# PHOTON INTERACTIONS IN SHERPA



Stefan Höche<sup>1</sup> Institute for Particle Physics and Phenomenology Durham University



<sup>1</sup> for Sherpa: Tanju Gleisberg, SH, Frank Krauss, Steffen Schumann, Marek Schönherr, Frank Siegert & Jan Winter



# OUTLINE



This talk does not contain much about photons ...

## So what is it about ?

- Matrix element (ME) generators
- Shower (PS) generators
- Merging of ME & PS (CKKW)
- Cluster fragmentation
- Hadron decays
- Multiple parton interactions

# Sherpa itself is the framework that combines all the above

## How do photons fit in ?

- Sherpa provides  $\gamma$  beamspectrum ( $p \rightarrow p\gamma$  by T. Pierzchala) & PDF
- all the rest is "standard", so let's talk about the rest first ...



![](_page_2_Picture_0.jpeg)

# WHAT IS CKKW AND WHY?

![](_page_2_Picture_2.jpeg)

# Matrix Elements

- Advantage
- Exact to fixed order
- Include all interferences
   Drawback
- Calculable only for low
   FS multiplicity (n≤6-8)

# Parton Showers

## Advantage

 Resum all (next-to) leading logarithms to all orders
 Drawback

# Interference effects only

through angular ordering

# **Combine both approaches: CKKW**

- Good description of hard radiation (ME)
- Correct intrajet evolution (PS)

## Strategy: Separate phase space Jet production region ME

■ Intrajet evolution region → PS

**Free parameter: Separation cut Q**cut (K<sub>T</sub>-type jet measure)

![](_page_3_Picture_0.jpeg)

# CKKW: Z+JETS@TEVATRON

![](_page_3_Picture_2.jpeg)

### The DØ collaboration, DØ note 5066-CONF

![](_page_3_Figure_4.jpeg)

Pythia 6.2

![](_page_3_Figure_5.jpeg)

# normalized to data

### Sherpa 1.0 normalized to data

![](_page_4_Picture_0.jpeg)

Jet- $p_T$ , jet 3

# CKKW: Z+JETS@TEVATRON

![](_page_4_Picture_2.jpeg)

### The DØ collaboration, DØ note 5066-CONF

![](_page_4_Figure_4.jpeg)

Pythia 6.2 normalized to data

### Sherpa 1.0 normalized to data

![](_page_5_Picture_0.jpeg)

 $\mathbf{\Delta \phi_{jet1, jet2}}$ 

# CKKW: Z+JETS@TEVATRON

![](_page_5_Picture_2.jpeg)

### The DØ collaboration, DØ note 5066-CONF

![](_page_5_Picture_4.jpeg)

![](_page_5_Figure_5.jpeg)

### Pythia 6.2 normalized to data

# Sherpa 1.0 normalized to data

![](_page_6_Picture_0.jpeg)

## CKKW EXTENSIONS

![](_page_6_Picture_2.jpeg)

2000002000000

### **Consider heavy flavour production**

Narrow width approximation 

 full ME factorises
 into production and decay parts
 into production

Schematically:  $\mathcal{A}^{(n)} = \mathcal{A}_{\text{prod}}^{(n_{\text{prod}})} \otimes \prod_{i \in \text{decays}} \mathcal{A}_{\text{dec},i}^{(n_i)}$ 

## How is it simulated in Sherpa?

- ME generator AMEGIC++ provides decay chains (projection onto relevant diagrams)
- PS generator APACIC++ provides production & decay shower off heavy partons (+ standard showering)
- CKKW ME-PS merging is applied separately and independent within production and each decay
   Method is fully general and applicable e.g. in SUSY production

![](_page_7_Figure_0.jpeg)

![](_page_8_Picture_0.jpeg)

# ME'S IN SHERPA: AMEGIC++

![](_page_8_Picture_2.jpeg)

