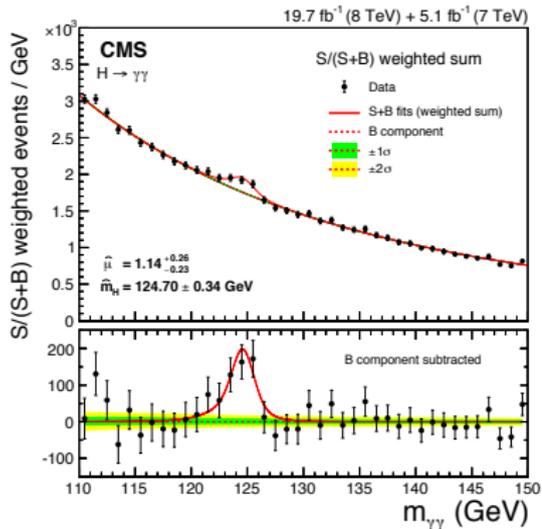
**Claude Duhr, Fabio Maltoni** and **Eleni Vryonidou**

LHCTheory meeting  
Wednesday, 22 March 2017



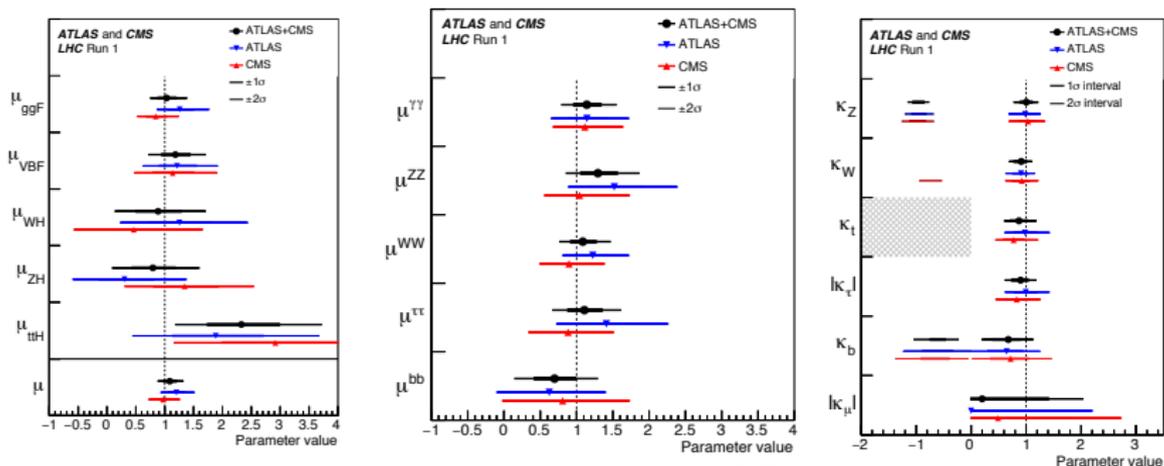
# Constraining the Higgs boson at the LHC

Convincing evidence for the new LHC boson to be a CP-even scalar with SM-Higgs like properties.



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Measurements of signal strengths and  $\kappa$  parameters from Run I

# Kappas VS couplings

The  $\kappa$  parameters are not the coefficients of a QFT, they are fit parameters:

$$\mathcal{N}_{\text{events}}(pp \rightarrow t\bar{t}H \rightarrow b\bar{b}) \xrightarrow{\text{fit}} \kappa_t^2 \kappa_b^2 (\sigma \times B) \in \mathcal{L} + \dots$$

Scaling based on LO description

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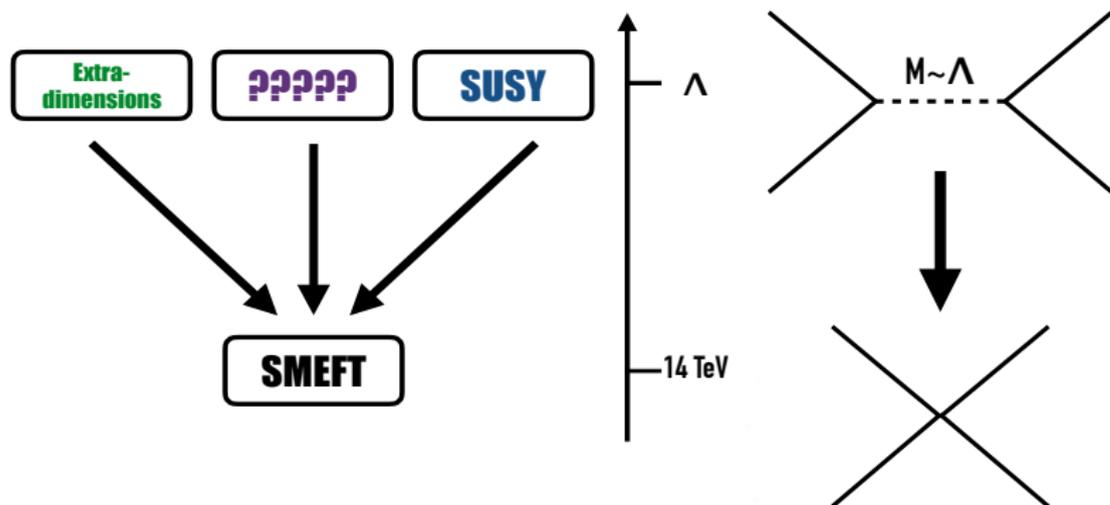
Scaling based on LO description

