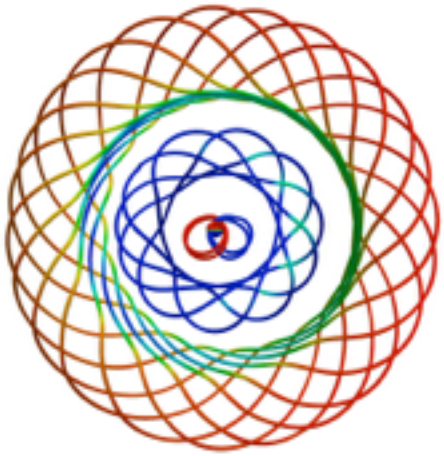
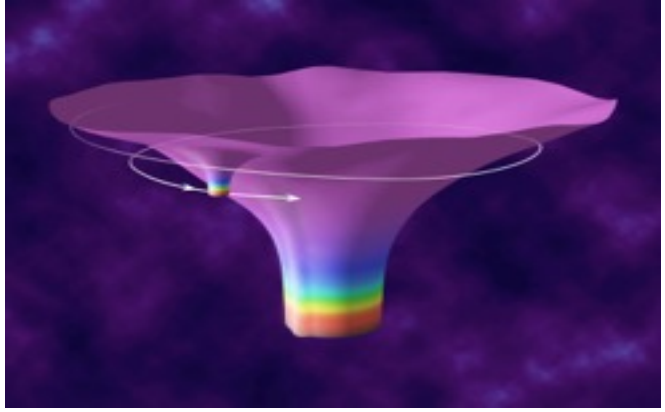
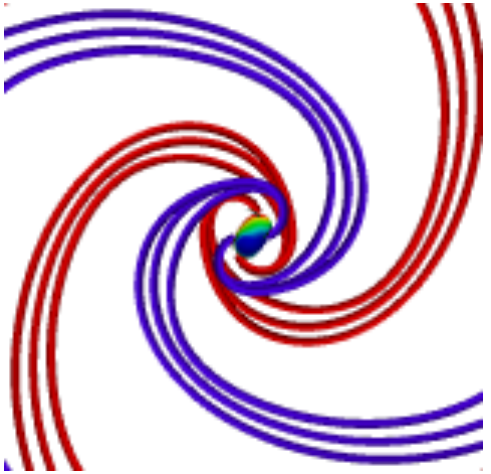




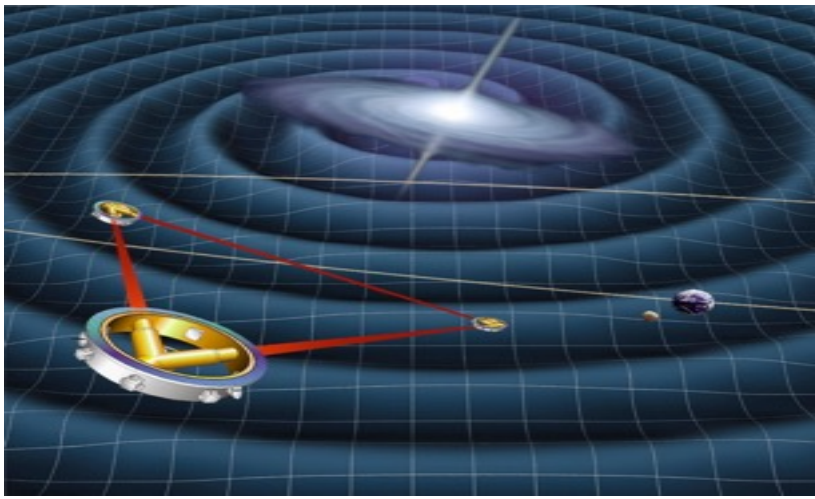


Geometrodynamics

Exploring the nonlinear dynamics of spacetime via computer simulations and gravitational wave observations

Kip Thorne





Université Catholique de Louvain, 18 October 2016

John Wheeler: Geometrodynamics

The Nonlinear Dynamics of Curved Spacetime



Nonlinear Dynamics Elsewhere in Physics

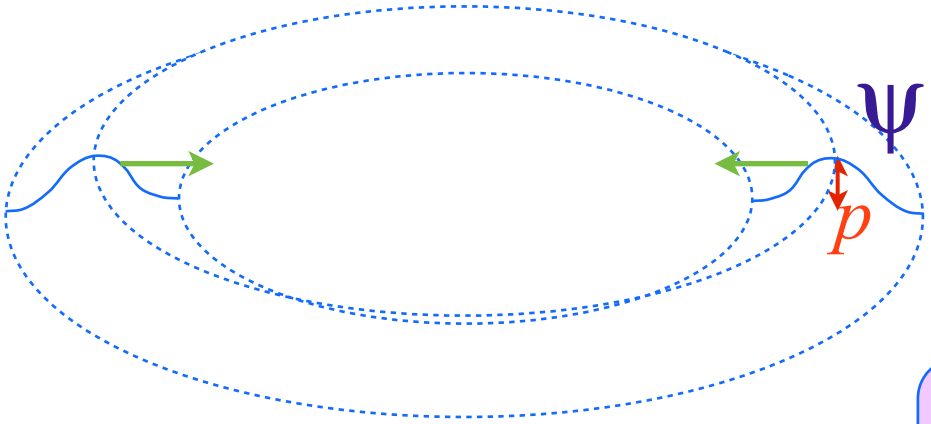
- fluid turbulence, tornados, …
- phase transitions in condensed matter
- nonlinear optics (modern optical technology)
- colliding solitons in fluids, plasmas, nonlinear crystals, optical fibers, …
- catastrophes, e.g. in optical caustics and phase transitions
- chaotic maps, strange attractors, …

Arenas for Geometrodynamics
probed by numerical & analytical relativity

- Gravitational waves: nonlinear self coupling in ***critical gravitational collapse***
- Spacetime dynamics near ***singularities***
  - cosmological singularities
  - singularities inside black holes
- ***Binary black hole*** mergers
- Gravitational-wave ***observations***

Gravitational Waves: Nonlinear Self-coupling

- Motivation: Choptuik's analysis of spherical scalar-wave implosion (1993 -)



scalar wave's energy generates spacetime curvature, then wave interacts with the curvature

$p > p_*$: Black hole forms

$$M_{BH} \propto (p - p_*)^{\beta} , \quad \beta \simeq 0.374$$

$p < p_*$: Wave disperses

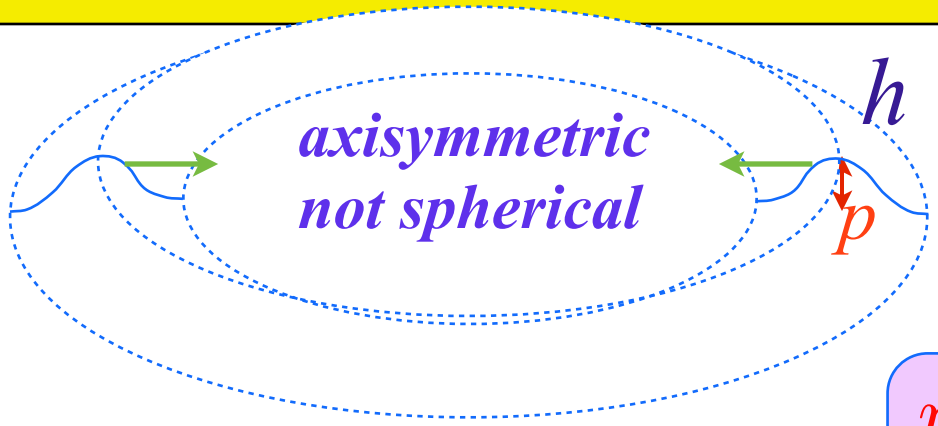
$$\left(R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}\right)^{-1/4}_{\max} \propto (p - p_*)^{\beta}$$

$p = p_*$:Discretely self-similar

Gravitational Waves: Nonlinear Self-coupling

- ***Imploding Gravitational Wave***

Abrahams & Evans (1993)
Evgeny Sorkin (2011)



wave self-coupling via
nonlinear Einstein equations

$p > p_*$: Black hole forms

$$M_{BH} \propto (p - p_*)^{\beta} \ , \quad \beta \simeq 0.38$$

$p < p_*$: Wave disperses

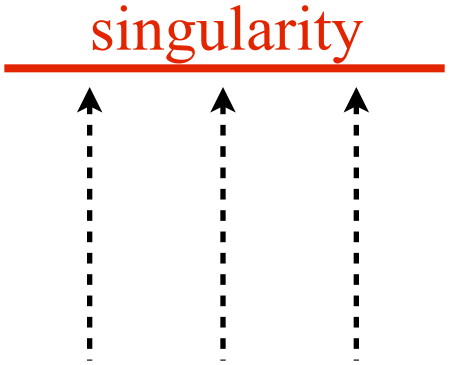
$$\left(R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}\right)^{-1/4}_{\max} \propto (p - p_*)^{\beta}$$

$p = p_*$: Moderately strong evidence for discrete self similarity

Numerical studies are in their infancy.
Great richness remains to be uncovered!

Geometrodynamics Near Singularities

- Some ancient history:
  - » 1960s: ***Singularity theorems*** - Penrose, Hawking, ...
  - » 1969 - 71: ***BKL*** approximate analysis (Belinsky, Khalatnikov, Lifshitz): ***geometrodynamics near generic spacelike singularity:***
    - – temporal dynamics is ***Mixmaster*** (Misner; Belinsky & Khalatnikov)
    - – (non-extreme) matter has no influence



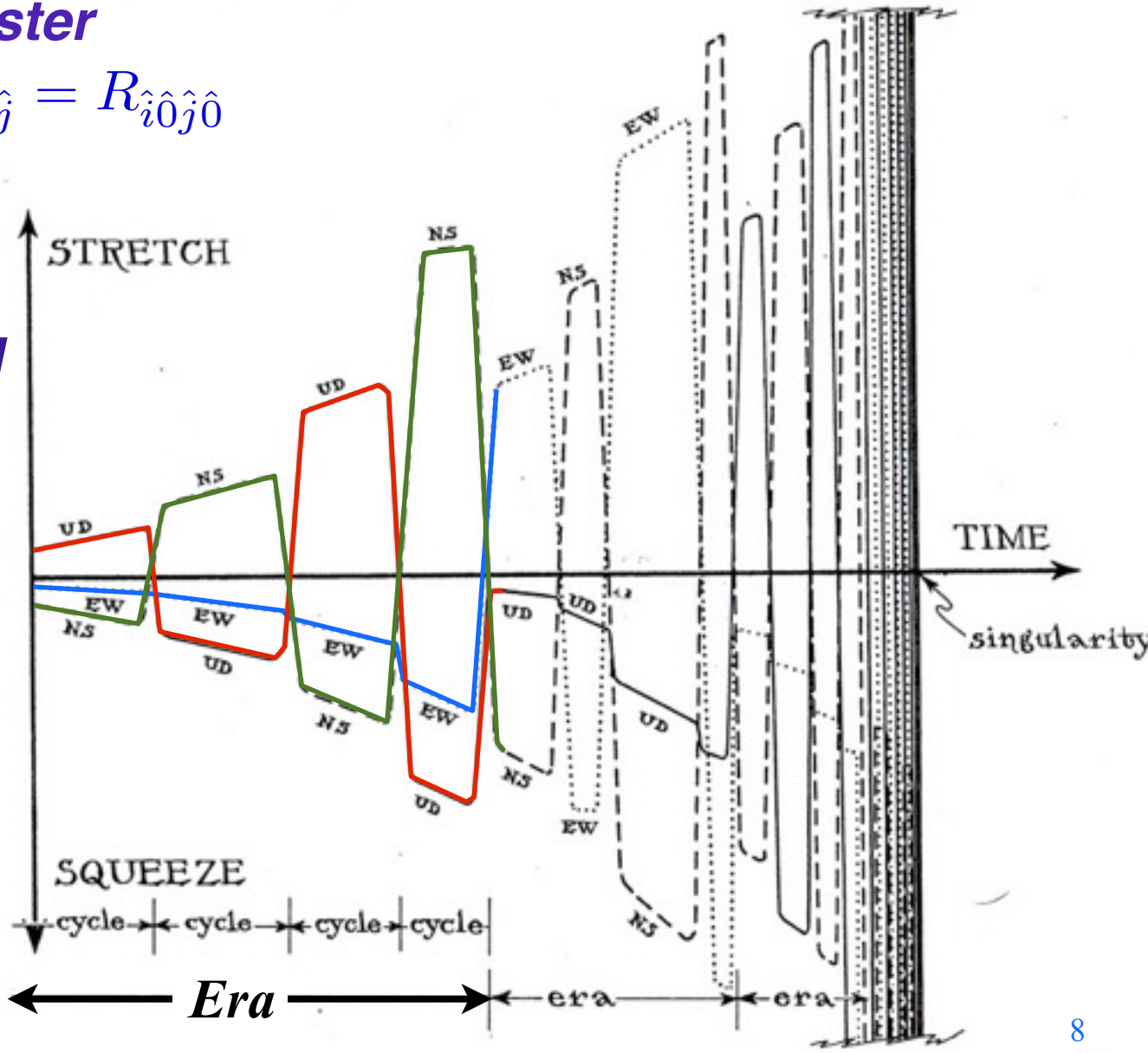
  - » ***Skepticism in the West:***
    - – BKL "conjecture" and "heuristic arguments"

Geometrodynamics Near Singularities

- Recall ***Mixmaster dynamics***: $\mathcal{E}_{\hat{i}\hat{j}} = R_{\hat{i}\hat{0}\hat{j}\hat{0}}$
  - » tidal field

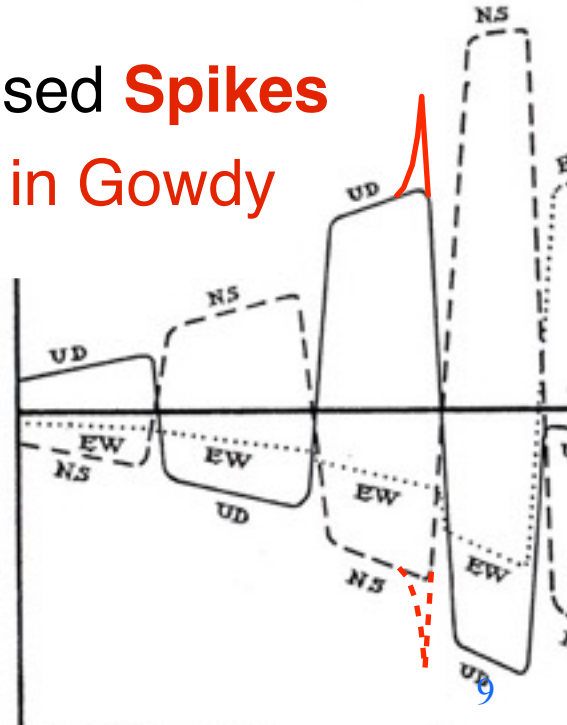
- ***Era transitions driven by spatial curvature***