R. Kuhn, F. Krauss, G. Soff JHEP 0202:044, 2002

### What does AMEGIC++ provide ?

Flexibility

- **Fully automated** calculation of (polarized) cross sections in the SM, MSSM and ADD model
- Expandability: FeynRules reader, dynamic add-on model libs
- Performance well comparable to that of dedicated codes
   Reliability
- $e^+e^- \rightarrow 6f$  comparison vs. HELAC/PHEGAS EPJ C34 (2004) 173  $\rightarrow$
- Comparison of arbitrary 2→2 MSSM processes vs. WHIZARD/O'Mega & SMadGraph Phys. Rev. D73(2006) 055005
- MC4LHC ME generator comparison http://mlm.home.cern.ch/mlm/mcwshop03/mcwshop.html

![](_page_8_Figure_12.jpeg)

n/a

# CSW RECURSION IN AMEGIC++

New twistor-inspired techniques (CSW vertex rules) help speeding up calculation of pure QCD ME's for higher multiplicites
 Advantage: Up to N<sub>out</sub> = 7 only up to 3 MHV-amplitudes must be sewed together

n/a

 $2j \rightarrow 5j$ 

Process	Time [s] for $10^5$	$10^5$ points		e [s] for $10^5$ points	Conventional /		
	Conventional		CSW rules		CSW-rules		
$2g \rightarrow 4g$	1977		19		104.1		
$2g \rightarrow 5g$	n/a			429	n/a		
$2q \rightarrow 4g$	124		14		8.9	<u> </u>	
$2q \rightarrow 5g$	43636	Norul	1	290	148.4		Significant
$2q \rightarrow 2q'+2g$	8	INEWI	y	6	1.33		
$2q \rightarrow 2q'+3g$	810	accessi	ble [	74	10.8		speedup
$2q \rightarrow 2q + 2g$	24	process	ses [	10	2.4	7	
$2q \rightarrow 2q+3g$	3923			118	33		
$2j \rightarrow 4j$	4082			202	20.2		

12103

![](_page_9_Picture_4.jpeg)

![](_page_9_Picture_5.jpeg)

![](_page_10_Picture_0.jpeg)

![](_page_10_Picture_2.jpeg)

T. Gleisberg, SH: in preparation

- Revisited "old-fashioned" Berends-Giele recursion JHEP 08(2006)062
  - → New ME generator **COMI**X
- Fully general implementation of SM interactions What you could do, for example:
  - $pp \rightarrow W/Z+N$  jets where so far N up to 6 (all partons !)
  - $pp \rightarrow N \text{ jets} + t [W^+b + M \text{ jets}] \overline{t} [W^-\overline{b} + M \text{ jets}]$ where so far {N,M} up to {2,1}
  - $pp \rightarrow N$  gluons where N up to 12 (QCD benchmark)
  - $pp \rightarrow N$  jets where N up to 8 (all partons !)
- Key point: Vertex decomposition of all four-particle vertices (Growth in computational complexity for CDBG
  - determined solely by number of external legs at vertices )
- The ME is ticked off, but how about the phasespace ?
   Recursive method analogous to ME calculation (see backup)

![](_page_11_Picture_0.jpeg)

![](_page_11_Picture_2.jpeg)

![](_page_11_Figure_3.jpeg)

#### Performance in QCD benchmarks World record ! Cross section [pb] $gg \to ng$ 8 12 9 10 11 n $\sqrt{s} \, [\text{GeV}]$ 5000 1500 2000 25003500 Comix 0.755(3)0.305(2)0.101(7)0.019(2)0.057(5)0.70(4)0.30(2)Phys. Rev. D67(2003)014026 0.097(6)Nucl. Phys. B539(1999)215 0.719(19)

