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Black holes in AdS and holographic applications

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0 – Outline

1 – Outline

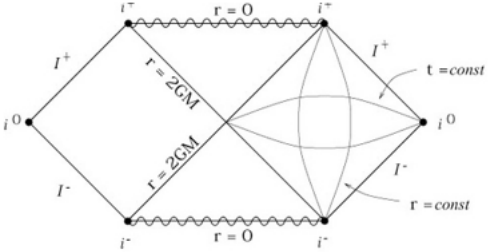
1. Motivations
2. Black branes in AdS and thermodynamics
3. Good singularity
4. Phase transition
5. Conclusions

1 – Black holes

- Classical Solutions of General Relativity

⇒ Black holes are the theoretical laboratory where to look for quantum gravity phenomena

  - Microstate origin of black hole entropy
  - Information paradox
  - Relations to chaotic systems



1 – String/M-Theory and holography

- String Theory is a framework that provides a UV completion of a unified theory of GR and QFT
- AdS/CFT is a tool to investigate quantum gravity

- Gravity solution in a space with boundary and negative cosmological constant
- Holographic dictionary relates fields in the bulk with operators of a (conformal) field theory

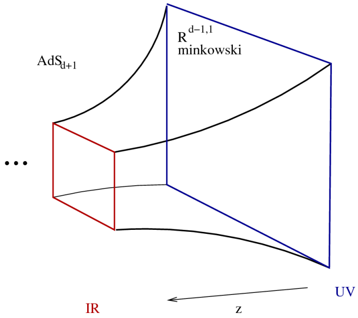
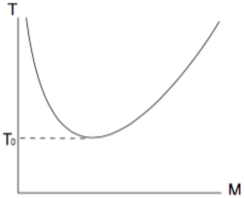


Fig. from McGreevy, 2009

1 – Holographic applications

- Black hole physics can teach about strongly coupled field theories
  - Investigate quantum critical phases of strongly coupled solid state systems

Hawking-Page transition for a black hole in anti de Sitter spacetime ['83]



⇒ Holographic interpretation as confinement/deconfinement phase transition

Witten ['98]

- Goal: construct an analytical example from black hole thermodynamics
  - In the dual field theory, states can undergo phase transitions
  - Study the phase space of the black hole solutions

2 – Outline

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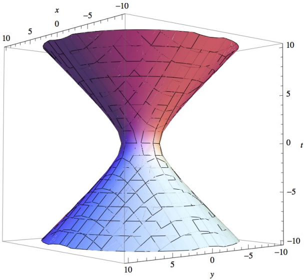
2 – Black holes in AdS - gauged Supergravity

- Holography applied to black holes requires asymptotically AdS space



- Solutions of *gauged* Supergravity theory - obtained from a higher dimensional String/M-theory.

- Generically they are configurations of fields $g_{\mu\nu}\ ,\ \phi^i\ ,\ A^\Lambda_\mu$:

$$S = \int \sqrt{-g}\, d^4x \left(\frac{R}{2\kappa^2} - \frac{1}{2\kappa^2}\partial_\mu\varphi\partial^\mu\varphi - e^{\sqrt{6}\varphi}\xi^3 F^0_{\mu\nu}F^{0\,\mu\nu} - \frac{3}{\xi} e^{-\sqrt{2/3}\varphi} F^1_{\mu\nu}F^{1\,\mu\nu} - \frac{V_g(\varphi)}{\kappa^2} \right) + S_{GH}\ .$$

- String/M-theory microscopic description [Strominger-Vafa, Maldacena-Strominger-Witten, Benini-Hristov-Zaffaroni]

2 – Bulk solutions

- Black holes have spherical horizon, which determines the topology of the boundary of spacetime.
- System in flat space: consider solutions with flat, $\mathbb{R}^2$, horizons "black branes".