- Good tools to test the SM
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Fit parameters  $\rightarrow$  Model parameter:  
precise calculation for each channel, for each model

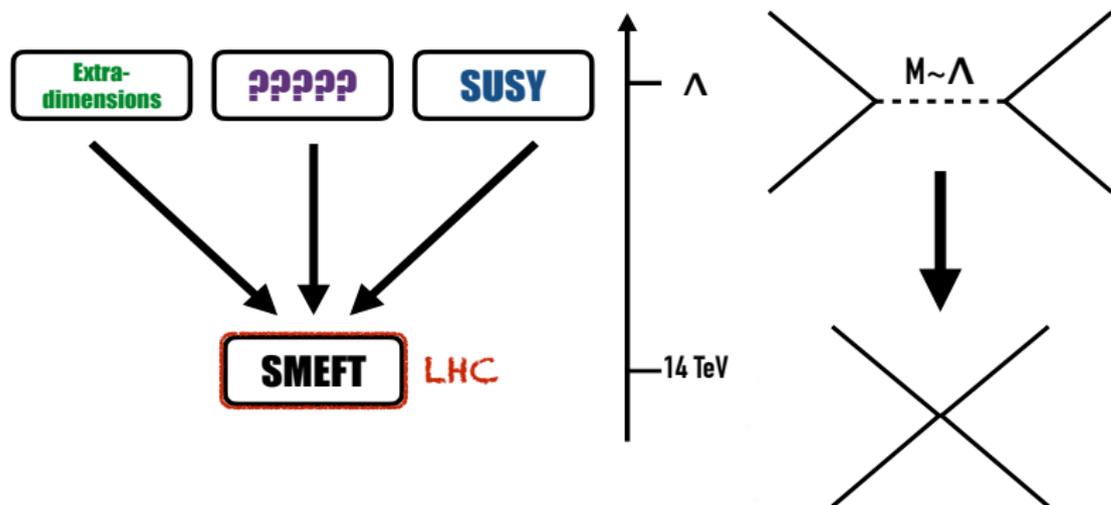
# Model independent BSM constraints

As long as new physics is heavy (no direct production), the SMEFT can encapsulate its effect on LHC physics.