## "Real life" example: b-pair + jets comparison with other ME generators

$\sigma \; [\mu \mathrm{b}]$	Number of jets						
$b\bar{b} + QCD$ jets	0	1	2	3	4	5	6
Comix	470.8(5)	8.83(2)	1.826(8)	0.459(2)	0.151(2)	0.0544(6)	0.023(2)
ALPGEN	470.6(6)	8.83(1)	1.822(9)	0.459(2)	0.150(2)	0.053(1)	0.0215(8)
AMEGIC++	470.3(4)	8.84(2)	1.817(6)	Children Terling			

Setup: http://mlm.home.cern.ch/mlm/mcwshop03/mcwshop.html

![](_page_12_Picture_0.jpeg)

# COMIX: PERFORMANCE

![](_page_12_Picture_2.jpeg)

 $\begin{array}{l} \mbox{Efficiencies: LHC @ 14 TeV} \\ \mbox{Cuts: 66 GeV} \leq m_{l\bar{l}} \leq 116 \mbox{GeV}, \\ \mbox{CDF Run II } K_T\mbox{-algo @ 20GeV} \end{array}$ 

Process	Efficiency		
Z+0 jet	8.50%		
Z+1 jet	1.05%		
Z+2 jets	0.60%		
Z+3 jets	0.15%		
Process	Efficiency		
W+0 jet	19.13%		
W+1 jet	1.50%		
W+2 jets	0.48%		
W+3 jets	0.16%		

T. Gleisberg, SH: in preparation **Also new:** HAAG-based QCD integrator for colour sampling

![](_page_12_Figure_6.jpeg)

![](_page_13_Picture_0.jpeg)

![](_page_13_Picture_2.jpeg)

### JHEP03(2008)038

k

Catani-Seymour subtraction terms General framework for QCD NLO calculations Splitting of parton **ij** into partons i and j, spectator k Advantages over Parton Shower  $\mathbf{V}_{ij,k}$ → Full phasespace coverage → Good approximation of ME Better analytic control e.g. final-final splitting: Implementation into Sherpa  $\left< V_{q_i,g_j,k} \right> (\tilde{z}_i, y_{ij}, k) =$  $\mathrm{C_F}\left(rac{2}{1- ilde{\mathrm{z}}_\mathrm{i}+ ilde{\mathrm{z}}_\mathrm{i}\mathrm{y}_\mathrm{ii,k}}-(1+ ilde{\mathrm{z}}_\mathrm{i})
ight)$ for the general case, i.e. final-final initial-final and initial-initial dipoles

Stefan Höche, yy Workshop CERN, 24.4.2008

 $\mathbf{y_{ij,k}} = \frac{\mathbf{p_i p_j}}{\mathbf{p_i p_k} + \mathbf{p_j p_k} + \mathbf{p_i p_j}}$ 

 $\mathbf{z_i} = \frac{p_i p_k}{p_i p_k + p_j p_k}$ 

 $\tilde{ij}$ 

![](_page_14_Picture_0.jpeg)

![](_page_14_Picture_2.jpeg)

### JHEP03(2008)038

![](_page_14_Figure_4.jpeg)

# DIPOLE SHOWER FOR HADRON COLLISIONS

![](_page_15_Picture_1.jpeg)

### arXiv: 0712.3913 [hep-ph]

- IS emission formulated completely perturbative
   Radiation associated to inital-inital, initial-final and final-final colour lines (dipoles)
  - Beam remnants kept outside
    Transverse momentum and rapidity defined through invariants, e.g. Drell-Yan:

$$\mathbf{p}_{\perp}^{2}=rac{\hat{\mathbf{u}}\hat{\mathbf{t}}}{\mathbf{m}_{\mathrm{B}}^{2}} \quad \mathbf{y}=rac{1}{2}\lnrac{\hat{\mathbf{u}}}{\hat{\mathbf{t}}}$$

![](_page_15_Figure_6.jpeg)

● pp→jets Phys. Rev. D50 (1994) 5562

![](_page_15_Figure_8.jpeg)

DIPOLE SHOWER FOR HADRON COLLISIONS

![](_page_16_Picture_1.jpeg)

