BERNHARD MISTLBERGER

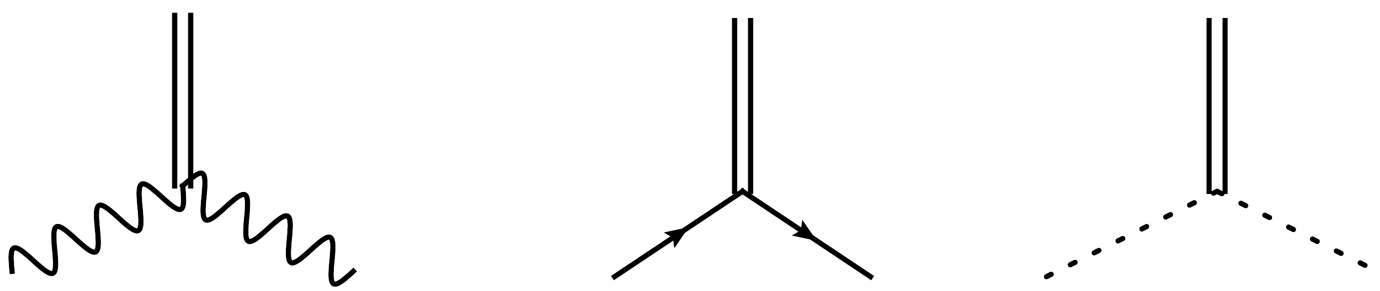
HIGGS-DIFFERENTIAL CROSS SECTIONS

STEPS TO DIFFERENTIAL N3LO

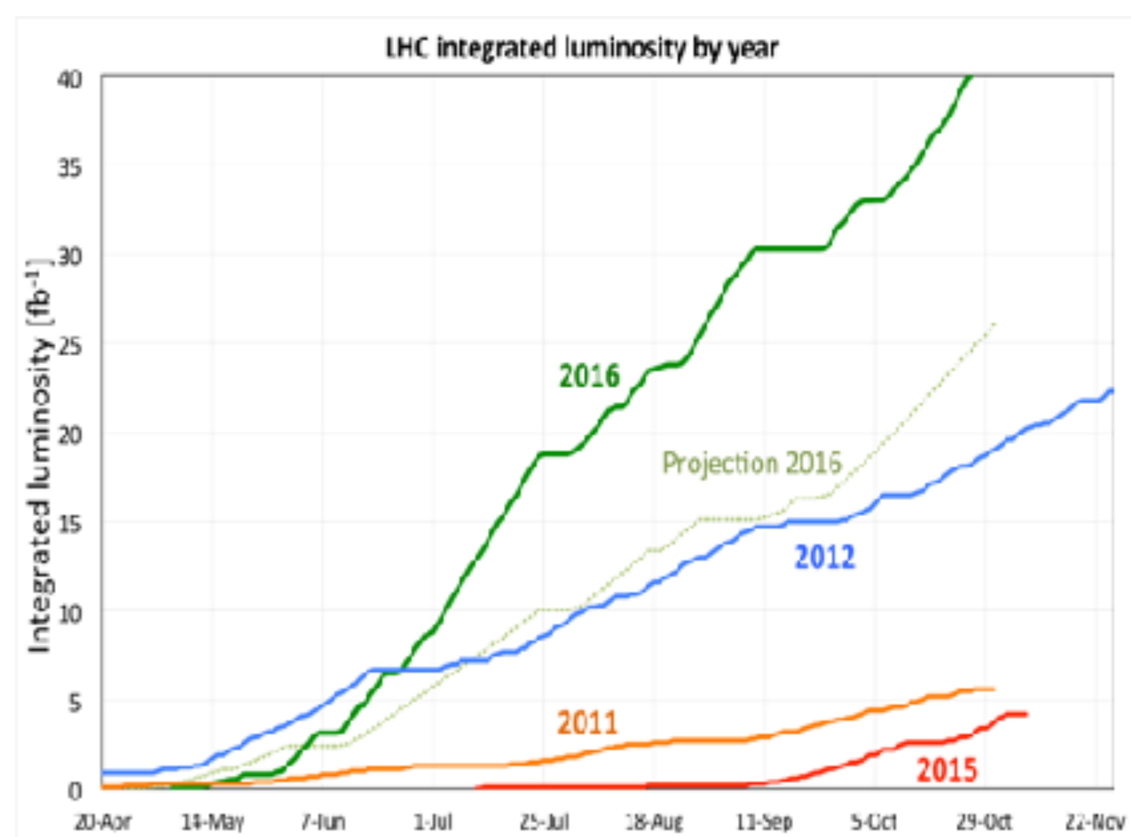
with Babis Anastasiou, Falko Dulat, Simone Lionetti, Andrea Pelloni, Caterina Specchia
[arXiv:1704.08220]

- 4th of July 2012: The begin of the precision physics age of Higgs boson phenomenology

- Immediately after the discovery of the Higgs boson we started to ask questions about it’s nature:
  Couplings, spin, parity, mass, cross sections …

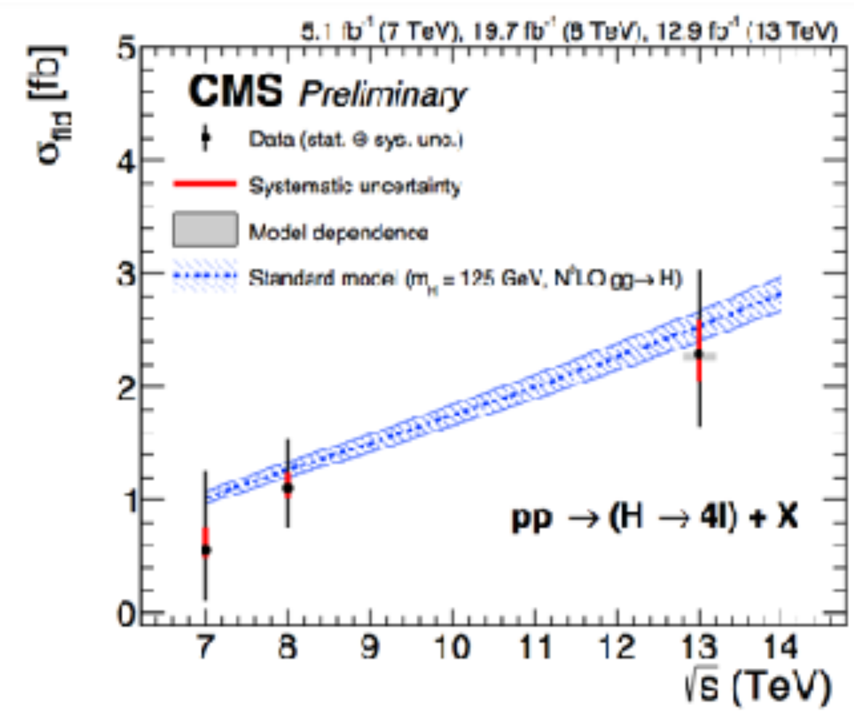


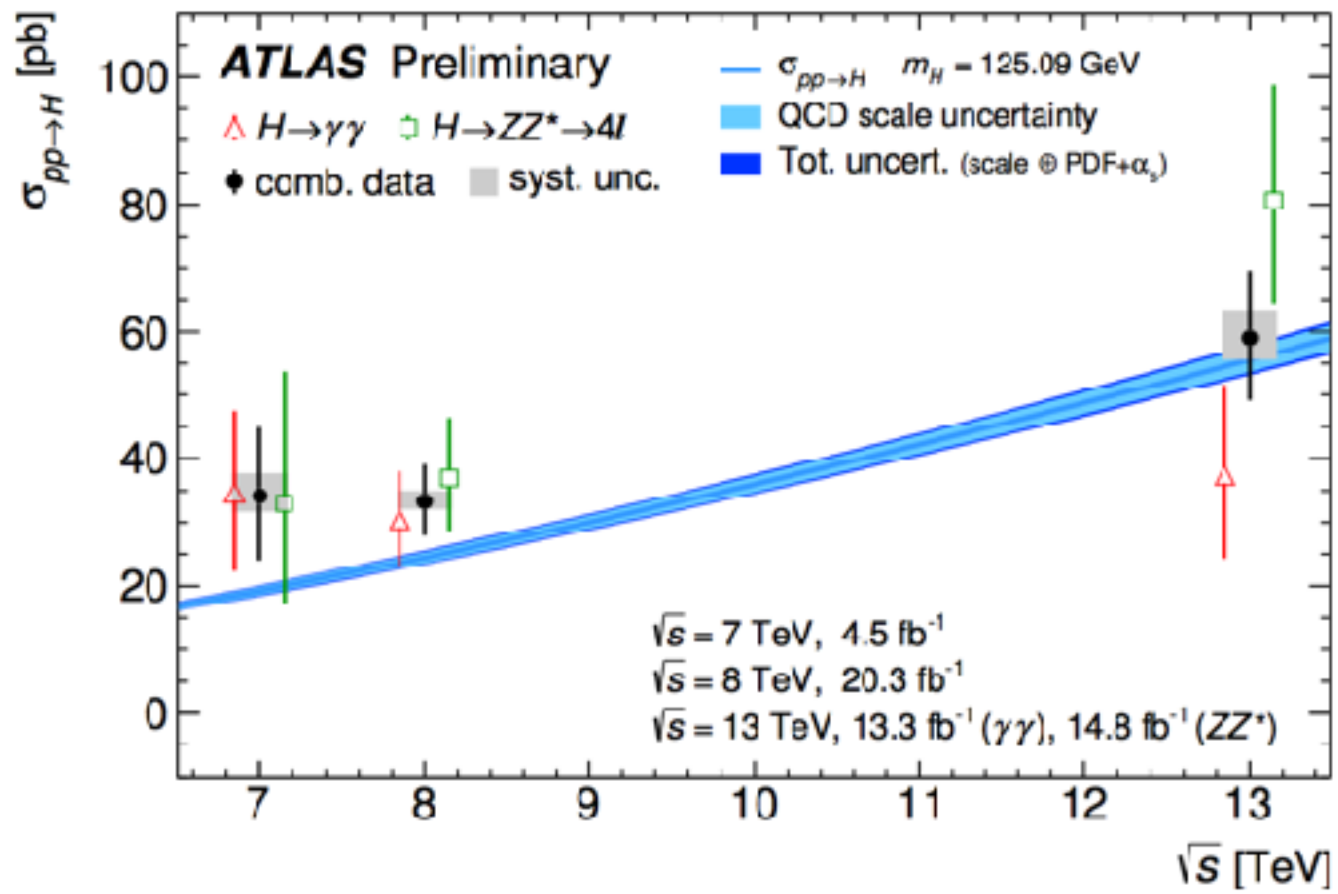
- The basis for testing our understanding of nature is on the one side precise measurements that are sensitive to the Higgs boson properties.

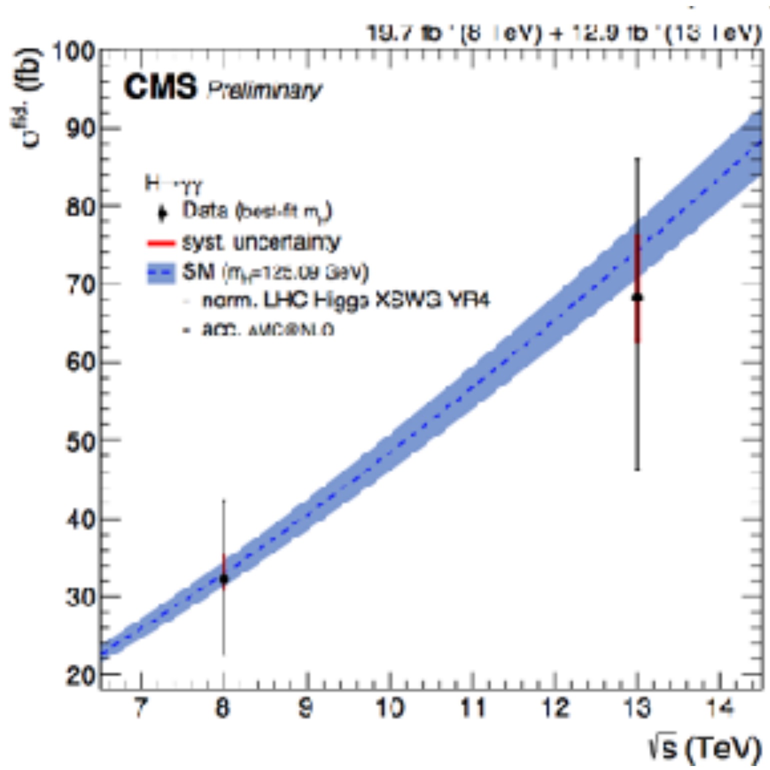


- LHC provides the input!
  Run 2: Data, data, data

ENTER THE AGE OF PRECISION HIGGS PHYSICS







- Incredible agreement of data and theory
- Triumph of SM predictions
- Higgs production ~10 sigma observed

DEMAND FOR PRECISION ON THEORY SIDE

- Testing our understanding of nature:
  Compare experiment and theory!

- The key to theoretical predictions at the LHC:

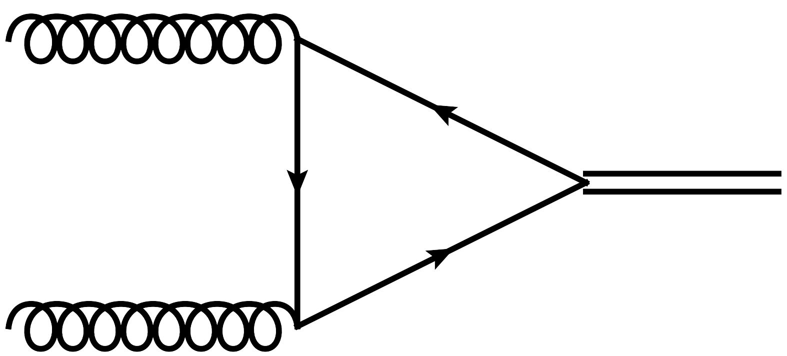
$$\sigma \sim \int dx dy f(x) f(y) \hat{\sigma} + \mathcal{O}\left(\frac{\Lambda}{Q}\right)$$

$$\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S^1 \hat{\sigma}^{(0)} + \alpha_S^2 \hat{\sigma}^{(2)} + \alpha_S^3 \hat{\sigma}^{(3)} \ldots$$

- Compute perturbativ corrections from first principle
  QFT: Standard Model

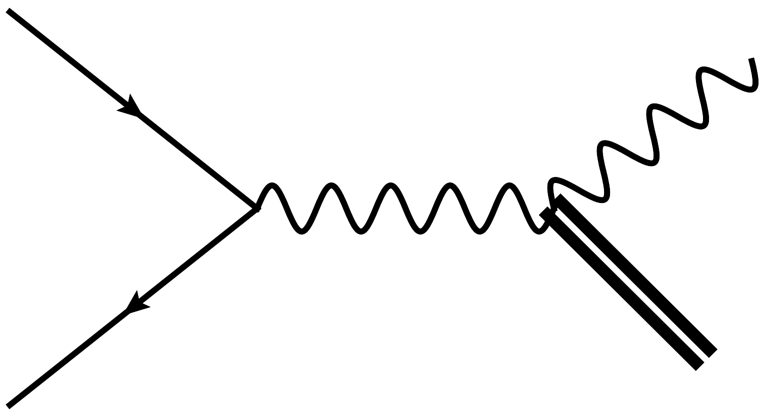
- Allows for % - level predictions for - experimental
  precision will reach comparable levels!