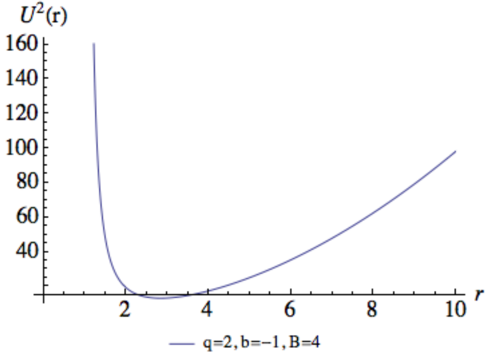
$$ds^2 = -U^2(r)dt^2 + \frac{dr^2}{U^2(r)} + e^{-K}\frac{r^2}{\ell^2_{AdS}}(dx^2+dy^2)\ ,$$

$$e^{-K} = \sqrt{H_0 H_1^3}\ ,$$

$$U^2(r) = e^K\left(\frac{c_1}{r} + \frac{c_2}{r^2} + \frac{r^2}{\ell^2_{AdS}}e^{-2K}\right)$$

$$A^\Lambda = \frac{1}{2}p^\Lambda(xdy - ydx)\ , \qquad \Lambda = 0,1\ .$$

$$e^{\sqrt{8/3}\varphi} = \frac{H_1}{H_0} = \frac{r+b}{r-3b}$$



2 – Holographic analysis

- r and $\tilde{r}$ related by $\frac{r}{\ell} = e^{\tilde{r}/\ell}$, define an asymptotic metric

$$ds^2 \sim d\tilde{r}^2 + e^{2\tilde{r}/\ell} h_{(0)ij}(x) dx^i dx^j \ .$$

- Field expansion at the boundary

$$\varphi \sim e^{-\Delta_- r/\ell}(\varphi_-(x) + ...) + e^{-\Delta_+ r/\ell}(\varphi_+(x) + ...) \ .$$

- Scalr mass $m_\varphi^2 = -\frac{2}{\ell_{AdS}^2}$
- in the window $-9/4 \leq m_\varphi^2 \ell_{AdS}^2 \leq -9/4 + 1$.
- Dual operator conformal dimensions are $\Delta_- = 1$, $\Delta_+ = 2$, both modes normalizable:

$$\varphi_+ = \lambda \varphi_-^2 \ ,$$

The solutions of electric and magnetic black holes are dual to marginal multitrace deformations

Corresponds to classically marginal deformation $\langle T_i^i \rangle = 0$. [Papadimitriou, 2007]

2 – Defining a thermodynamic ensemble

Euclidean path integral formulation of gravity at the semiclassical:
[Gibbons, Hawking '76 , York, '86]

$$Z = \int d[g_{\mu\nu}]d[\phi]\exp\{iI_e[g_{\mu\nu},\phi]\}\ .$$

For a system like black holes and black branes, exhibiting a thermodynamic behaviour, the partition function defines a free energy, which, within a saddle point approximation, corresponds to the Euclidean on-shell action

$$-\beta F = \ln Z = iI_e[g^*,\phi^*]\ ,$$

with $\beta = T^{-1}$.

$\Rightarrow$ What are the thermodynamic variables?

2 – Electric vs magnetic charges

Analyze which quantities are kept fixed at the boundary:

$$\begin{aligned} \delta I &= \text{(terms giving the equations of motion)} \\ &\quad + \text{(gravitational boundary terms)} + \frac{1}{4\pi}\int_\Sigma d^3x\sqrt{h}F^{\mu\nu}n_\mu\delta A_\nu , \end{aligned}$$

In this case the ensemble that extremizes I has a fixed gauge field A_μ.
$\rightarrow$ fixing the gauge potential fixes the charge of the black brane/black object.

Different meaning depending on the charges of the configuration:

- electric gauge field $\rightarrow$ fix the chemical potential χ,
- magnetic configuration $\rightarrow$ fix the charge p

$$p = \frac{1}{4\pi}\int_{S^2_\infty} F_{\theta\phi}$$

2 – Thermodynamic potentials

[Hawking, Ross, '95]

- **Electric configuration**: the bare the on-shell action corresponds to the free energy for the
  grand canonical ensemble $F(T, \chi)$. For a system with n charges q_Λ $(\Lambda = 1, .., n)$

$$F(T, \chi) = M - TS - q_\Lambda \chi^\Lambda$$

- **Magnetic configuration**: the on-shell action gives the free energy for the canonical ensemble $F(T, p)$:

$$F(T, p^\Lambda) = M - TS$$

Adding boundary terms on the action change the boundary conditions: **Légendre transformations**, change the ensemble.