### arXiv: 0712.3913 [hep-ph]

### First emission by construction ME-corrected

![](_page_16_Figure_4.jpeg)

![](_page_17_Picture_0.jpeg)

# MPI SIMULATION IN SHERPA

![](_page_17_Picture_2.jpeg)

hep-ph/0601012

### Sherpas current multiple parton interaction (MPI) module

- Based on the PYTHIA model
   T. Sjöstrand & M. van Zijl, PRD36(1987)2019
- Parton showers (PS) attached to secondary interactions

![](_page_17_Figure_7.jpeg)

### Combination of MPI's with hard processes and CKKW matching

- Hard processes with final state multiplicity different from two require unique definition of starting scale for MI evolution, µ<sub>MI</sub>
- Sherpa algorithm (works for arbitrary n-jet ME):
  - Employ  $K_T$ -algorithm to define 2+2 core process
  - Set starting scale µ<sub>MI</sub> to p<sub>T</sub> of final state QCD parton(s) from this process and veto partons harder than µ<sub>MI</sub> (from PS) in secondary interactions

![](_page_18_Picture_0.jpeg)

# MPI RESULTS FROM SHERPA

![](_page_18_Picture_2.jpeg)

### hep-ph/0601012

### Our current "best fit" for CDF

- Lower  $p_T$  cutoff •  $p_{T,min} \approx 2.4 \text{ GeV}$
- Moderate interaction number due to additional multiplicity from PS
   → < N<sup>2→2</sup><sub>hard</sub> >≈ 2.08

### To take home ...

- Highly dependent on p<sub>T,min</sub> and PDF
- Does not give any prediction for the LHC (naive scaling)

![](_page_18_Figure_10.jpeg)

![](_page_19_Picture_0.jpeg)

![](_page_19_Picture_2.jpeg)

### Shortcomings of the current MPI model

arXiv: 0705.4577 [hep-ph]

- Lower p<sub>T</sub> cutoff defines total cross section
- Energy extrapolation depends on tuning parameter
   We try to solve part of this by ...
  - Definiton of hard cross section through BFKL kernel convoluted with DUPDF's  $\Rightarrow$  can be extended into diffractive region  $\sigma = \frac{\pi^2}{2S} \sum_{a^{(1)}} \int dy_1 \int dk_{1\perp}^2 \int d\phi_1 \int dy_n$  $\times f^{(1)}(x^{(1)}, z^{(1)}, k_{1\perp}^2, \bar{k}_{2\perp}^{(1)2}) f^{(2)}(x^{(2)}, z^{(2)}, k_{n\perp}^2, \bar{k}_{n-1\perp}^{(2)2}) \frac{1}{2\xi^{(1)} 2\xi^{(2)} 2S} \frac{1}{\Delta_{a_1}(y_1, y_2)}$  $\times \left[ \prod_{i=2}^n \int \frac{d\phi_i}{2\pi} \int dy_i \int \frac{dk_{i\perp}^2}{k_{i\perp}^2} \frac{\alpha_s(k_{i\perp}^2)}{\pi} \sum_{a_i} C_{a_{i-1}a_i}(q_{i-1}, k_i) \Delta_{a_i}(y_i, y_{i-1}) \right]$ Markovian algorithm to generate splittings
    - from  $\Delta_{a_i}(y_i, y_{i-1})$  in the spirit of a parton shower
    - number of emissions determined on the flight

![](_page_20_Picture_0.jpeg)

# TOWARDS A NEW MPI MODEL

![](_page_20_Picture_2.jpeg)

### • Jet - p<sub>T</sub> spectra PRD75(2007)092006

arXiv: 0705.4577 [hep-ph]

![](_page_20_Figure_5.jpeg)

Azimuthal decorrelation of widely separated jets PRL77(1996)595

![](_page_20_Figure_7.jpeg)

![](_page_21_Picture_0.jpeg)

![](_page_21_Picture_2.jpeg)