4 WAYS TO PRODUCE A HIGGS



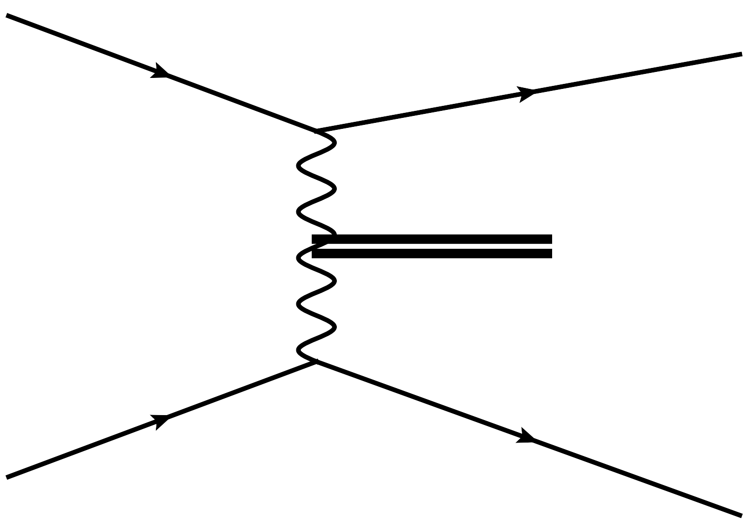
ggF

~88.2%



Associated

~4.1%



VBF

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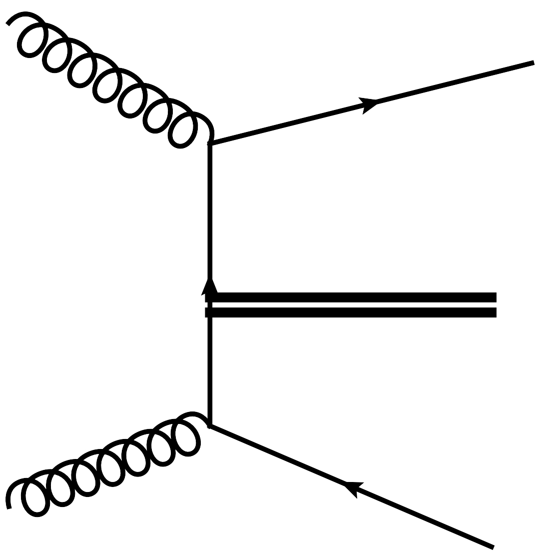
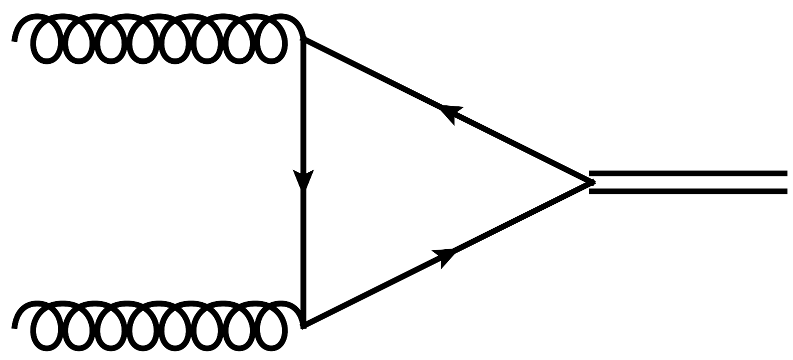


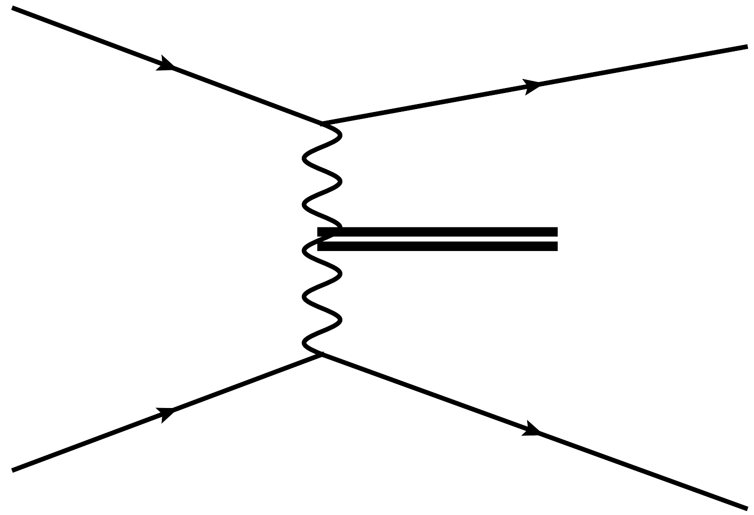
ttH

~0.9%

DOMINANT QCD CORRECTIONS

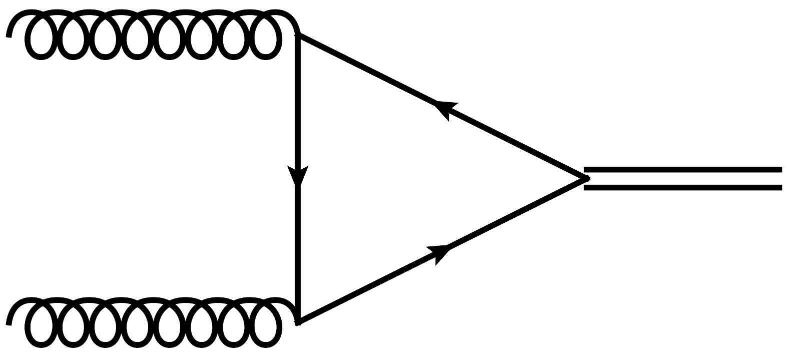


- Inclusive cross section: N3LO
- Differential cross section: NNLO
- H+J: NNLO

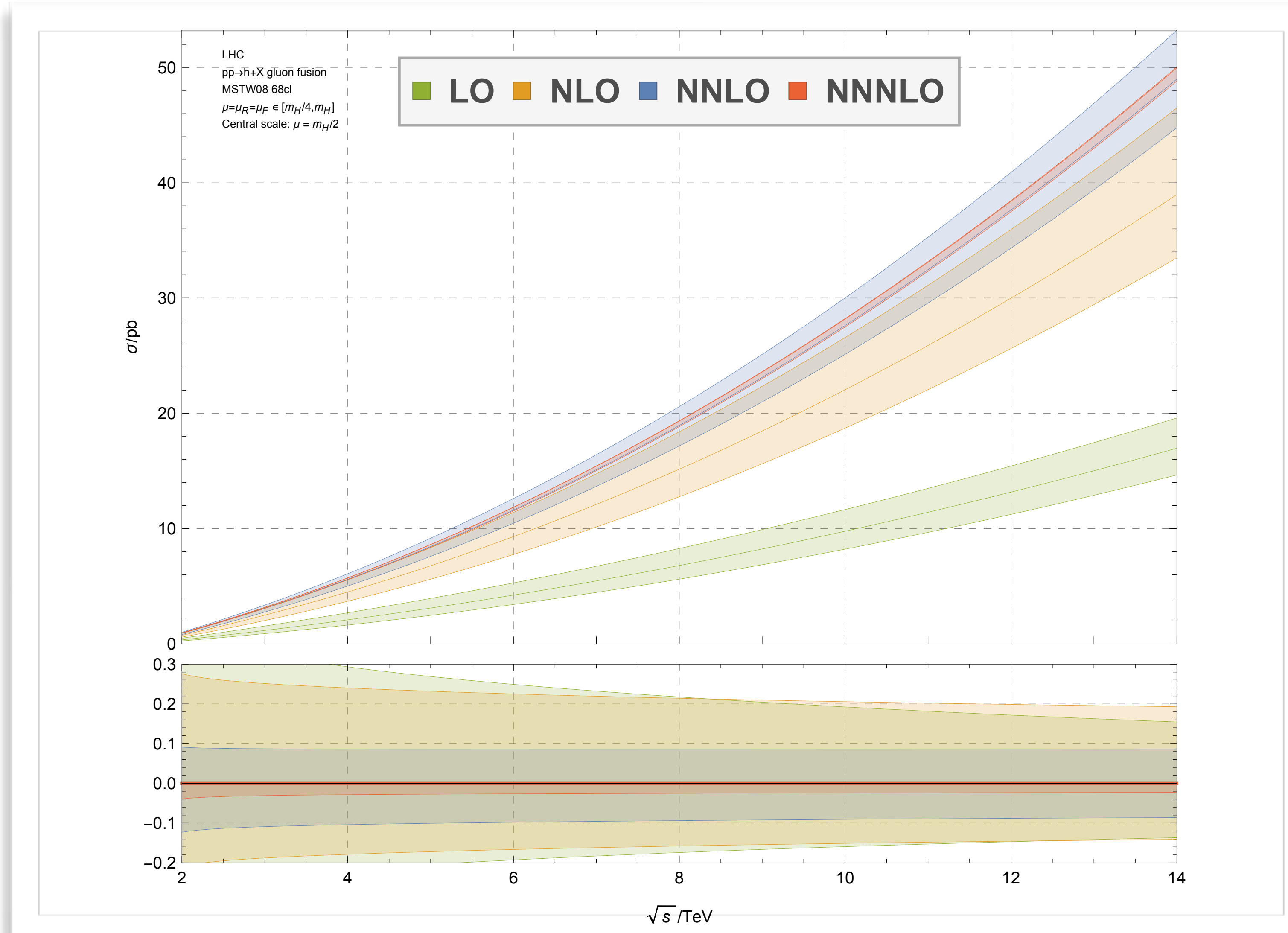


- Inclusive cross section: N3LO
- Differential cross section: NNLO

MOTIVATION FROM INCLUSIVE CROSS SECTION



- N3LO corrections stabilise perturbative expansion.
- We would like to see a similar pattern for differential cross sections.



CHALLENGES OF PERTURBATIVE PREDICTIONS

- Analytic complexity of high order perturbative computation
  - Complicated mathematical structures: Elliptic / multiple polylogarithms, couple differential equations, algebraic complexity, …
- Numerical integration over complicated and "divergent" final state configurations:
  - Infrared subtraction at 2-loops and beyond.
  - Main challenge of the last couple of years.
  - Many methods available now.

- **Sector decomposition**
- **Non-Linear Mappings**
- **qT**
- **FKS+**
- **N-Jettiness**
- **Antenna**
- **Colourful**
- **Projection-To-Born**
- **…**

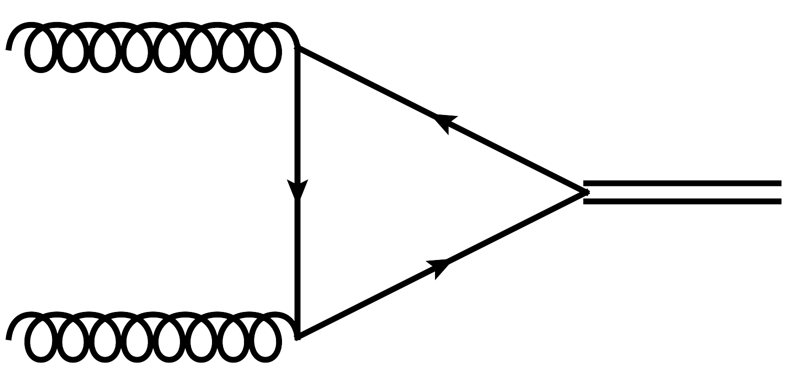
H+J

VBF

HIGGS - DIFFERENTIAL CROSS SECTIONS

- Introduce a framework that allows to compute differential cross sections at N3LO.
- Circumvent problems of NNLO infrared subtraction.
- Applicable for real life observables at the LHC.

Specifically: Differential Higgs Production in QCD



$$P\ P \to H + X \to \gamma\gamma + X$$

$$P\ P \to H + X \to 4l + X$$

- Today: Recent Progress, NNLO, Obstacles, Method

HIGGS – DIFFERENTIAL CROSS SECTIONS

- **Focus on the degrees of freedom of the Higgs boson:**

$$p_h \equiv (E,\, p_x,\, p_y,\, p_z) = \left(\sqrt{p_T^2 + m_h^2}\cosh Y,\ p_T\cos\phi,\ p_T\sin\phi,\ \sqrt{p_T^2+m_h^2}\sinh Y\right)$$

- Entirely described in terms of pT and Y (and a trivial azimuthal angle).

$$Y = \frac{1}{2}\log\left(\frac{E+p_z}{E-p_z}\right)$$

$$p_T = \sqrt{E^2 - p_z^2 - m_h^2}.$$

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- Higgs-differential cross section:

$$\sigma_{PP\to H+X}\,[\mathcal{O}] = \sum_{i,j}\int_{-\infty}^{+\infty} dY \int_0^{\infty} dp_T^2 \int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^1 dx_1 dx_2 f_i(x_1) f_j(x_2)$$

$$\times\frac{d^2\hat{\sigma}_{ij}}{dY\,dp_T^2}(S, x_1, x_2, m_h^2, Y, p_T^2)\,\mathcal{J}_{\mathcal{O}(Y, p_T^2, \phi, m_h^2)}$$

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XS for observable O

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Integrate over Higgs degrees of freedom

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Partonic Higgs-differential cross section →

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Definition of observable - measurement function

For example: $\mathcal{J}_{(Y,p_T^2,\phi,m_h^2)} = \theta(p_T^2 > 20GeV)$

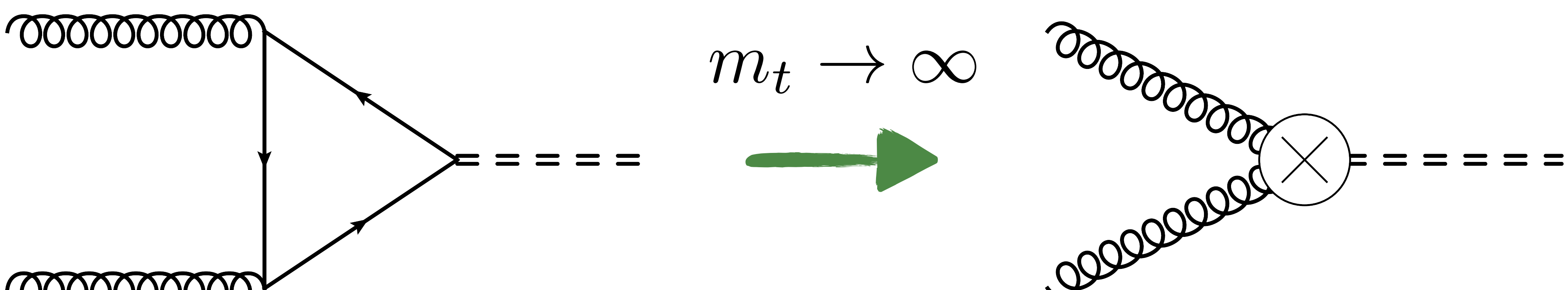
PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

$$\frac{d^2\hat{\sigma}_{ij}}{dY\,dp_T^2} \sim \sum_X \int d\Phi_n \left|\mathcal{M}_{ij\to H+X}\right|^2$$

PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

$$\frac{d^2\hat{\sigma}_{ij}}{dY\, dp_T^2} \sim \sum_X \int d\Phi_n \left| \mathcal{M}_{ij\to H+X} \right|^2$$

- Compute all required matrix elements of different final states X to a given order in perturbation theory.
- Work in effective theory: Excellent approximation!



$$\mathcal{L} = \mathcal{L}_{QCD,5} - \frac{1}{4v} C_1\, H\, G^a_{\mu\nu}\, G_a^{\mu\nu}$$

PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

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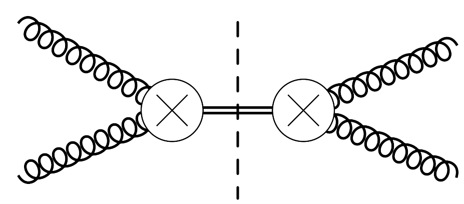
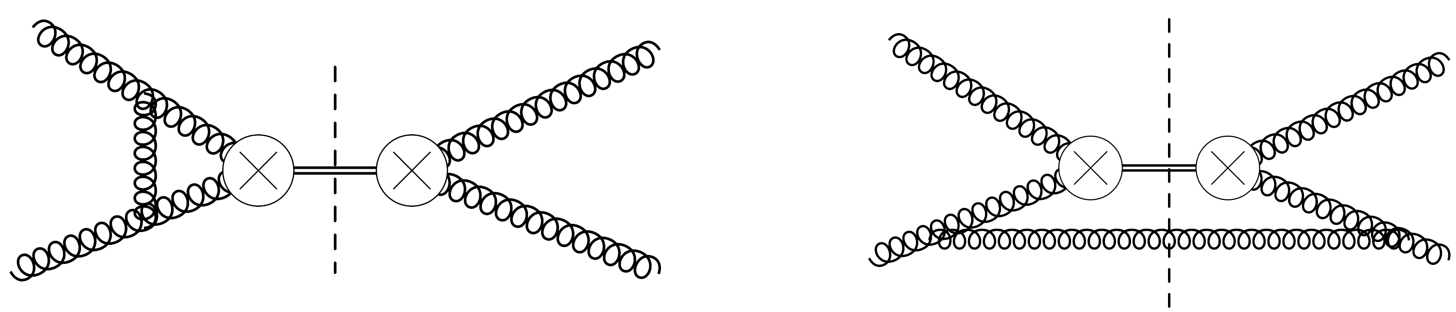
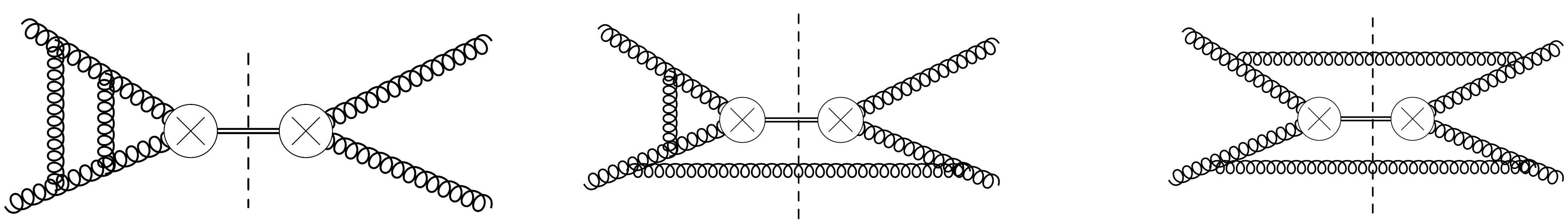
LO:



NLO:



NNLO:



PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

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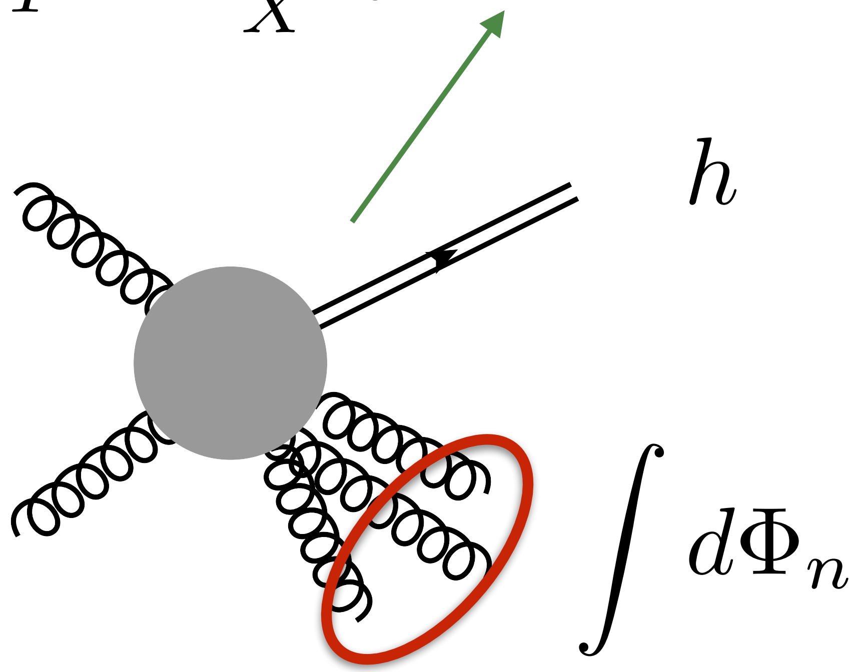
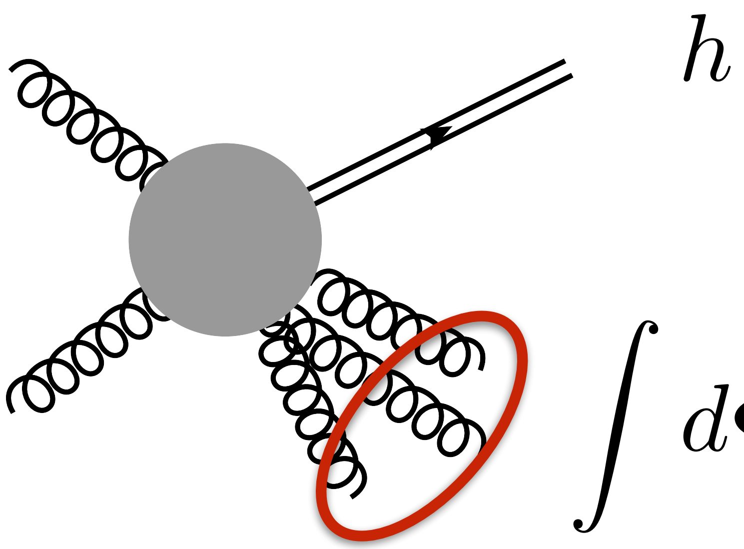


- Phase space integral over partonic final state phase space momenta
- Integrate over as many partons as there are in X.
- Integration over fixed multiplicity matrix elements is divergent! (KLN).

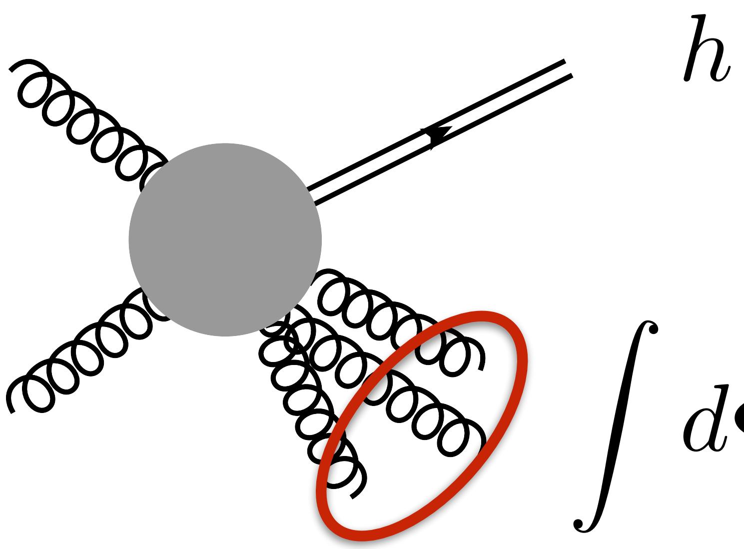
PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS



$$\int d\Phi_n$$

- Perform integration over parton phase space analytically
- Everything else effectively covered by H+J
- Rely on tools to perform analytic computation learned from inclusive N3LO
- Make singularities of final state parton integrations manifest using dimensional regularisation. $d = 4 - 2\epsilon$

PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS



$$\int d\Phi_n \sim \int \prod_{i=1}^{n} d^d p_i \delta_+(p_i^2)$$

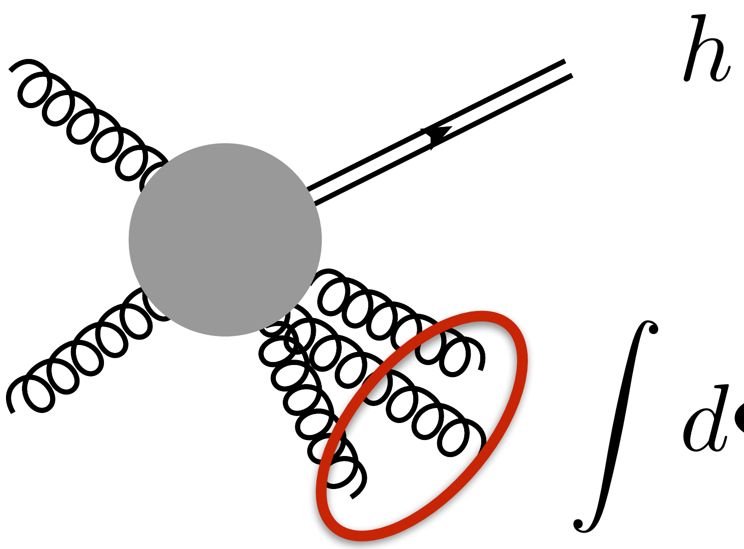
with $\delta_+(p_i^2) = \theta(E_i)\delta(p_i^2)$

REVERSE UNITARITY FRAMEWORK:

- Replace on-shell constraints with cut propagators

$$\delta_+(p_i^2) \sim \lim_{\delta \to 0} \left[\frac{1}{p_i^2 + i\delta} - \frac{1}{p_i^2 - i\delta} \right] = \left[\frac{1}{p_i^2} \right]_c$$

REVERSE UNITARITY FRAMEWORK:



$$\int d\Phi_n \sim \int \prod_{i=1}^{n} d^d p_i \left[\frac{1}{p_i^2}\right]_c$$

- Opens the door to large variety of loop integral technology!
    - IBPs + Differential equations
- Key observation: Cut propagators can be differentiated similar to usual propagators.

REVERSE UNITARITY FRAMEWORK:

$$\frac{d^2\hat{\sigma}_{ij}}{dY\,dp_T^2} \sim \sum_X \int d\Phi_n \Big|\mathcal{M}_{ij\to H+X}\Big|^2$$
$$= \sum_X \sum_i c_{X,\,i} F_{X,\,i}(S, p_T, Y, m_h^2)$$

- Coefficient: Rational function of remaining kinematic variables.
- Master Integral: Integrated Feynman integrals: Polylogarithms, rational functions of remaining kinematic variables.
- Explicit Laurent series in dimensional regulator.

HIGGS - DIFFERENTIAL CROSS SECTIONS

$$\sigma_{PP\to H+X}\left[\mathcal{O}\right] = \sum_{i,j}\int_{-\infty}^{+\infty} dY \int_0^{\infty} dp_T^2 \int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^1 dx_1 dx_2 f_i(x_1) f_j(x_2)$$

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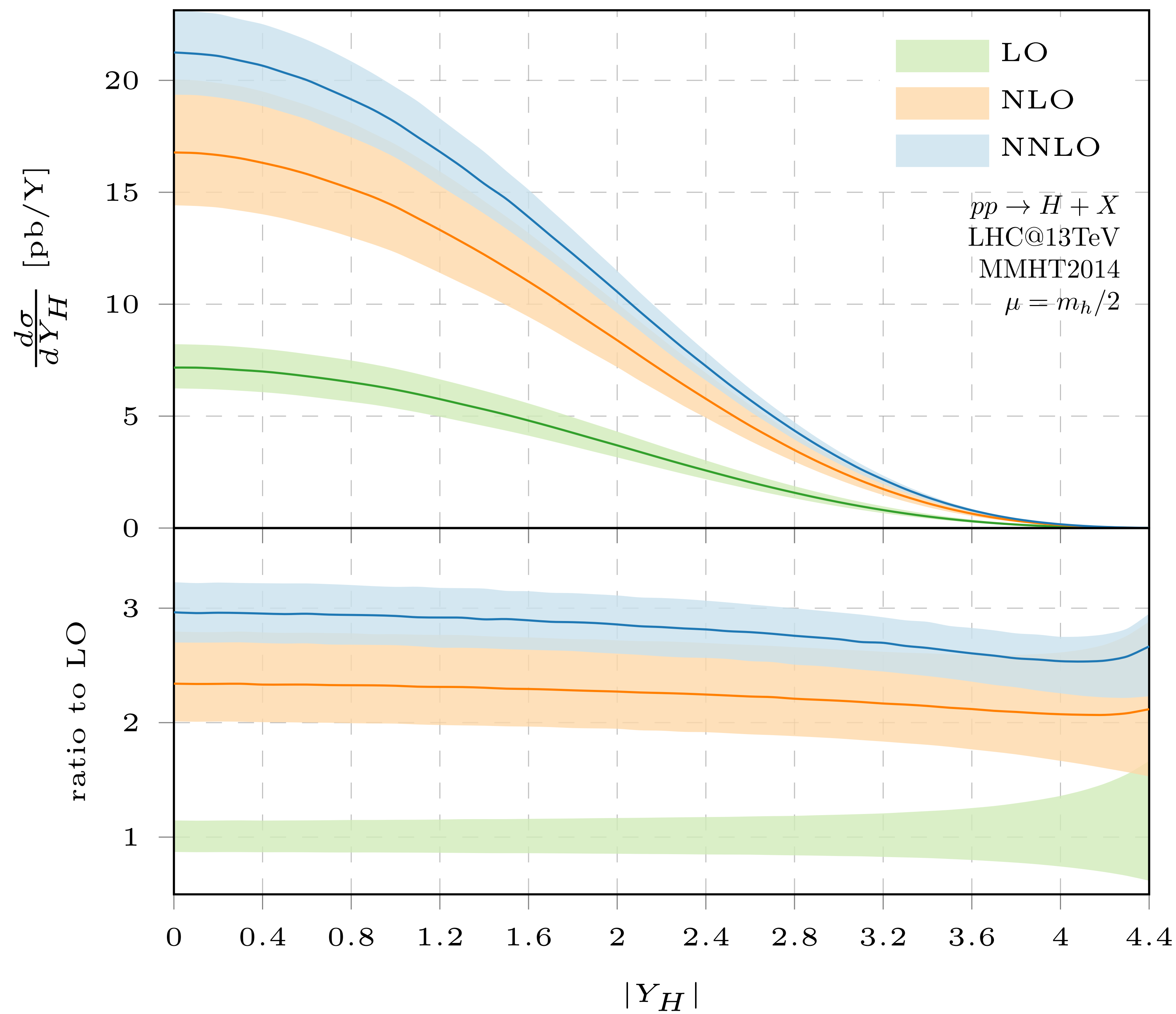
- UV - renormalisation ($\alpha_S \to \alpha_S^R Z$, etc.)
- Initial state collinear singularities: Redefine parton distributions functions: $f(x) \to f_R(x) \circ \Gamma$