- Chaotic map

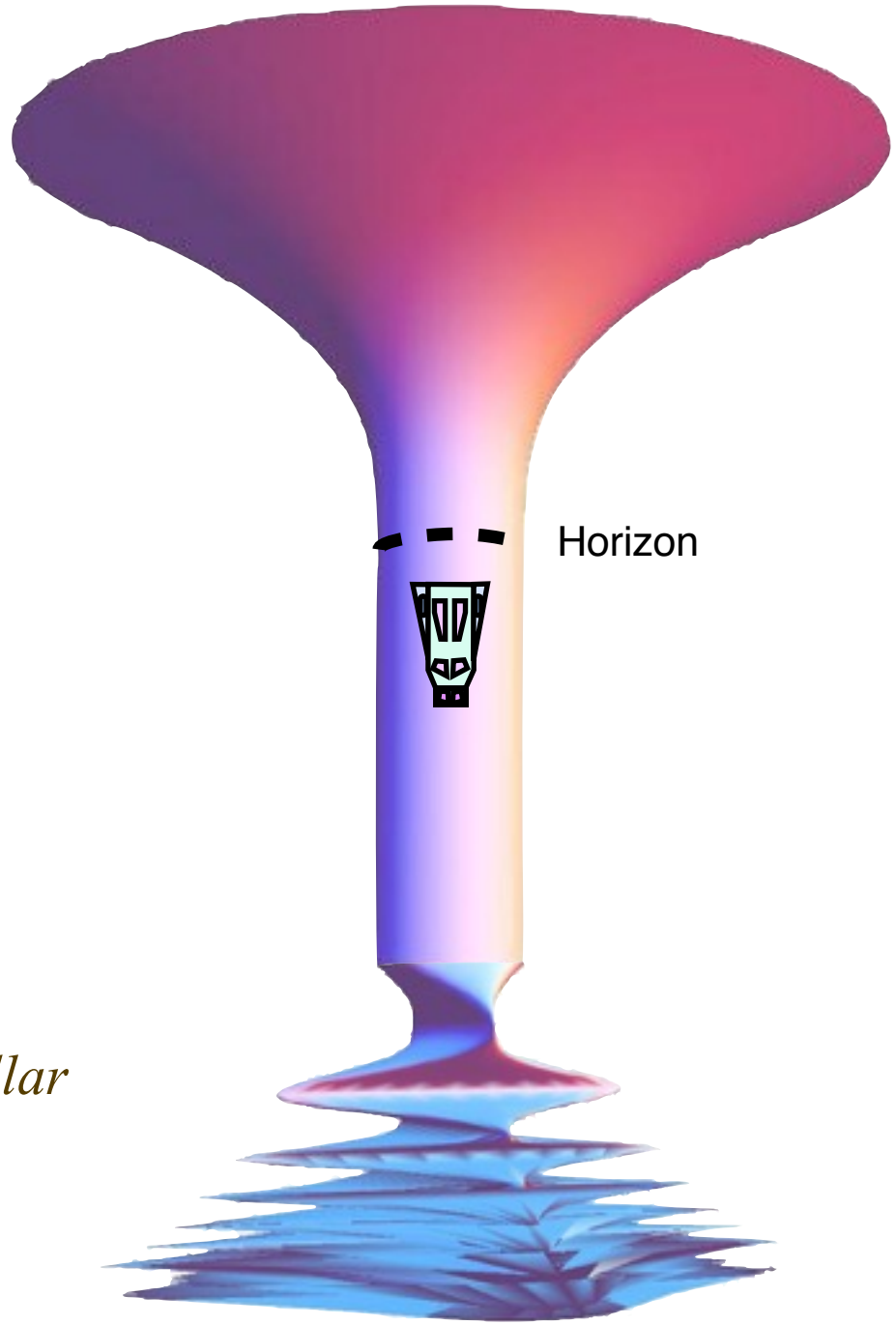


Geometrodynamics Near Singularities

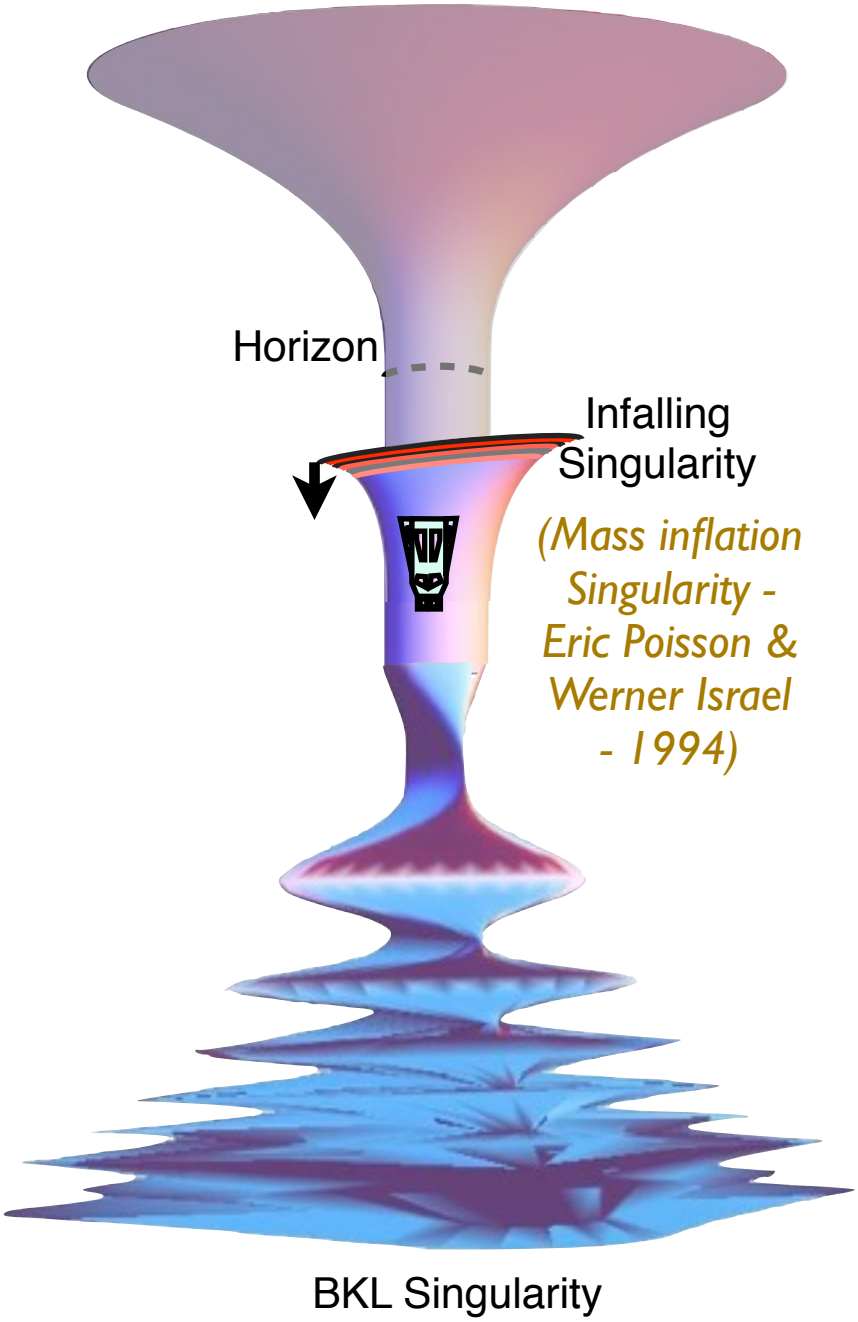
- Program to test BKL via numerical simulations:
  - » Formulated by Beverly Berger and Vince Moncrief (1994)
  - » Carried out by Berger, Moncrief, Garfinkle, Isenberg, Weaver: 1994 - ....
  - » Analytic studies motivated by simulations: Rendall, Weaver, ... 2001 - ....
- ***BKL largely confirmed, Except:*** BKL missed **Spikes**
  - » Discovered by Berger & Moncrief (1994) in Gowdy
  - » ***Triggered by spatial inhomogeneity***
  - » On spatial 2-surfaces in 3D space
  - » Recur; sharper at later times
  - » Modify the chaotic map (Lim, Andersson, Garfinkle, Pretorius, 2009)

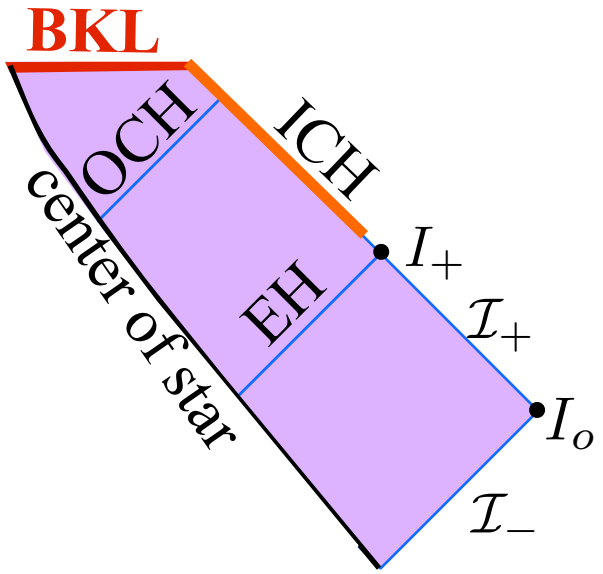


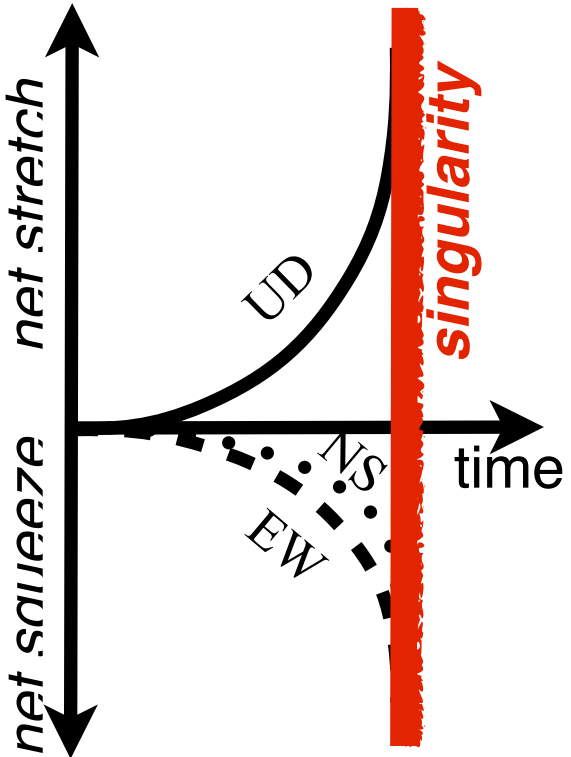
Generic Singularity Inside a Black Hole

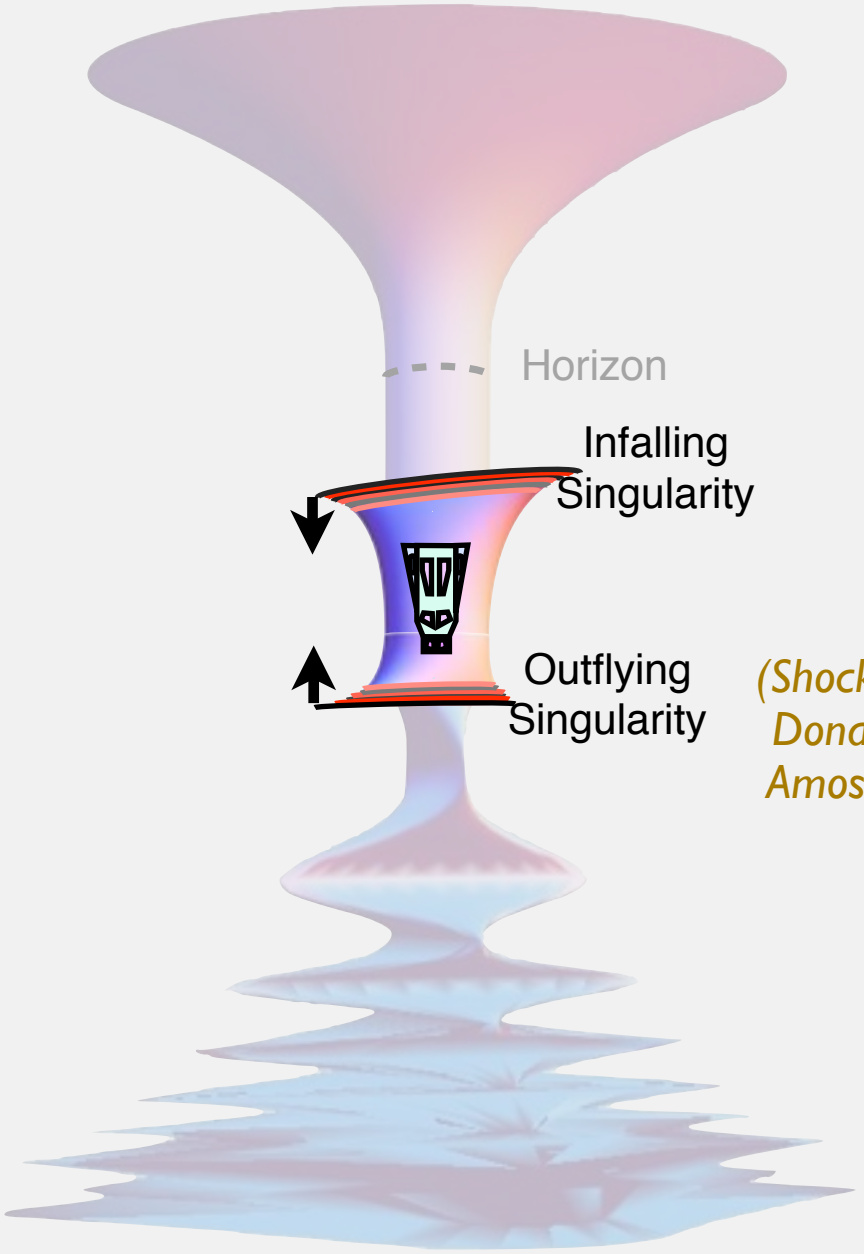


from my book *The Science of Interstellar*

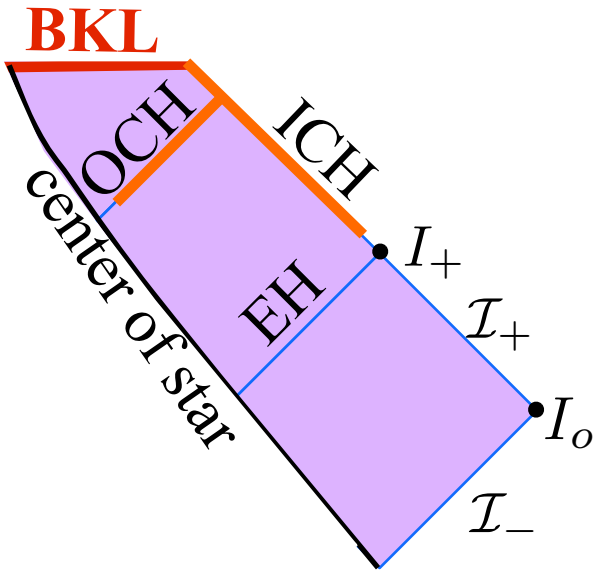

Horizon
Infalling
Singularity
(Mass inflation
Singularity -
Eric Poisson &
Werner Israel
- 1994)
BKL Singularity

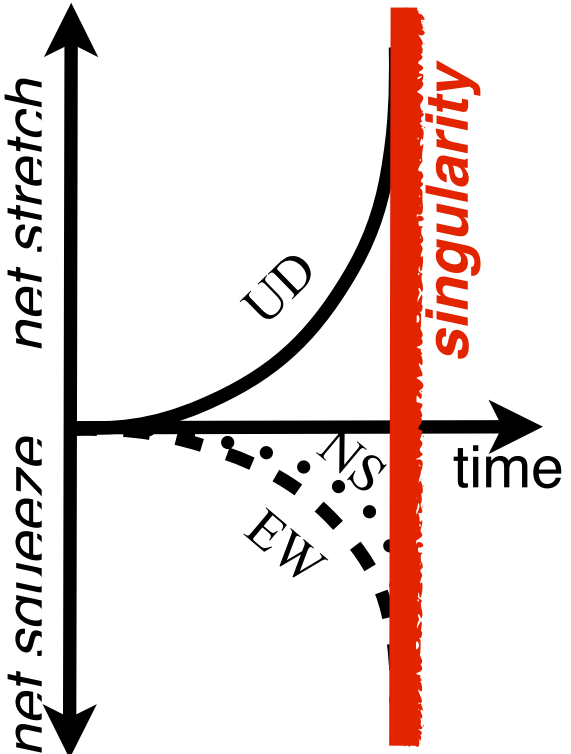

BKL
OCH
ICH
EH
center of star
I_+
$\mathcal{I}_+$
I_o
$\mathcal{I}_-$


net stretch
net squeeze
UD
NS
EW
time
singularity



(Shock Singularity - Donald Marolf & Amos Ori - 2012)





Geometrodynamics in Binary Black Holes

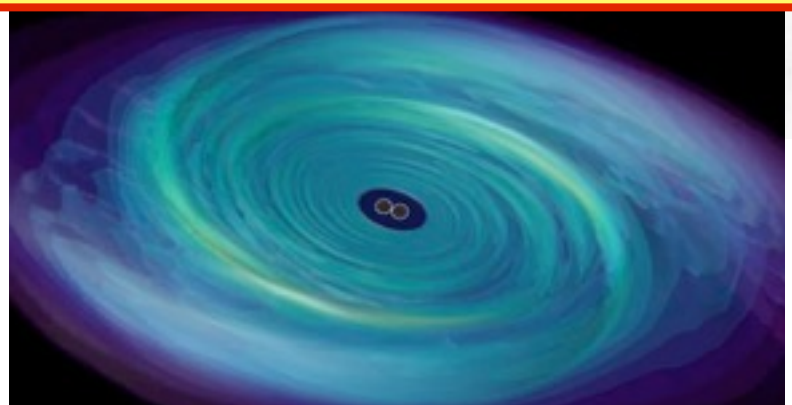
Collisions of Black Holes:
The most violent events in the Universe

Power output in gravitational waves:
~ 50 times luminosity of all stars in Universe