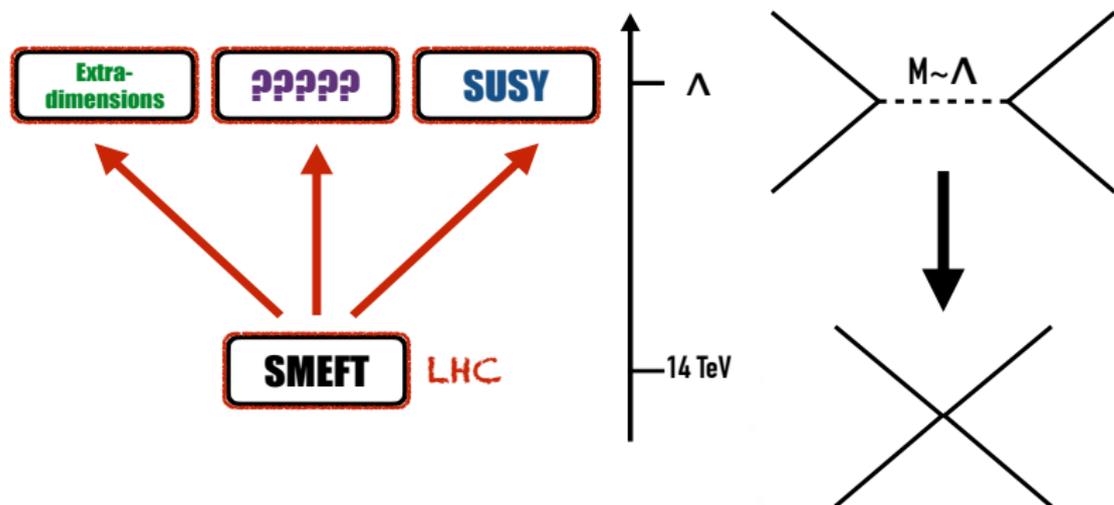
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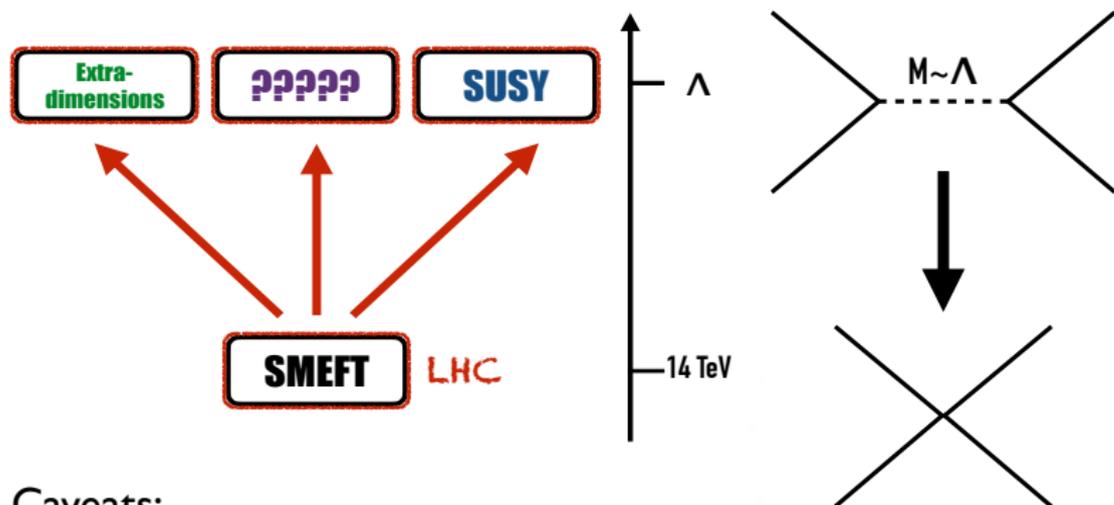
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Caveats:

- Standard Model processes (no searches)
- Small deviations:  $14 \text{ TeV}/\Lambda \ll 1$

# The SMEFT framework

A consistent QFT for parametrizing small BSM effects using higher-dimensional operators with SM fields:

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{k=1}^N \frac{1}{\Lambda^k} \sum_i C_i \mathcal{O}_i^{[k+4]} \quad (1)$$

Consistent approach to radiative corrections: truncate  $\mathcal{L}_{\text{SMEFT}}$  and observables at a finite order.

- Dimension 5: 1 operator (neutrino masses)
- Dimension 6: 59 operators or 63, or 84

[Weinberg]

[Buchmuller, Wyler]

[Grzadkowski, Iskrzynski, Misiak, Rosiek]

# NLO corrections to SMEFT processes

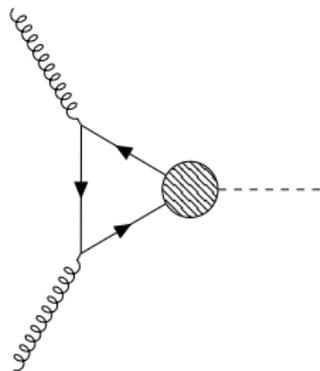
The SMEFT parametrizes possible deviations on precision LHC measurements.

Need for NLO predictions:

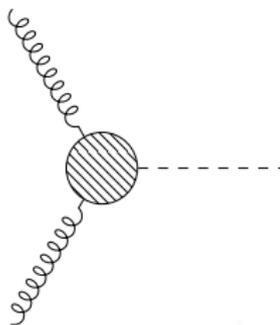
- Better accuracy: SM Higgs cross-section changes by 100% from LO to NLO
- Better bounds: typically increased rates at NLO (e.g. bound on one operator involved in  $t\bar{t}$  improved by  $\times 1.5$  [Buarque Franzosi,Zhang] )
- Better precision: reduction of scale uncertainties.  
SMEFT@LO for  $gg \rightarrow H$ :  $15 \sim 25\%$  [Maltoni,Vryonidou,Zhang]