Currently: **Complete NNLO Higgs-differential cross section.**

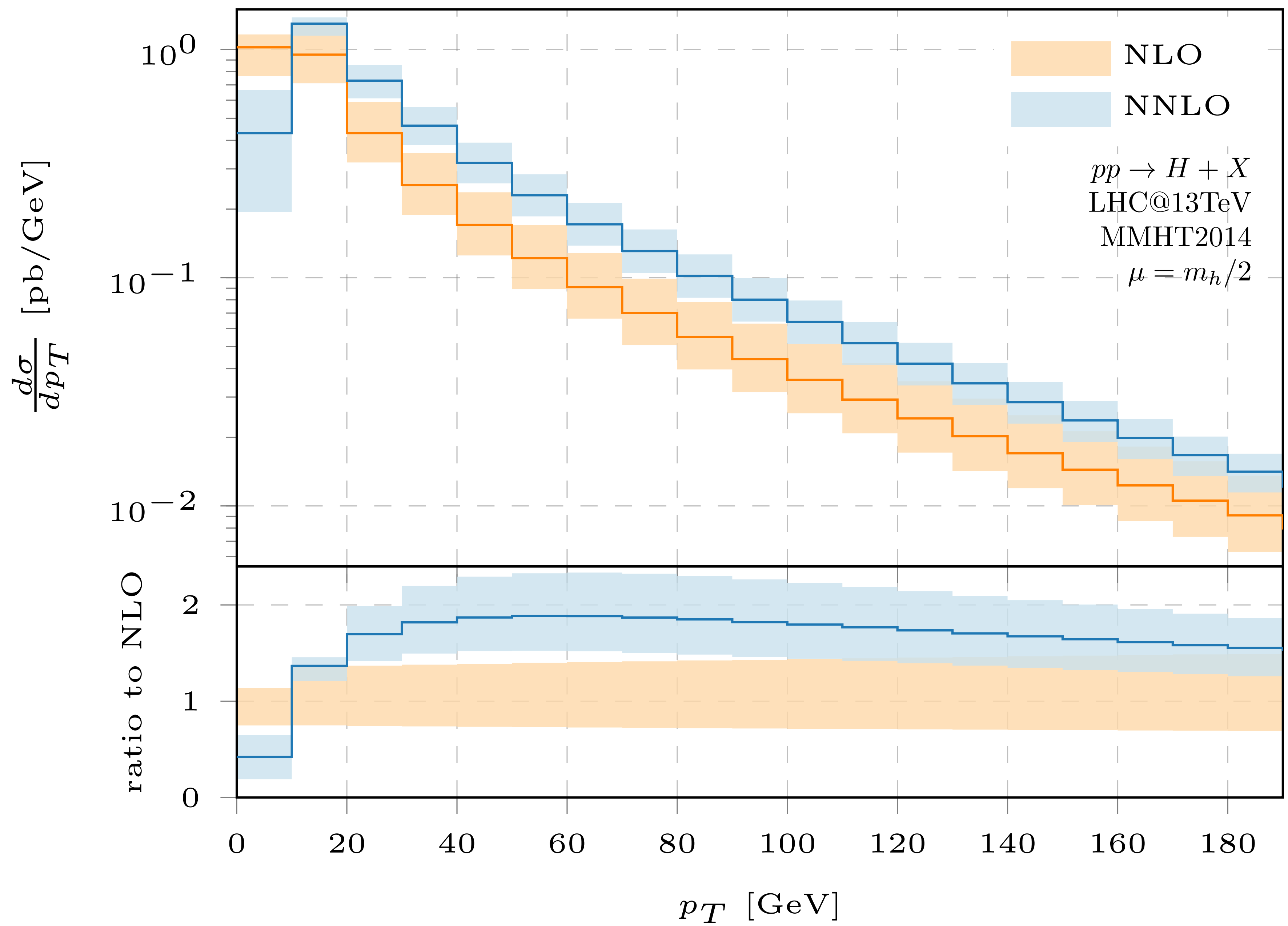
HIGGS – DIFFERENTIAL CROSS SECTIONS: RAPIDITY

- Inclusive rapidity distribution
- Large K-factors



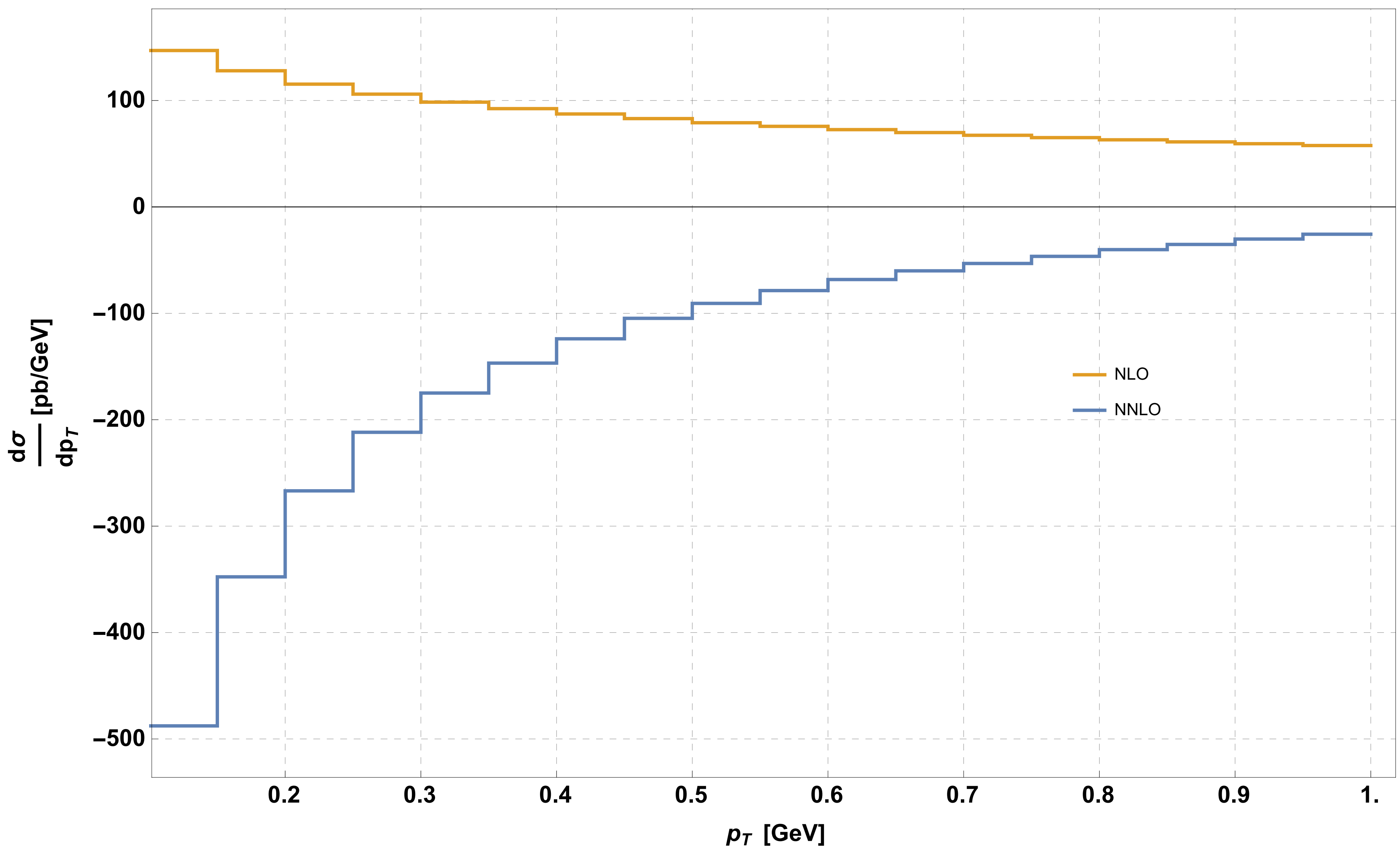
HIGGS – DIFFERENTIAL CROSS SECTIONS: PT

- Inclusive PT distribution



HIGGS - DIFFERENTIAL CROSS SECTIONS: PT

- Inclusive PT distribution - extreme regime



HIGGS – DIFFERENTIAL CROSS SECTIONS: FIDUCIAL XS

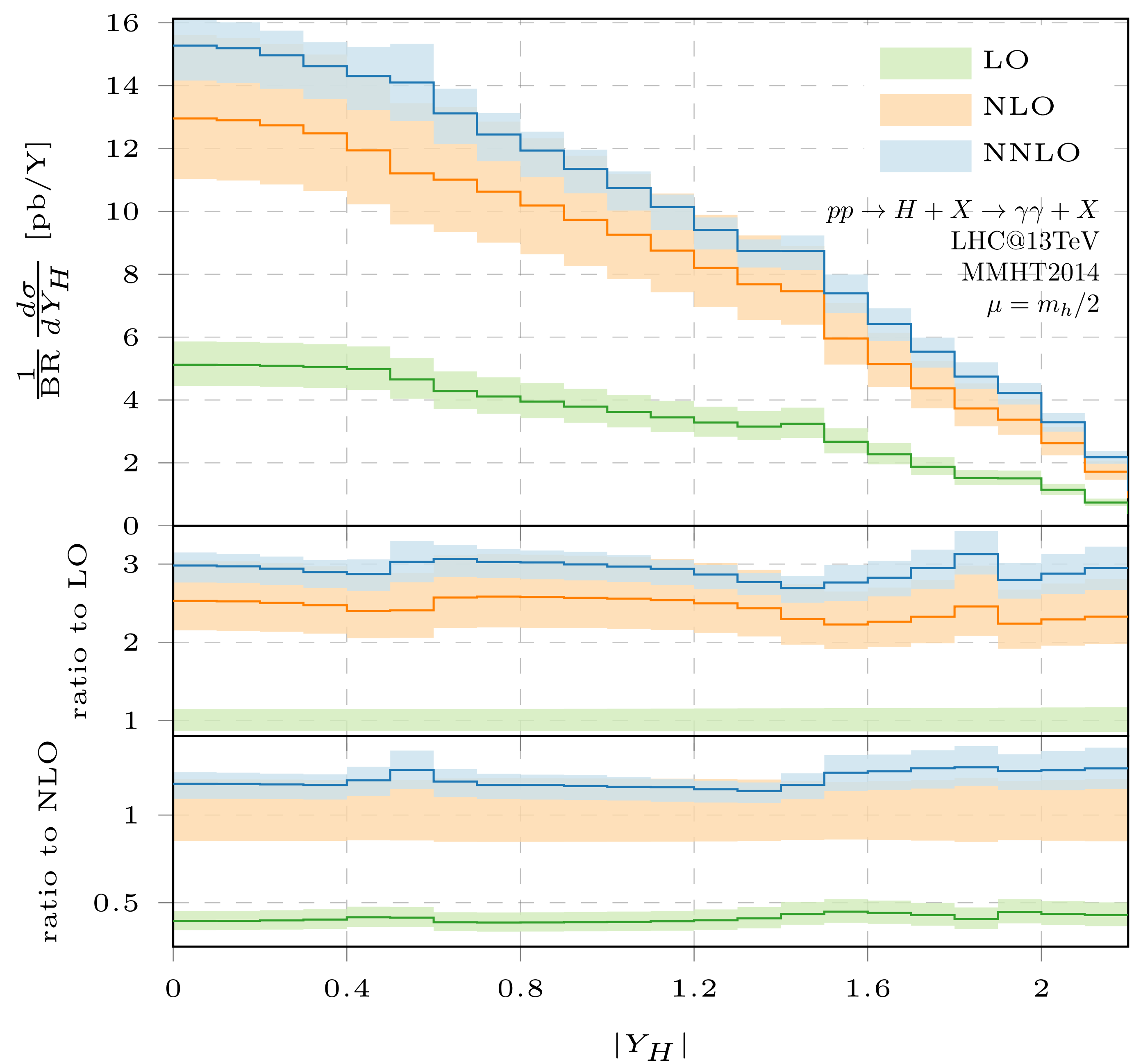
- Decay of the Higgs boson to two photons
  (Narrow width approximation)

- Employ realistic selection cuts (ATLAS)

$$\eta_\gamma < 2.37$$
$$\eta_\gamma \notin [1.37, 1.52]$$
$$p_{T,\,\gamma_1} > 0.35\, m_h$$
$$p_{T,\,\gamma_2} > 0.25\, m_h$$

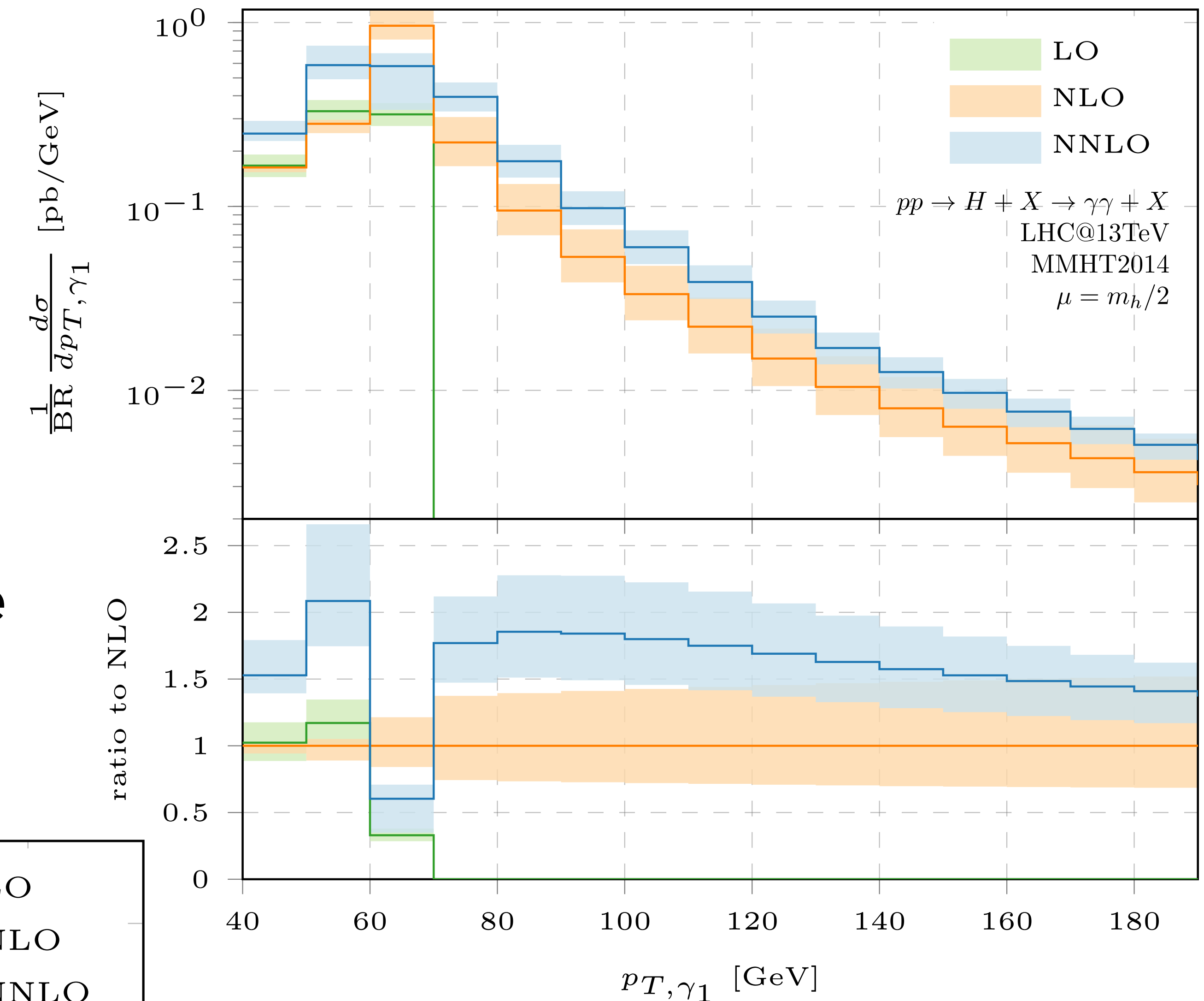
HIGGS – DIFFERENTIAL CROSS SECTIONS: FIDUCIAL XS

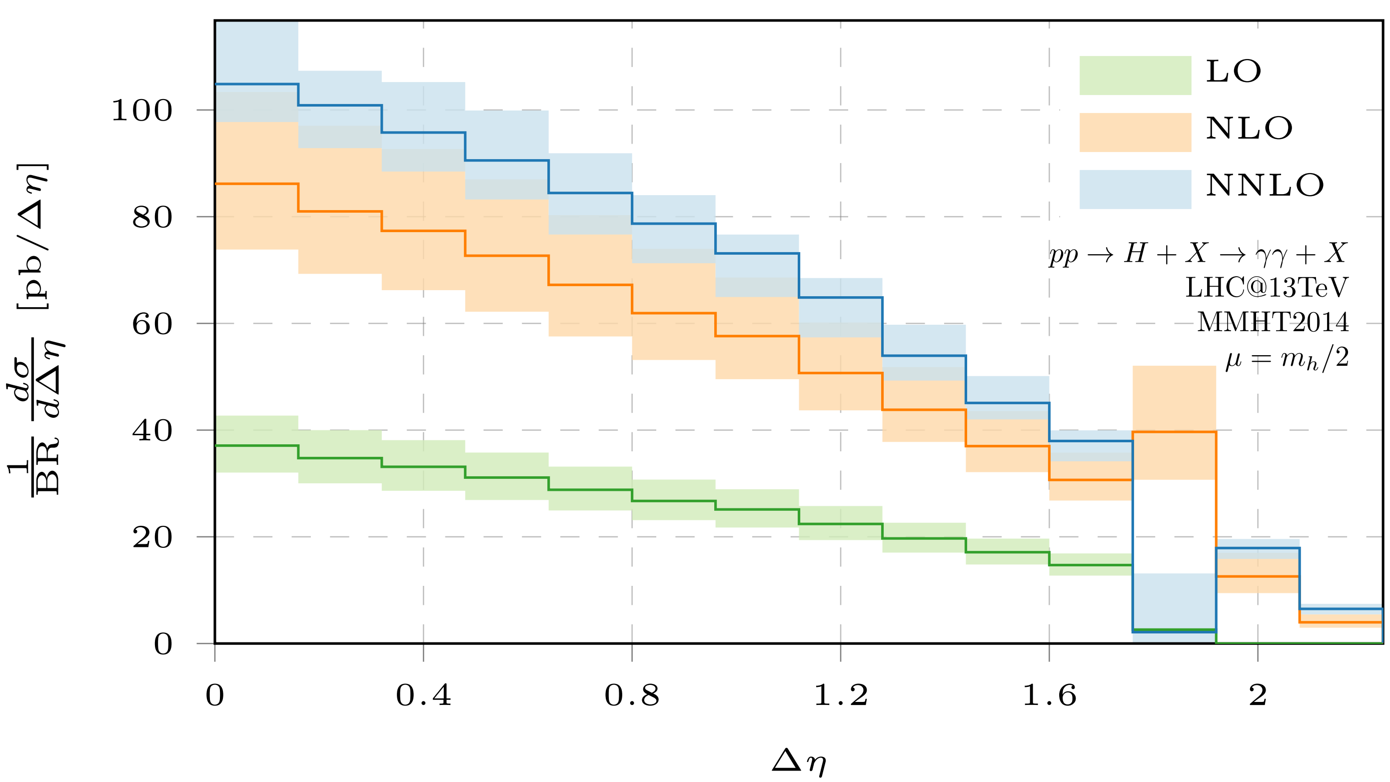
- Fiducial rapidity distribution.
- Non-trivial features due to selection criteria.
- Relatively flat K-factors
- Similar perturbative behaviour as inclusive distribution