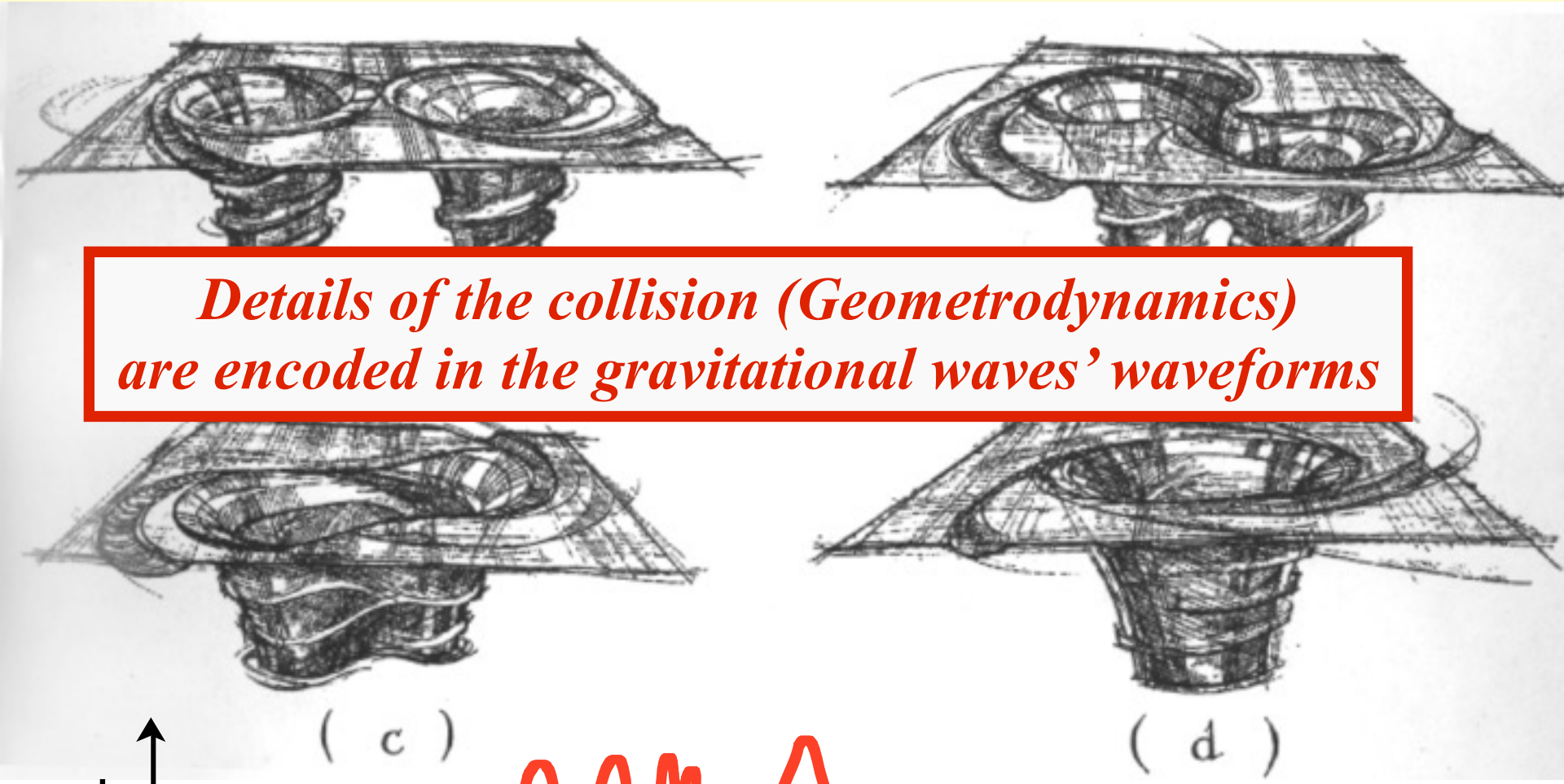
No Electromagnetic Waves emitted whatsoever

except from disturbed accretion disks





Collisions of Black Holes: The most violent events in the Universe



Details of the collision (Geometrodynamics) are encoded in the gravitational waves' waveforms



Example of Numerical Simulation

GW150914

SXS* Collaboration:

[Project to Simulate eXtreme Spacetimes]

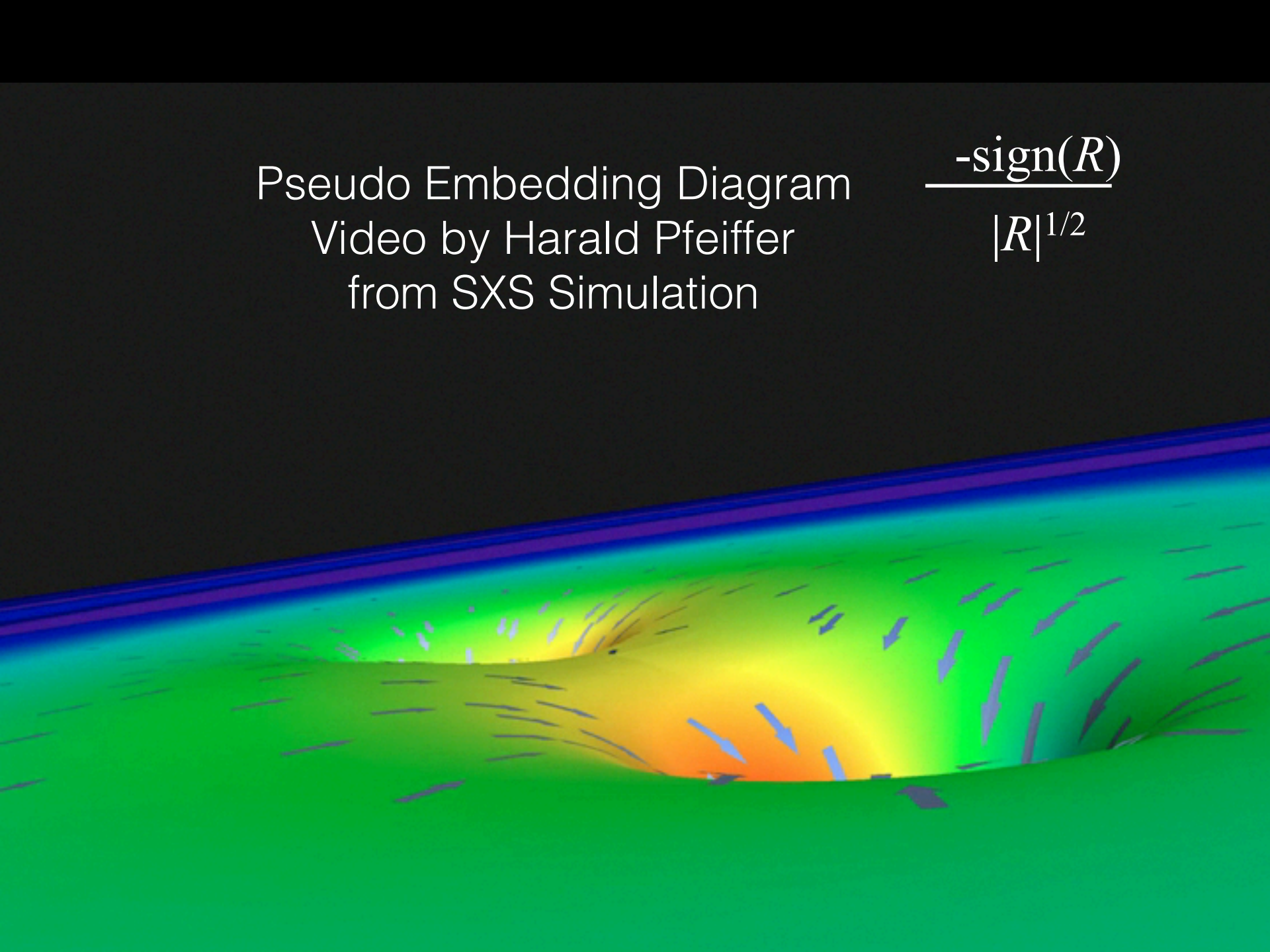
Cornell/Caltech/CITA/CalState Fullerton/Oberlin/WSU

[Kidder, Pfeiffer, Scheel, Teukolsky,…]

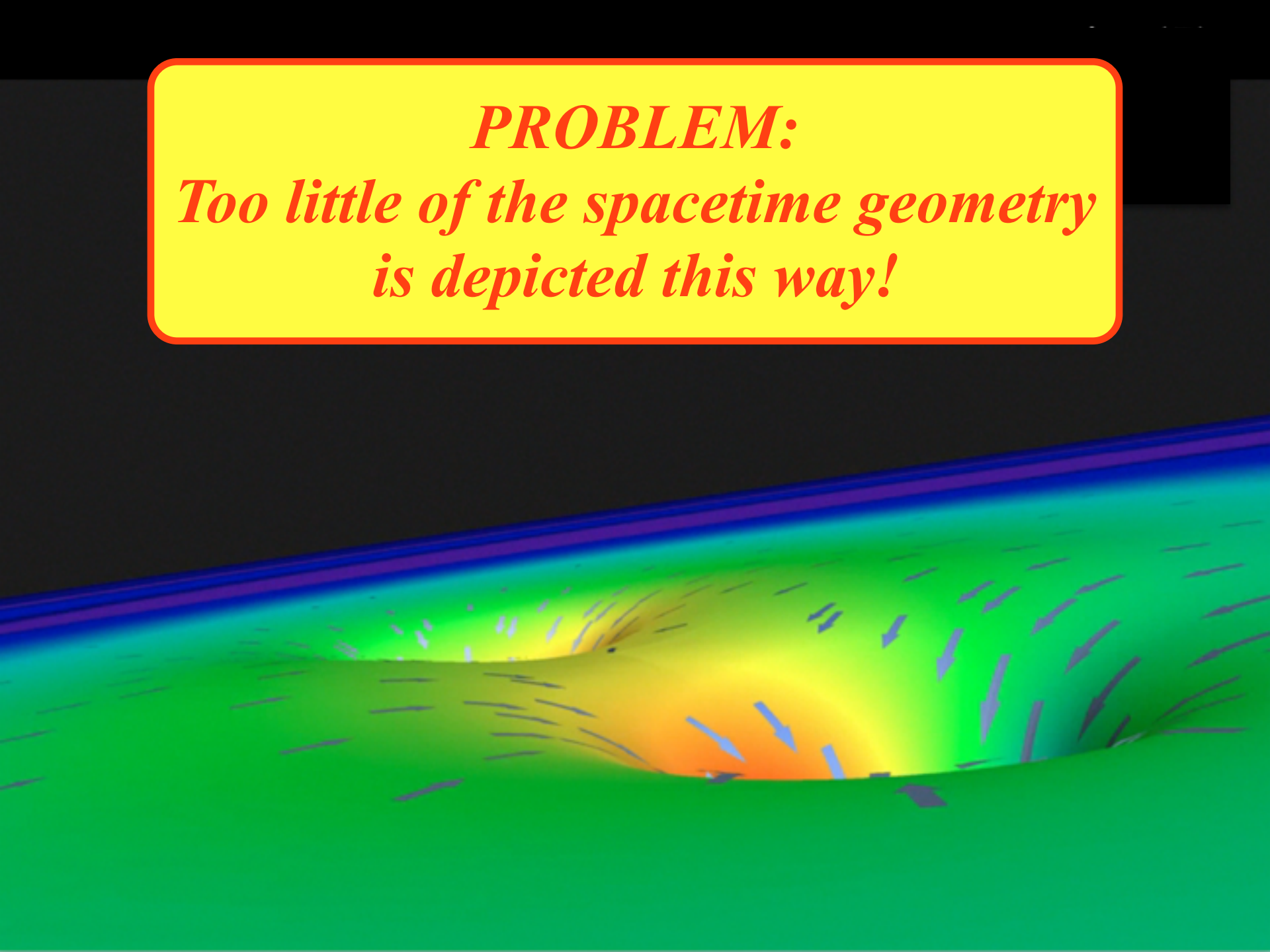
Depiction of spacetime metric (geometry) in orbital plane

Pseudo Embedding Diagram
Video by Harald Pfeiffer
from SXS Simulation

$$\frac{-\mathrm{sign}(R)}{|R|^{1/2}}$$



PROBLEM:
Too little of the spacetime geometry is depicted this way!



Visualizing the Vacuum Riemann Curvature Tensor

Rob Owen, Jeandrew Brink, Yanbei Chen, Jeff Kaplan, Geoffrey Lovelace, Keith Matthews, David Nichols, Mark Scheel, Fan Zhang, Aaron Zimmerman, and Kip Thorne

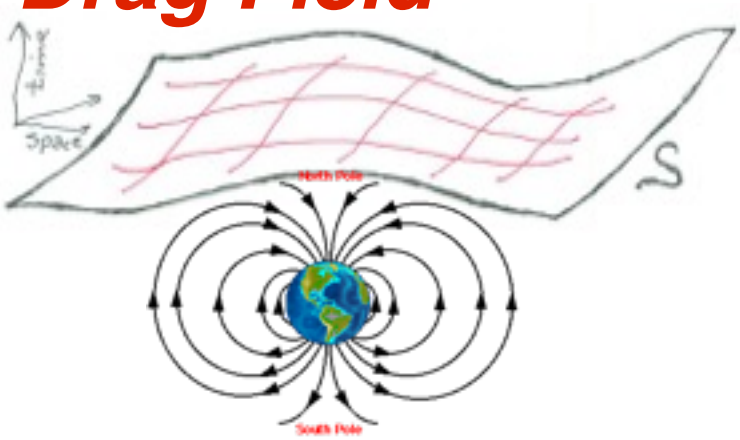
Caltech, Cornell, and NiTheP (South Africa)

Physical Review Letters , **106,** 151101 (2011)

arXiv:1012.4869

Tidal Field & Frame-Drag Field

- Slice spacetime into space plus time
- EM field tensor ***F*** → Electric field

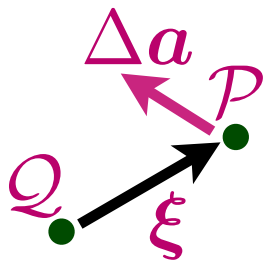


- Weyl curvature tensor → **"electric" part** $\mathcal{E}_{jk}$ and **"magnetic" part** $\mathcal{B}_{jk}$

$$\mathcal{E}_{jk} = C_{0j0k} \qquad \mathcal{B}_{jk} = \frac{1}{2}\epsilon_{jpq}C^{pq}{}_{k0}$$

Symmetric, Trace-Free (STF) tensors

- $\mathcal{E}_{jk}$ describes **tidal accelerations** $\Delta a_j = -\mathcal{E}_{jk}\,\xi^k$



We call $\mathcal{E}_{jk}$ the ***tidal field***

- $\mathcal{B}_{jk}$ describes **differential frame dragging**: Gyroscope at P precesses relative to inertial frames at Q with angular velocity

$$\Delta\Omega_j = \mathcal{B}_{jk}\xi^k$$

We call $\mathcal{B}_{jk}$ the ***frame-drag field***

Horizon Tendicity

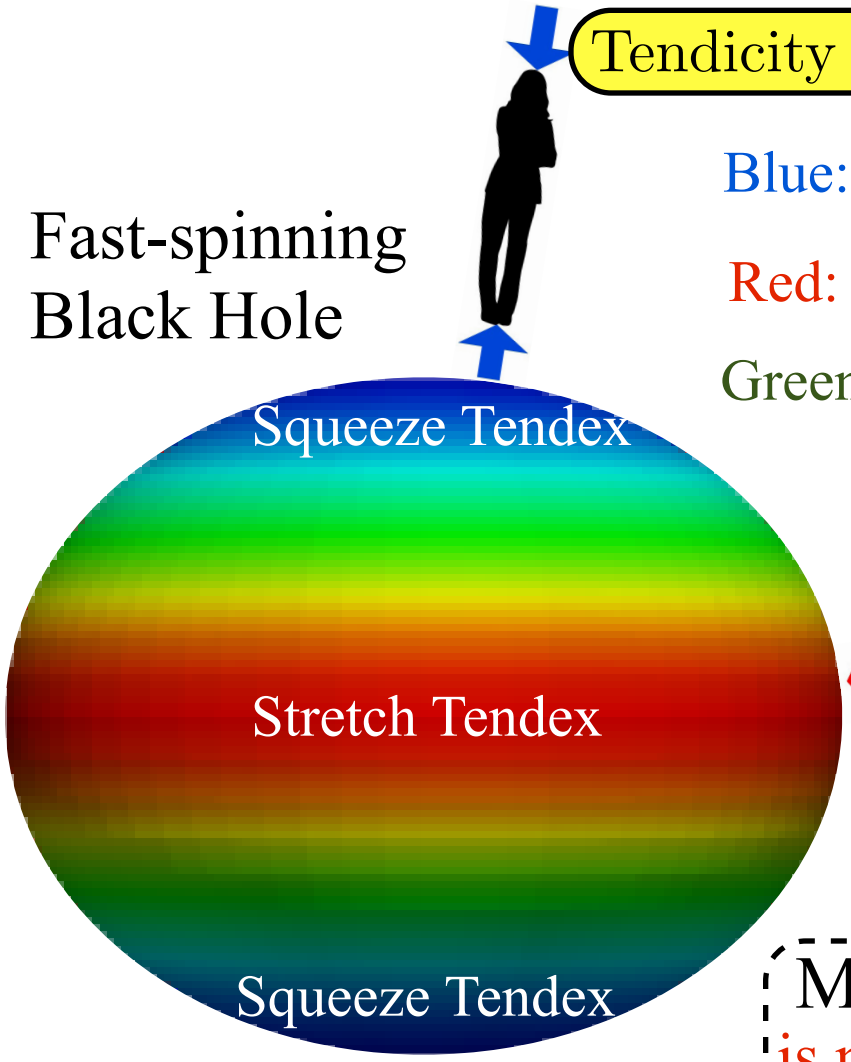
Tendicity $= \mathcal{E}_{nn} =$ (RelativeAcceleration)/height

