Status report on HH

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LHCTheory ERC Meeting
CP3 Louvain-La-Neuve, Mar 23th 2017

Probing the nature of the Higgs boson

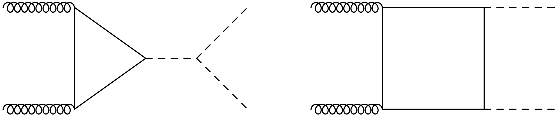
- Higgs boson discovered
- Couplings to fermions so far consistent with SM prediction
- Does the predicted SM Brout-Englert-Higgs potential

$$V(\phi^\dagger\phi) = -\,\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2\,, \quad \lambda, \mu > 0$$

  match what we observe?
- deduction of SM trilinear coupling $\lambda_{HHH} = \frac{m_H^2}{2v}$ directly probes the structure of the potential
- largest Higgs pair production cross section in gluon fusion
- still low experimental precision expected at high-luminosity LHC BUT accurate differential prediction still needed

Higgs-boson pair production in gluon fusion

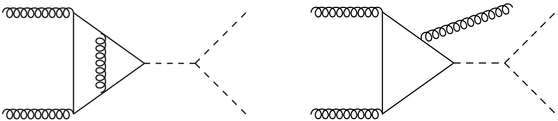
- the leading order is loop induced Glover, van der Bij '88



- Higher-order corrections were as of Apr 2016 only known in approximations

$gg \to hh$ @ NLO with full top mass dependence

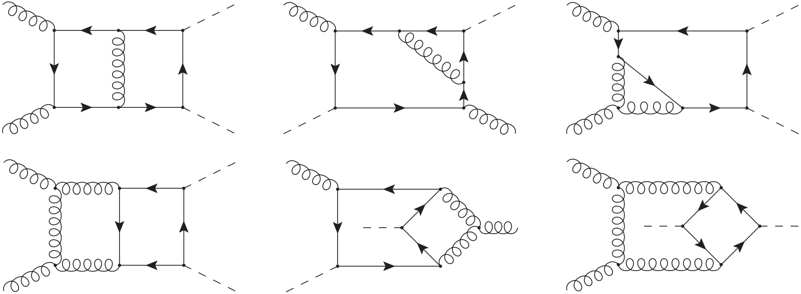
- NLO with full top mass dependence involves the computation of two-loop triangles and one-loop box diagrams (known from single Higgs production), e.g.



$gg \to hh$ @ NLO with full top mass dependence

Higher order correction to box-type LO diagram also needed:

- one-loop pentagon integrals
- unknown two-loop integrals (with 4 independent mass scales: $\hat{s}$, $\hat{t}$, m_t^2, m_h^2 or 3 ratios), e.g.



with up to 4 scalar products $\to$ reduction of virtual 2-loop amplitude to master integrals is highly non-trivial

Higher-order approximations for $gg \to hh$

- NLO $m_t \to \infty$ limit
  Plehn, Spira, Zerwas '96; Dawson, Dittmaier, Spira '98
- NLO $m_t \to \infty$, supplemented with $1/m_t$ expansion
  Grigo, Hoff, Melnikov, Steinhauser '13; Degrassi, Giardino, Gröber '16
- NNLO $m_t \to \infty$ limit De Florian, Mazzitelli '13
- NNLO $m_t \to \infty$ + all matching coefficients
  Grigo, Melnikov, Steinhauser '14
- Full mass dependence in real radiation part + matching to parton shower Frederix, Hirschi, Mattelaer, Maltoni, Torrielli, Vryonidou, Zaro '14; Maltoni, Vryonidou, Zaro '14
- NNLO $m_t \to \infty$ + all matching coefficients + top quark mass effects Grigo, Hoff, Steinhauser '15
- NNLO $m_t \to \infty$ + NNLL threshold resummation
  De Florian, Mazzitelli '15
- NNLO $m_t \to \infty$ (differential) De Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev '16

