An overview of $\mathcal{N} = 4$ SYM

LHCTheory ERC Meeting

UCLouvain - Mar 23rd, 2017

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CERN

Aim

- What is $\mathcal{N} = 4$ SYM?
- Why study it?

Field content

Maximal supersymmetry in $d = 4$

$$Q^I, \quad \bar{Q}_I \quad I = 1, 2, 3, 4$$

Field content

Maximal supersymmetry in $d = 4$

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g

$h = \quad +1$

Field content

Maximal supersymmetry in $d = 4$

$$Q^I, \quad \bar{Q}_I \quad I = 1, 2, 3, 4$$

$$\xrightarrow{Q^I}$$

$$g \qquad \psi^1 \quad \psi^2 \quad \psi^3 \quad \psi^4$$

$$h = \quad +1 \qquad\qquad +\frac{1}{2}$$

Field content

Maximal supersymmetry in $d = 4$

$$Q^I, \quad \bar{Q}_I \quad I = 1, 2, 3, 4$$

Q^I → Q^J →

		ϕ^{12}
	ψ^1	ϕ^{13}
	ψ^2	ϕ^{14}
g	ψ^3	ϕ^{23}
	ψ^4	ϕ^{24}
		ϕ^{34}

$$h = \quad +1 \qquad\qquad +\frac{1}{2} \qquad\qquad 0$$

Field content

Maximal supersymmetry in $d = 4$

$$Q^I, \quad \bar{Q}_I \quad I = 1, 2, 3, 4$$

	$\xrightarrow{Q^I}$	$\xrightarrow{Q^J}$	$\xrightarrow{Q^K}$
		ϕ^{12}	
	ψ^1	ϕ^{13}	$\bar{\psi}^{123}$
	ψ^2	ϕ^{14}	$\bar{\psi}^{124}$
g			
	ψ^3	ϕ^{23}	$\bar{\psi}^{134}$
	ψ^4	ϕ^{24}	$\bar{\psi}^{234}$
		ϕ^{34}	
$h = +1$	$+\frac{1}{2}$	0	$-\frac{1}{2}$

Field content

Maximal supersymmetry in $d = 4$

$$Q^I, \quad \bar{Q}_I \quad I = 1,2,3,4$$

$$g \xrightarrow{Q^I} \psi^1, \psi^2, \psi^3, \psi^4 \xrightarrow{Q^J} \phi^{12}, \phi^{13}, \phi^{14}, \phi^{23}, \phi^{24}, \phi^{34} \xrightarrow{Q^K} \bar{\psi}^{123}, \bar{\psi}^{124}, \bar{\psi}^{134}, \bar{\psi}^{234} \xrightarrow{Q^L} g^{1234}$$

$$h = \quad +1 \qquad +\frac{1}{2} \qquad 0 \qquad -\frac{1}{2} \qquad -1$$

Field content

$$\Phi = g^+ + \eta_I \psi^I + \frac{1}{2}\eta_I \eta_J \phi^{IJ} + \frac{1}{3!}\eta_I \eta_J \eta_K \bar{\psi}^{IJK} + \eta_1 \eta_2 \eta_3 \eta_4 g^{1234}$$

All particles combined into one superfield – Nair '88 –

η_I is a fermionic variable

$$A_n = A(p_i, \eta_I^i), \quad i = 1, \ldots, n$$

Properties

- Only one multiplet, $m^2 = 0$
- Gauge group $SU(N)$, coupling constant g_{YM}
- $\beta_{g_{\mathrm{YM}}} = 0$ to all loops $\Rightarrow$ Quantum conformal symmetry
- Conformal symmetry + SUSY = Superconformal symmetry

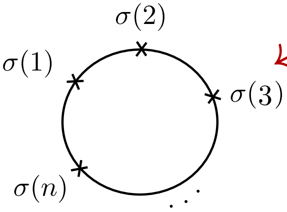
Properties

- "Planar limit" $N \to \infty$
- AdS/CFT correspondence – Maldacena '97 –
- Integrability

generators of $SU(N)$

$$\mathcal{A}_n^{\text{planar}} \propto \sum_{\sigma \in S_n/\mathbb{Z}_n} \mathrm{Tr}(t^{a_{\sigma(1)}} t^{a_{\sigma(2)}} \dots t^{a_{\sigma(n)}}) \underbrace{A_n(\sigma(1), \sigma(2), \dots, \sigma(n))}_{\text{partial amplitude (colour ordered)}}$$

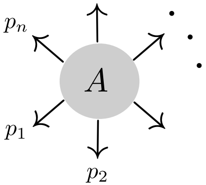
partial amplitude
(colour ordered)