HIGGS – DIFFERENTIAL CROSS SECTIONS: FIDUCIAL XS

- Distributions of the photon momenta:
  - Leading Photon pT
  - Pseudo - rapidity difference





$$\Delta\eta = |\eta_{\gamma_1} - \eta_{\gamma_2}|$$

BEYOND NNLO

WHAT DID WE LEARN FROM NNLO

- Higgs-differential cross sections: fast and stable framework for fiducial cross sections.
- Analytic computation at NNLO comparably simple.

MAIN CHALLENGES FOR N3LO

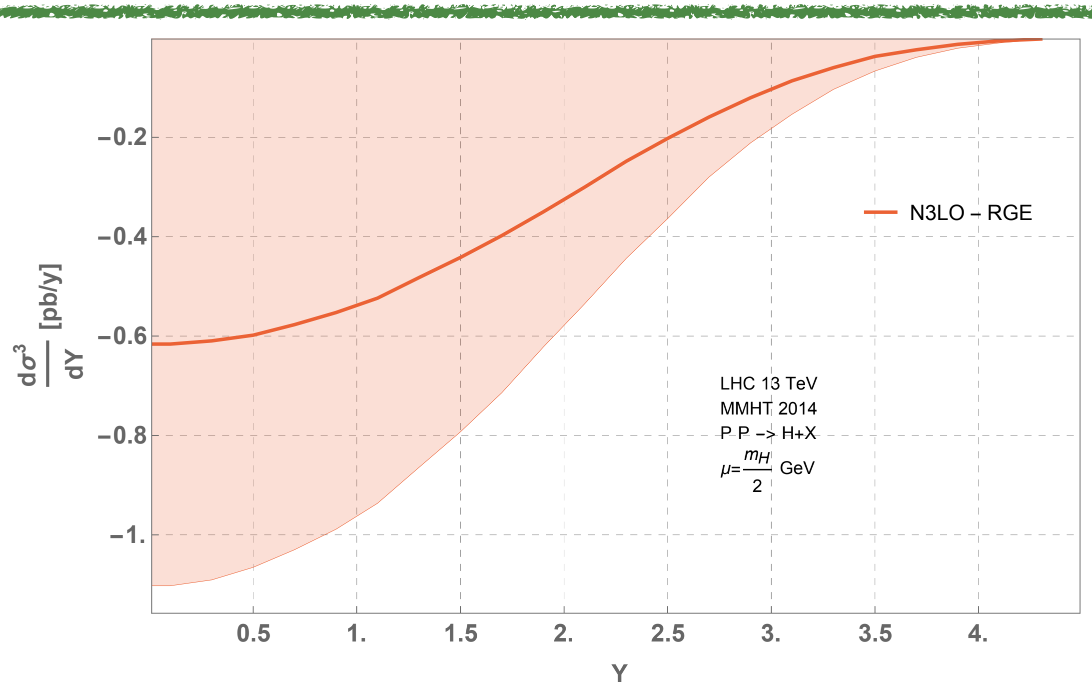
- Rapid growth in analytic complexity: Many more integrals to compute, large rational expressions as a result
- Numerical stability vs. speed in evaluation of analytic coefficients.

UV RENORMALISATION AND IR FACTORISATION

- To derive UV counter terms and IR subtraction terms we require NNLO cross sections computed beyond the finite term in ϵ

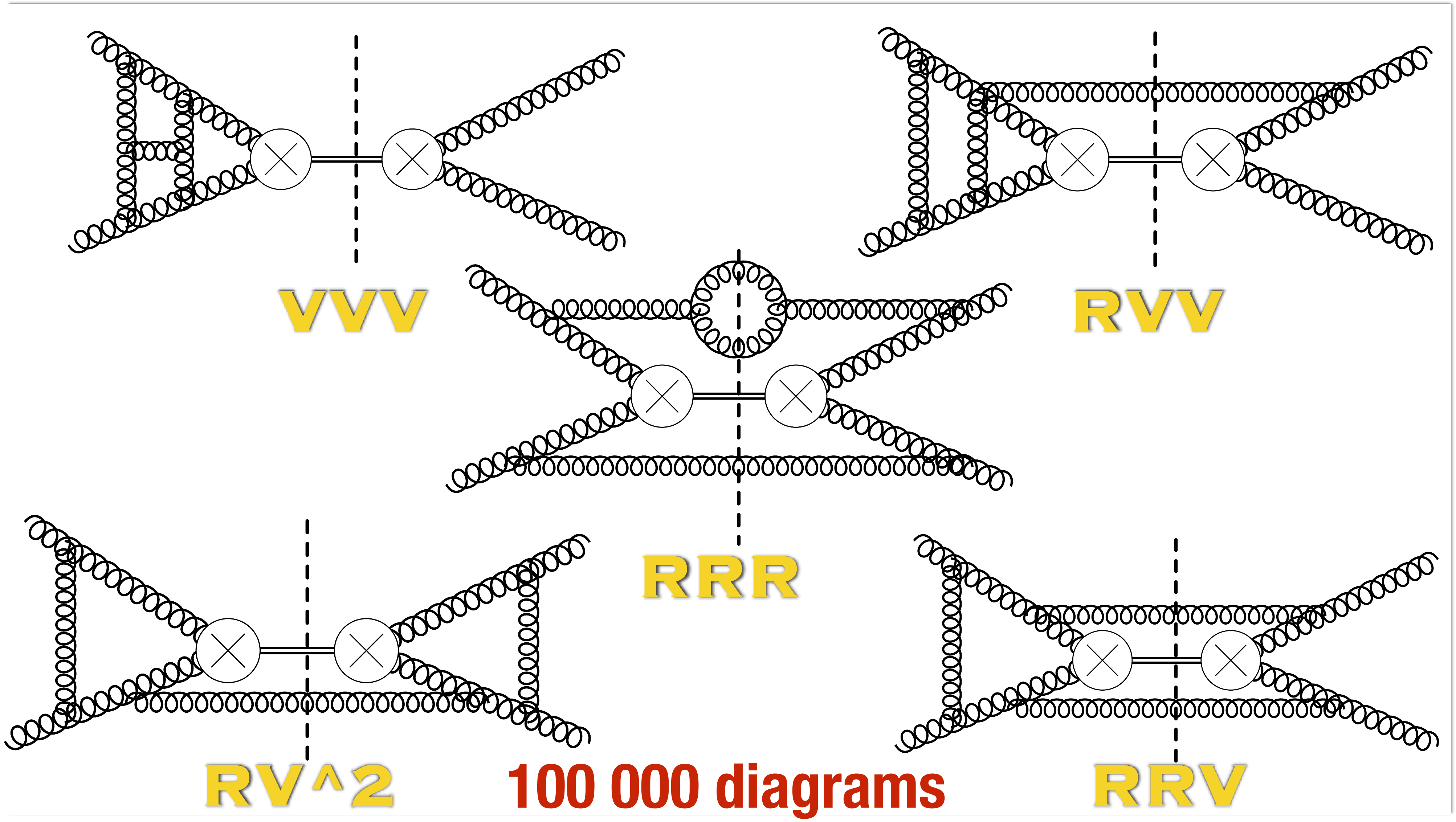
- Allow to derive complete N3LO scale variation from DGLAP

$$\hat{\sigma}^{(3)} = \hat{\sigma}_0^{(3)} + \hat{\sigma}_1^{(3)} \log\left(\frac{m_h^2}{\mu^2}\right) + \hat{\sigma}_2^{(3)} \log^2\left(\frac{m_h^2}{\mu^2}\right) + \hat{\sigma}_3^{(3)} \log^3\left(\frac{m_h^2}{\mu^2}\right)$$



FIXED ORDER MATRIX ELEMENTS

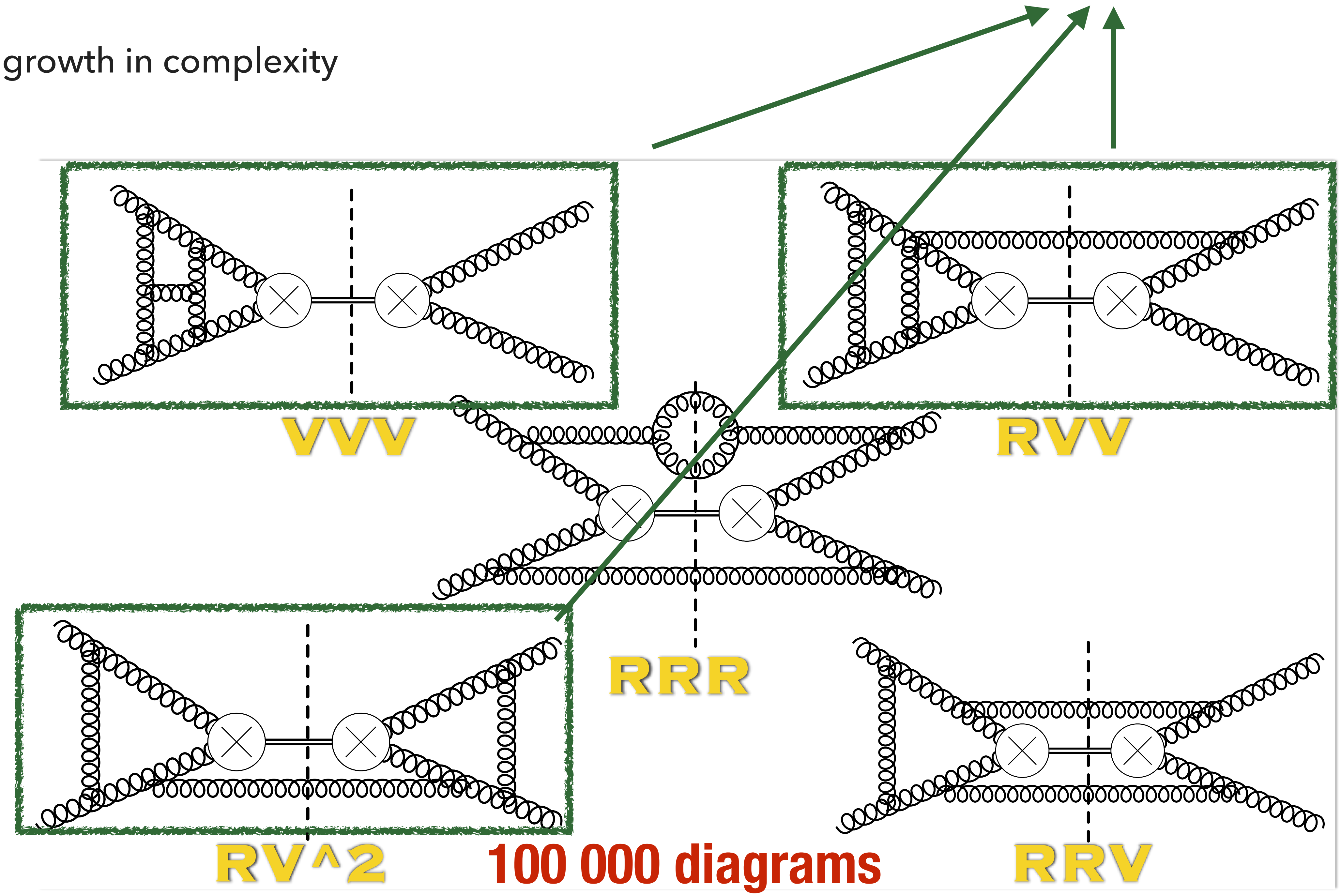
Rapid growth in complexity



FIXED ORDER MATRIX ELEMENTS

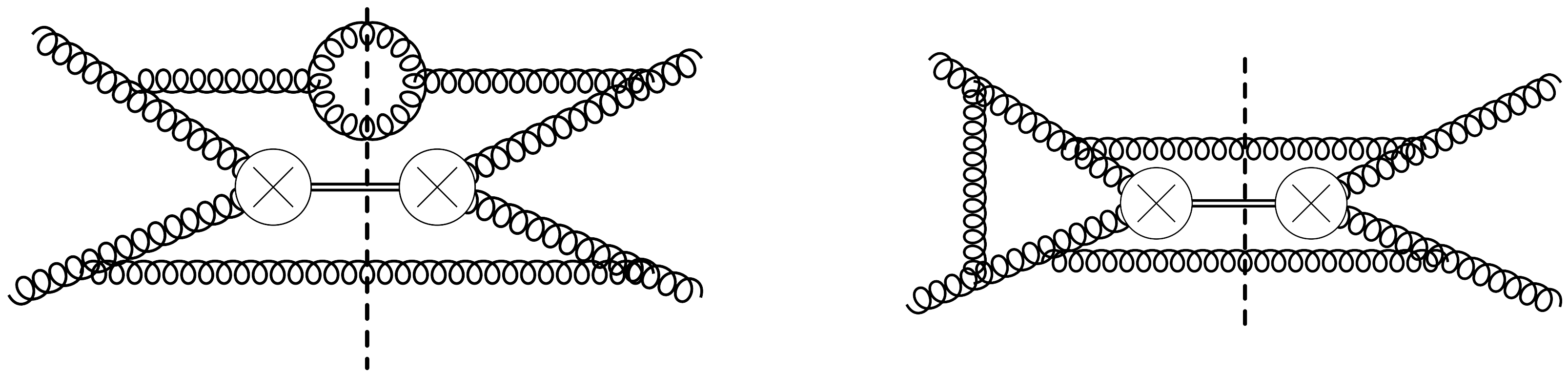
Known already! (Inclusive / H+J @NNLO)

Rapid growth in complexity



1000 @ NNLO

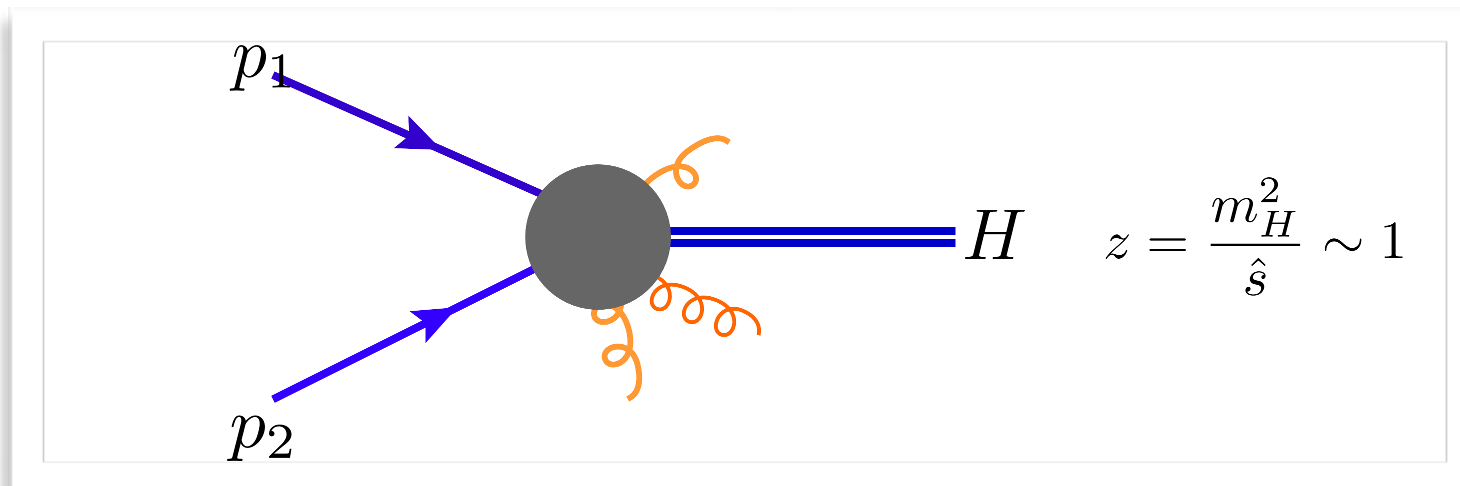
FIXED ORDER MATRIX ELEMENTS



- Missing matrix elements with 2 or 3 final state partons.
- Same strategy as for NNLO: Analytic computation using reverse unitarity, master integrals and differential equations.
- Number of master integrals required: 100 x NNLO.
- Solving differential equations for master integrals:
  Need boundary conditions = Master integrals evaluated at one single point.