Blue: Squeeze; $\mathcal{E}_{nn} > 0$

Red: Stretch; $\mathcal{E}_{nn} < 0$

Green: small tendicity; $\mathcal{E}_{nn} \simeq 0$

Fast-spinning Black Hole



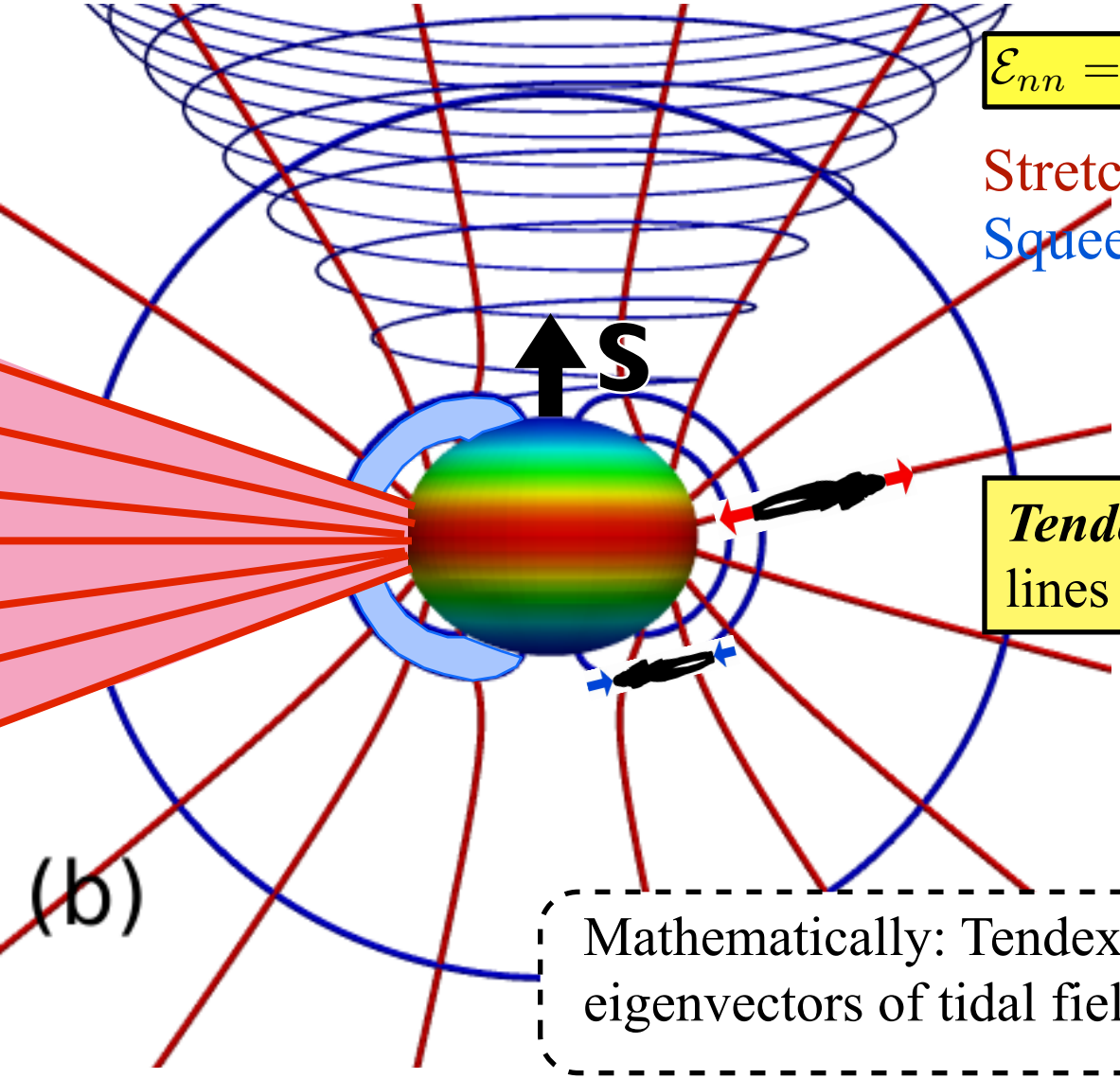
Horizon Tendex: Region with large tendicity

Mathematically: $\mathcal{E}_{nn}$ is normal-normal component of "electric" part of Weyl Curvature Tensor ***(tidal field)***

Tendex Lines and their Tendicities

Fast Spinning Black Hole

Each tendex line has a tendicity $\mathcal{E}_{nn}$



$\mathcal{E}_{nn} = (\text{RelativeAcceleration})/\text{height}$

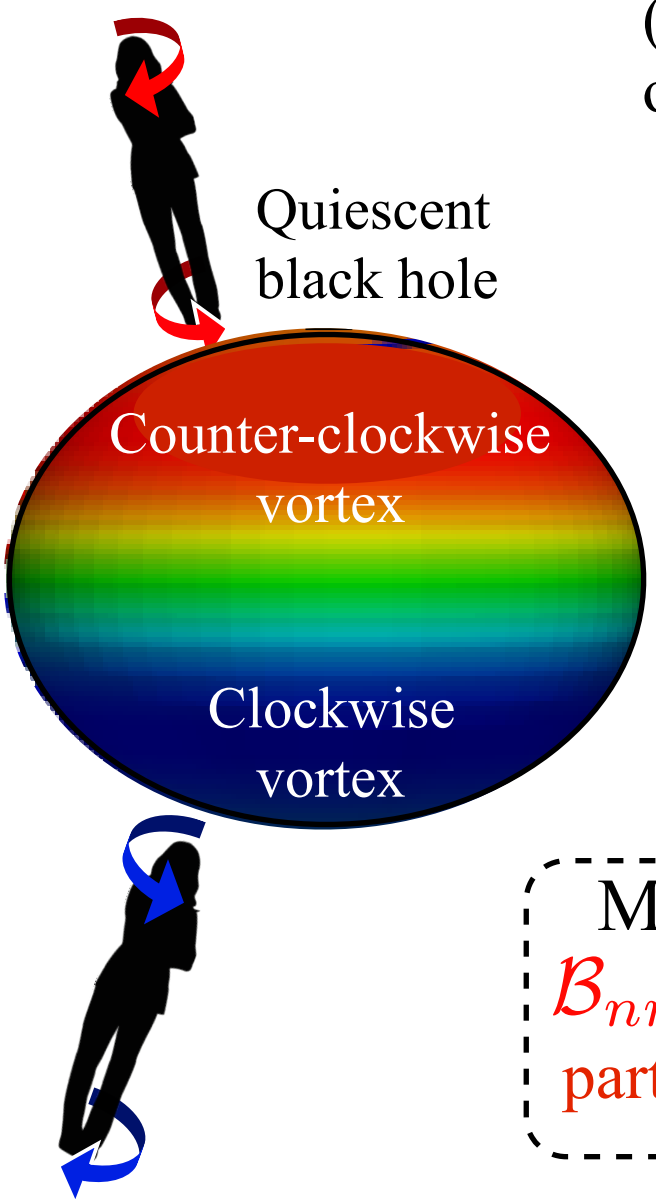
Stretch along red lines $\mathcal{E}_{nn} < 0$

Squeeze along blue lines $\mathcal{E}_{nn} > 0$

Tendex: a collection of tendex lines with large tendicity

Mathematically: Tendex lines are Integral curves of eigenvectors of tidal field $\mathcal{E}_{ij}$; tendicity is eigenvalue

Horizon Vorticity



(Angular velocity of feet as seen by head, or head as seen by feet) = Ω

$$\text{HorizonVorticity} = \mathcal{B}_{nn} = \Omega/\text{height}$$

Blue: Clockwise vorticity; $\mathcal{B}_{nn} > 0$

Red: Counter-clockwise vorticity; $\mathcal{B}_{nn} < 0$

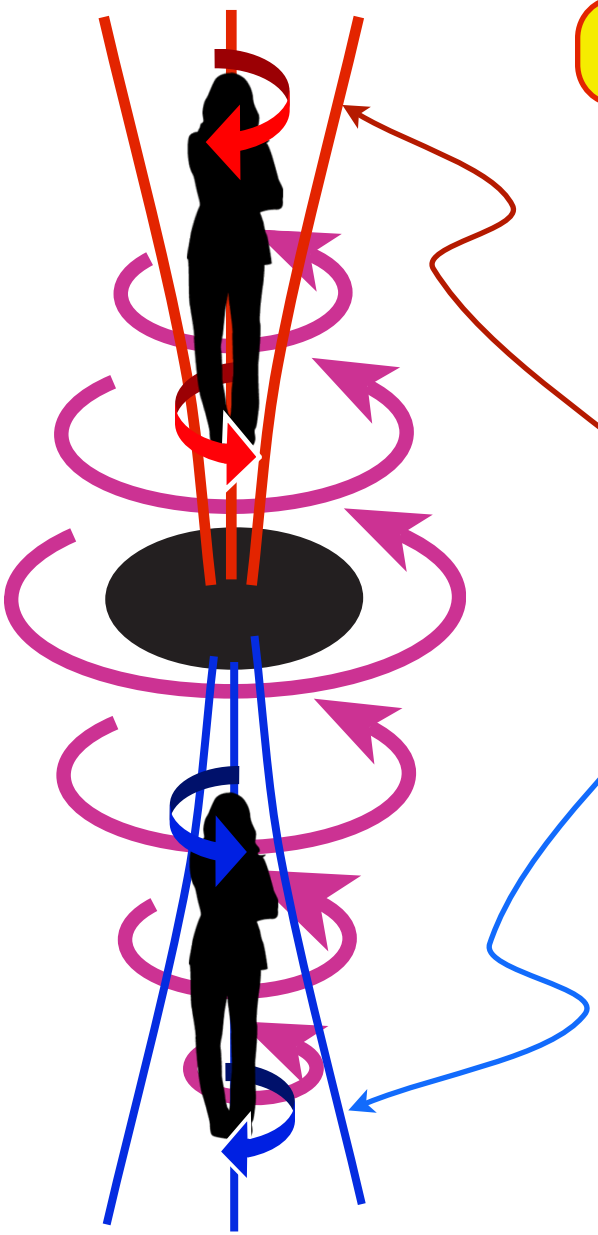
Green: small vorticity; $\mathcal{B}_{nn} \simeq 0$

Horizon Vortex: Region with high vorticity

Mathematically:

$\mathcal{B}_{nn}$ is normal-normal component of "magnetic" part of Weyl Curvature Tensor ***(frame-drag field)***

Vortex Lines Outside Black Hole



Each vortex line has a vorticity

Vorticity $= \mathcal{B}_{nn} = \Omega/\text{height}$

Blue: Clockwise vorticity; $\mathcal{B}_{nn} > 0$

Red: Counter-clockwise vorticity; $\mathcal{B}_{nn} < 0$

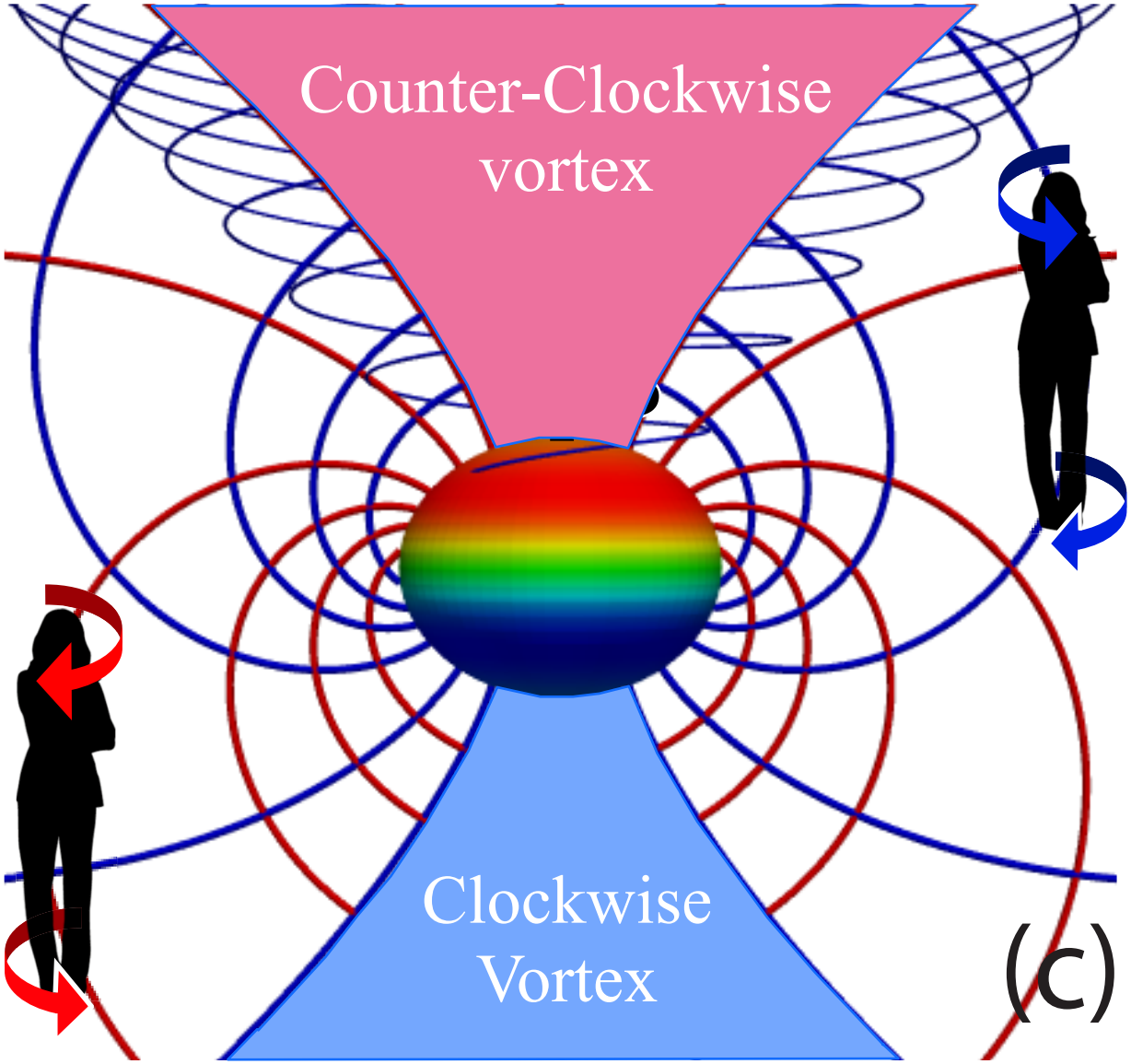
Vortex lines guide the whirling vortex

Mathematically: ***Vortex line*** is Integral curve of an eigenvector ***n*** of frame-drag field $\mathcal{B}_{ij}$

Vorticity $\mathcal{B}_{nn}$ is eigenvalue of $\mathcal{B}_{ij}$

Vortex lines & their vorticities completely characterize the frame-drag field $\mathcal{B}_{ij}$

Kerr Black Hole

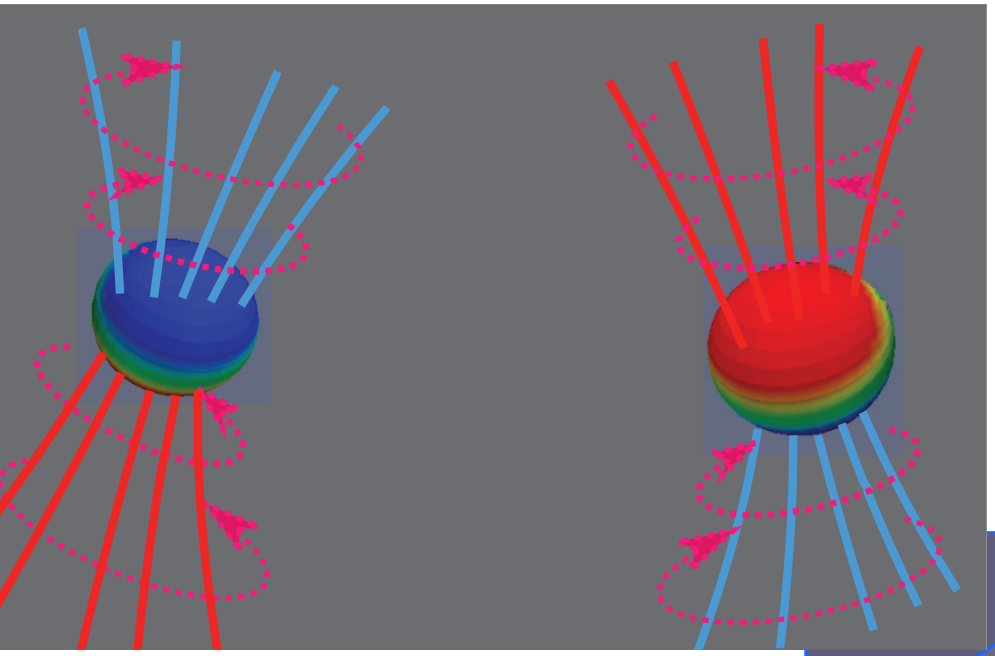


Vortex
A collection of vortex lines with large vorticity

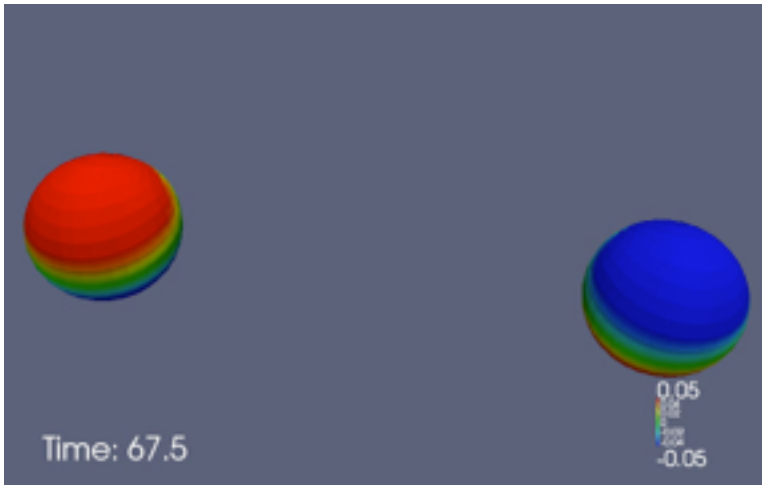
Physically:
A strong "tornado" of twisting space