Virtual two-loop amplitude

- virtual amplitude generated with GoSam-XL
  - diagram generation with Qgraf Nogueira '93 (≈10000 different integrals before accounting for symmetries)
  - further processed with FORM Vermaseren '00; Kuipers, Ueda, Vermaseren '12
  - python interface to Reduze von Manteuffel, Studerus '12
- reduction up to non-planar 6- and planar 7-propagator topologies with Reduze, partially transformed into finite basis Panzer '14; von Manteuffel, Panzer, Schabinger '14
  $\rightarrow$ 228 planar master integrals
- 99 non-planar integrals
- integrals calculated numerically with SecDec 3 SB, Heinrich, Jones, Kerner, Schlenk, Zirke '15 using a dedicated integration setup Jones '16, Kerner '16

Numerical integration of two-loop amplitude

Idea and method of sector decomposition pioneered by
Hepp '66, Denner & Roth '96, Binoth & Heinrich '00

So far SecDec has been used for...

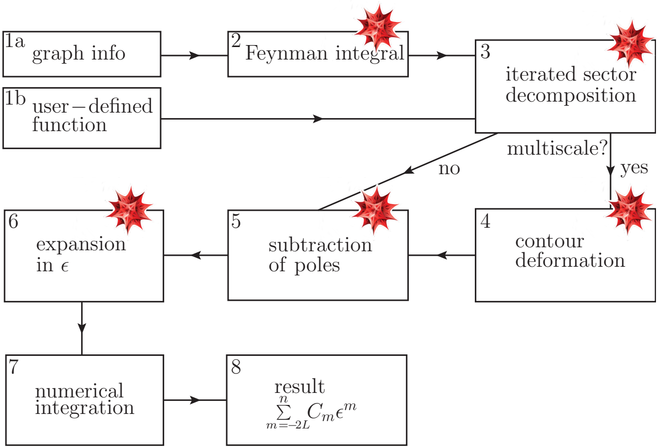
- fast evaluation of massive bubbles (34 mass topologies, up to 5 scales) to calculate $\mathcal{O}(\alpha_s \alpha_t)$ self-energy contributions to the MSSM Higgs-boson masses

  SB, Hahn, Heinemeyer, Heinrich, Hollik '14

- checks of analytically calculated integrals

NEW:
use SecDec as a library to numerically compute all 327 integrals contributing to the Higgs-pair production amplitude

Outline of the program SecDec



numerical integration: CUBA library Hahn '04, NIntegrate Wolfram

NEW: QMC Jones '16

Further important prerequisites

Important new steps:

- use a quasi-finite basis where possible
  Panzer '14; von Manteuffel, Panzer, Schabinger '14
- evaluate integrals numerically with Quasi Monte Carlo integrator Dick, Kuo, Sloan '13; Li, Wang, Zan, Zhao '15 and using GPUs
- only integrate up to the necessary accuracy
  - set number of sampling points dynamically for each integral
  - target accuracy is set at amplitude level (3% for one form factor, ≈10% for the other, depending on the ratio of the two)

Also:

- be pragmatic: leave some Feynman integrals unreduced and compute them individually

Real radiation at NLO

- four real radiation channels

$$gg \to hh + g \;, \quad gq \to hh + q \;,$$
$$g\bar{q} \to hh + \bar{q} \;, \quad q\bar{q} \to hh + g$$

- 1-loop amplitudes generated and processed with GoSam
  Cullen, van Deurzen, Greiner, Heinrich, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Reiter, Schlenk, von Soden-Fraunhofen, Tramontano; '11 '14
- Catani-Seymour dipole formalism used for IR subtraction
  Catani, Seymour '96
- numerical integration using Vegas Lepage '80 of CUBA library Hahn '04

Checks

- independent calculation of (unreduced) amplitude
- checked invariance under exchange of $\hat{t}$ and $\hat{u}$
- several two-loop integrals recomputed with Vegas
- single Higgs production reproduced, comparison to SusHi
  Harlander, Liebler, Mantler '13 '16
- numerical pole cancellation (tested for up to 5 digits accuracy)
- independence of dipole parameter α Nagy '03
- comparison of $1/m_t$ expansion with Jens Hoff
  Grigo, Hoff, Steinhauser '15
- comparison of HEFT result to MG5_aMC@NLO
  Maltoni, Vryonidou, Zaro '14 '15