EXPAND CROSS SECTION AROUND PRODUCTION THRESHOLD

- Inclusive Cross Section: Computed as a Threshold Expansion



$$\bar{z} = 1 - z \quad \Rightarrow \quad \hat{\sigma}(\bar{z}) = \sigma^{SV} + \sigma^{(0)} + \bar{z}\sigma^{(1)} + \ldots$$

- Served as an excellent approximation for inclusive cross section.

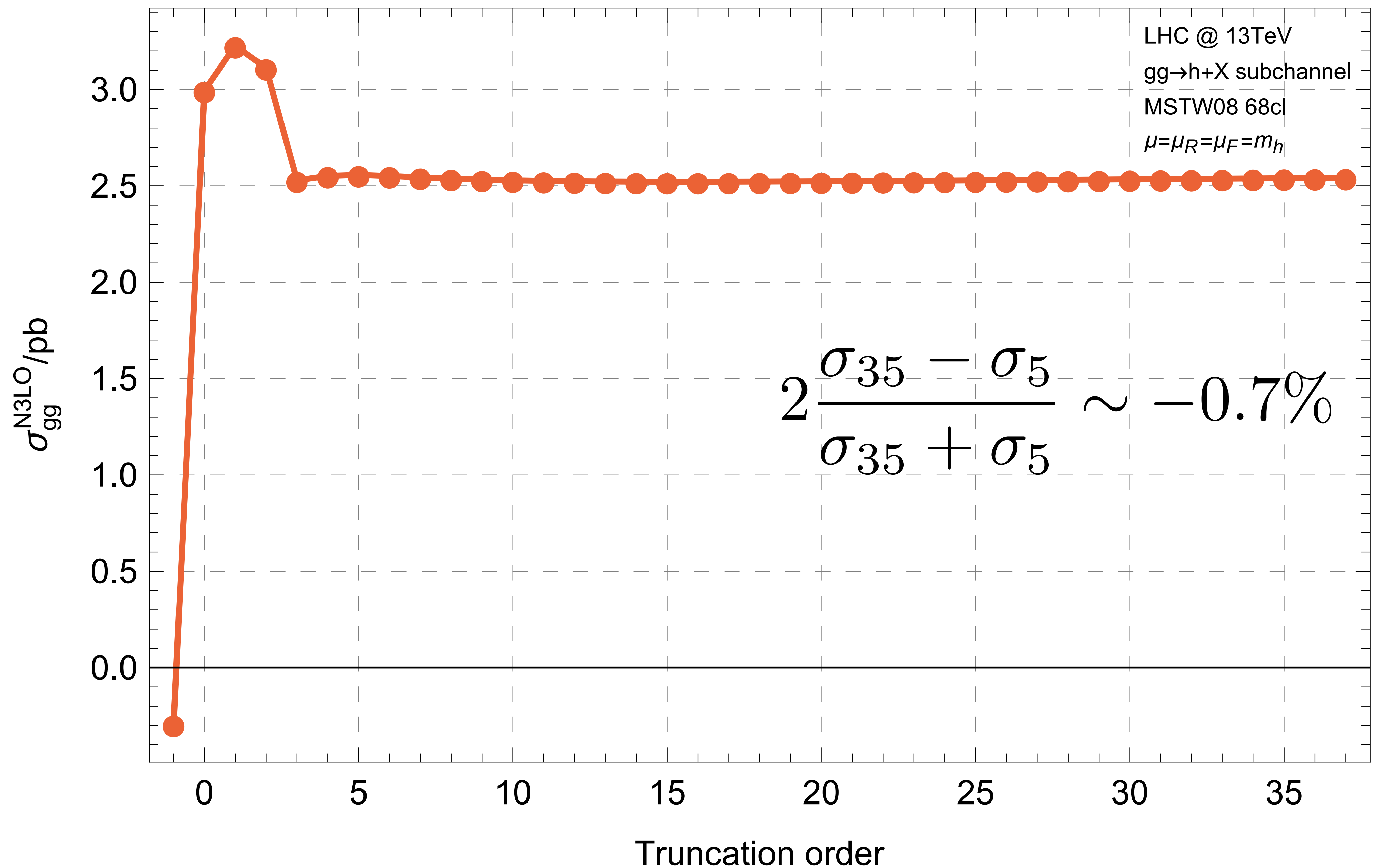
- Reason **Nr.1**:
  Crucial analytic information a full calculation relies on - boundary conditions.
  + checks, testing ground for technology, etc.

- Reason Nr. 2: Can we use it for phenomenology?

TRUNCATION ORDER @ N3LO: INCLUSIVE

- How well did it work for N3LO inclusive?

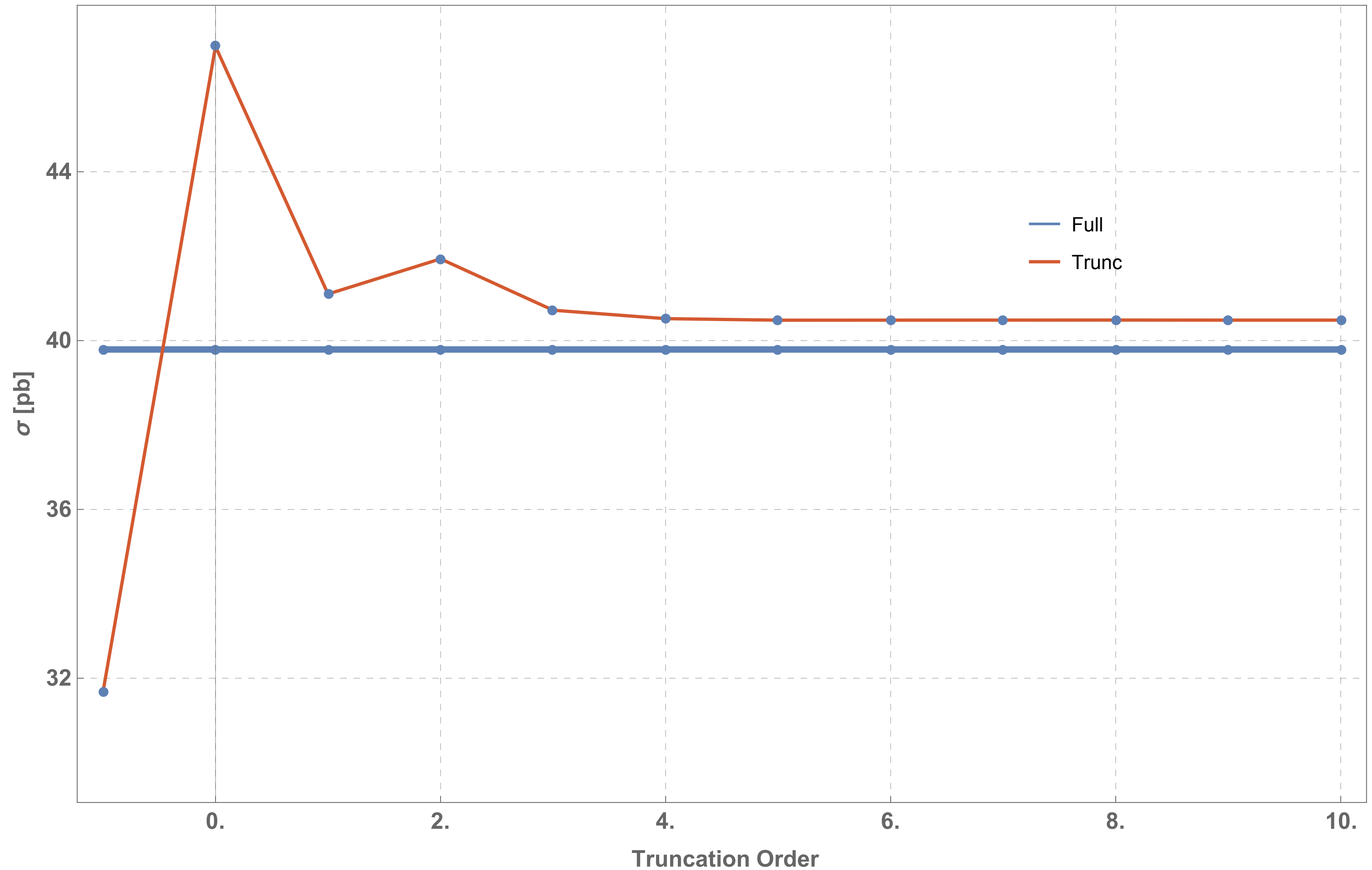
$$\delta\sigma^{N3LO} =\sim 0.5\%$$



$$2\frac{\sigma_{35}-\sigma_5}{\sigma_{35}+\sigma_5} \sim -0.7\%$$

TRUNCATION ORDER @ NNLO: INCLUSIVE

- Let's test at NNLO:

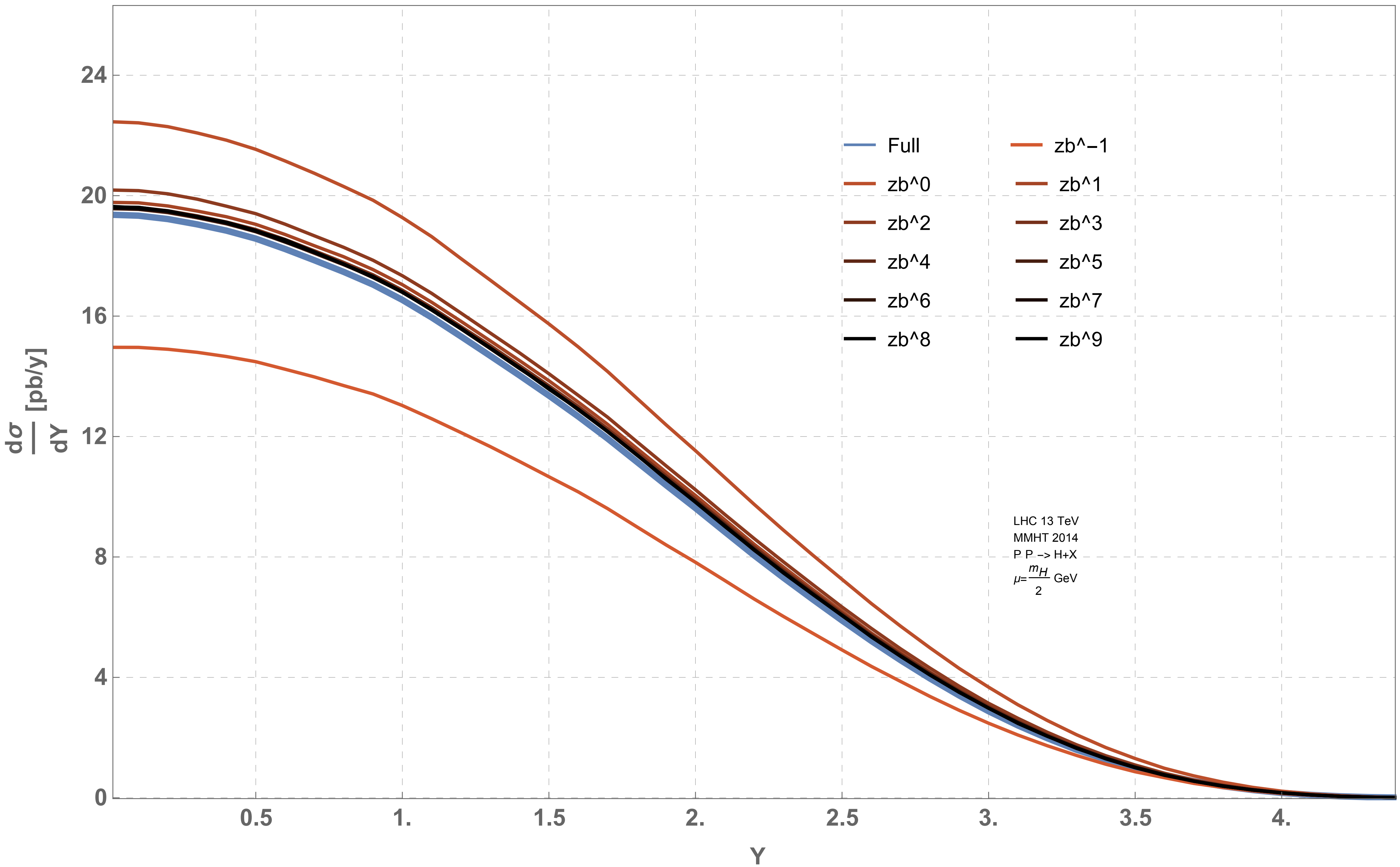


- Good approximation reached with only 5-10 terms
- ~3% off-set from full result for the NNLO correction -> ~0.5 % on the total

- How well does it work differentially?