# The top-Higgs sector of the SMEFT

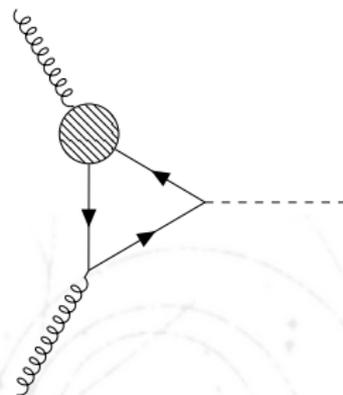
Yukawa correction  
 $C_1 \bar{Q} \sigma^{\mu\nu} t_R \phi (\phi^\dagger \phi)$



Gluon fusion  
 $g_s^2 C_2 \phi^\dagger \phi G_{\mu\nu} G^{\mu\nu}$



Chromomagnetic  
 $g_s C_3 \bar{Q} \sigma^{\mu\nu} t_R \phi G_{\mu\nu}$



# Structure of the LO amplitude

Two unusual features:

- Mix of tree-level and one-loop
- UV divergence in loop contribution

$$\mathcal{A}_{\text{EFT}}^{(0)} = C_1 \times \text{diagram}_1 + C_3 \times \text{diagram}_2 + C_2 \times \text{diagram}_3$$

[Degrande, Gérard, Grojean, Maltoni, Servant]

# Structure of the LO amplitude

Two unusual features:

- Mix of tree-level and one-loop
- UV divergence in loop contribution

$$\mathcal{A}_{\text{EFT}}^{(0)} = C_1 \times \text{Finite} + C_3 \times \text{UV divergent} + C_2 \times \text{Finite}$$

The diagram illustrates the structure of the LO amplitude  $\mathcal{A}_{\text{EFT}}^{(0)}$  as a sum of three terms. The first term,  $C_1 \times$ , is a tree-level diagram with two incoming lines and one outgoing line, labeled "Finite". The second term,  $+ C_3 \times$ , is a one-loop diagram (triangle loop) with two incoming lines and one outgoing line, labeled "UV divergent". The third term,  $+ C_2 \times$ , is another tree-level diagram with two incoming lines and one outgoing line, labeled "Finite".

[Degrande, Gérard, Grojean, Maltoni, Servant]

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Two unusual features:

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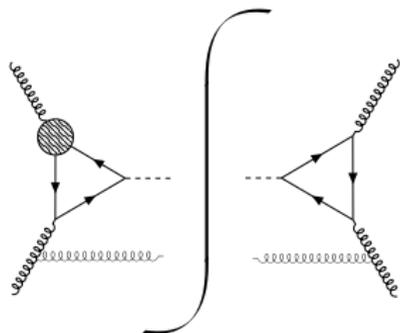
$$\mathcal{A}_{\text{EFT}}^{(0)} = C_1 \times \text{Finite} + C_3 \times \text{UV divergent} + C_2 \times \text{Finite} + \delta Z_{23} C_3 \times \text{Counter-term}$$

[Degrande, Gérard, Grojean, Maltoni, Servant]

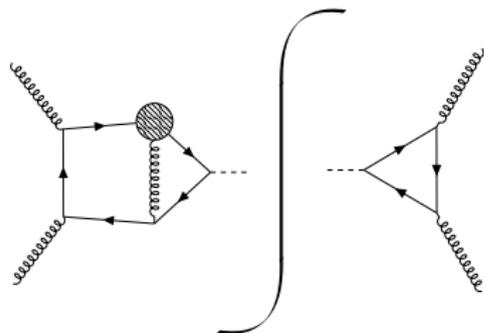
# NLO correction to the amplitude

The NLO contribution to the cross-section is composed of

Real emissions



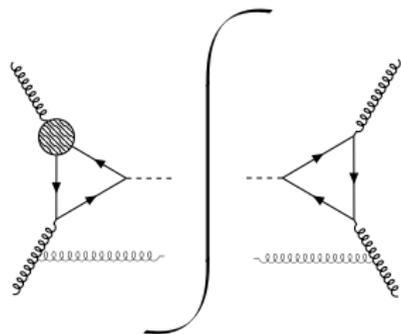
Virtual corrections



# NLO correction to the amplitude

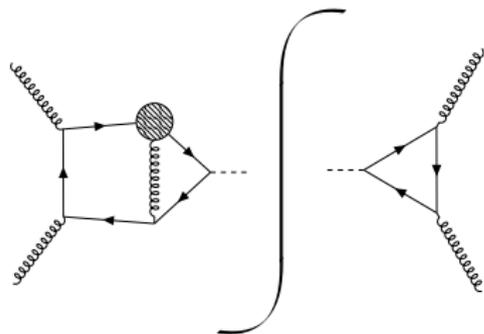
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1-loop: automated

