The alternative approach to QCD analysis of the structure function F_2^γ

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CERN, 2008

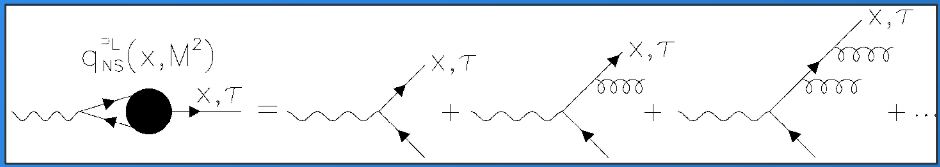
Outline

- Basic notation
- Conventional formulation
- Alternative formulation
  - Numerical results
  - Global analysis of F_γ^2
- Conclusion

Evolution equations

System of inhomogeneous evolution equations:

$$\frac{dq_{NS}(x,M)}{d\ln M^2} = \delta_{NS} k_q + P_{NS} \otimes q_{NS}$$



$$\frac{d\Sigma(x,M)}{d\ln M^2} = \delta_{\Sigma} k_q + P_{qq} \otimes \Sigma + P_{qG} \otimes G$$

$$\frac{dG(x,M)}{d\ln M^2} = k_G + P_{Gq} \otimes \Sigma + P_{GG} \otimes G$$

where

$$q_{NS}(x,M) \equiv \sum_{i=1}^{n_f} \left(e_i^2 - \langle e^2 \rangle^2 \right) \left(q_i(x,M) + \overline{q_i}(x,M) \right),$$

$$\Sigma(x,M) \equiv \sum_{i=1}^{n_f} \left[q_i(x,M) + \overline{q_i}(x,M) \right]$$

$$\delta_{NS} = 6 n_f \left(\langle e^4 \rangle - \langle e^2 \rangle^2 \right), \qquad \delta_{\Sigma} = 6 n_f \langle e^2 \rangle$$

Evolution equations

…and

$$k_q(x,M) = \frac{\alpha}{2\pi}\left[k_q^{(0)}(x) + \frac{\alpha_S(M)}{2\pi}k_q^{(1)}(x) + \left(\frac{\alpha_S(M)}{2\pi}\right)^2 k_q^{(2)}(x) + \ldots\right]$$

$$k_G(x,M) = \frac{\alpha}{2\pi}\left[\frac{\alpha_S(M)}{2\pi}k_G^{(1)}(x) + \left(\frac{\alpha_S(M)}{2\pi}\right)^2 k_G^{(2)}(x) + \ldots\right]$$

$$P_{ij}(x,M) = \frac{\alpha_S(M)}{2\pi}P_{ij}^{(0)}(x) + \left(\frac{\alpha_S(M)}{2\pi}\right)^2 P_{ij}^{(1)}(x) + \ldots$$

← Splitting functions

Coefficient functions →

$$C_q(x,M) = \delta(1-x) + \frac{\alpha_S(M)}{2\pi}C_q^{(1)}(x) + \left(\frac{\alpha_S(M)}{2\pi}\right)^2 C_q^{(2)}(x) + \ldots$$

$$C_G(x,M) = \frac{\alpha_S(M)}{2\pi}C_G^{(1)}(x) + \left(\frac{\alpha_S(M)}{2\pi}\right)^2 C_G^{(2)}(x) + \ldots$$

$$C_\gamma(x,M) = C_\gamma^{(0)}(x,M) + \frac{\alpha_S(M)}{2\pi}C_\gamma^{(1)}(x) + \left(\frac{\alpha_S(M)}{2\pi}\right)^2 C_\gamma^{(2)}(x) + \ldots$$

Photon structure F_2^γ

Photon stucture function

$$\frac{1}{x}F_2^\gamma(x,Q^2) = \underbrace{q_{NS}(x,Q^2)\otimes C_q(x)}_{\text{Non-singlet}} + \underbrace{\langle e^2\rangle \Sigma(x,Q^2)\otimes C_q(x)}_{\text{Singlet}} + \underbrace{\langle e^2\rangle G(x,Q^2)\otimes C_G(x)}_{\text{Gluon}} + \underbrace{\frac{\alpha}{2\pi}\left(\delta_{NS} + \langle e^2\rangle \delta_\Sigma\right) C_\gamma(x)}_{\text{Photon}}$$

contribution

Solution of evolution equations

$$q(x,Q^2) = q^{PL}(x,Q^2) + q^{HAD}(x,Q^2)$$

Pointlike solution LO

$$q_{PL}^{\gamma}(n,Q^2) = \frac{4\pi}{\alpha_S(Q^2)}\left(1-\left(\frac{\alpha_S(Q^2)}{\alpha_S(Q_0^2)}\right)\right)\frac{\alpha}{2\pi\beta_0}\frac{k^{(0)}(n)}{1-\frac{2}{\beta_0}P^{(0)}(n)}$$

