Higher dimensionnal AdS Black Strings Stability

Térence DELSATE

Physique Théorique et Mathématique
Université de Mons-Hainaut

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Content

1. Introduction
   - Extradimensions
   - Cosmological Constant
   - Experimental Interest
2. 4 Dimensional Black Holes
   - Variationnal Principle
   - Schwarzschild Solution
   - *dS* - *AdS* Schwarzschild
3. Higher Dimensional Black Objects
   - Black Holes
   - Black Strings
4. Stability and The Gubser-Mitra Conjecture
   - Gregory laflamme Instability
   - The Gubser Mitra Conjecture
   - Stability of *AdS* Black Strings
   - Evidence for the Gubser-Mitra Conjecture
5. Conclusion and Outlook

Introduction

Brane Worlds

Higher dimensional models are more and more taken seriously nowaday. The reason is essentially that

- These higher dimensional models provide a nice solution to the hierarchy problem.
- String Theories (ST) all predict the existence of extradimensions.
- ST $\Rightarrow$ higher dimensional gravity as effective low energy ST.
- In the multidimensional picture, our 4 Dim spacetime is seen as a 4 Dim slice of a d dimensional manifold on wich Standard Model (SM) fields are stuck (@ our level of energy)

Introduction
Brane Worlds

d-dimensionnal theory $\Rightarrow$ 4 Dim theory... In particular, 4 Dim gravity and black holes.

$\hookrightarrow$ Higher dimensional theories must then contain black holes (BH). But

- Nothing guaranties that these BH are localised on the brane
- Neither that they will have a spherical horizon topology in higher dimensions.

$\Rightarrow$ lot of different horizon topologies as long as the intersection with the brane is of spherical topology (Unicity theorem, ...).

Introduction
Cosmological Constant

- Astronomical observations points to a positive cosmological constant (CC) : Λ.
- ST predicts *AdS/CFT* duality : relation between some conformal fields theory on the conformal boundary of an asymptotically AdS spacetime and the gravitational property of this spacetime.

$$\text{Experiments} \Rightarrow \Lambda > 0$$

$$AdS/CFT \Rightarrow \Lambda < 0$$

$\hookrightarrow$ study of Black Objects with both sign of the CC.

Introduction
Signature of Extra Dimensions

Consider gravity in d dimensions, with $n = d - 4$ flat transverse directions of finite extension L :

$$ds^2 = \gamma_{\mu\nu} dx^\mu dx^\nu + \delta_{ij} dy^i dy^j \quad (1)$$

It is possible to have d dimensional Planck Mass $\approx$ TeV scale :

$$\mathcal{L} = \frac{M_d^{d+2}}{16\pi} \int \sqrt{-g} R d^4x d^n y = \frac{V_n M_d^{d+2}}{16\pi} \int \sqrt{-\gamma} \mathcal{R} d^4 x \quad (2)$$

$\Rightarrow M_P^2 = V_{d-4} M_d^{d+2}$, recall that $V_n \propto L^n$:

$$M_P = M_d (M_d L)^{\frac{n}{2}}$$

Fixing $M_d \approx 1$ TeV restricts L (Deviation from Newton's Law should appear @ L)

Introduction
Signature of Extra Dimensions

- Gravity is non renormalizable $\Rightarrow$ Semi Classical approximation, tree level...
- Linearisation of Einstein Tensor about flat space time implies

$$ds^2 = (\eta_{AB} + \epsilon h_{AB})\, dx^A dx^B \;,\; \mathcal{L}_I \propto h^{AB} T_{AB} \tag{3}$$

- Roughly speaking, one has to add a graviton propagator to each Feynmann diagramm of a theory $T \approx \partial\mathcal{L} - \mathcal{L}$

$\hookrightarrow$ Emergence of new process @ M_d

NB : Presence of a brane breaks translational symetrie in transverse directions.

Introduction
Micro Black Holes

- *BH* should radiate and microblack holes (μBH) should form (for example @ LHC).
- μBH are supposed to decay via Hawking radiation, actually all kinds of particules-antiparticule should be produced.
- Signature : "mess" of particules going in every direction and comming from one point.