Amplitude

$A(p_1, \ldots, p_n)$

$p_i^2 = 0$



Notation:

$$p^2 = 0 \Rightarrow p = \lambda\tilde{\lambda}$$

$$\langle ij \rangle = \lambda_i^a \lambda_j^b \epsilon_{ab}$$

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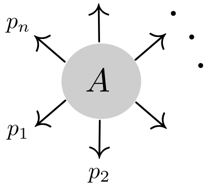

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$$p^2 = 0 \Rightarrow p = \lambda\tilde{\lambda}$$

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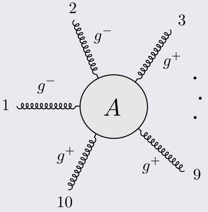
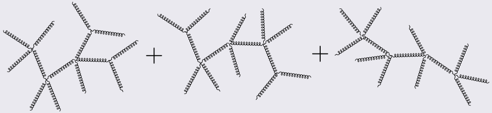
Hidden simplicity: – Parke, Taylor '86 / Mangano, Parke, Xu '88 –



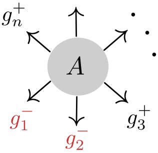
Feynman diagrams



+10 million terms

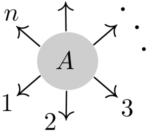
Result

$$\frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle \cdots \langle 9\,10\rangle\langle 10\,1\rangle}$$

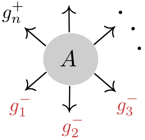


= Maximally Helicity Violating (MHV) = $\dfrac{\langle 12\rangle^2}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$

SUSY $\rightarrow$

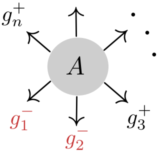


$$= \frac{\delta^8\left(\sum_i \lambda_i \eta_i\right)}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$



= Next-to-MHV (NMHV)

. . .



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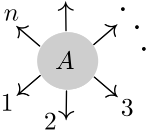
SUSY →



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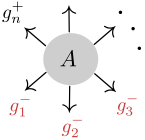


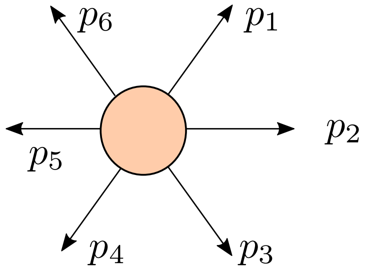
= Next-to-MHV (NMHV)

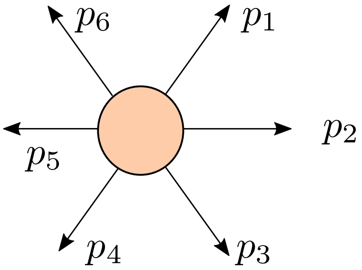
. . .

Recursion relations: – Britto, Cachazo, Feng, Witten '05 / Cachazo, Svrček and Witten '04 –

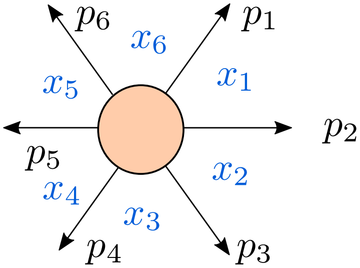
BCFW: N^kMHV amplitude in terms of A_3^{MHV} and $A_3^{\overline{\mathrm{MHV}}}$

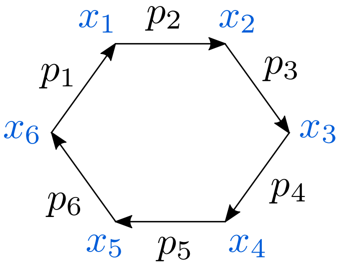
CSW: N^kMHV amplitude in terms of A_n^{MHV}