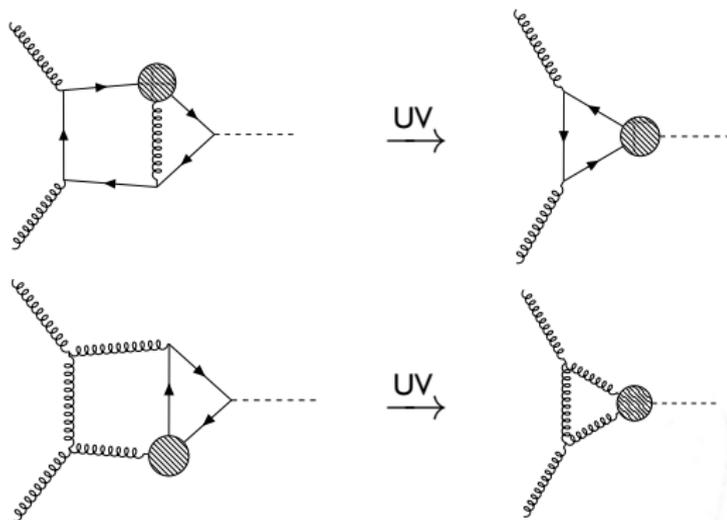
Virtual corrections



2-loop: by hand

# Divergence structure: operator mixing

The chromomagnetic operator requires counter-terms from both other operators:



Renormalization matrix:

$$C_i^0 = Z^{ij} C_j^R$$

[Jenkins, Manohar, Trott]

# Divergence structure: IR divergences

The Infrared divergences factorize:

$$\mathcal{A}_R^{(1)} = \mathcal{A}_{\text{finite}}^{(1)} + \hat{I}_1 \mathcal{A}_R^{(0)}$$

$\hat{I}_1$  is a universal operator encapsulating the IR divergences.

For  $gg \rightarrow H$ , 
$$\hat{I}_1 = -\frac{e^{\epsilon\gamma}}{\Gamma(1-\epsilon)} \left( \frac{C_A}{\epsilon^2} + \frac{\beta_0}{\epsilon} \right) \left( \frac{\mu^2}{-s} \right)^\epsilon$$

Our LO amplitude already has a pole so an unusual divergence appears :

$$\text{Tree-level diagram} \dots \propto \frac{1}{\epsilon} \quad \Rightarrow \quad \hat{I}_1 \text{Tree-level diagram} \dots \propto \frac{1}{\epsilon^3}$$

[Catani, Seymour]

# Structure of the NLO amplitude

Simplest part of the amplitude:  $C_2$

The diagram illustrates the decomposition of a bare 1-loop amplitude into its finite, UV divergent, and IR divergent components. On the left, a vertex with a shaded circle is connected to three external wavy lines, with a bracket underneath labeled "Bare 1-loop". This is set equal to the sum of three terms: 1)  $\mathcal{A}_{\text{finite}}^{(1)}|_{C_2}$ , 2) a vertex with a shaded circle and a dashed line labeled  $\delta Z_{22}$  inside, with a bracket underneath labeled "UV divergence", and 3) a vertex with a shaded circle and a dashed line labeled  $\hat{I}_1$  inside, with a bracket underneath labeled "IR divergence".

$$\underbrace{\text{Bare 1-loop}} = \mathcal{A}_{\text{finite}}^{(1)}|_{C_2} + \underbrace{\delta Z_{22}}_{\text{UV divergence}} + \underbrace{\hat{I}_1}_{\text{IR divergence}}$$

# Structure of the NLO amplitude

SM-like two-loop amplitude:  $C_1$

$$\underbrace{\text{Bare 2-loop}} = \mathcal{A}_{\text{finite}}^{(1)} \Big|_{C_1} + \underbrace{\text{UV divergence}} + \underbrace{\text{IR divergence}}$$

# Structure of the NLO amplitude

SM-like two-loop amplitude:  $C_1$

$$\underbrace{\text{Bare 2-loop}}_{\text{Hard}} = \mathcal{A}_{\text{finite}}^{(1)} \Big|_{C_1} + \underbrace{\text{UV divergence}} + \underbrace{\text{IR divergence}}$$