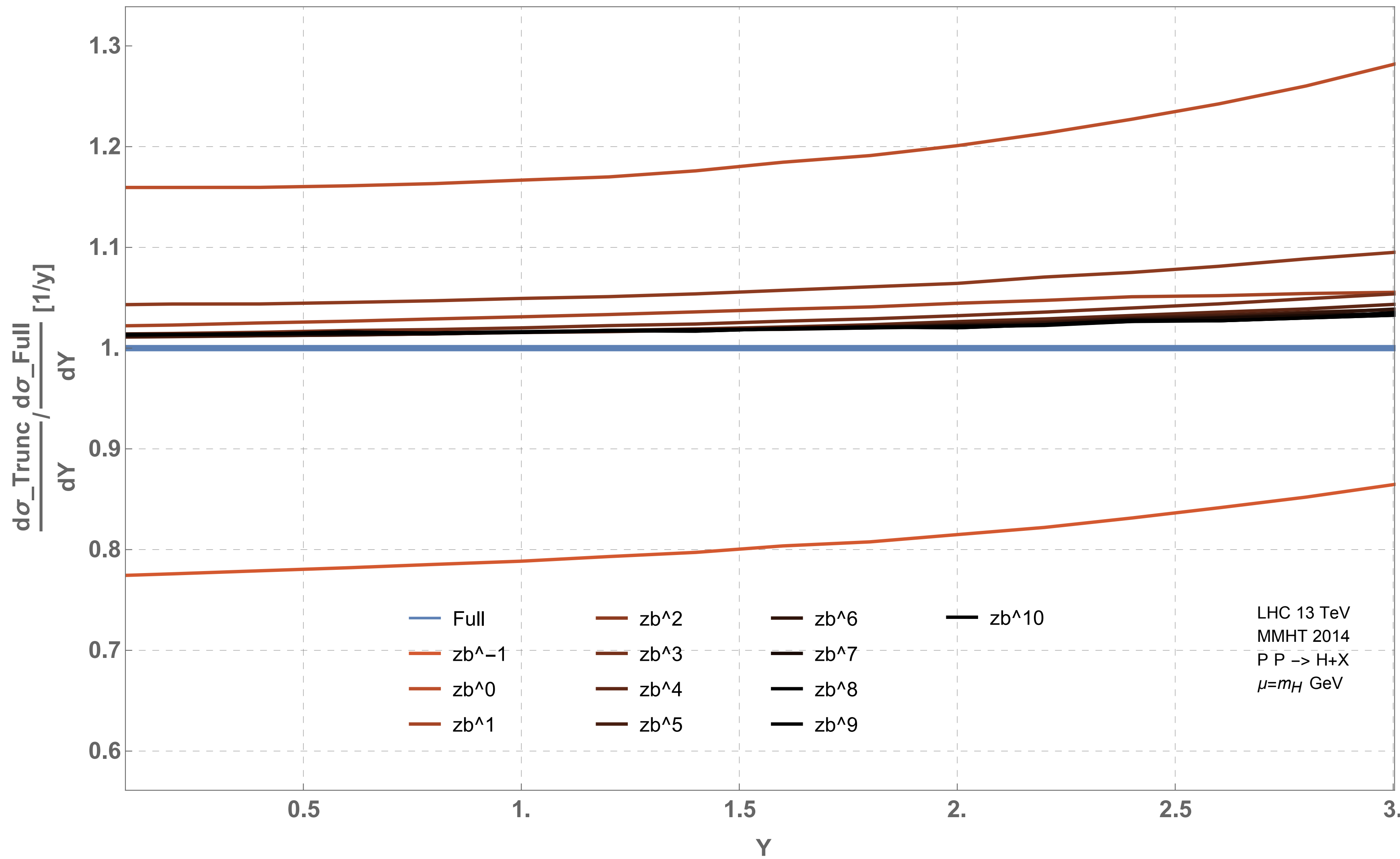
THRESHOLD EXPANSION @ NNLO DIFFERENTIAL

- Rapidity distribution



THRESHOLD EXPANSION @ NNLO DIFFERENTIAL

- Rapidity distribution normalised to true value.



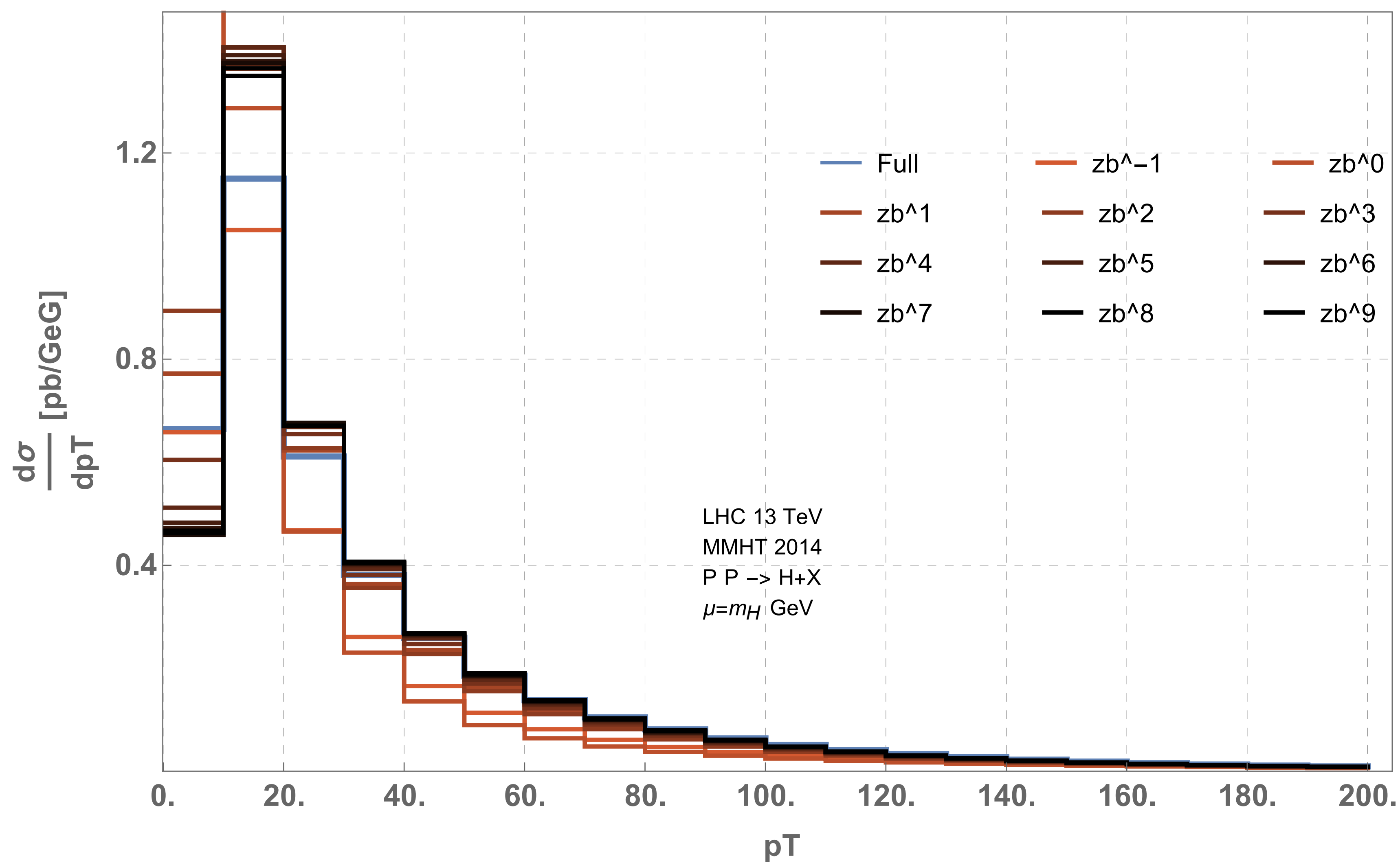
- Bulk of XS is described well with a couple of terms
- Systematic improvement possible

THRESHOLD EXPANSION @ NNLO DIFFERENTIAL

- PT distribution

- Bad convergence at low pT

- On-set of distribution at NLO while threshold limit is tree level.

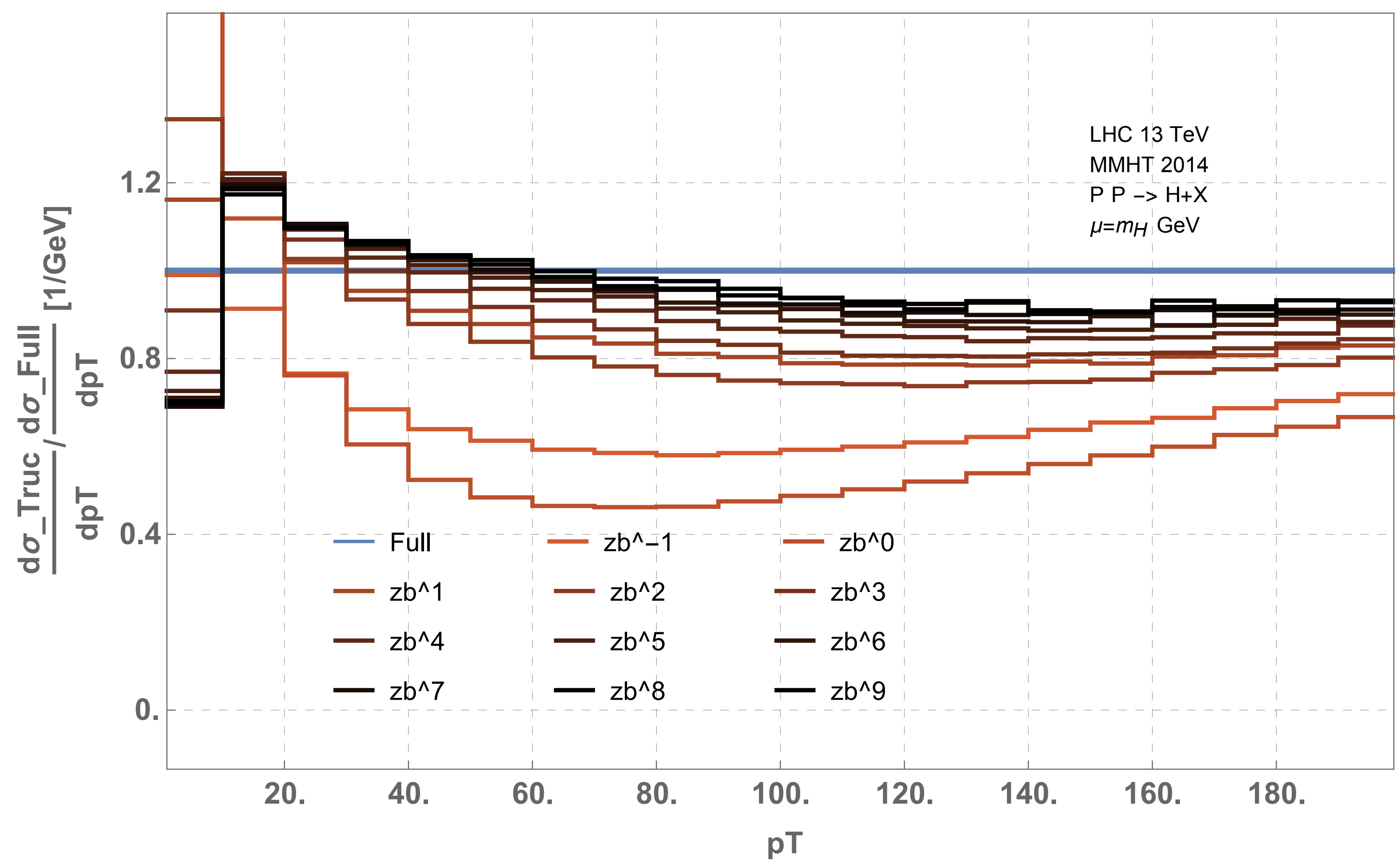


THRESHOLD EXPANSION @ NNLO DIFFERENTIAL

- PT distribution normalised to true value.

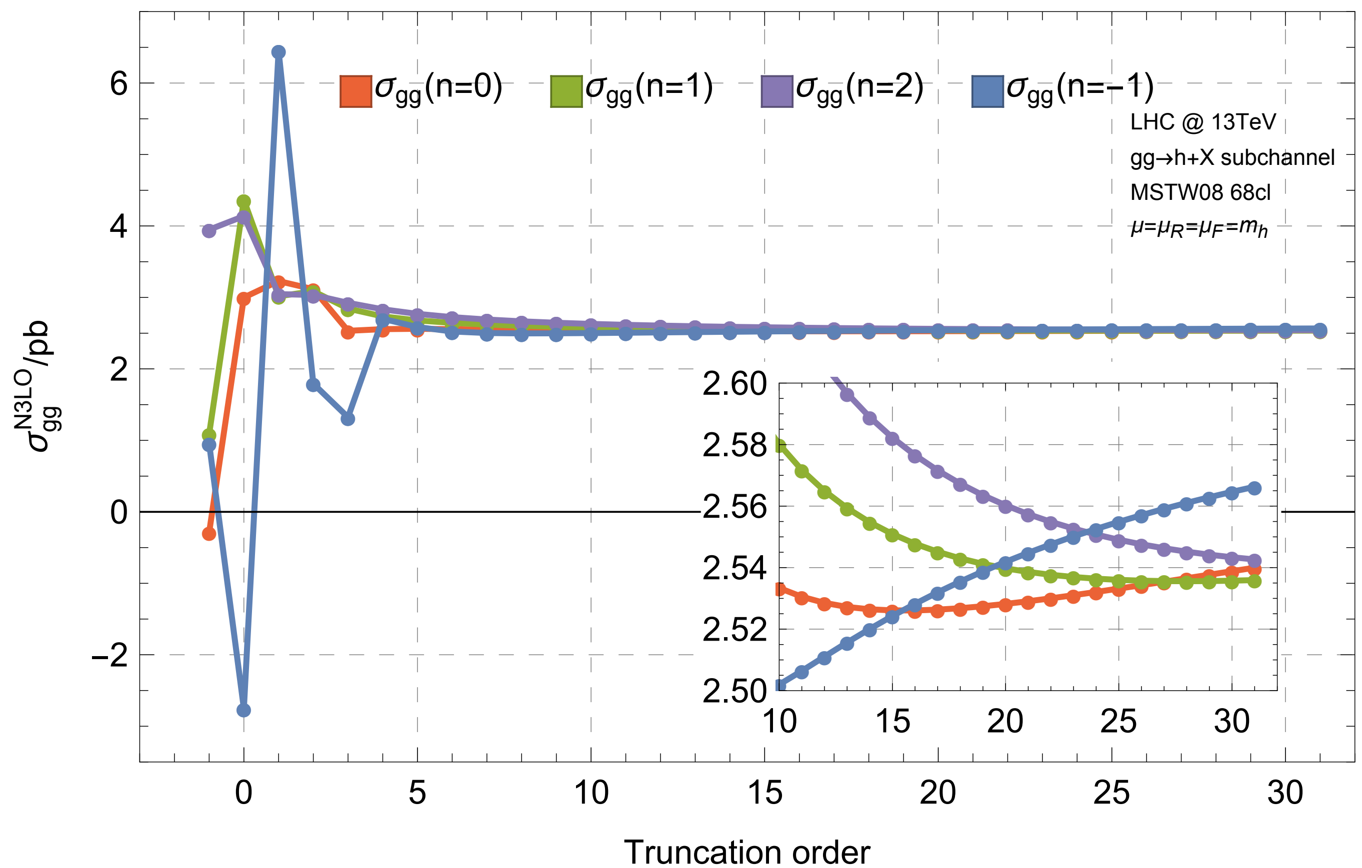
- Even with 10 terms marginally within 20 %