Total cross section @ 14 TeV by courtesy of Stephen Jones

	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)
HEFT	$17.07^{+30.9\%}_{-22.2\%}$	$31.93^{+17.6\%}_{-15.2\%}$	$37.52^{+5.2\%}_{-7.6\%}$
B.I. HEFT	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$	$43.63^{+5.2\%}_{-7.6\%}*$
FTapprox	$19.85^{+27.6\%}_{-20.5\%}$	$34.26^{+14.7\%}_{-13.2\%}$	–
Full Theory	$19.85^{+27.6\%}_{-20.5\%}$	$32.91^{+13.6\%}_{-12.6\%}$	–
N.I. HEFT	–	$32.91^{+13.6\%}_{-12.6\%}$	$38.67^{+5.2\%}_{-7.6\%}*$

PDF4LHC15_nlo_30_pdfas
$m_H = 125$ GeV
$m_T = 173$ GeV
Uncertainty:
$\mu_R = \mu_F = \frac{m_{HH}}{2}$
$\mu \in \left[\frac{\mu_0}{2}, 2\mu_0\right]$ (7 − point)

* re-weighted on total cross-section level

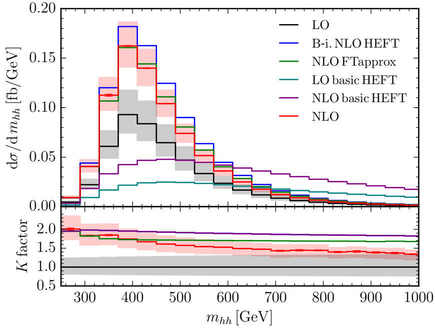
de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16;
Maltoni, Vryonidou, Zaro 14 (recalculated by us); Borowka, Greiner, Heinrich, Kerner, Schlenk, Schubert, Zirke 16;
Dawson, Dittmaier, Spira 98 (recalculated by us); Glover, van der Bij 88 (recalculated by us)

Comparison to Full Theory

	$\Delta\sigma^{\mathrm{Full}}_{\mathrm{LO}}$	$\Delta\sigma^{\mathrm{Full}}_{\mathrm{NLO}}$
HEFT	−14%	−3.0%
B.I. HEFT	0%	+16%
FTapprox	0%	+4.1%

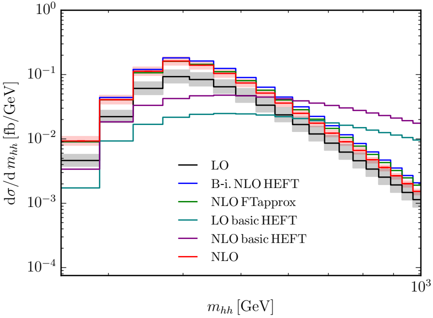
Can do a similar exercise @ 100 TeV, differences typically larger

Comparison I: full result to HEFT approximations



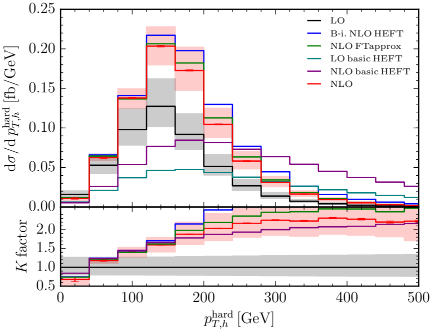
- 913 phase-space points with ∼16 dual NVIDIA TESLA K20X GPGPU nodes → total of ∼9 days runtime (median GPU time per PS point: 2 hours)
- NLO HEFT good approximation for $m_{hh} < 2m_t$
- scale uncertainties of HEFT and FT_{approx} do not enclose central value of full result in m_{hh} tail → HEFT breaks down