# Structure of the NLO amplitude

Two-loop and mixing:  $C_3$

Bare 2-loop

$$= \mathcal{A}_{\text{finite}}^{(1)} \Big|_{C_1}$$

+ 1-loop UV divergences

+ 2-loop UV divergence

+ IR divergence

# Structure of the NLO amplitude

Two-loop and mixing:  $C_3$

Bare 2-loop

Hard

$$= \mathcal{A}_{\text{finite}}^{(1)} \Big|_{C_1}$$

1-loop UV divergences

2-loop UV divergence

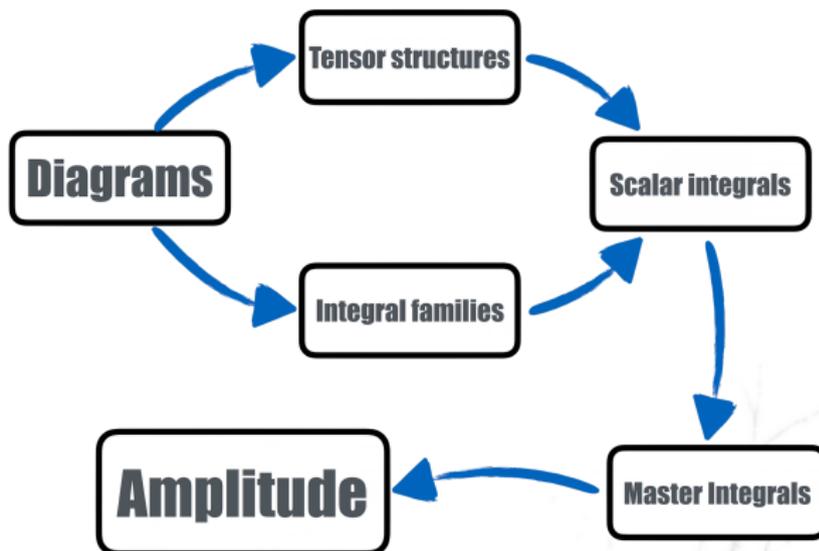
New!

IR divergence

$$\hat{I}_1$$

# Introduction to modern multi-loop techniques

Workflow of a multi-loop calculation



# Integral families

Multi-loop amplitudes contain thousands of loop integrals: need for an organisational principle

## Definition: Family

A family of integrals is given by a set of propagators. Scalar integrals with arbitrary powers of this propagators are in the same family

$$I(a_1, \dots, a_k) = \int d^d k_1 \dots d^d k_l \frac{1}{D_1^{a_1} \dots D_k^{a_k}}$$

Arbitrary numerators are included using negative powers

# Family bases

**Theorem:** all integrals in a family can be expressed in terms of a finite basis (!)

How can we reach it?

Integration-by-parts identities (IBP)

In dimensional regularization:

$$\int \prod d^d k_i \frac{\partial}{\partial k_j^\mu} \frac{v_\mu}{D_1^{a_1} \dots D_k^{a_k}} = 0$$

Families are stable under differentiation: generates an infinite linear system

[Chetyrkin, Tkachov]

# Going down the ladder

The IBP relation provide a way to reach a basis with low powers

**Example:** massless bubble integral

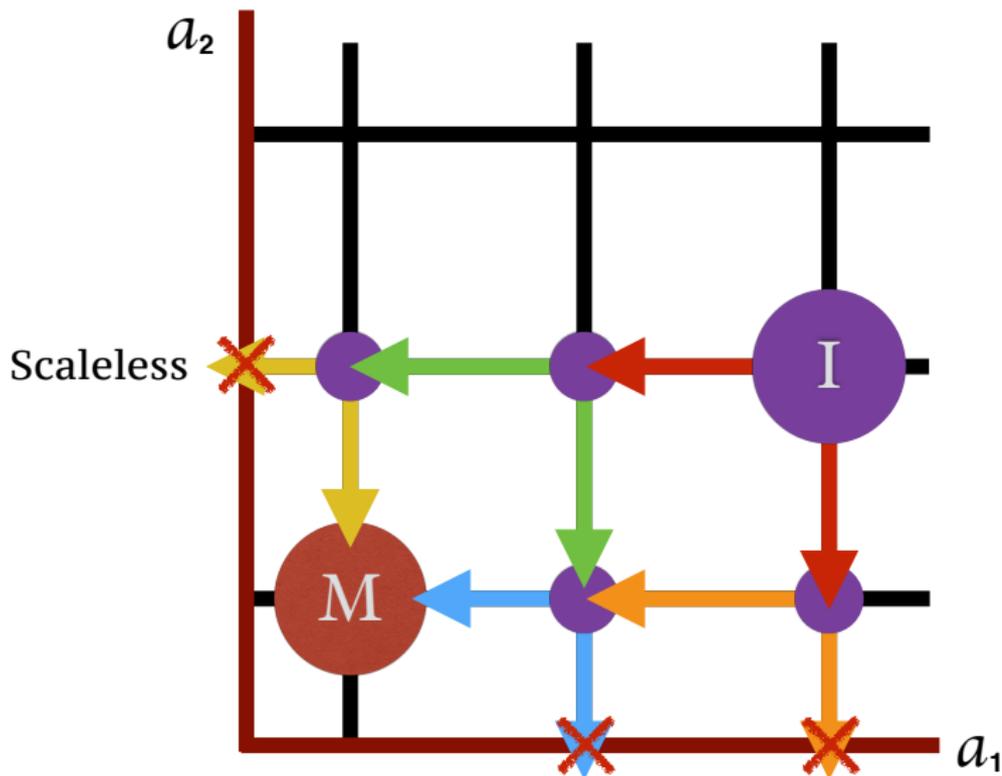
$$I(n_1, n_2) = \int d^d k \frac{1}{(k^2)^{n_1} ((k+p)^2)^{n_2}}$$