Eur. Phys. J. C36 (2004) 381

Sherpas cluster fragementation model:

- Colour ordered partons transformed into primary clusters according to combination of
  - kinematical weight

$$\mathbf{W_{ij,\,kl}} = \ \frac{\mathbf{t_0}}{\mathbf{t_0} + 4(\mathbf{w_{ij}} + \mathbf{w_{kl}})^2}$$

![](_page_21_Picture_8.jpeg)

- Clusters decayed according to overlap between cluster mass and hadron mass spectrum
  - cluster mass in hadron regime 

    transition to hadron
  - else → 2-body decay
     C→HH, C→CH or C→CC
     combined weight applied again<sup>1</sup>

<sup>1</sup> with  $t_0$  replaced by  $Q_0$  (hadronic scale)

![](_page_21_Picture_13.jpeg)

![](_page_22_Picture_0.jpeg)

## CLUSTER FRAGMENTATION

![](_page_22_Picture_2.jpeg)

Eur. Phys. J. C36 (2004) 381

![](_page_22_Figure_4.jpeg)

![](_page_23_Picture_0.jpeg)

# HADRON DECAYS

![](_page_23_Picture_2.jpeg)

## Features of Sherpas hadron decay package

- Full flexibility, all information is read from parameter files
   (branching ratios, decay channels, form factors, integrators)
- Extremely easy to extend with specific decay modes / models (feel free to add your favourite decay ...)
- Spin correlation algorithm with full spin information from AMEGIC++ matrix element
- Extensively tested in  $\tau$  and hadron decays
- B-mixing implemented in full generality
- First fully functional release with version 1.1

![](_page_24_Picture_0.jpeg)

## HADRON DECAYS: RESULTS

![](_page_24_Picture_2.jpeg)

F. Siegert, F. Krauss: in preparation PYTHIA+TAUOLA: hep-ph/0101311

### Many models: e.g. $\tau \rightarrow \nu_{\tau} \pi^{-} \pi^{-} \pi^{+}$

### Spin correlations: e.g. $\mathbf{Z} \rightarrow \mathbf{W}^+ \mathbf{W}^-, \mathbf{W}^- \rightarrow \tau^- \overline{\nu}_{\tau} \rightarrow \nu_{\tau} \pi \overline{\nu}_{\tau}$

![](_page_24_Figure_6.jpeg)

![](_page_25_Picture_0.jpeg)

![](_page_25_Picture_2.jpeg)

F. Siegert, F. Krauss: in preparation

B-mixing: e.g. Decay rate asymmetry  $B_0 \rightarrow J/\Psi K_s \leftrightarrow \bar{B}_0 \rightarrow J/\Psi K_s$ in  $\Upsilon(4s) \rightarrow B_0 \overline{B}_0$  events

![](_page_25_Figure_5.jpeg)

![](_page_26_Figure_0.jpeg)

# PHOTON INTERACTIONS

![](_page_26_Picture_2.jpeg)

### Now that we know the rest ...

 Sherpa provides LASER backscattering beam spectrum acc. to Acta Phys. Polon. B34 (2003) 2741

cross section in  $\gamma\gamma \rightarrow \tilde{\mu}^+\tilde{\mu}^-$ 

![](_page_26_Figure_6.jpeg)

![](_page_26_Figure_7.jpeg)

Prog. Part. Nucl. Phys. 53 (2004) 329

Beam spectrum for  $p \rightarrow p\gamma$ acc. to Phys. Lett. C15 (1975) 181 implemented by T. Pierzchala ported into v1.1 during this WS

![](_page_27_Picture_0.jpeg)

![](_page_27_Picture_2.jpeg)

Sherpa is much more than what I talked about ...