In Supergravity, that corresponds to an **electric-magnetic duality rotation**

2 – Changing ensemble for magnetic configurations

- Start from magnetic black brane (p^0, p^1)
  - Full Legendre transform of the on-shell action:

    $$\mathcal{L}^{on-shell}_{p^0,p^1} \to \mathcal{L}^{on-shell}_{p^0,p^1} + p\chi$$

    In the gravity setup: electric-magnetic duality transformation on the black brane solution.

- Duality transformation that involves only the charge p^0

  $$(p^0, p^1) \qquad \to \qquad (q_0, p^1) \equiv (q, B)$$

  Em-duality is a symmetry of the equations of motion $\to$ new solution is physically equivalent to the original one.

- The on-shell action of the dual configuration now gives the free energy of the grand ensemble wrt the charge $q \to$ **mixed ensemble.**

  $$F = M - TS - q\chi \ .$$

3 – Outline

3 – Good singularity

There exist competing solutions in phase space

- Black brane limit in which **the horizon coincides with the singularity**: "good" singularity [Gubser,2000].

$$g_{tt}(r_h) = 0 \ , \qquad as \quad r_h \to r_s$$

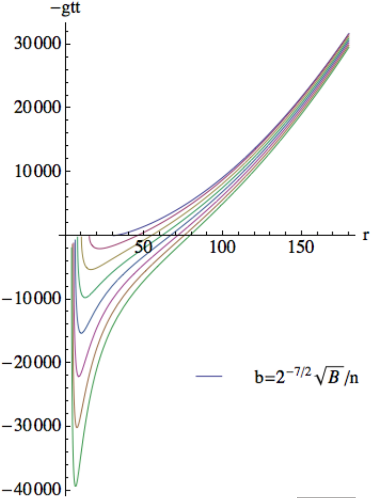
- $g_{xx} = g_{yy} = \sqrt{(r-3b)(r+b)^3}$
- Family of black branes with horizon $r_h = 3b + \epsilon$

$$g_{tt}(3b+\epsilon) = 0$$

- For epsilon $\epsilon \ll 1$

$$|B| = 8\sqrt{2}b^2 + \frac{(6b^2+\chi^2)\epsilon}{\sqrt{2}b} + \mathcal{O}(\epsilon^2) \ .$$

[Gnecchi et al, '16]



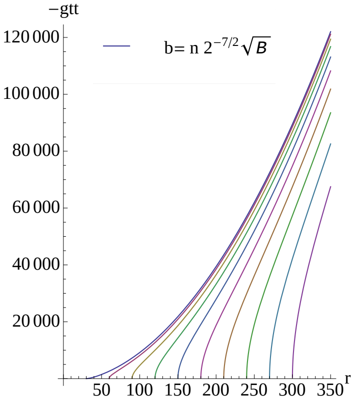
3 – Good singularity

Define the thermal gas as the subset in parameter space (B, b, χ) defined by

$$|B| = 8\sqrt{2}b^2 \ , \qquad \chi \quad \text{finite}$$

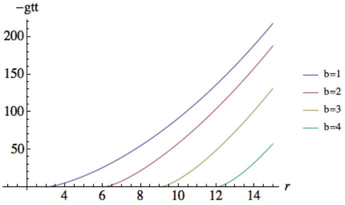
- This implies $q = 0$.
- The thermal gas the dependence on χ drops from the metric:

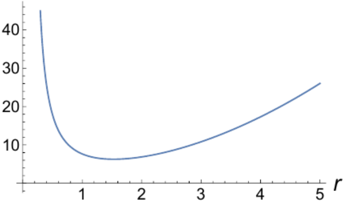
$$\begin{aligned} ds^2_{TG} &= e^{-\sqrt{6}\varphi}(r^2 + 6br + 21b^2)dt^2 + \\ &\quad -e^{\sqrt{6}\varphi}dr^2(r^2 + 6br + 21b^2)^{-1} + \\ &\quad -e^{\sqrt{6}\varphi}(r - 3b)^2(dx^2 + dy^2) \ , \\ e^{\sqrt{6}\phi} &= \left(\frac{r+b}{r-3b}\right)^{3/2} \ . \end{aligned}$$