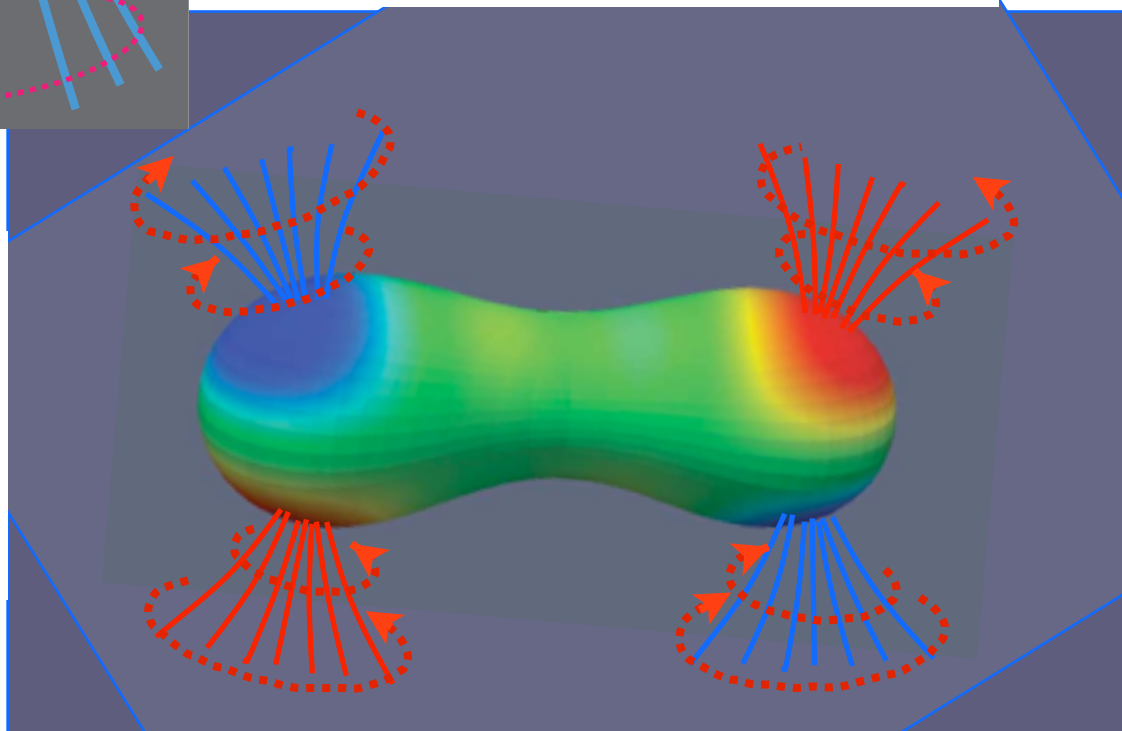
Head-On Collision with Transverse Spin

Keith Matthews, Geoffrey Lovelace, Mark Scheel



Vortexes robustly retain their individuality



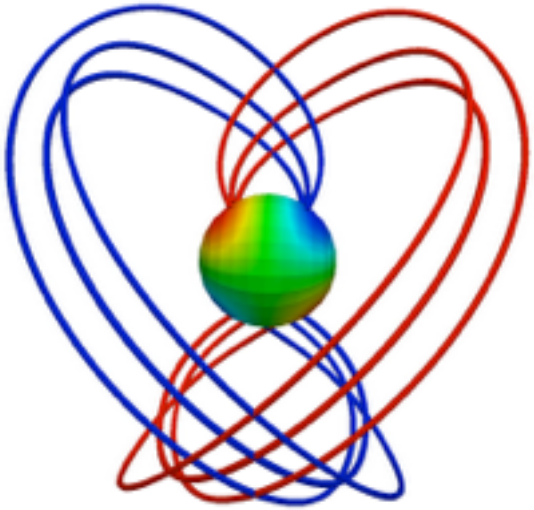


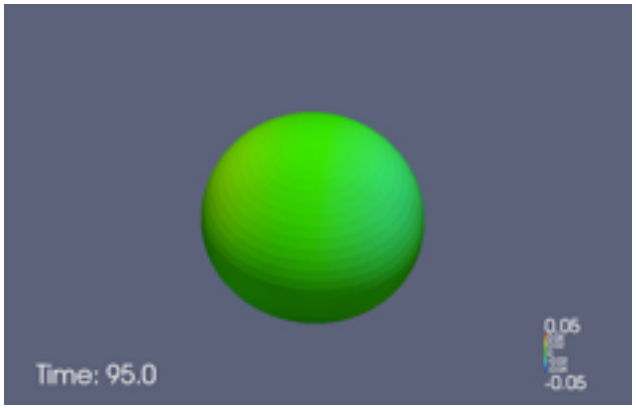
Head-On Collision with Transverse Spin

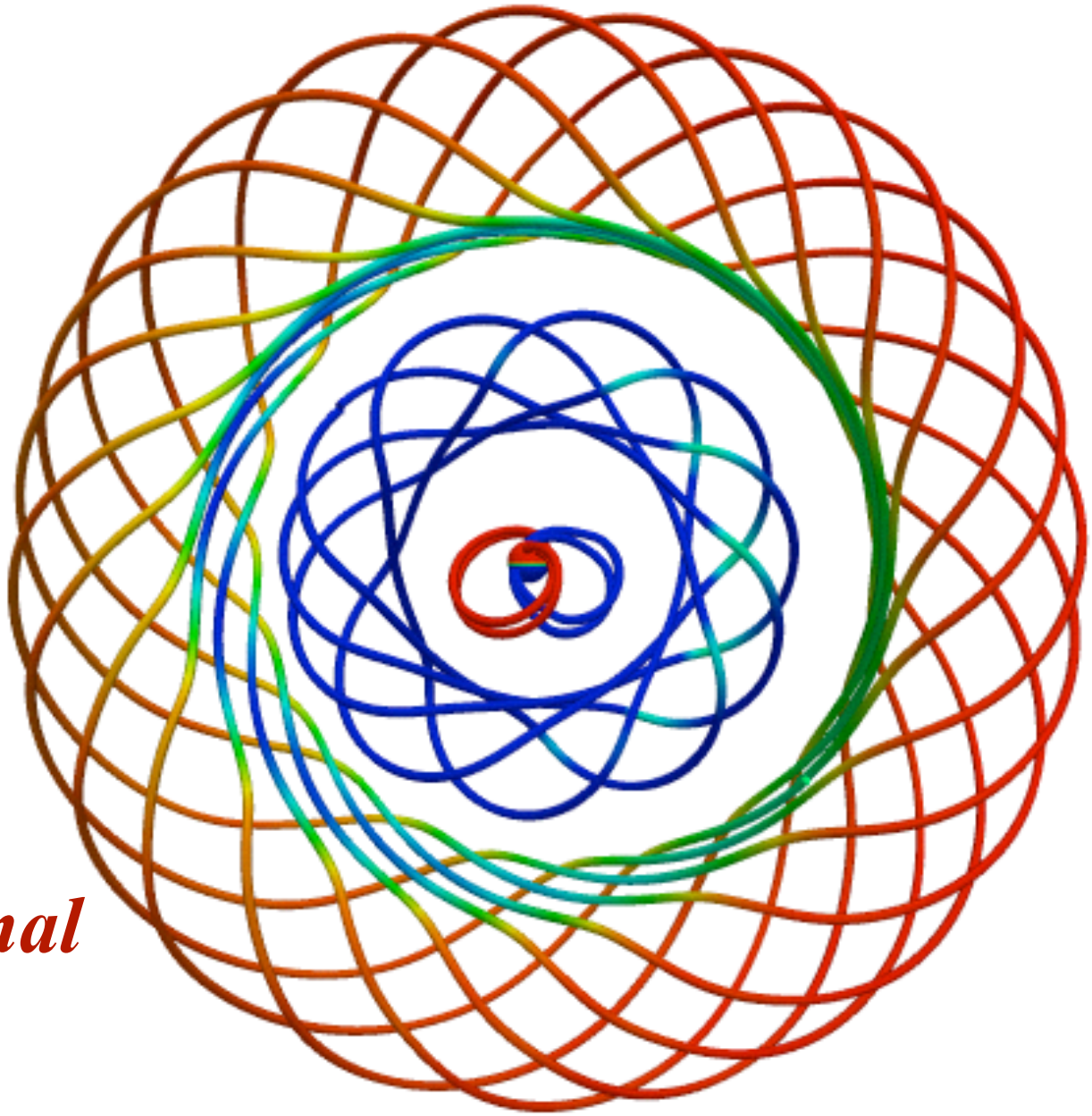
Keith Matthews, Geoffrey Lovelace, Mark Scheel