## Sherpas and collaborators currently also work on:

- Preparing the two new showers for ME-Shower merging
   systematics studies with different shower prescriptions
   BSM beyond the MSSM:
  - Little Higgs, MWTC 

    J. Ferland (ATLAS, Montreal), ...
- Interfaces to Athena 
   J. Ferland (ATLAS, Montreal) and CMS software 
   M. Merschmeyer (CMS, Aachen) and LHCb software 
   SH, F. Siegert, J. Stieglitz (Durham/Dortmund)
- Grid support: At the IPPP, we run Sherpa on the Grid ! Multithreading: Speed up your computation with more CPU's !

Latest release: Version 1.1.0 available on Genser and HepForge

Updates on Sherpa can be found on

![](_page_28_Picture_1.jpeg)

WWW.SHERPA-MC.DE

![](_page_28_Picture_3.jpeg)

INFO@SHERPA-MC.DE

![](_page_28_Picture_5.jpeg)

![](_page_29_Figure_0.jpeg)

# CKKW IN A NUTSHELL

![](_page_29_Picture_2.jpeg)

JHEP 0111 (2001) 063

JHEP 0208 (2002) 015

 $cut,Q_1)$ 

**RS** Domain

• Define jet resolution parameter  $Q_{cut}$  (Q-jet measure) divide phase space into regions of jet production (ME) and jet evolution (PS) Select final state multiplicity and kinematics  $\Delta_q(Q_{\mathrm{cut}}, Q$ 

according to  $\sigma$  'above' Q<sub>cut</sub>

- K<sub>T</sub> -cluster backwards (construct PS-tree) and identify core process
- Reweight ME to obtain exclusive samples at Q<sub>cut</sub>
- Start the parton shower at the hard scale Veto all PS emissions harder than Q<sub>cut</sub>

This yields the correct jet observables ! Generic example: 2-jet rate in  $ee \rightarrow qq$  $\mathbf{R_2}(\mathbf{q}) = \left( \boldsymbol{\Delta}(\mathbf{Q_{cut}}, \mu_{\mathbf{hard}}) \frac{\boldsymbol{\Delta}(\mathbf{q}, \mu_{\mathbf{hard}})}{\boldsymbol{\Delta}(\mathbf{Q_{cut}}, \mu_{\mathbf{hard}})} \right)$ 

Stefan Höche, yy Workshop CERN, 24.4.2008

 $\Delta_{ar{q}}(Q_{ ext{cut}},\mu_H)$ 

**ME** Domain

![](_page_30_Picture_0.jpeg)

# PS IN SHERPA: APACIC++

![](_page_30_Picture_2.jpeg)

R. Kuhn, F. Krauss, G. Ivanyi, G. Soff CPC 134 (2001) 223 F. Krauss, A. Schälicke, G. Soff, hep-ph/0503087

### Basic features of APACIC++ :

- Virtuality ordered parton cascade, colour coherence imposed by angular veto
- Final & initial state showering in e<sup>+</sup>e<sup>-</sup> & hadron collisions
   ( no DIS-like situations )
- Algorithm similar to virtuality ordered PYTHIA parton shower
- Extensively tested, e.g. vs. LEP data (hadronisation: PYTHIA)

![](_page_30_Figure_9.jpeg)

![](_page_30_Figure_10.jpeg)

![](_page_31_Picture_0.jpeg)

- In quasi-collinear limit (b  $\leftrightarrow$  heavy quark) ME factorises  $|\mathbf{M}(\mathbf{b}, \mathbf{c}, \dots, \mathbf{n})|^2 \rightarrow |\mathbf{M}(\mathbf{a}, \dots, \mathbf{n})|^2 \frac{8\pi\alpha_s}{\mathbf{t} \mathbf{m}_a^2} \mathbf{P}_{\mathbf{a} \rightarrow \mathbf{b} \mathbf{c}}(\mathbf{z})$
- Virtuality ordered PS  $\rightarrow$  evolution variable t changes to  $t m_a^2$
- Splitting functions P<sub>ab</sub>(z) become those for massive quarks Nucl. Phys. B627(2002)189