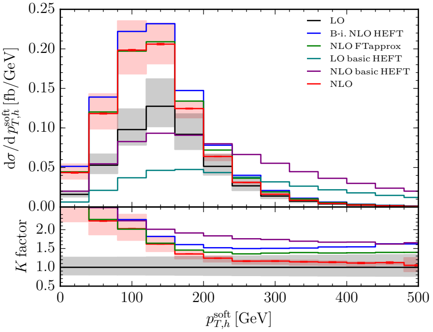
Comparison I: Scaling behavior



- Scaling behavior at LO
  - exact top-mass dependence: $\hat{\sigma} \sim \hat{s}^{-1}$
  - Higgs EFT: $\hat{\sigma} \sim \hat{s}$
- Scaling behavior at NLO matches the one at LO

Comparison I: full result to HEFT approximations





- larger discrepancies between HEFT and exact result for $p_{T,h}^{\text{soft}}$

Comparison II: virtual amplitude vs. $1/m_t^{2\rho}$ approximations

- Expansion of the virtual NLO matrix element

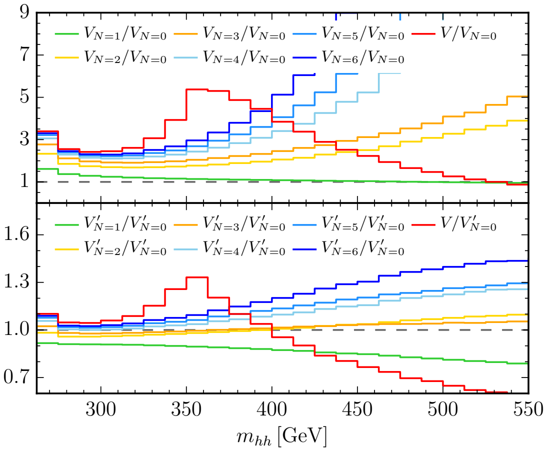
$$\mathrm{d}\hat{\sigma}^V_{\exp,N} = \sum_{\rho=0}^{N} \mathrm{d}\hat{\sigma}^{V(\rho)} \left(\frac{\Lambda}{m_t}\right)^{2\rho}, \quad \Lambda \in \left\{\sqrt{\hat{s}}, \sqrt{\hat{t}}, \sqrt{\hat{u}}, m_H\right\}$$

- Born-improved finite part of virtual amplitude

$$\mathrm{d}\sigma^V_{\exp,N} \frac{d\sigma^{LO}(\varepsilon)}{d\sigma^{LO}_{\exp,N}(\varepsilon)} + \mathrm{d}\sigma^{LO}(\varepsilon) \otimes \mathbf{I}$$

$$= \underbrace{\underbrace{\left(\mathrm{d}\sigma^V_{\exp,N} + \mathrm{d}\sigma^{LO}_{\exp,N}(\varepsilon) \otimes \mathbf{I}\right)}_{\equiv V_N} \frac{d\sigma^{LO}}{d\sigma^{LO}_{\exp,N}}}_{\equiv V'_N} + \mathcal{O}(\varepsilon).$$