Sloshing Ejects Vortexes







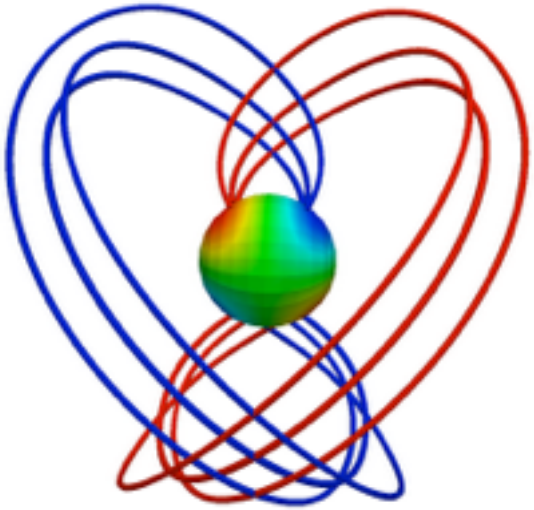
gravitational waves

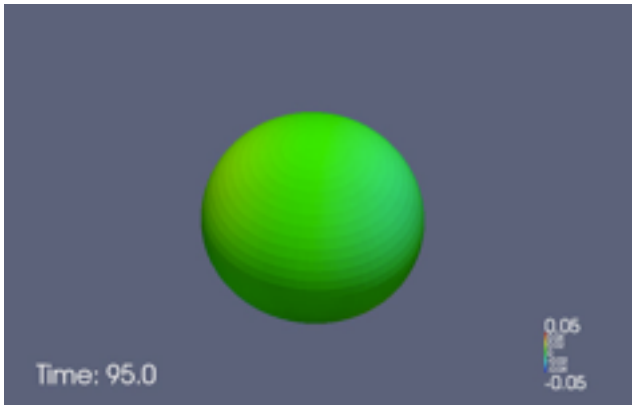
Sloshing Ejects Vortexes

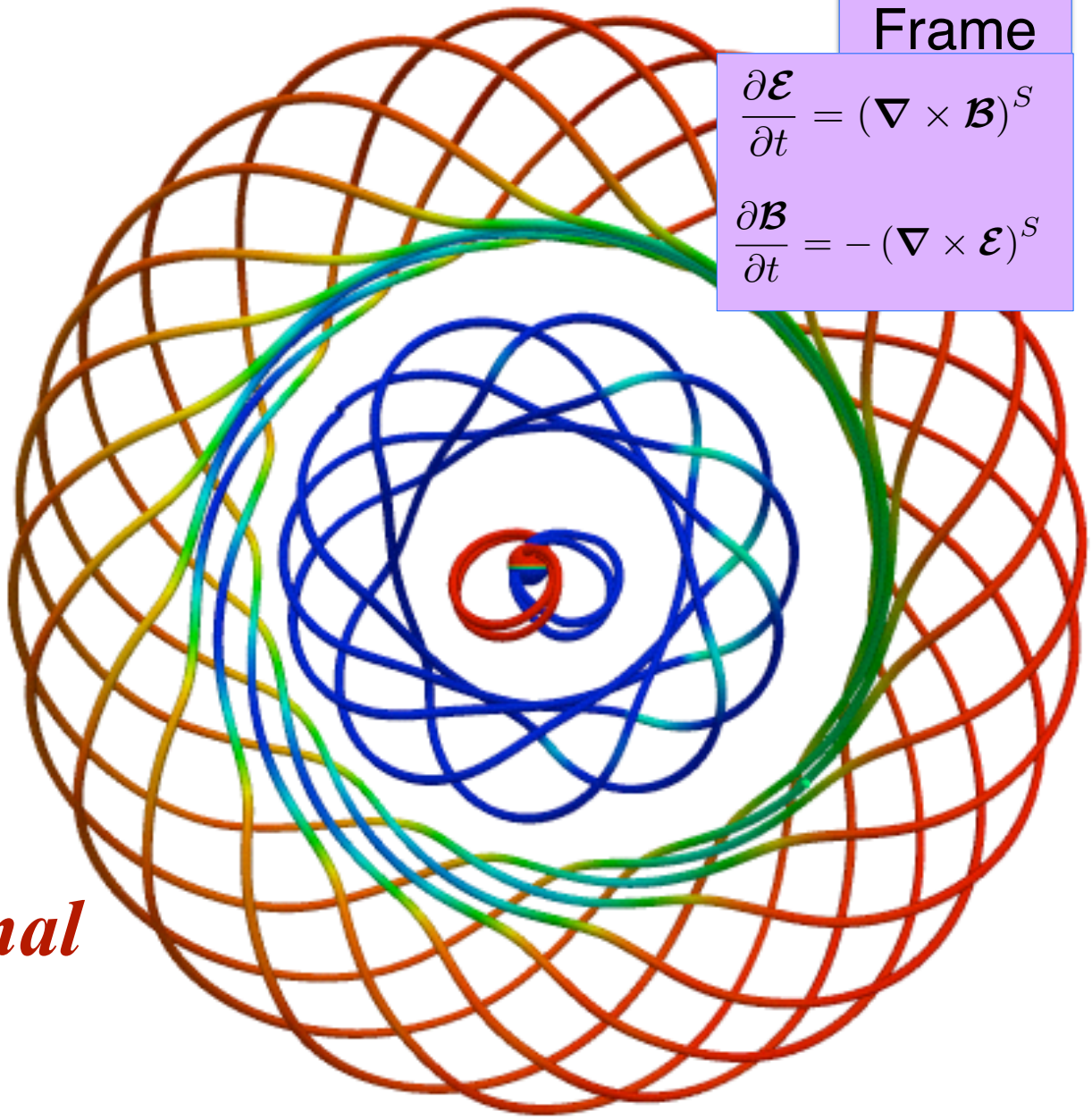
In Local Lorentz Frame

$$\frac{\partial \mathcal{E}}{\partial t} = (\nabla \times \mathcal{B})^S$$

$$\frac{\partial \mathcal{B}}{\partial t} = -(\nabla \times \mathcal{E})^S$$

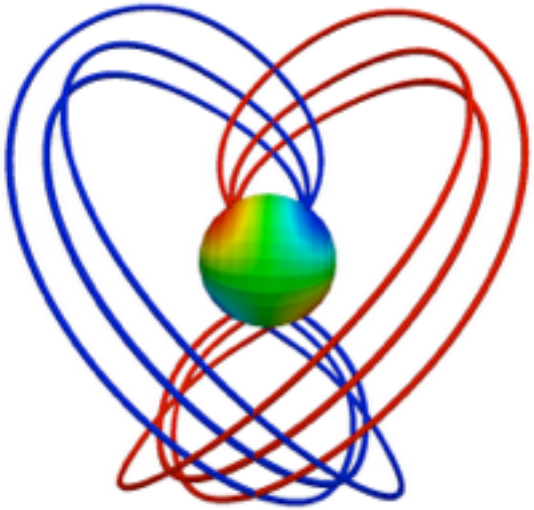


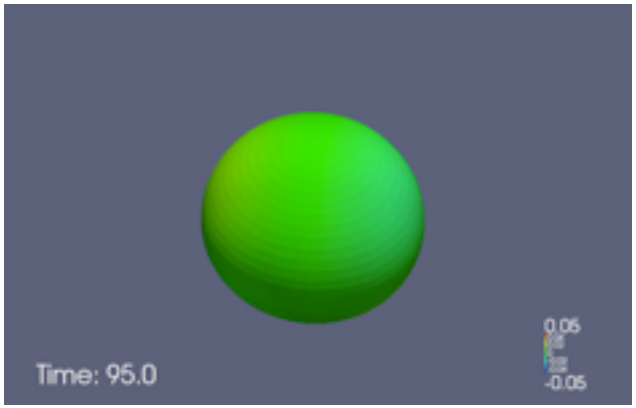




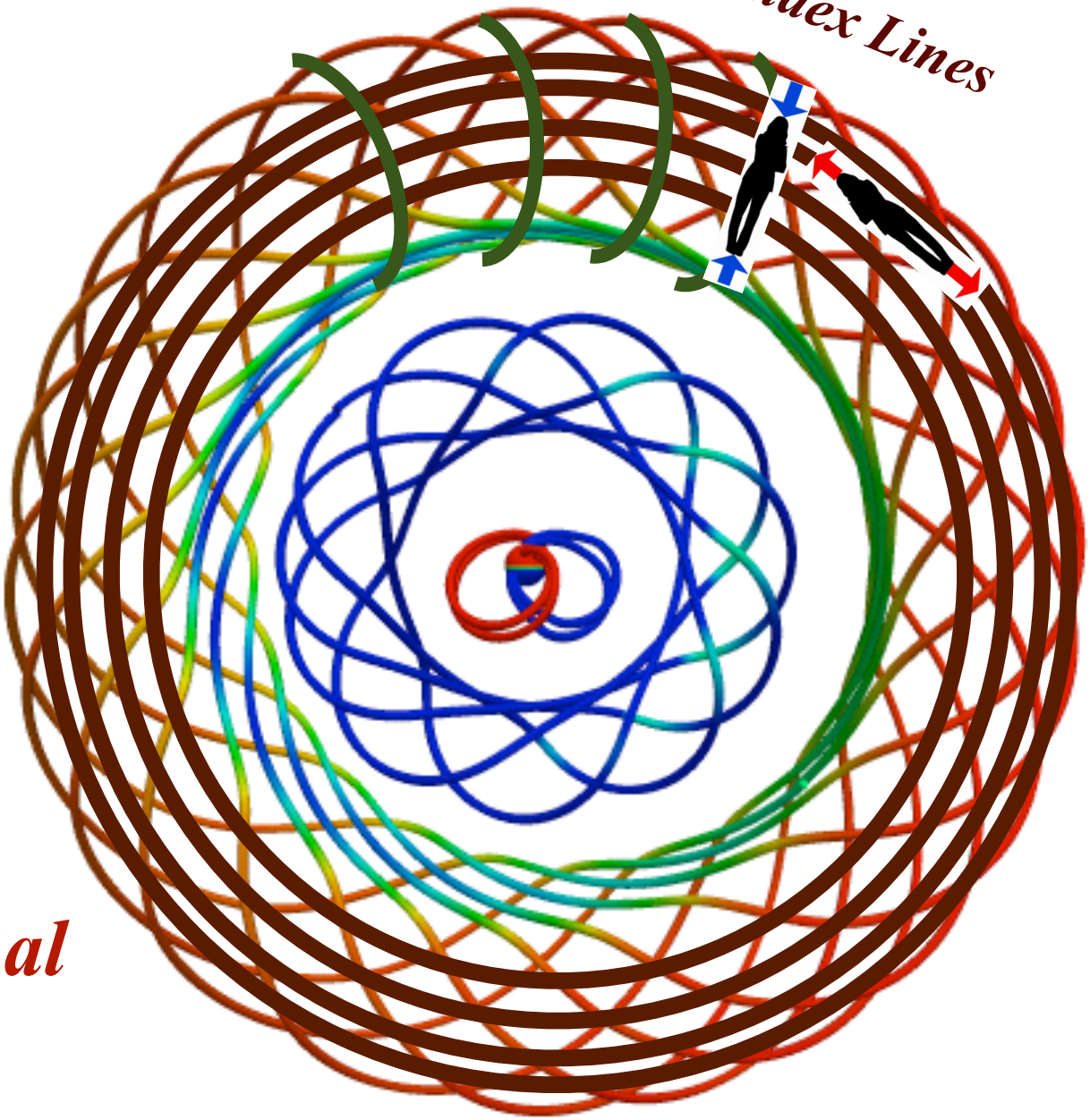
gravitational waves

Sloshing Ejects Vortexes





gravitational waves

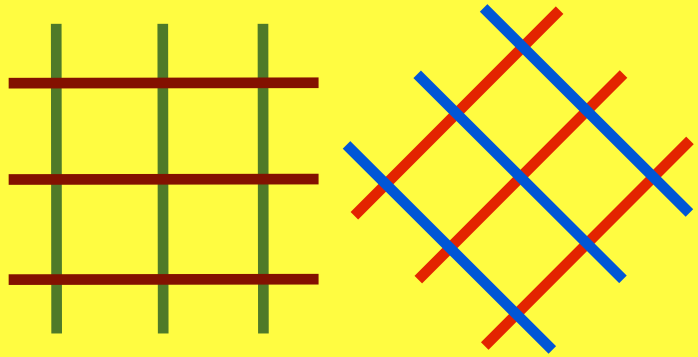


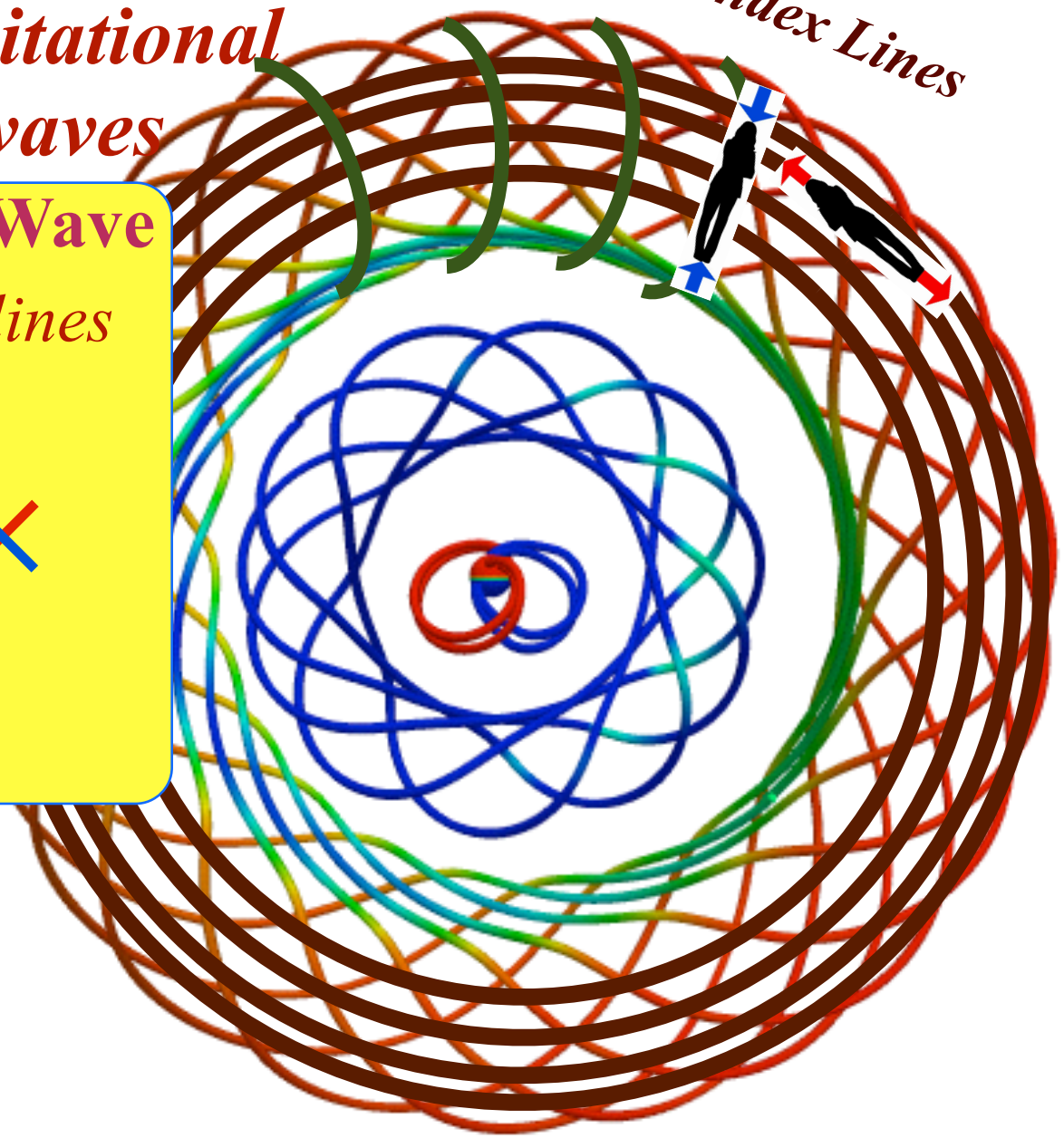