$\hookrightarrow$ Multidimensional BH should radiate too $\Rightarrow$ possible radiations in a transverse direction $\Rightarrow$ missing energy.
NB : Precise signature depends on d, horizon topology, fundamental Planck Mass

4D Black Holes

Variationnal Principle

One starts with the Einstein Hilbert action, with Hawking Gibbons terms :

$$S = \frac{1}{4\pi G}\int_{\mathcal{M}} \sqrt{-g}\,(R - 2\Lambda)\, d^d x - \frac{1}{8\pi G}\int_{\partial\mathcal{M}} \sqrt{-h} K d^{d-1} x$$

G :Newton's constant
$\mathcal{M}$: the Manifold
h : the induced metric on $\partial\mathcal{M}$
R : intrinsic curvature of $\mathcal{M}$

g : det of the metric
$\partial\mathcal{M}$ the boundary of $\mathcal{M}$
Λ : Cosmological constant
K : extrinsic curvature of $\partial\mathcal{M}$ in $\mathcal{M}$

4 Dimensional Black Holes

Schwarzschild

The simplest case, vacuum and non rotating with zero CC is well known (Schwarzschild, 1916 ; $G = c = k_B = \hbar = 1$)

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\Omega^2 \tag{4}$$

- This represents the metric outside a Black Hole of Mass M (conservered charge)
- Its horizon radius is $2M$, the area is $A = 16\pi M^2$.
- Asking regularity in the euclidian sections $\Rightarrow$ Temperature $T_H = \frac{1}{8\pi M}$

4 Dimensional Black Holes

Schwarzschild

Let's see what happens if we increase the mass by δM $(M \rightarrow M + \delta M)$

- $A = 16\pi M^2 \rightarrow 16\pi(M^2 + 2M\delta M) \;\Rightarrow\; \delta A = 32\pi M\delta M$
- Recall that $T_H = \frac{1}{8\pi M} \Rightarrow \delta M = T_H \delta \frac{A}{4}$.
- Regarding the first thermodynamical low $\delta E = T\delta S$ suggests to identify (See Hawking,...)

$$S = \frac{A}{4}$$

Finally, in any classical physically relevant process, $M \nearrow \;\Rightarrow\; \delta S > 0$

NB : Specific heat : $C_p = \frac{\partial S}{\partial T_H} < 0$

4D Black Holes

dS - *AdS* Schwarzschild

This solution extends in asymptotically *dS*, *AdS* spacetime :

$$ds^2 = -\left(1 - \frac{r_0}{r} + \frac{\Lambda}{3}r^2\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)} + r^2 d\Omega^2 \qquad (5)$$

$\hookrightarrow$Existence of a cosmological horizon for $\Lambda > 0$
$\hookrightarrow$In *AdS*, two phases of black holes :

- Small BH ($\sqrt{|\Lambda|}r_h << 1$), properties essentially like Schwarzschild
- Large BH ($\sqrt{|\Lambda|}r_h \approx 1$) : the specific heat becomes positive !

Higher dimensional black objects

Tangherlini black holes

Schwarzschild like solution (S_{d-2} horizon topology) are known in more than 4 Dim (Tangherlini, 1963)

$$ds_d^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-2}^2 \qquad (6)$$

where $d\Omega_{d-2}^2$ $(d-2)$ unit sphere line element and $f(r) = 1 - r_0/r^{d-3} \pm \frac{r^2}{l^2}$, depending on the CC sign.

$$\text{with } \Lambda = \pm\frac{(d-1)(d-2)}{2l^2}$$

Higher dimensional black objects

Other Black Holes

Some generalisation (non exhaustive) :

- Rotating : Myers and Perry, 1986
- Rotating with CC : Pope, Page and Gibbons, 2004
- Charged : Bronnikov, 2005
- Charged and rotating : Kunz, Navarro Lérida and Petersen, 2007
- Charged and rotating with CC : Brihaye, Delsate, 2007