Pointlike solution NLO

$$q_{PL}^{\gamma}(n,Q^2) = \frac{4\pi}{\alpha_S(Q^2)}\left(1-\left(\frac{\alpha_S(Q^2)}{\alpha_S(Q_0^2)}\right)\right)\frac{\alpha}{2\pi\beta_0}\frac{k^{(0)}(n)}{1-\frac{2}{\beta_0}P^{(0)}(n)}\left(1+\frac{\alpha_S(Q^2)}{2\pi}U\right)$$

$$+\left[1-L^{-(2/\beta_0)P^{(0)}(n)}\right]\frac{1}{-P^{(0)}(n)}\frac{\alpha}{2\pi}\left(k^{(1)}(n)-\frac{\beta_1}{2\beta_0}k^{(0)}(n)-Uk^{(0)}(n)\right)$$

Conventional formulation

$$\frac{1}{x}F_2^{\gamma}(x,Q^2) = \left[q_{NS}(x,Q^2) + \frac{\alpha_S(Q^2)}{2\pi} q_{NS}(x)\otimes C_q^{(1)}(x) + \cdots\right] +$$

$$\langle e^2\rangle\left[\Sigma(x,Q^2) + \frac{\alpha_S(Q^2)}{2\pi}\Sigma(x)\otimes C_q^{(1)}(x) + \cdots\right] +$$

$$\langle e^2\rangle\left[\frac{\alpha_S(Q^2)}{2\pi}G(x)\otimes C_G^{(1)}(x) + \cdots\right] +$$

$$\delta_{\gamma}\left[\frac{\alpha}{2\pi}C_{\gamma}^{(0)} + \frac{\alpha_S(Q^2)}{2\pi}\frac{\alpha}{2\pi}C_{\gamma}^{(1)} + \cdots\right] +$$

Leading order

$$\frac{1}{x}F_{2,LO}^{\gamma}(x,Q^2) = q_{NS}(x,Q^2) + \Sigma(x,Q^2)$$

Next to leading order

$$\frac{1}{x}F_{2,NLO}^{\gamma}(x,Q^2) = \frac{1}{x}F_{2,LO}^{\gamma} + \frac{\alpha_S(Q^2)}{2\pi}q_{NS}(x)\otimes C_q^{(1)}(x) + \frac{\alpha_S(Q^2)}{2\pi}\Sigma(x)\otimes C_q^{(1)}(x) + \frac{\alpha_S(Q^2)}{2\pi}G(x)\otimes C_G^{(1)}(x) + \frac{\alpha}{2\pi}C_{\gamma}^{(0)}$$

Next-to-next to leading order

$$\frac{1}{x}F_{2,NNLO}^{\gamma}(x,Q^2) = \cdots + \frac{\alpha_S(Q^2)}{2\pi}\frac{\alpha}{2\pi}C_{\gamma}^{(1)}$$

Conventional formulation

Summary: the conventional approach is based on the assumptions:

- $F^{\gamma}_{2,LO}$ is related to $q(x,Q^2)$ in the same way as for hadrons
- $q_{LO}(x,Q^2)$ are solutions of evolution equations including $k_q^{(0)}$ and $P_q^{(0)}$ only

- This leads to mixing the terms of α and α_S order in QCD expansion

$$\frac{1}{x}F^{\gamma}_{2,LO}(x,Q^2) = q_{LO}(x,Q^2)$$

$$\frac{1}{x}F^{\gamma}_{2,NLO}(x,Q^2) = q + \frac{\alpha_S}{2\pi} q \otimes C_q^{(1)} + \frac{\alpha}{2\pi} C_{\gamma}^{(0)}$$

For example the **pure QED** quantity $C_\gamma^{(0)}$ is assigned to NLO order of QCD!

- Moreover, consistency with the factorization scale independence of F_2^γ requires that:

$$q^{PL}_{LO}(x,Q^2) = O(\alpha/\alpha_S)$$

!

Note, that provided M_0 is kept fixed when $\alpha_S \to 0$

$$q^{PL}_{NS}(x,M,M_0) \to \frac{\alpha}{2\pi} k^{(0)}_{NS}(x) \ln\frac{M^2}{M_0^2}$$

…pure QED term describing splitting $\gamma \to q\bar{q}$

Alternative formulation

Alternative approach is based on:

- Systematic separation of genuine QCD effects from those of pure QED origin, which leads to unambiguous definition of the concepts "leading" and "next-to-leading"
- Accepting the fact, that PL distribution functions of the photon are proportional to α