Sloshing Ejects Vortexes

gravitational waves

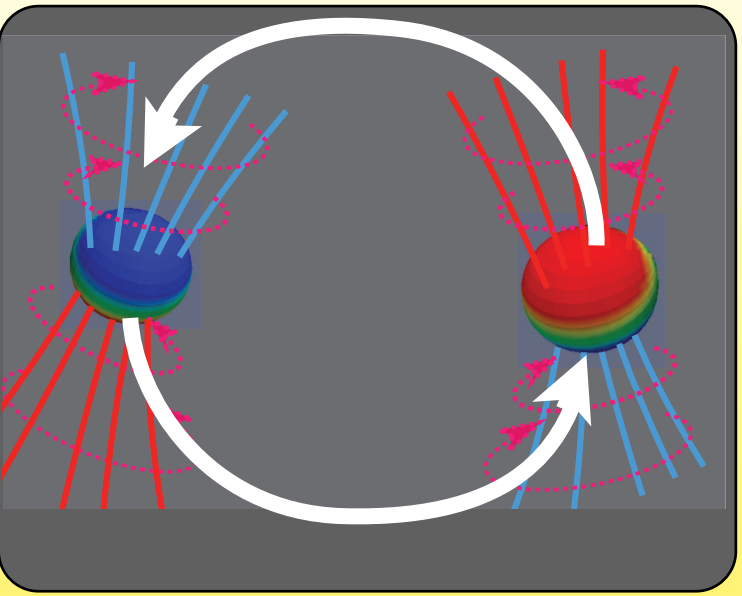
Plane Gravitational Wave

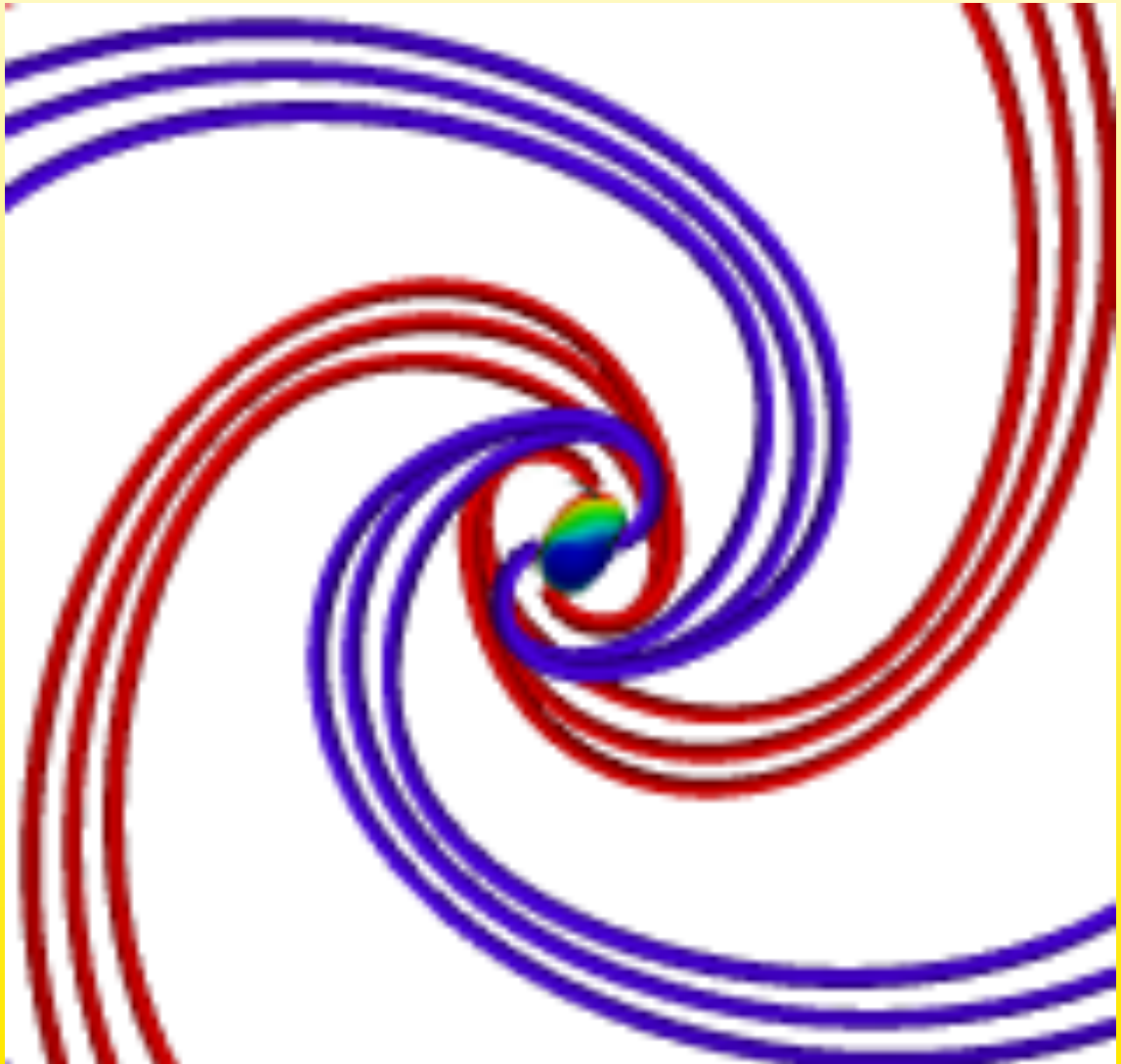
tendex lines *vortex lines*





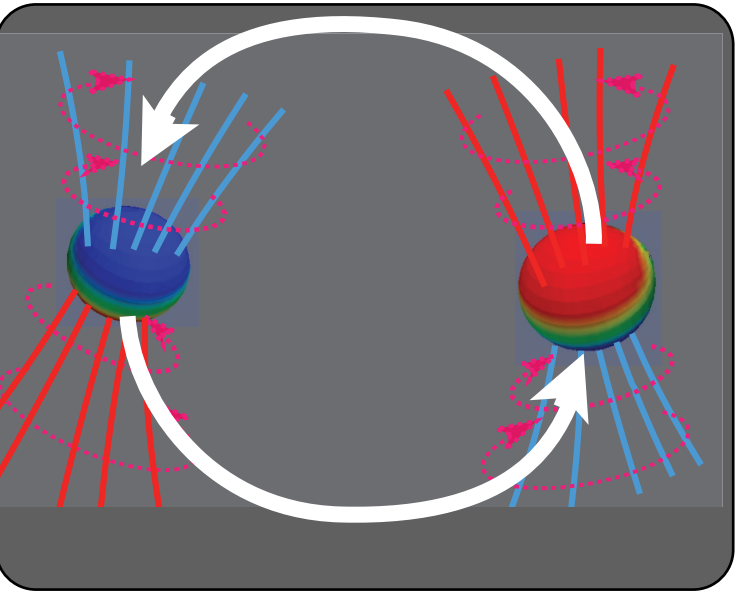
Orbiting Collision

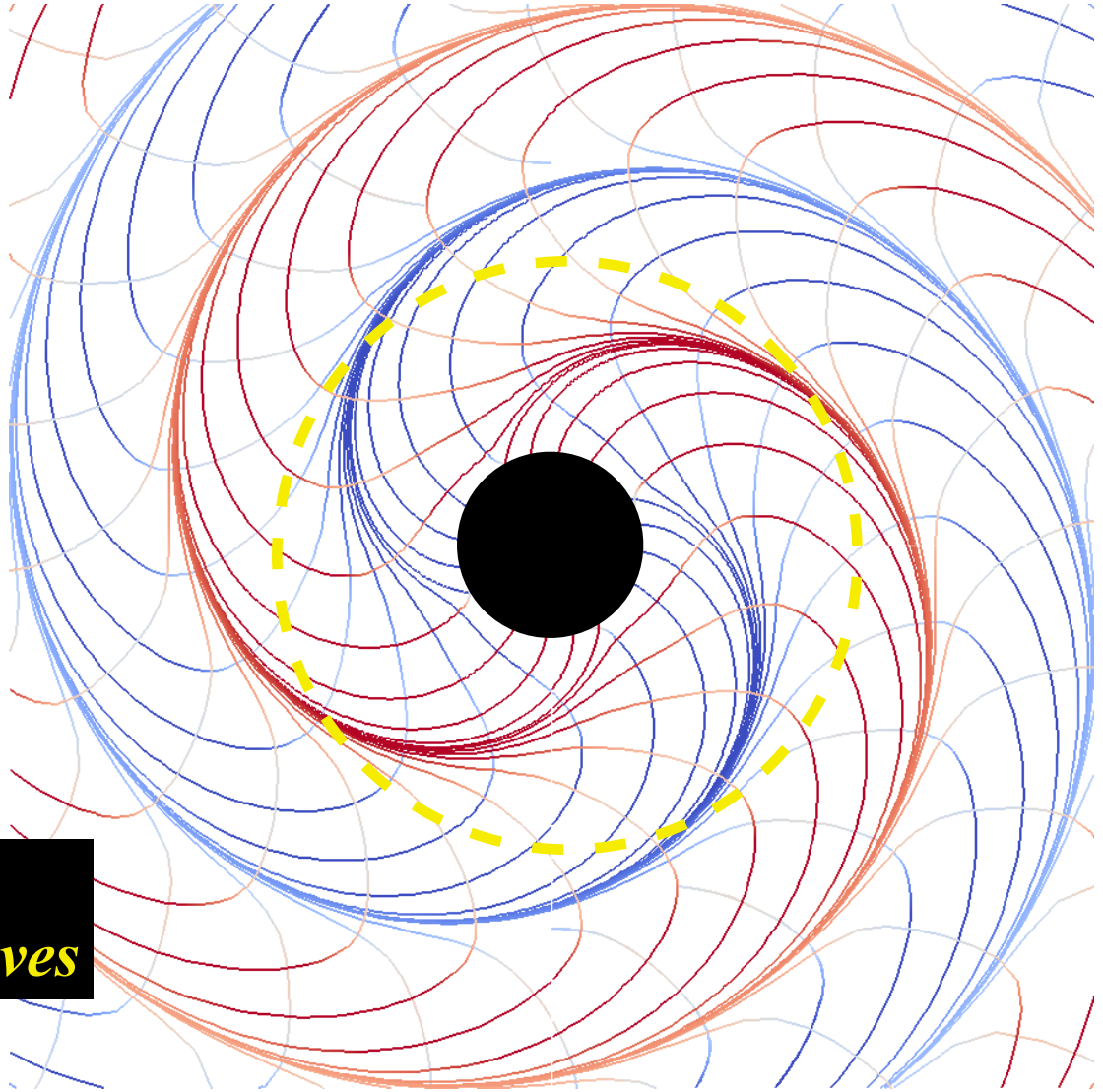




gravitational waves

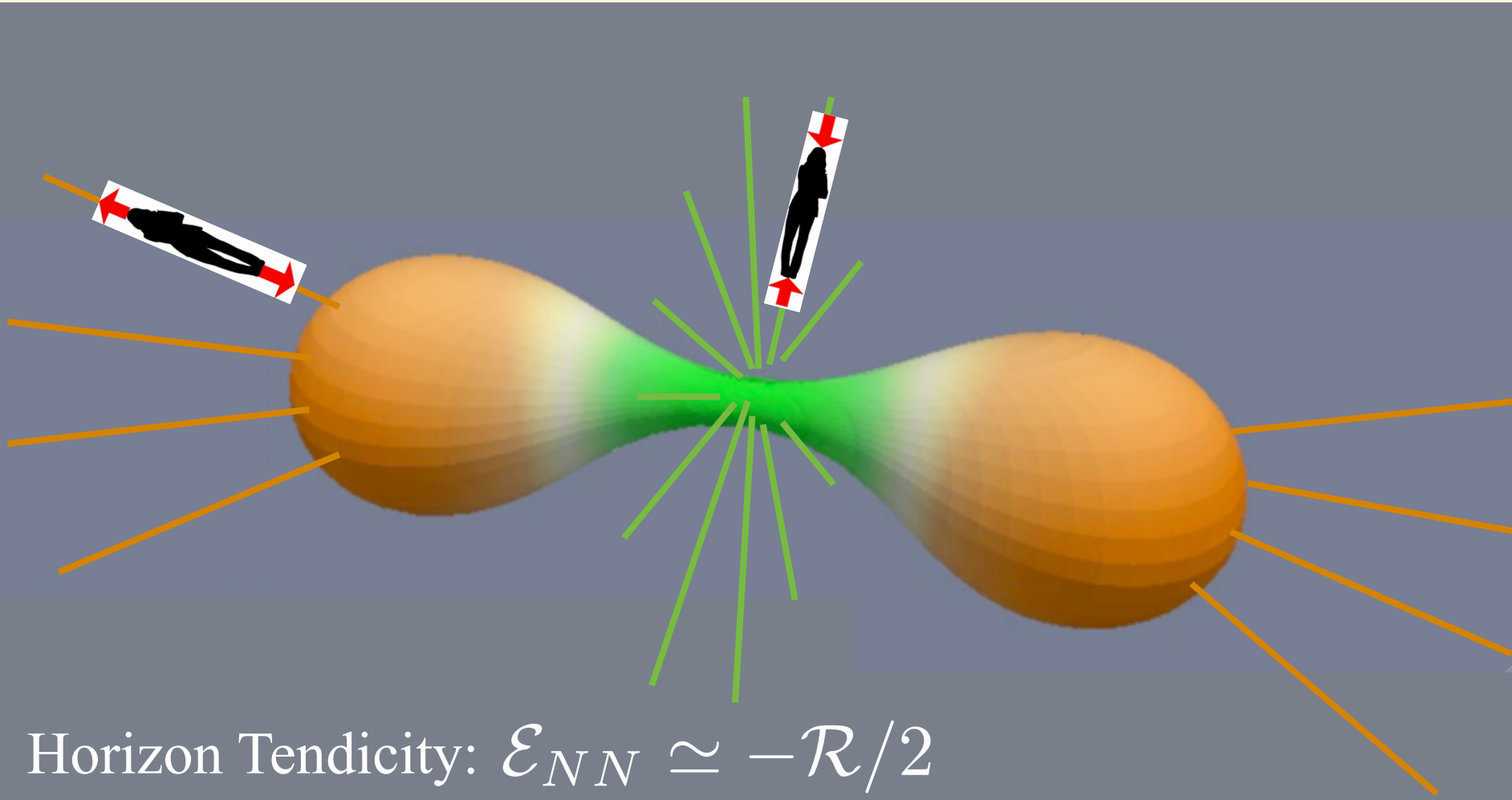
Vortex Lines in Orbital Plane at Late Times





Near-hole vortexes generate gravitational waves

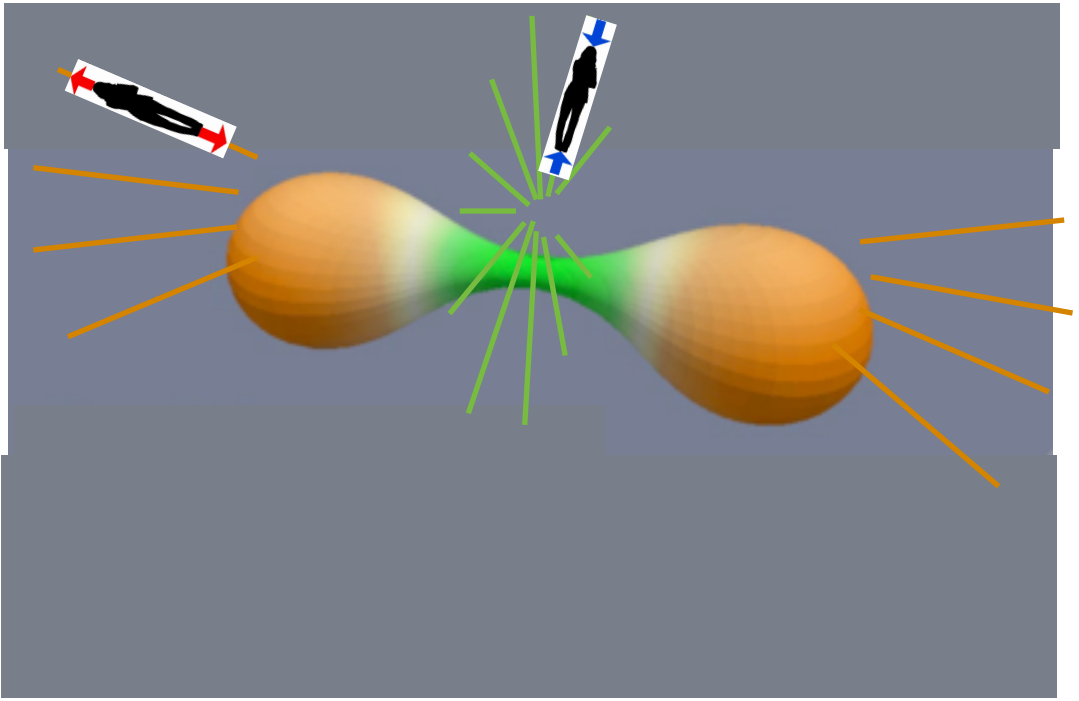
Horizon Tendicity and Tendex Lines



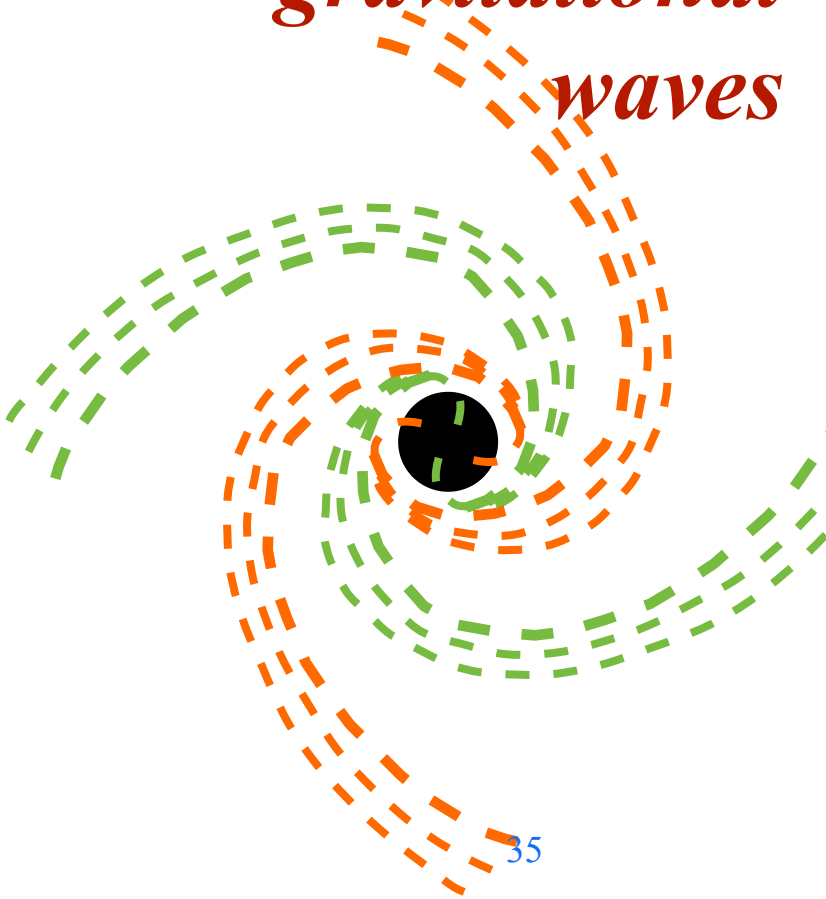
Horizon Tendicity: $\mathcal{E}_{NN} \simeq -\mathcal{R}/2$