Comparison II: full result to $1/m_t^{2\rho}$ approximations

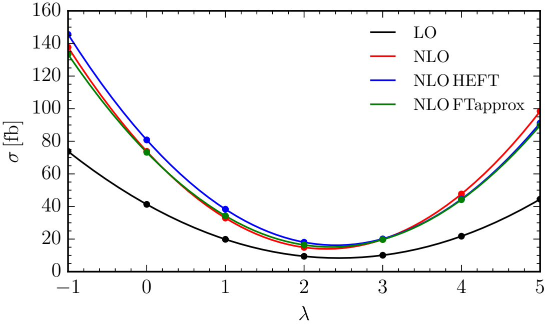


$N < 4$ Tom Zirke,

$N = 4, 5, 6$ thanks to Jens Hoff

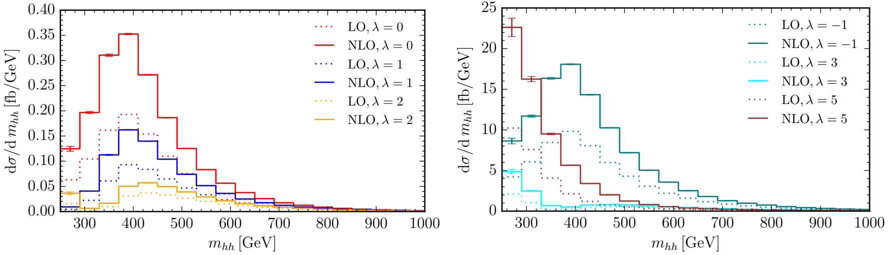
$\rightarrow$ shapes very different beyond $m_{hh} \sim 2m_t$

Dependence on trilinear coupling l at 14 TeV



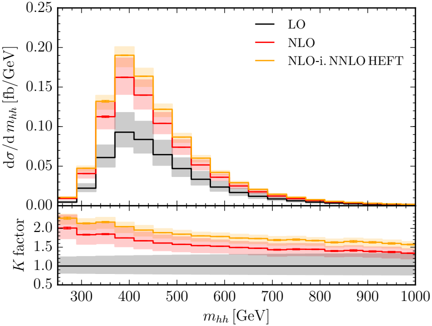
- variation of total cross section with $\lambda = \frac{\lambda_{HHH}}{3m_h^2}$ is $\sim$ 120fb
- $\lambda = 2$: Destructive interference largest at LO Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira '12, same for NLO

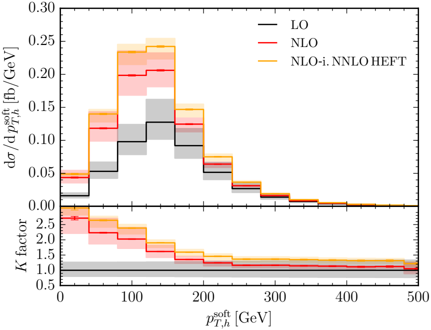
Dependence on trilinear coupling II at 14 TeV



- $\lambda = 0$: no triple Higgs coupling, $\lambda = 1$: Standard Model
- $\lambda = 2$: destructive interference maximal
- $\lambda = 5$: differential XS dominated by self-coupling contributions

Comparison III: full result to HEFT approximations





$$d\sigma^{\text{NLO-i. NNLO HEFT}} = d\sigma^{\text{NLO}} \frac{d\sigma^{\text{NNLO basic HEFT}}}{d\sigma^{\text{NLO basic HEFT}}}$$

NNLO to NLO ratio from de Florian, Grazzini, Hanga, Kallweit, Lindert, Meierhöfer, Mazzitelli, Rathlev '16

Summary

- We calculated the total cross section and distributions for Higgs pair production in gluon fusion at NLO with full top-quark mass dependence.
- Evaluation of integrals done fully numerically using SecDec in dedicated integration setup
- HEFT does not reproduce shapes beyond top-threshold
- tail of m_{HH} distributions: differences wrt. Born-improved HEFT approximation $\gtrsim 50\%$, and $\sim 20\%$ wrt. FT_{approx} result
- variation of shapes with λ analyzed
- inclusion of the full top-quark mass dependence vital for reliable Higgs-boson pair production predictions over full invariant mass range

Outlook

- apply setup to other processes (e.g. pp $\rightarrow$ Hg, gg$\rightarrow$ ZZ, gg$\rightarrow \gamma\gamma$, gg$\rightarrow$ WW)
- make PySecDec publicly available
  - generate integral libraries
  - efficiency gains: Mathematica (& Perl) code replaced with Python code

Thank you.