Higher dimensional black objects
Black Strings

Simplest other non trivial horizon topology ($\Lambda = 0$) : $S_{d-3} \times S_1$

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{d-1}\right) dt^2 + \frac{dr^2}{\left(1 - \left(\frac{r_0}{r}\right)^{d-1}\right)} + r^2 d\Omega^2_{d-3} + dz^2 \quad (7)$$

with $z \in [0, L]$.
Remarks :

- For $r_0/L << 1$, essentially same property as Tangherlini
- For $r_0/L \approx 1$ some special properties (Instability, phase transitions,...) (Gregory Laflamme, 1993)

Higher dimensional black objects
Other Black Strings

For instance, given a BH, one can "add a flat extradimension" :

$$ds^2_{d,BH} \quad \Rightarrow \quad ds^2_{d+1,BS} = ds^2_{d,BH} + dz^2 \tag{8}$$

In the same way, adding a Ricci flat manifold $\Rightarrow$ Black n-brane :

$$ds^2_{d,BH} \quad \Rightarrow \quad ds^2_{d+n,BS} = ds^2_{d,BH} + \gamma_{ij}(y) dy^i dy^j \tag{9}$$

with $\tilde{R}_{ij}(\gamma_{kl}) = 0$

Higher dimensional black objects

Other types of Black Strings

More difficult : warped geometries...

$$ds^2 = ds^2_{BH}(x_\mu) + a(x_\mu)dy^2 \tag{10}$$

For example, *AdS* Black Strings (Radu, Mann and Stelea, 2006) :

$$ds^2 = -b(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2_{d-3} + a(r)dz^2 \tag{11}$$

Charged and Rotating BS with $\Lambda > 0$ (Brihaye and Delsate, 2007)

Higher dimensional black objects

Charged-Rotating Black Holes with Cosmological Constant

No exact Solutions $\Rightarrow$ Numerical construction : Let $(d = 2N + 1)$

$$
\begin{aligned}
ds^2 &= -b(r)dt^2 + \frac{dr^2}{f(r)} + g(r)\sum_{i=1}^{N-1}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right)d\theta_i^2 \\
&+ h(r)\sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k\left(d\varphi_k - \omega(r)dt\right)^2 \\
&+ p(r)\left\{\sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k^2 d\varphi_k^2\right. \\
&- \left.\left[\sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k^2 d\varphi_k\right]^2\right\}
\end{aligned}
$$

Higher dimensional black objects

Charged-Rotating Black Holes with Cosmological Constant

Cosmological horizon $\Rightarrow$ technical difficulties $\Lambda > 0$:

- Fix the event horizon r_h, and the cosmological horizon r_c "by hand".
- Regularity conditions @ $r_h, r_c \rightarrow$ Integration over $[r_h, r_c]$ (Boundary Value Problem)
- But what fixes r_h is mainly $\Lambda \Rightarrow$ No *a priori* value...
- Also, too many BC $\rightarrow$ add an equation for Λ : $\frac{d\Lambda}{dr} = 0$.
- Big Deal : find a sufficiently "good" starting profile (Newton Raphson based algorithm).
- Integration over $[r_c, \infty]$: $\Lambda(r_c)$ and fields value @ r_c known $\Rightarrow$ Cauchy Problem.

Higher dimensional black objects

Charged-Rotating Black Holes with Cosmological Constant

The $U(1)$ vector potential is given by :

$$A_\mu dx^\mu = V(r)dt + a_\varphi(r) \sum_{k=1}^{N} \left(\prod_{l=0}^{k-1} \cos^2 \theta_l \right) \sin^2 \theta_k d\varphi_k$$

Note that

- Consistency of the ansatz $\Rightarrow p = g - h$, $g := r^2$.
- $\omega(r)$ is the angular velocity in the different rotation plans.