Tendex Lines: Integral Curves of Eigenvectors of $\mathcal{E}_{ij}$

Orbiting Collision



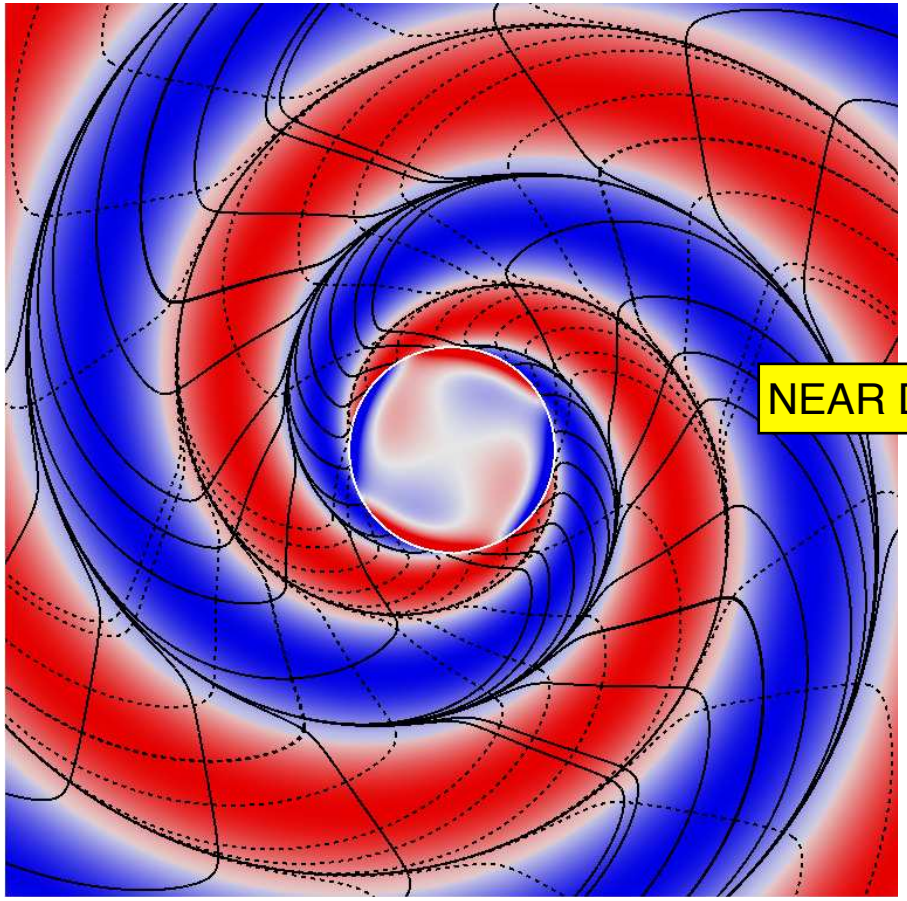
Tendex-generated gravitational waves



At late times, a/M=0.945

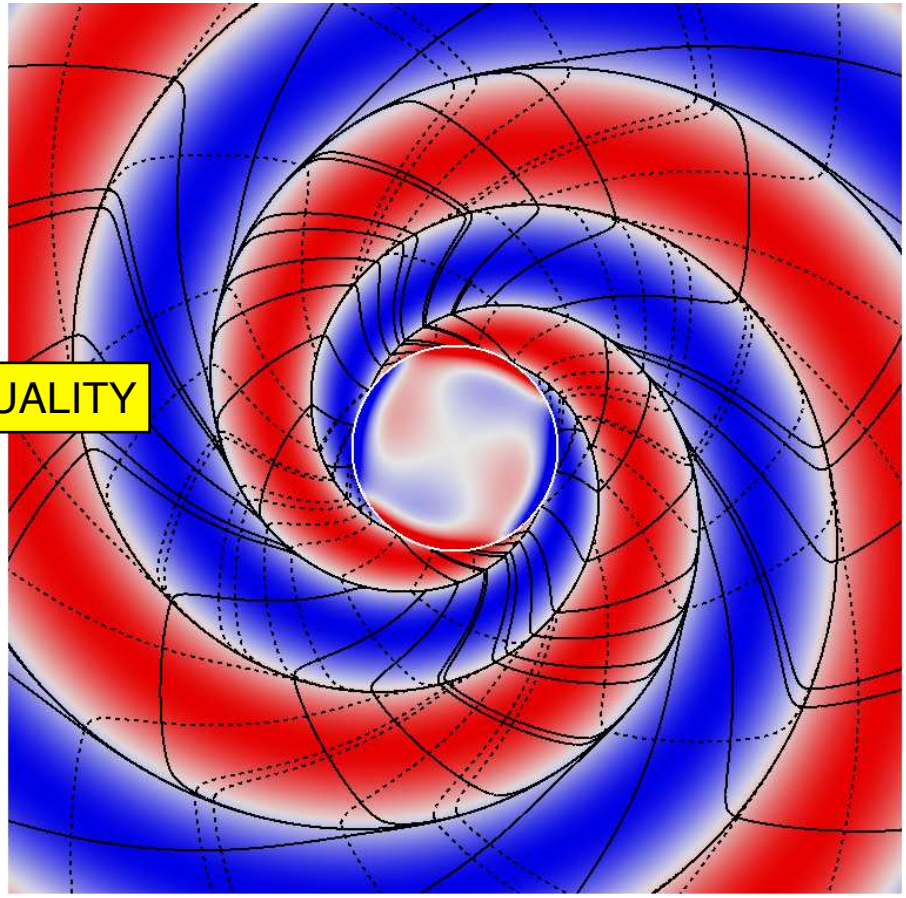
Vortex-generated waves

Equatorial vortex lines and vortices



Tendex-generated waves

Equatorial tendex lines and tendices



NEAR DUALITY

Super Kicks

“Extreme-Kick” Collision of Black Holes

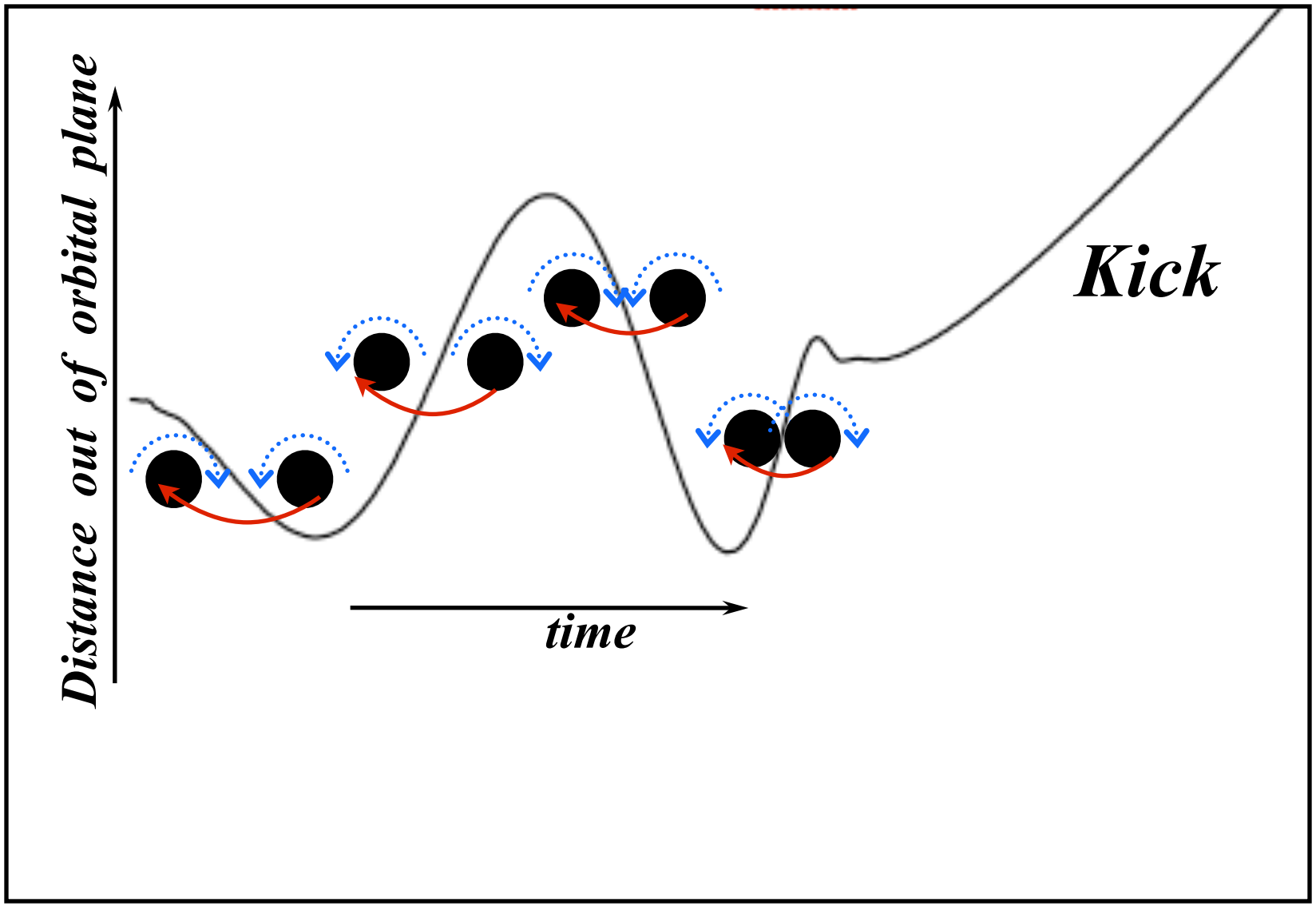
Rochester Institute of Technology:
Campanelli, Lousto, Zlochower




Simulation:
Manuela Campanlli
Carlos Lousto
Yosef Zlochower
Visualization:
Hans-Peter Bischof
CCRG
RIT
Copyright - CCRG - 2009

Explanation of Bobbing

[Pretorius]



Analogous to 2 Vortexes in a Fluid

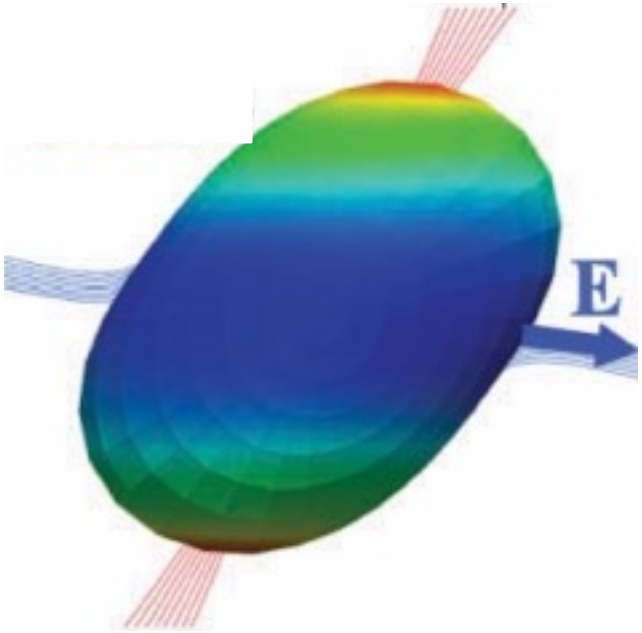
From film ***Vorticity*** by **Ascher H. Shapiro** (Educational Services Inc, 1961)



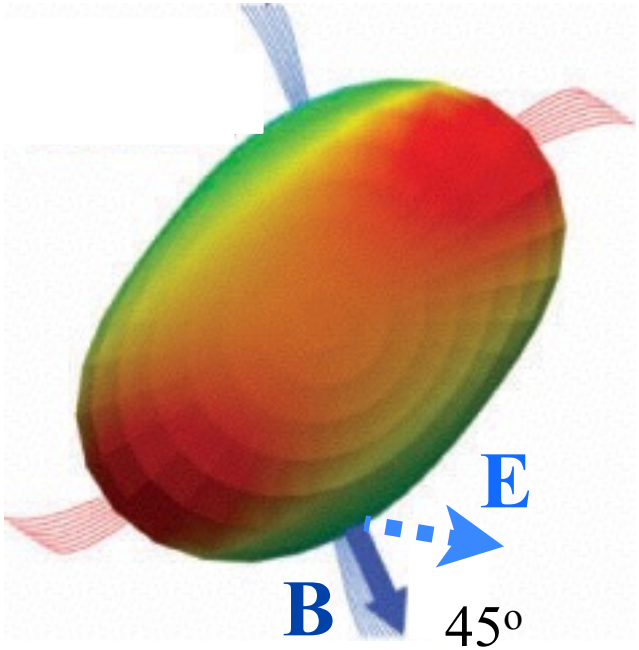
What causes the Kick?

Immediately After Merger

Near-Hole Tendexes



Near-Hole Vortexes



For waves going into screen: Waves from vortexes ADD to those from tendexes

For waves going out of screen: Waves CANCEL

Merged hole is kicked out of screen

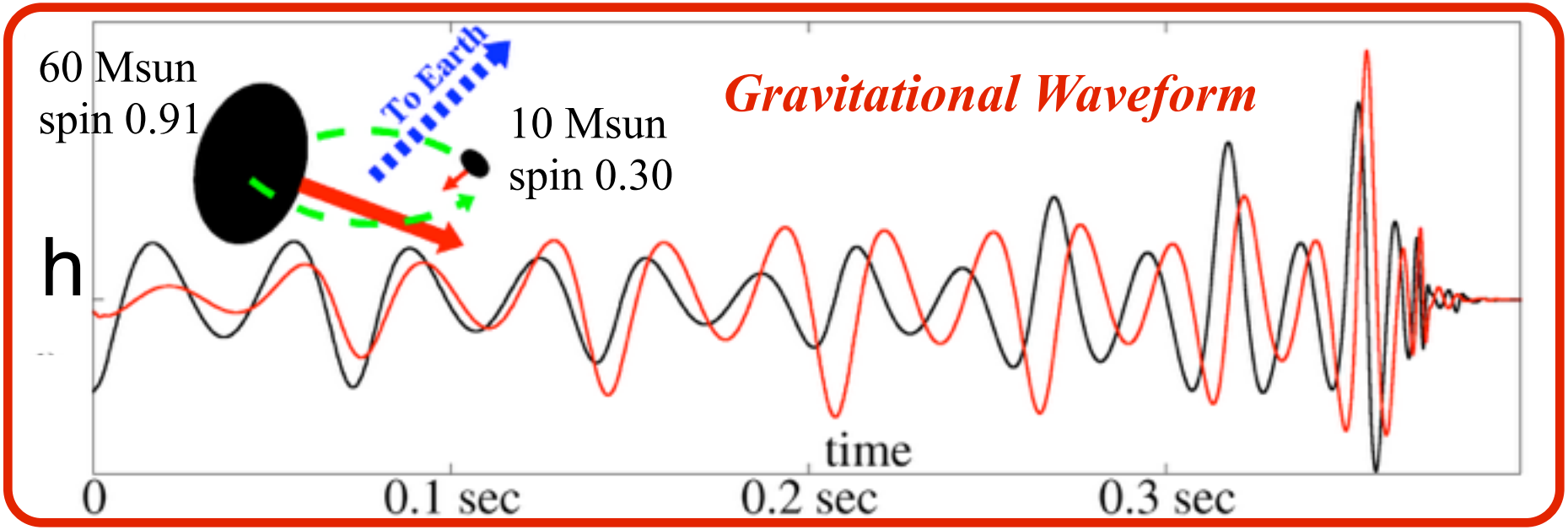
Gravitational-Wave Observations

- ***Challenge:*** From observed BBH gravitational waveforms, how can we read off the near-zone geometrodynamics?
  - » i.e., the dynamics of near-zone vortexes and tendexes?



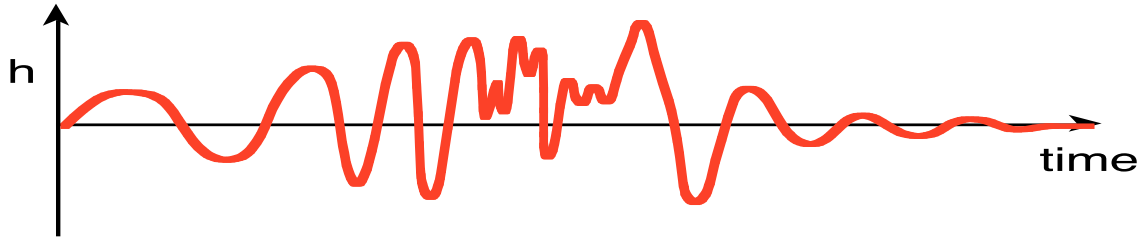
Gravitational-Wave Observations

- ***Challenge:*** From observed BBH gravitational waveforms, how can we read off the near-zone geometrodynamics?
  - » i.e., the dynamics of near-zone vortexes and tendexes?
- ***Answer:*** Identify BBH parameters by comparing observed waveforms with waveforms from simulations; then look at the simulations' dynamics

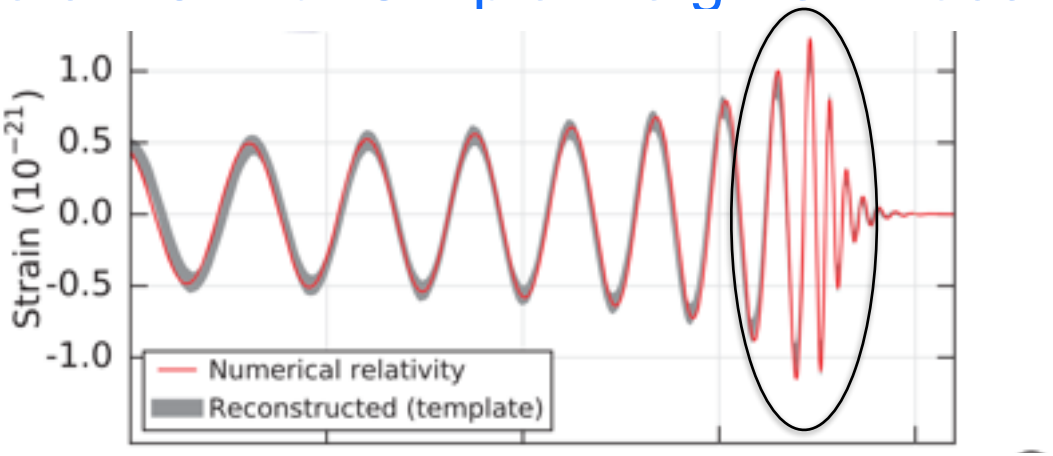


Simplicity of BBH Collision Waveforms

- My speculation in 1984



- Actual Waveforms: Far Simpler - e.g. GW150914:

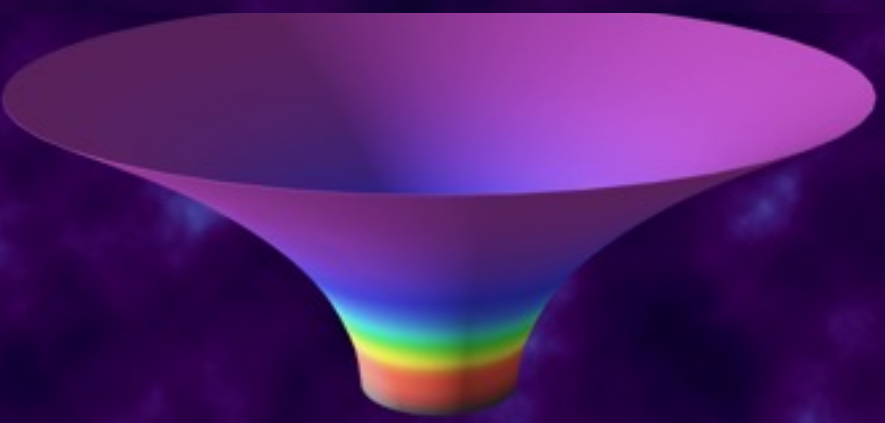


- Disturbances depart very quickly!

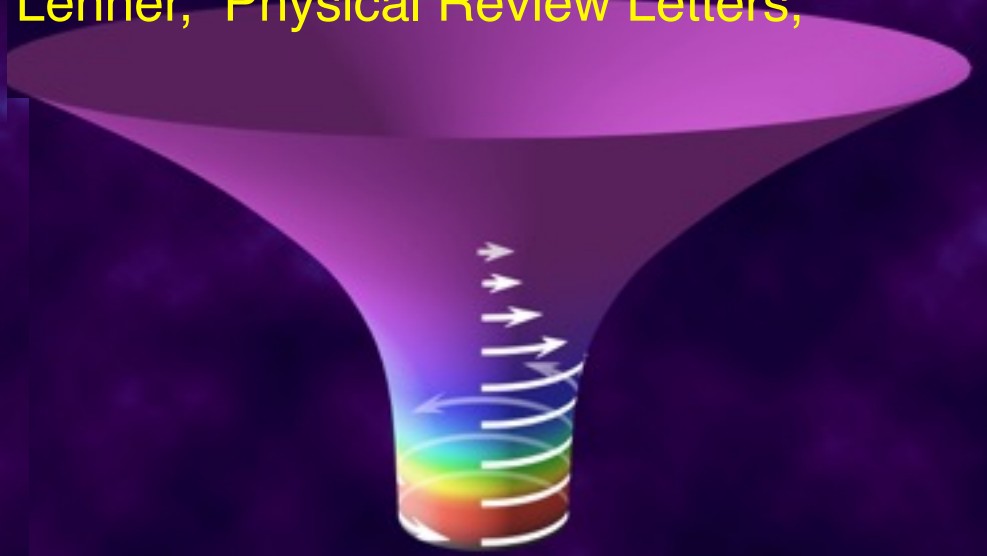
Increase a (black hole spin).

Trap Dynamical disturbances long enough for mode-mode coupling - 2D Turbulence!

Huan Yang, Aaron Zimmerman, Luis Lehner, Physical Review Letters, 114, 081101 (2015)



$a/M = 0$



$a/M = 0.998$

family of modes with zero damping in limit

$a \rightarrow M$ (ZDMs)

Summary

- Five current arenas for geometrodynamics
  - » critical collapse
  - » singularities
  - » black-string instability
  - » binary black holes
  - » gravitational-wave observations
- In all, I suspect we have barely scratched the surface.