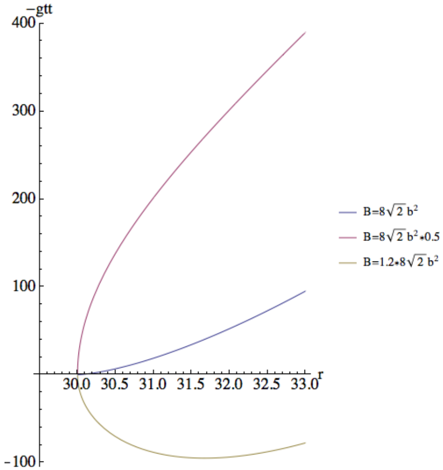


- Set of parameters for the thermal gas: (B, χ, T),
- Temperature T and electric potential χ are **moduli** of the thermal gas solution. solution.

3 – Good vs bad singularities







$$U^2_{good}(r) \quad = \quad \left(\frac{r-3b}{r+b}\right)^{3/2} (r^2 + 6br + 21b^2)$$

4 – Outline

4 – Thermodynamics - mixed canonical ensemble

Black brane

$$F_{bb} = M_{bb} - TS_{bb} + q_{bb}\chi_{bb}\,,$$

- M is the mass of the black brane,

$$M = \frac{B^2 - q^2}{4b}\,,$$

- Thermodynamic potential

$$dF_{bb} = -S_{bb}dT + q_{bb}d\chi_{bb} + m_{bb}dB$$

Thermal gas

$$b_{TG} = +2^{-\frac{7}{4}}\sqrt{|B|}\,.$$

- The free energy for the thermal gas is a function of B only, at any temperature

$$\begin{aligned} F_{TG} &= M_{TG} = \frac{B^2}{4b_{TG}} = 2^{-\frac{1}{4}}|B|^{\frac{3}{2}}\,, \\ dF_{TG} &= m_{TG}dB \end{aligned}$$

The magnetization is qualitatively different for BB and TG

$$m_{bb} = \left.\frac{\partial F_{bb}}{\partial B}\right|_{\chi,T} = 3\sqrt{\frac{3}{2}}\,\frac{B}{|\chi|}\,, \qquad m_{TG} = 3\,2^{-\frac{5}{4}}\sqrt{|B|}\,\mathrm{sgn}(B)\,.$$

4 – Phase transition

$$\Delta F = F_{bb} - F_{TG} = \frac{27B^2 + 32\chi^4}{24\sqrt{6}|\chi|} - 2^{-\frac{1}{4}}|B|^{\frac{3}{2}}\,.$$

- The critical point is given by

$$|B_*| = \frac{4\sqrt{2}}{3}\chi^2$$

$\Rightarrow$ for every value of χ (except $\chi = 0$) there is a phase transition at a constant magnetic field.

- ***Second order*** phase transition:

$$\Delta F = 3\sqrt{\frac{3}{2}}\frac{(B - B_*)^2}{|\chi|} + \mathcal{O}\left((B - B_*)^3\right)$$

The solutions are completely specified by S, m and q, which become equal for TG and BB at the critical point: non trivial check!

4 – Phase transition

This is a second order phase transition hence *it corresponds to an interesting quantum critical point that should be described by a scale invariant conformal field theory on the boundary*.