- Quality of expansion is subject to observable: Threshold sensitivity



THRESHOLD EXPANSION

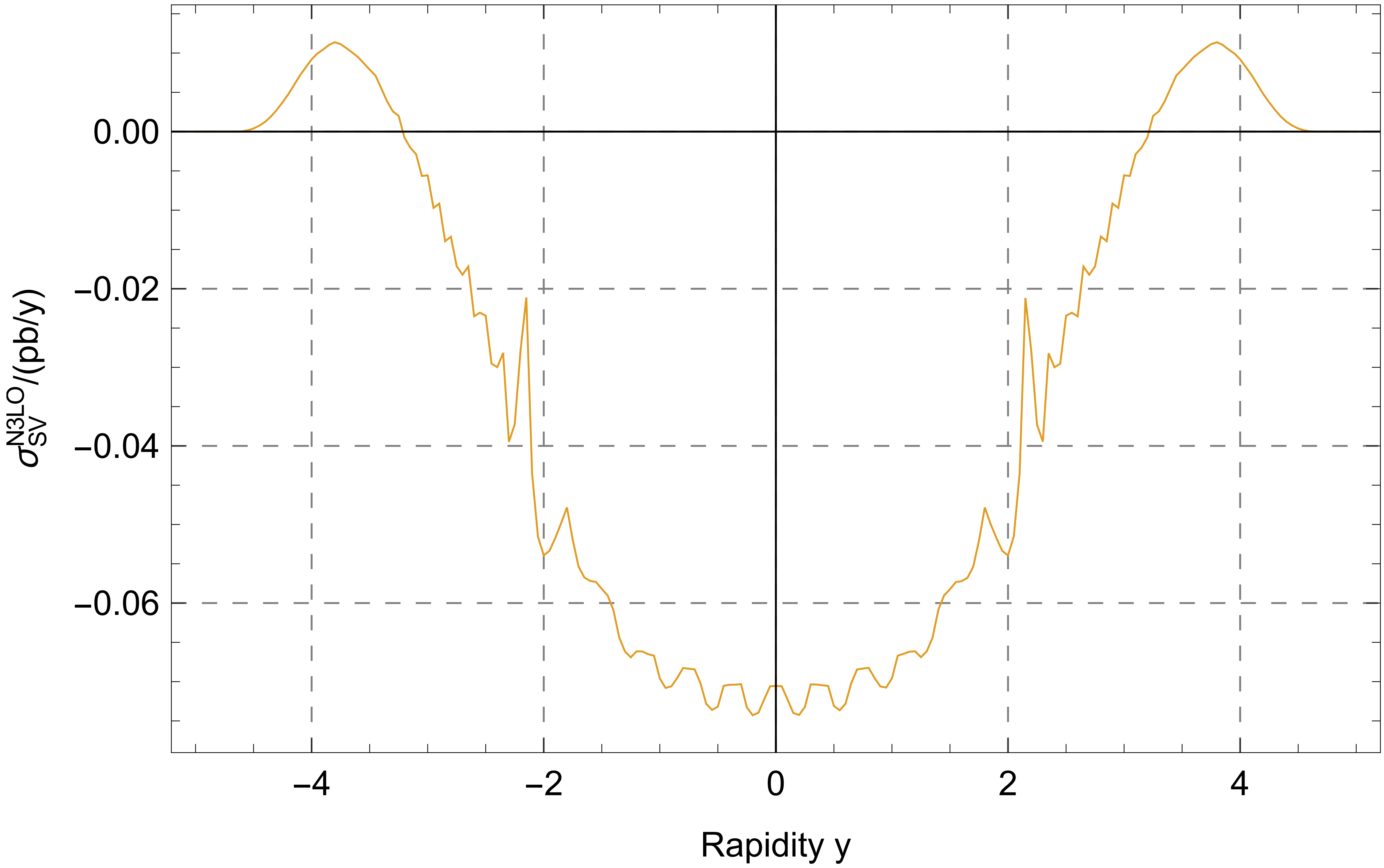
- Systematically improvable approximation.
- Soft expansion gives the opportunity to study differential distribution
- Doing phenomenology in this approximation requires careful case by case analysis to see if the approximation is valid!
- Ambiguity of higher order terms has to be controlled.
  Example:
  Inclusive N3LO:



THE ROAD TO N3LO

- Extend analytic techniques
  for automatic soft amplitude expansions.
- Apply reverse unitarity, differential equations,
  Multiple PolyLog, IBPs, Symbol tools, ….
- Compute 110 new double differential soft master integrals.
- Compute the first terms (Soft-Virtual SV) at N3LO
- Put into code and look at the N3LO corrections to the rapidity distribution and …

N3LO CORRECTIONS TO THE RAPIDITY DISTRIBUTION



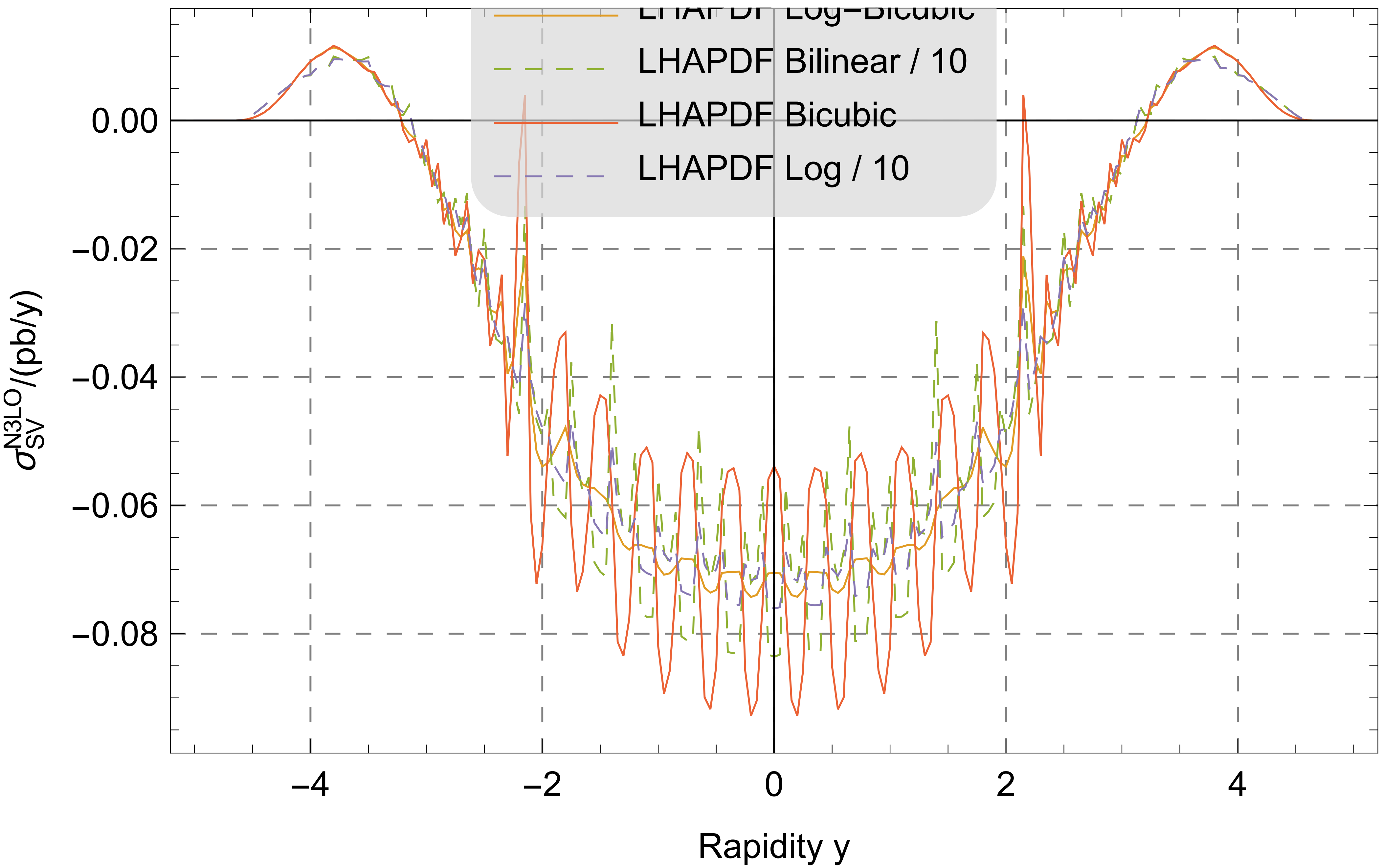
BE CAREFUL WHEN YOU DO SOMETHING NEW

$$\sigma \sim \int dz \mathcal{L}_{gg}(z) \left[\frac{log^{5}(1-z)}{1-z} \right]_{+}$$

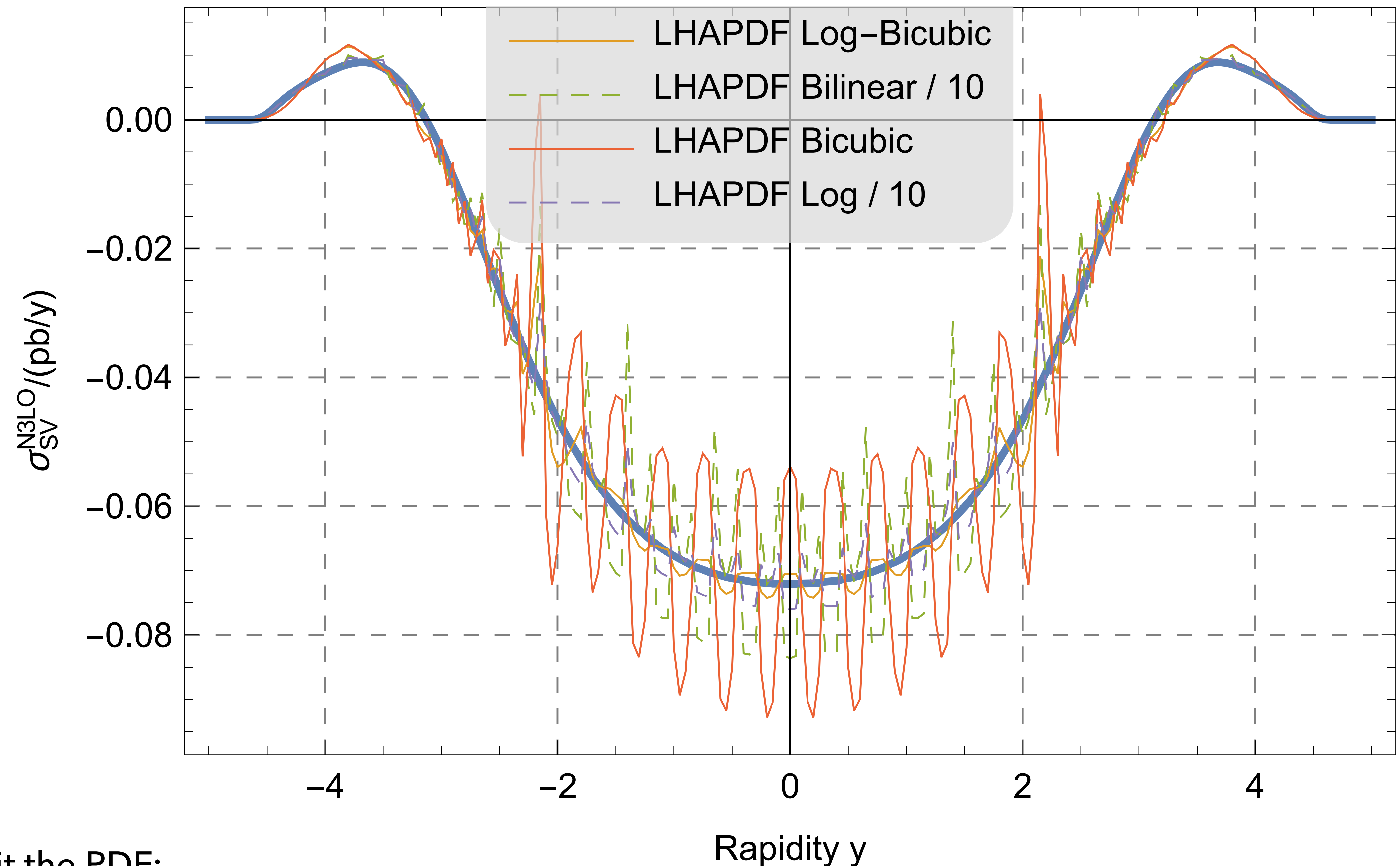
LHAPDF

- LHAPDF: Grid of points for PDFs in x and Q
- Interpolation between points with certain precision
- Not meant to be precise enough for N3LO plus distributions yet ….
- Improvements required: New interpolator, evolve from smooth PDF …. ?

N3LO CORRECTIONS TO THE RAPIDITY DISTRIBUTION



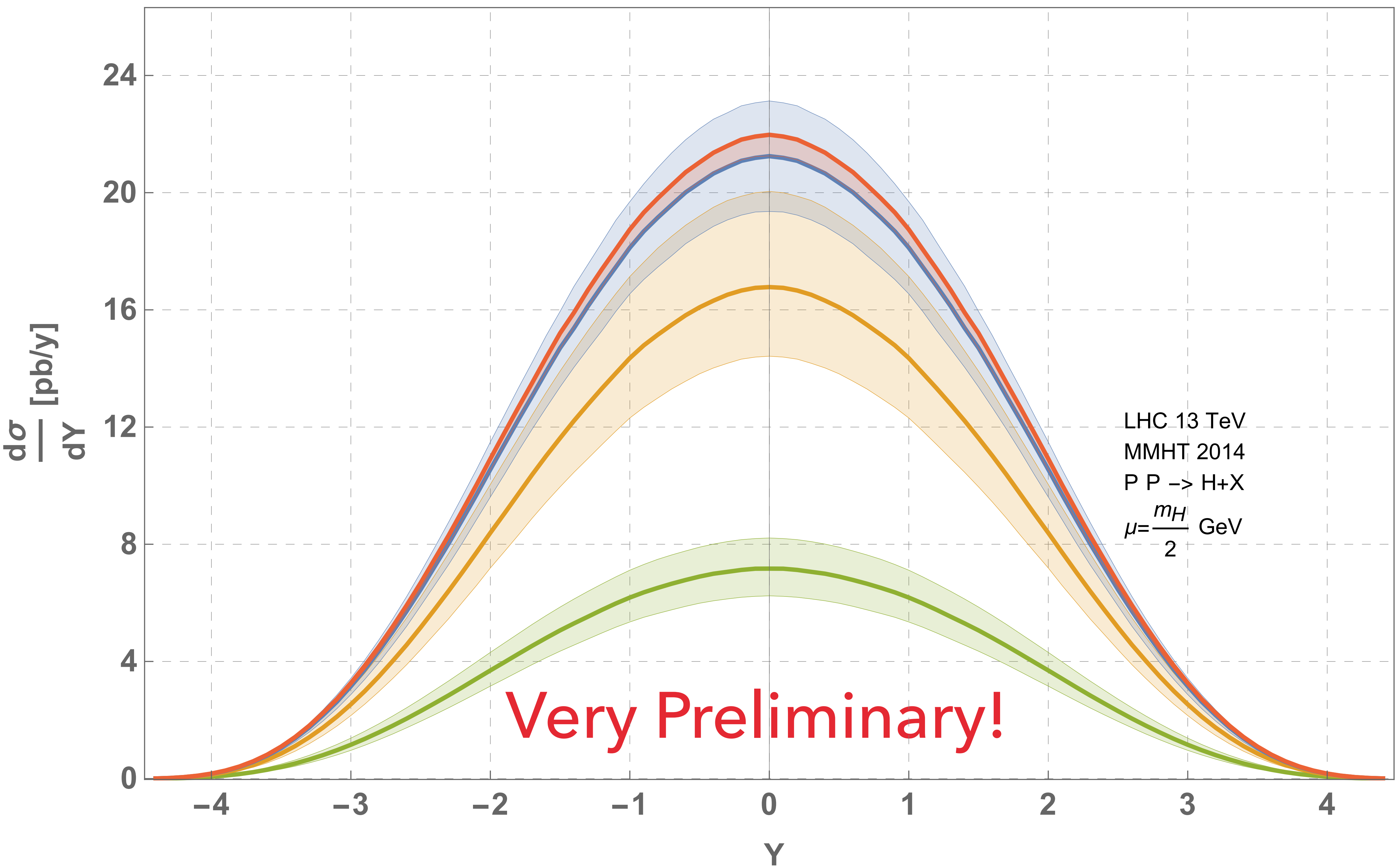
N3LO CORRECTIONS TO THE RAPIDITY DISTRIBUTION



Fit the PDF:

$$f(x, Q = 125) = c_1(1-x)^{e_1}x^{e_2} + c_2(1-x)^{e_3}(1 - c_3\sqrt{x}) + c_4 x + c_5 \log^2(x) + c_6 \log^4(x) + c_7 \log^4(x)$$

INCLUDE RESCALED SV CORRECTIONS TO RAPIDITY AT N3LO



CONCLUSIONS

- Progress towards differential N3LO.
- Higgs-differential framework for realistic final states looks promising.
- Threshold expansions provide a key ingredient for analytic computation.
- Threshold expansion can be used at the differential level to approximate differential cross section predictions.
- Many interesting things to be encountered when going to higher order.

Thank you!