Two relations:

$$\begin{aligned} I(n_1, n_2) &= \frac{n_1 + n_2 - d - 1}{p^2(n_2 - 1)} I(n_1, n_2 - 1) + \frac{1}{p^2} I(n_1 - 1, n_2) \\ &= \frac{n_1 + n_2 - d - 1}{p^2(n_1 - 1)} I(n_1 - 1, n_2) + \frac{1}{p^2} I(n_1, n_2 - 1) \end{aligned}$$

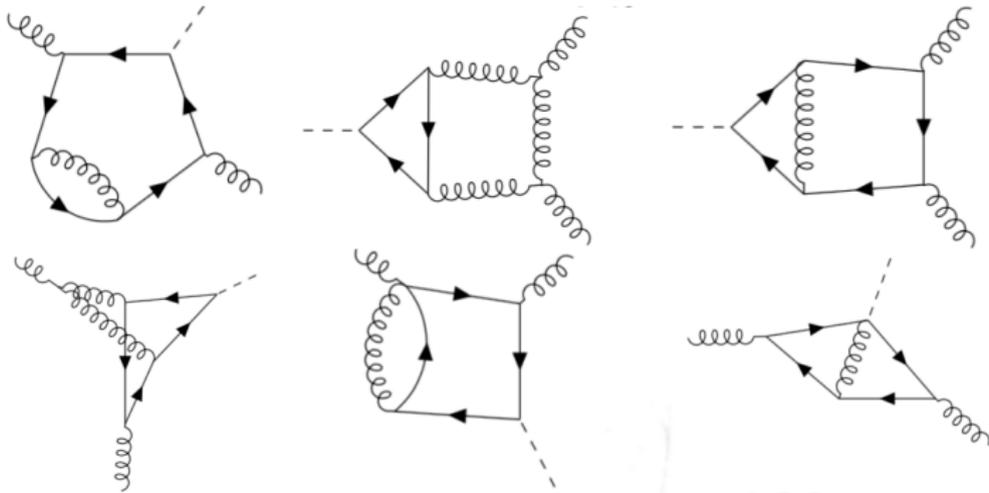
# Going down the ladder

Starting from one integral, one can always go back to  $I(1, 1)$ .



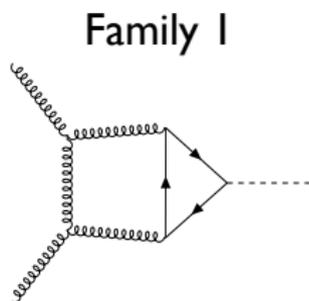
# Our calculation in numbers: diagrams

21 diagrams for  $C_1$  and 75 for  $C_3$

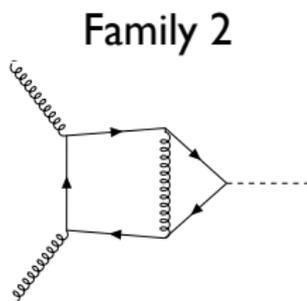


# Our calculation in numbers: integrals and masters

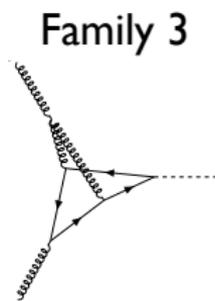
The integrals in our amplitude fall into three families:



821 Integrals



635 Integrals

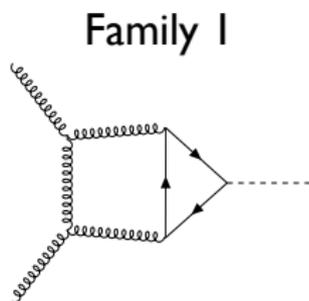


513 Integrals

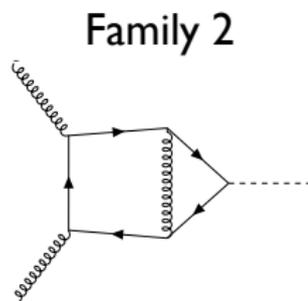
Total number of master integrals : 17

# Our calculation in numbers: integrals and masters

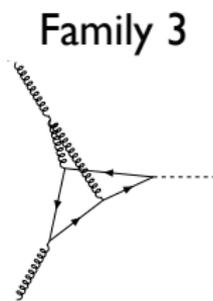
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All known from SM calculation

[Anastasiou, Beerli, Bucherer, Daleo, Kunstz]

[Aglietti, Bonciani, Degrassi, Vicini]

# Master integrals in terms of polylogarithms

Many master integrals can be expressed in terms of iterated integrals called multiple polylogarithms:

$$G(a_1, \dots, a_n; x) = \int_0^x \frac{dt}{t - a_1} G(a_2, \dots, a_n, dt)$$

With  $G(a; x) = \log(1 - x/a)$

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With  $G(a; x) = \log(1 - x/a)$

All our integrals are expressed in terms of MPLs with entries in  $\{0, \pm 1\}$ , depending on the dimensionless variable

$$x = \frac{\sqrt{4m_t^2 - s} - \sqrt{s}}{\sqrt{4m_t^2 - s} + \sqrt{s}}$$

# Conclusion and outlook

We have reached the final stages of the calculation of the NLO correction to the Higgs production cross-section by gluon fusion in the SMEFT

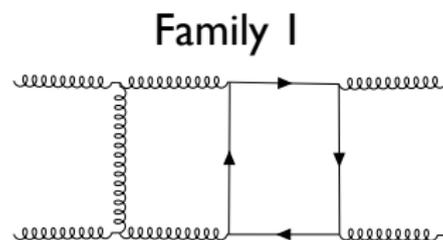
- Finished the calculation of the two-loop renormalized amplitude
- Obtained the new two-loop counter-term
- On our way to combine the virtuals with real emissions to compute the cross-section

Expected improvement on the precision and accuracy of the prediction, paving the way for future investigations of the Higgs boson properties.

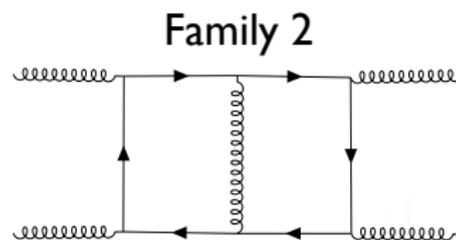
# Thank you for your attention

# Precise definition of the families

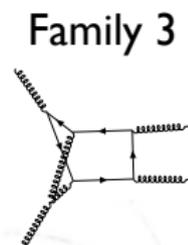
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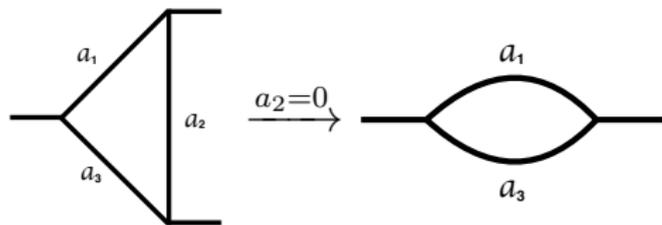
513 Integrals

# Integral families: example

One-loop triangle scalar integral family:

$$I(a_1, a_2, a_3) = \int d^d k \frac{1}{(k_1^2)^{a_1} ((k_1 + p_1)^2)^{a_2} ((k_1 + p_1 + p_2)^2)^{a_3}}$$

Contains sub-topologies:



Numerator example:

$$\int d^d k \frac{k_1 \cdot p_1}{(k_1^2)(k_1 + p_1)^2(k_1 + p_1 + p_2)^2} = \int d^d k \frac{1}{2} \frac{1}{(k_1^2)(k_1 + p_1 + p_2)^2} - \frac{1}{2} \frac{1}{(k_1 + p_1)^2(k_1 + p_1 + p_2)^2}$$