$$\xrightarrow{\mathbf{cr}} \mathbf{C}_{\mathbf{F}} \left( \frac{1+\mathbf{z}^2}{1-\mathbf{z}} - \frac{2\mathbf{z}(1-\mathbf{z})\mathbf{m}^2}{\mathbf{q}^2 + (1-\mathbf{z})^2\mathbf{m}^2} \right)$$

$$\xrightarrow{\mathbf{T}} \left( 1 - 2\mathbf{z}(1-\mathbf{z}) + \frac{2\mathbf{z}(1-\mathbf{z})\mathbf{m}^2}{\mathbf{z}^2 + (1-\mathbf{z})\mathbf{m}^2} \right)$$

$$\rightarrow T_R \left( 1 - 2z(1-z) + \frac{2z(1-z)m}{q^2 + m^2} \right)$$

Cross-check: 2- and 3-jet fraction in  $e^+e^- \rightarrow t\bar{t}$ , PS vs. ME, weighted with NLL Sudakov form factors Phys. Lett. B576(2003)135

![](_page_31_Figure_7.jpeg)

APACIC++: HEAVY QUARK PRODUCTION

![](_page_32_Picture_1.jpeg)

## PS in production

![](_page_32_Figure_3.jpeg)

- On-shell daughter partons
   New decay kinematics via Lorentz transformation Choice: Boost into new (daughter) cms
- FSR-like situation
- Evolution stops once diced virtuality reaches on-shell mass of heavy quark

## PS in decay

![](_page_32_Figure_8.jpeg)

- Off-shell daughter partons
   Decay kinematics need to be reconstructed
  - Choice: Reconstruct in cms of decayed quark, such that p/|p| is preserved
- ISR-like situation
- Evolution stops if p<sub>⊥</sub> reaches width of decaying quark

![](_page_33_Picture_0.jpeg)

![](_page_33_Picture_1.jpeg)

Nucl. Phys. B9 (1969) 568

- State-of-the art approach for general phasespace generation: Factorise PS using
  - $\mathrm{d}\Phi_{\mathbf{n}}\left(\mathbf{a},\mathbf{b};\mathbf{1},\ldots,\mathbf{n}\right)=\mathrm{d}\Phi_{\mathbf{m}}\left(\mathbf{a},\mathbf{b};\mathbf{1},\ldots,\mathbf{m},\bar{\pi}\right)\,\mathbf{d}\mathbf{s}_{\pi}\,\mathrm{d}\Phi_{\mathbf{n}-\mathbf{m}}\left(\pi;\mathbf{m}+\mathbf{1},\ldots,\mathbf{n}\right)$
  - Remaining basic building blocks of the phasespace:

![](_page_33_Figure_6.jpeg)

Arrows → Momentum flow

# COMIX: PHASESPACE RECURSION

![](_page_34_Picture_1.jpeg)

 $\hat{S}^{\,\rho,\pi\setminus
ho}_{\pi}$ 

 $\pi$ 

 $\hat{T}^{\pi,\overline{\alpha b\pi}}_{\alpha}$ 

 Basic idea: Take above recursion literally and "turn it around" S-channel phasespace (schematically)

 $d\Phi_{S}(\pi) = \left[\sum_{\alpha} \alpha \left(S_{\pi}^{\rho,\pi\setminus\rho}\right)\right]^{-1} \times \left[\sum_{\alpha} \alpha \left(S_{\pi}^{\rho,\pi\setminus\rho}\right) S_{\pi}^{\rho,\pi\setminus\rho} P_{\rho} d\Phi_{S}(\rho) P_{\pi\setminus\rho} d\Phi_{S}(\pi\setminus\rho)\right] -$ 

T-channel phasespace (schematically) Weights for adaptive multichanneling

"b" is fixed → Every PS-weight is unique !
Arrows → Weight flow !
Factorial growth of PS-channels tamed

![](_page_35_Picture_0.jpeg)

![](_page_36_Picture_0.jpeg)