p_1
p_2
p_3
p_4
p_5
p_6


p_6
p_1
p_2
p_5
p_4
p_3
p_2
p_1
p_3
p_4
p_6
p_5

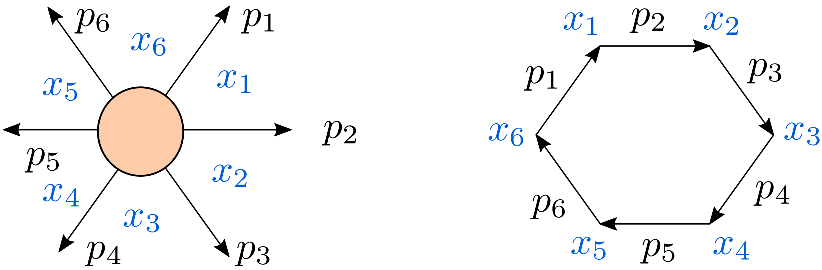
Symmetries and Dualities





$$p_i = x_{i+1} - x_i$$

Symmetries and Dualities

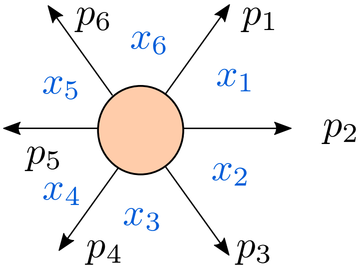


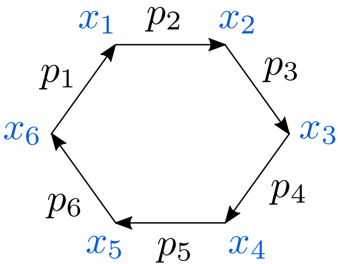
$$p_i = x_{i+1} - x_i$$

Dual conformal symmetry – Drummond, Henn, Korchemsky, Sokatchev '08 –

superconformal + dual superconformal = ∞-dimensional symmetry algebra (Yangian)
– Drummond, Henn, Plefka '09 –

Symmetries and Dualities



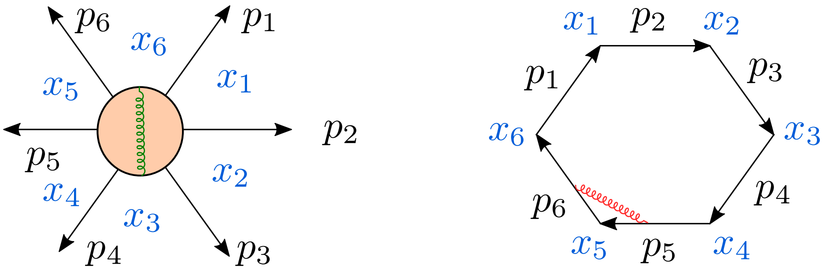


$$p_i = x_{i+1} - x_i$$

Amplitude - Wilson loop duality – Drummond, Korchemsky, Sokatchev / Brandhuber, Heslop, Travaglini '07 –

$$\log \frac{A_n^{\text{MHV}}}{A_n^{\text{MHV,tree}}} \quad \leftrightarrow \quad \log \mathcal{W}_{\mathcal{C}} \equiv \log \left\langle \mathrm{Tr} P \exp ig \oint_{\mathcal{C}} A_\mu dx^\mu \right\rangle$$

Symmetries and Dualities



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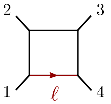
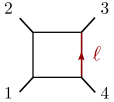
IR divergences $(m^2 = 0)$ $\leftrightarrow$ UV divergences (cusps)

Consequences: Integrands

Dual conformal symmetry is preserved at the integrand level ⇒ Definition of *Integrand*



$$\leftrightarrow \quad \int d^4\ell \frac{1}{\ell^2(\ell+p_1)^2(\ell+p_1+p_2)^2(\ell-p_4)^2}$$



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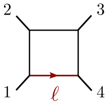

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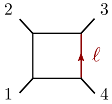

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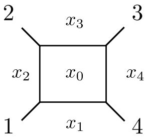
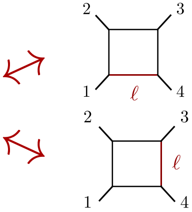


$$= \int d^4x_0 \frac{1}{x_{01}^2 x_{02}^2 x_{03}^2 x_{04}^2}$$



$\ell = x_{01}$

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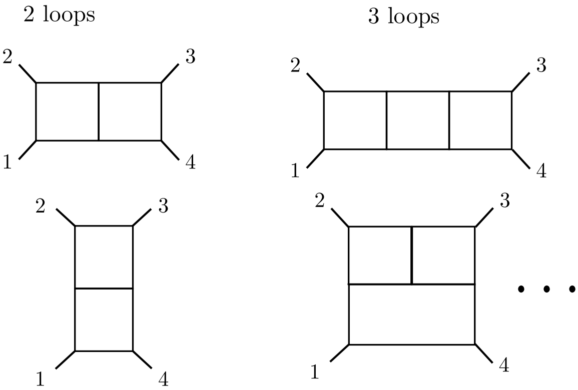
Consequences: Integrands



Topologies are constrained ⇒ Generalised unitarity methods

Consequences: Integrands

- Integrand → All-loop recursion relation
  – Arkani-Hamed, Bourjaily, Cachazo, Caron-Huot, Trnka '10 –
- Alternative geometric representation:
  - Positive Grassmannian – Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, Trnka '12 –
  - Amplituhedron – Arkani-Hamed, Trnka '13 –

Consequences: Integrals

- IR divergent part of the planar MHV amplitude resummed to all loops:

  – Bern, Dixon, Smirnov / Anastasiou, Bern, Dixon, Kosower –

$$\mathcal{M}_n^{\mathrm{MHV}} = \exp\Big[\sum_{L=1}^{\infty} a^L \big(f^{(L)}(\epsilon) M_n^{(1)}(L\epsilon) + C^{(L)} + \mathcal{O}(\epsilon)\big)\Big]$$

$$M_n^{(L)} \equiv \frac{A_n^{\mathrm{MHV}(L)}}{A_n^{\mathrm{MHV}(0)}}, \quad d = 4 - 2\epsilon$$

- $f^{(L)}$ function of cusp and collinear anomalous dimensions

Iterative structure at higher loops

$$\mathcal{M}_n^{\mathrm{MHV}} = \exp\Big[\sum_{L=1}^{\infty} a^L \big(f^{(L)}(\epsilon) M_n^{(1)}(L\epsilon) + C^{(L)} + \mathcal{O}(\epsilon)\big)\Big]$$

- Captures all IR divergent behaviour
- Exact for $n = 4, 5 \Rightarrow$ Amplitudes constrained to all orders
- Starting at $n = 6$ $\rightsquigarrow$ Remainder function R_n^{MHV}
- R_6^{MHV} is a function of 3 cross ratios:

$$u = \frac{s_{12}s_{45}}{s_{123}s_{345}}, \quad v = \frac{s_{23}s_{56}}{s_{234}s_{456}}, \quad w = \frac{s_{45}s_{16}}{s_{345}s_{561}}$$

Amplitude six-point two-loop remainder

$$\mathcal{R}_6^{(2)} = M_6^{(2)} - \underbrace{\left[\tfrac{1}{2}(M_6^{(1)}(\epsilon))^2 + f^{(2)}(\epsilon)M_6^{(1)}(2\epsilon) + C^{(2)}\right]}_{\text{BDS ansatz}}$$

$$f^{(2)}(\epsilon) = -2(\zeta_2 + \epsilon\zeta_3 + \epsilon^2\zeta_4)\,, \quad C^{(2)} = -\zeta_2^2$$

Del Duca, Duhr and Smirnov $\rightsquigarrow$ 17-page-long combination of transcendental functions

Goncharov, Spradlin, Vergu, Volovich $\rightsquigarrow$ Identities and cancellations give a single line expression with only classical polylogs

Ex: 2 loops: $\mathrm{Li}_4(x),\ \log^4(x),\ (\mathrm{Li}_2(x))^2$

$\rightarrow$ More hidden simplicity

Bypassing integrands and integrals: Bootstrap approach

Assumption: MHV remainder functions at L loops is a combination of *transcendental functions* of weight $2L$

This + physical constraints $\Rightarrow$ results up to $L = 5$, $n = 6$ and $L = 4$, $n = 7$

– Caron-Huot, Dixon, McLeod, von Hippel '16 /

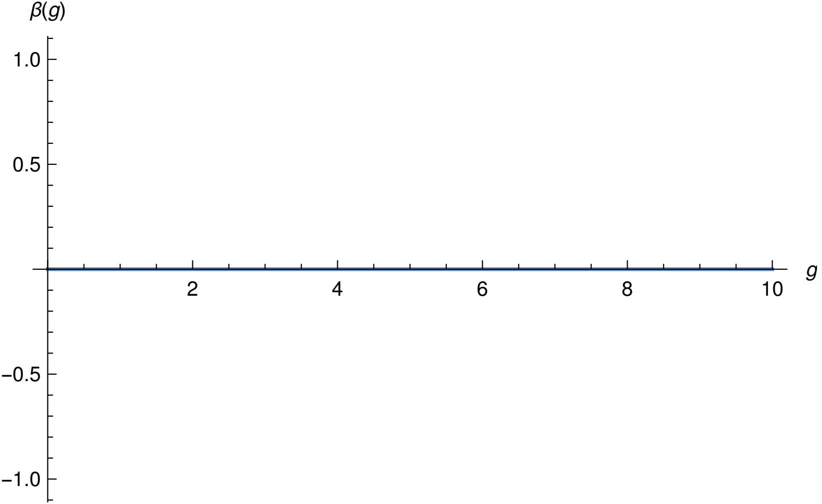
Dixon, Drummond, Harrington, McLeod, Papathanasiou, Spradlin '17 –

(also results beyond MHV)

Summary & take home message

$\mathcal{N} = 4$ SYM is a QFT with very special properties:

- Simple enough so that analytical results are manageable, but not too simple to be trivial
- Great context on which to test new ideas
- "Toy model" for QCD
- Promote development of methods applicable for general field theories
- Interesting from mathematical perspective


β(g)
1.0
0.5
−0.5
−1.0
2
4
6
8
10
g