Backup

Quasi-finite Basis Panzer '14; von Manteuffel, Panzer, Schabinger '14

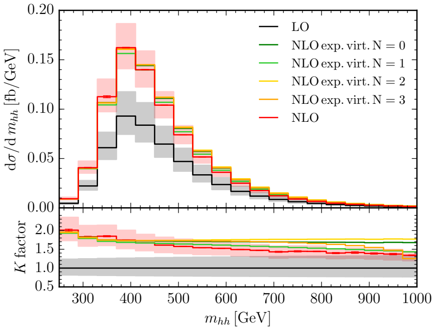
(Scalar) Multi-loop Feynman integral in Feynman parametrization:

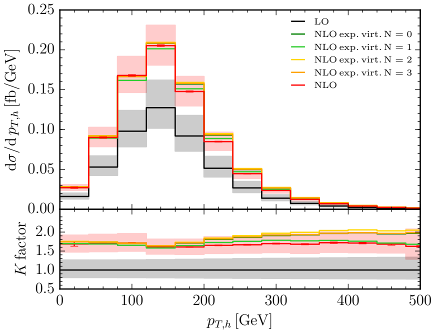
$$G = \frac{(-1)^{N_\nu}}{\prod_{j=1}^{N}\Gamma(\nu_j)}\Gamma(N_\nu - LD/2)\int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j\; x_j^{\nu_j-1}\,\delta(1-\sum_{l=1}^{N} x_l)\frac{\mathcal{U}(\vec{x})^{N_\nu-(L+1)D/2}}{\mathcal{F}(\vec{x},s_{ij})^{N_\nu-LD/2}}$$

with $N_\nu = \sum_{j=1}^{N}\nu_j$ in D dimensions with L loops, N propagators to power ν_j

- Appearance of UV and IR divergences depend on N_ν, L and D
- Add dots on propagators and/or shift the dimension until the integral is finite
- facilitates numerical integration due to lack of subtraction terms

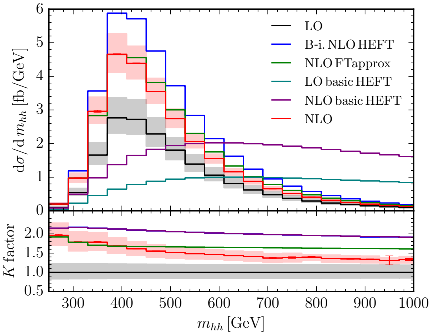
Improved FT_{approx}

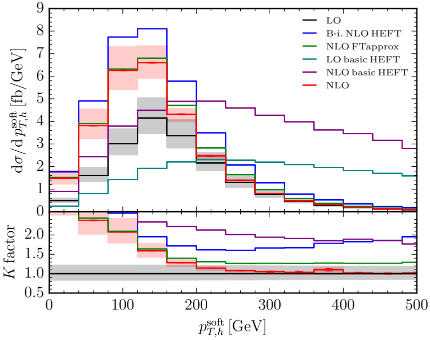




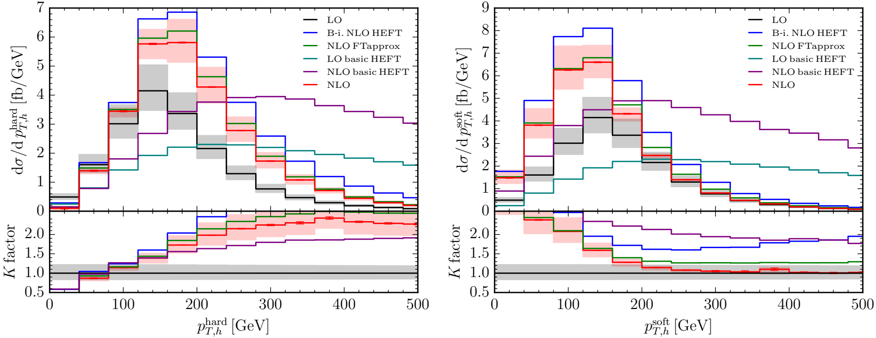
- FT_{approx} is supplemented with $1/m_t^2$ corrections
- No improvement beyond the top threshold

m_{HH} distributions at 100 TeV





p_T^H distributions at 100 TeV



Variation of $\lambda_{HHH} = \lambda$ at 100 TeV