Higher dimensional black objects

Charged-Rotating Black Holes with Cosmological Constant

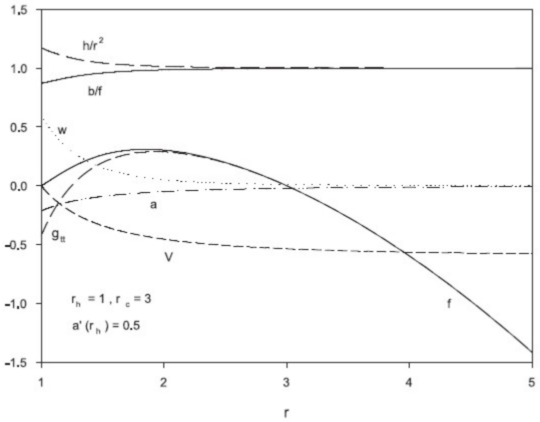
Ansatz $\Rightarrow$ 5 coupled ODE (*hep-th/0703146*) for $b'', \omega'', V'', a''_\varphi, f', h''$

- The solutions are asymptotically *dS* (resp. *AdS*) for $\Lambda > 0$ (resp. $\Lambda < 0$)
- $\omega \to 0$ when $r \to \infty$.

Higher dimensional black objects

Charged-Rotating Black Holes with Cosmological Constant

A plot of a numerical solution



Higher dimensional black objects

Charged-Rotating Black Strings with Cosmological Constant

From BH to BS :

$$ds^2_{d,BH} \Rightarrow ds^2_{d+1,BS} = ds^2_{d,BH} + a(r)dz^2$$

- Extra equation for a.
- *AdS* rotating charged BS (Radu-Brihaye) : essentially same properties as charged rotating BH.
- Surprisingly, no asymptotically *dS* BS ! ! non rotating and uncharged sufficent to see this.

Higher dimensional black objects

Naked Singularities for $\Lambda > 0$ Black Strings

Consider the asantz $ds^2 = -b(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2_{d-3} + a(r)dz^2$ ans the Einstein equations with positive CC.

- Natural evolution of $a, b, f \Rightarrow$ Singularity @ finite proper distance

$$a(r) \rightarrow r^{\alpha} \quad , \quad b(r) \rightarrow r^{\alpha} \quad , \quad f(r) \rightarrow r^{\phi}, \tag{12}$$

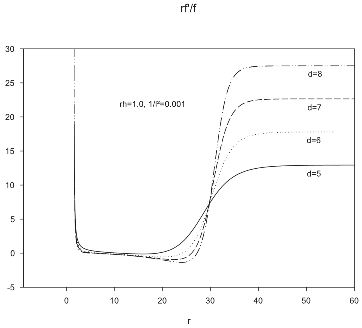
for $\alpha = -2(d-3) - \sqrt{2(d-2)(d-3)}$, $\phi = 2(d-2) + \sqrt{2(d-2)(d-3)} > 4\ \forall d$

- Worst : No possible regular matching between near horizon and asymptotically dS ! ! !

Higher dimensional black objects

Naked Singularities for $\Lambda > 0$ Black Strings

A plot of rf'/f for $\Lambda > 0$ BS :



Stability and The Gubser-Mitra Conjecture

Gregory Laflamme Instability

BS and Black Branes present a long wavelength instability (R. Gregory and R. Laflamme, 1993).

- This instability shows up @ linearized level :

$$ds^2 = (g_{MN} + \epsilon h_{MN})\, dx^M dx^N \tag{13}$$

  with $\epsilon << 1$.

- Let the time dependance of h be of the form $e^{\Omega t}$ and Fourier transform in the transverse direction ($h \propto e^{ikz}$).

$\hookrightarrow$ for each Ω, one gets an eigen value problem for k. When $\Omega = 0$, one gets a stationnary mode with $k = k_c$.

Stability and The Gubser-Mitra Conjecture
Gregory Laflamme Instability

Thermodynamical argument :

$$S(\text{array BH}) > S(\text{BS}) \quad @ \text{ same M} \tag{14}$$

Evolution to an array of BHs : infinite proper time $\Rightarrow \exists$ Non Uniform Black Strings (NUBS : $g_{AB}(x^\mu) \rightarrow g_{AB}(x^\mu, y^i)$)
See (T. Wiseman, 2004 ; Gubser, 2001 ; ...)