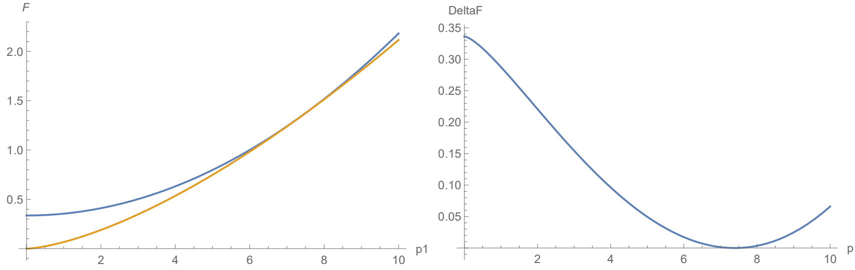


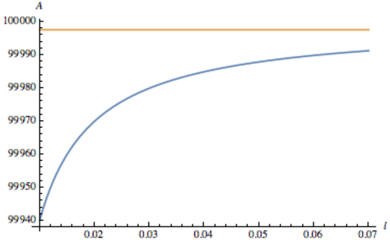
Figure: (L) Plot of the free energies of the black brane and the solitonic solution for $\chi_0 = 1.98$, as functions of p_1, the blue line is the black brane and the yellow line is the soliton. (R) Plot of the difference of the the two free energies for $\chi_0 = 1.98$, as function of p_1.

4 – Holographic considerations

- Quantum criticality emerges at the locus $B = B_c(\chi)$.
- No confinement/deconfinement phase transition
- String attached at the boundary of AdS_4 into the bulk. The area of its worldsheet can be connected or disconnected

$$A^{con} = \frac{L}{2G_{N4}} \int_{r^*}^{r^\infty} dr \frac{h(r)^3}{h(r^*)^2 U(r)} \frac{1}{\sqrt{\frac{h(r)^4}{h(r^*)^4} - 1}} ,$$

$$A^{dis} = \frac{L}{2G_{N4}} \int_{r_0}^{r^\infty} dr \frac{h(r)}{U(r)} .$$



$\rightarrow$ Entanglement entropy is not the order parameter of the phase transition.

4 – Phase transition

- The spectrum of the dual theory can tell us information of what happens at the critical point. [Gursoy, Kiritsis, Nitti '07]
- The spectrum of fluctuations of the background: acting with a bosonic operator $\mathcal{O}_\Delta$ on vacuum, holographically induce fluctuations of the corresponding bosonic bulk field with mass m^2_Δ on the vacuum background.
- For $m^2 = 0$ $\phi(r, x) = \xi(r) e^{-i\omega t + \vec{k}\cdot\vec{x}}$, the effective action is

$$\begin{aligned} S_{fluc} &= \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^* = \\ &= \int drd^3x \sqrt{-g} \left\{ g^{rr} |\partial_r \xi_\omega(r)|^2 + \omega^2 g^{00} |\xi_\omega(r)|^2 \right\} , \end{aligned}$$

Look for eigenvalues ω of solutions, normalizable both in the UV, $r \to \infty$ and in the IR $r \to r_s$.

4 – Phase transition

Solve the fluctuation equation with normalizability condition:

$$\xi''_\omega(r) + \frac{d}{dr}\log(\sqrt{-g}g^{rr})\xi'_\omega(r) - \omega^2 g^{00} g_{rr}\xi_\omega(r) = 0$$

- The thermal gas has no normalizable modes for arbitrary small omega: gapped system $\xi_0 \sim \epsilon^{-1}$, with $\epsilon = r - r_s$.
  Notice: Releasing the condition $|B| = 8\sqrt{2}b^2$ introduces normalizable modes!
- The black brane has QNM with spectrum given by discrete frequencies, the lowest being $|\omega| \sim T$. Lowering the temperature one can reach arbitrary small energies $|\omega| \propto \epsilon$, with separations also $|\Delta\omega| \sim \epsilon$.
  One can study the $T = 0$ identically in a separate calculation, one finds $(x = r - r_{-h})$

$$\xi_\omega(x) \sim De^{i\frac{\omega}{f_0 x}} \; , \qquad f(r) = f_0(B,\chi)(r-r_h)^2 + \mathcal{O}(r-r_h)^3$$

- The near-B_c behaviour of TG and BB also match.

5 – Outline

5 – Conclusions and outlook

- Analysis of black branes phase space in AdS_4
- Construction of a “thermal gas” configuration and identification of a “good singularity” solution
- Identification of a *second order* phase transition
- Interpretation of the QCP as a gapless phase in the spectrum of fluctuation of the BB.

- To do:
  - Extensions to finite temperature.
  - Possible supersymmetry enhancement for the thermal gas solution
  - Resolution of the singularity by embedding in String/M-Theory

Thank you!