Stability and The Gubser-Mitra Conjecture

The Gubser Mitra Conjecture

The Gubser Mitra conjecture states that

"For a black brane solution to be free of dynamical instabilities, it is necessary and suficient for it to be locally thermodynamically stable"

S.S Gubser and I. Mitra, 2001 :
The evolution of unstable black holes in anti-de Sitter space

Basicaly, GM conjecture states that

$$\text{Gregory Laflamme Instability} \Leftrightarrow \text{Thermodynamical Instability} \tag{15}$$

Stability and The Gubser-Mitra Conjecture

Stability of *AdS* BS

We have studied the stability of warped *AdS* BSs :

- Start with a general Non Uniform BS (NUBS) ansatz
- "Fourier Transform" the extra dimension dependance
- Solve the linearized equation $\Rightarrow$ Eigen Value Problem

Note that we study static perturbation ($\Omega = 0$ in GL ansatz).

Stability and The Gubser-Mitra Conjecture

Stability of *AdS* BS

The metric ansatz :

$$ds^2 = -b(r)e^{2A(r,z)}dt^2 + e^{2B(r,z)}\left(\frac{dr^2}{f(r)} + a(r)dz^2\right) + r^2 e^{2F(r,z)} d\Omega^2_{d-3} \tag{16}$$

Let

$$X(r,z) = \epsilon X_1(r)\cos(kr) + o(\epsilon^2) \tag{17}$$

where X is A, B, F, $k = 2\pi/L$, L : length of coordinate z.
Possible to solve for $B = \mathcal{B}(A_1, A_1', F_1, F_1', a, b, f, r)$
$\Rightarrow$ Two second order coupled ODE

Stability and The Gubser-Mitra Conjecture

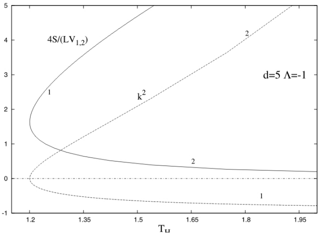
Stability of *AdS* BS

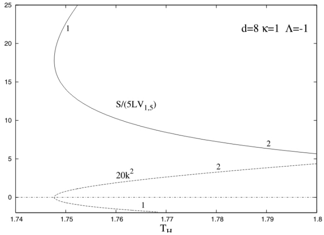
Numerical Method :

- Solve for background (a,b,f)
- Solve linearized eigen value (k^2) problem
- BCs : $A_1, F_1 \to 0$ @ $r \to \infty$, regularity @ horizon (A_1', F_1')
- Add an extra equation for k^2 : $k^{2'} = 0 \Rightarrow$ Extra BC
  Expected result :
  - $k^2 > 0 \Rightarrow \exists$ GL instability
  - $k^2 < 0 \Rightarrow$ Stable.

Stability and The Gubser-Mitra Conjecture
Evidence for the Gubser-Mitra Conjecture

Plot of k^2, S as a function of T_H





Stability and The Gubser-Mitra Conjecture

Evidence for the Gubser-Mitra Conjecture

We have also considered so called "Topological BS"
Replace $d\Omega^2_{d-3}$, curvature $\kappa = 1$ by $d\sigma^2_{d-3}$ ($\kappa = 0$) or dH_{d-3} ($\kappa = -1$).

Topological BSs ($\kappa = -1, 0$) are thermodynamically stable
We find $k^2 < 0$ for all values we considered...

$\Rightarrow$ Accordance with GM conjecture holds for topological BSs.
(Brihaye-Delsate, 2007)

Conclusion and Outlook

- We construct numerically rotating charged Black Holes and Black Strings with Cosmological Constant.
- These present a non trivial, singular non de Sitter geometry for $\Lambda > 0$.
- We presented numerical arguments for the Gubser Mitra Conjecture for warped *AdS* BS and topological *AdS* Black Strings.

Outlook

- Study stability of Charged rotating *AdS* Black Strings
- Second order in perturbation theory (perturbative NUBS) $\Rightarrow$ Black Strings Phase diagram in *AdS*.