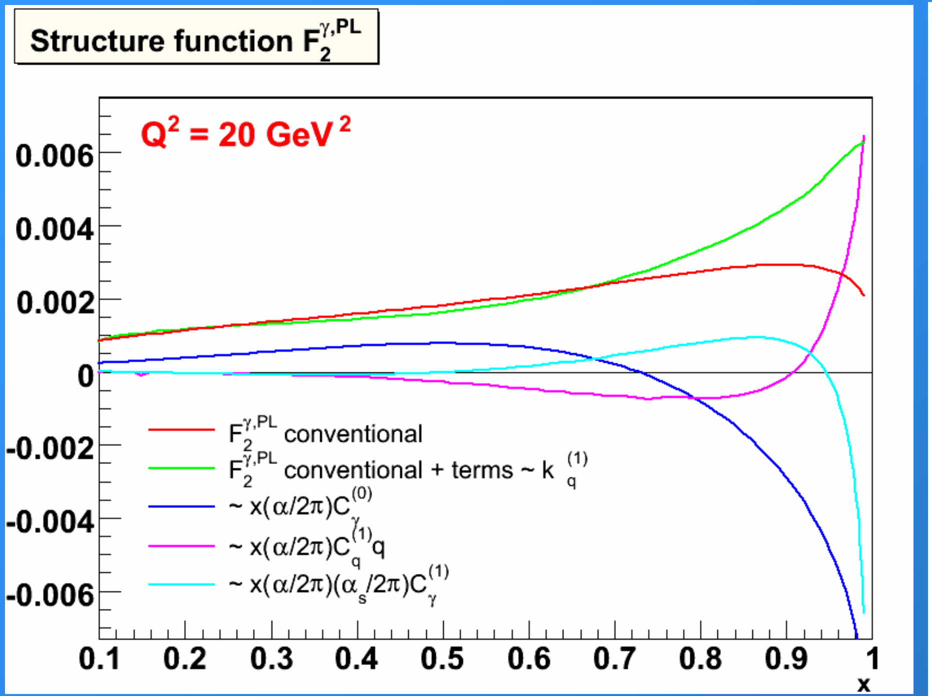
$$\frac{1}{x}F_{2,LO}^{\gamma,PL}(x,Q^2) = q_{NS}^{PL}(x,Q^2) + \frac{\alpha}{2\pi}C_{\gamma}^{(0)} + \frac{\alpha_S(Q^2)}{2\pi}q_{NS}^{PL}(x)\otimes C_q^{(1)}(x) + \frac{\alpha}{2\pi}\frac{\alpha_S(Q^2)}{2\pi}C_{\gamma}^{(1)}$$
$$+\langle e^2\rangle\left(q_{\Sigma}^{PL}(x,Q^2) + \frac{\alpha}{2\pi}C_{\gamma}^{(0)} + \frac{\alpha_S(Q^2)}{2\pi}q_{\Sigma}^{PL}(x)\otimes C_q^{(1)}(x) + \frac{\alpha}{2\pi}\frac{\alpha_S(Q^2)}{2\pi}C_{\gamma}^{(1)}\right)$$

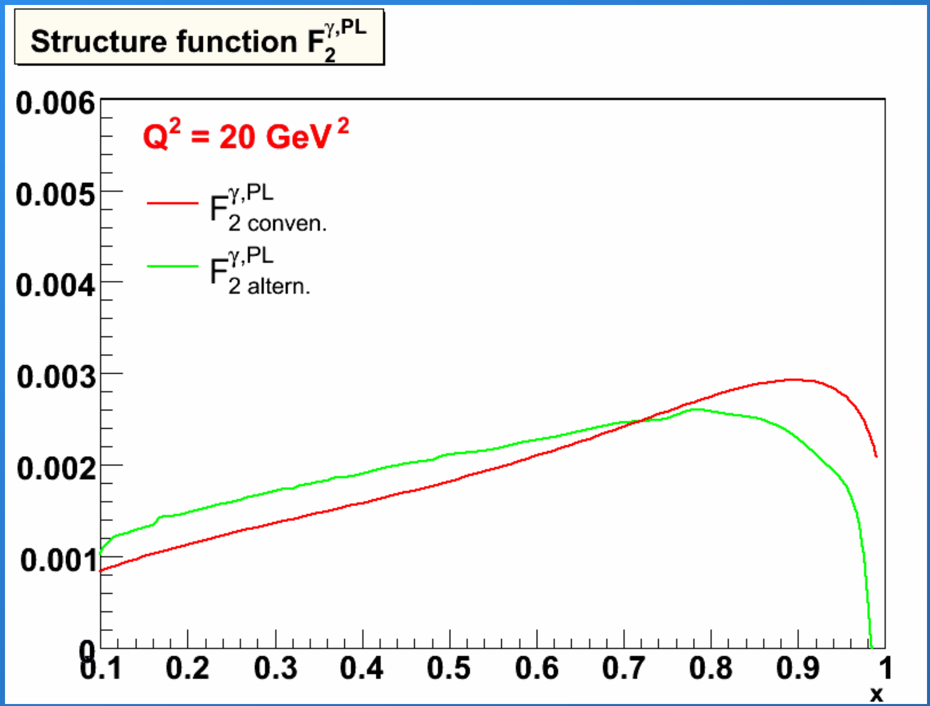
We can conclude that in LO the alternative approach differs from the conventional:

- by the presence of the photonic coefficient functions $C_{\gamma}^{(0)}$ and $C_{\gamma}^{(1)}$
- by the presence of the convolution of quark coefficient function $C_q^{(1)}$ with $q^{PL}(x,Q^2)$
- by the fact $k_q^{(1)}$ is included in the evolution equation for $q^{PL}(x,Q^2)$

Alternative formulation in LO

Pointlike structure function $F_2^{\gamma,PL}$

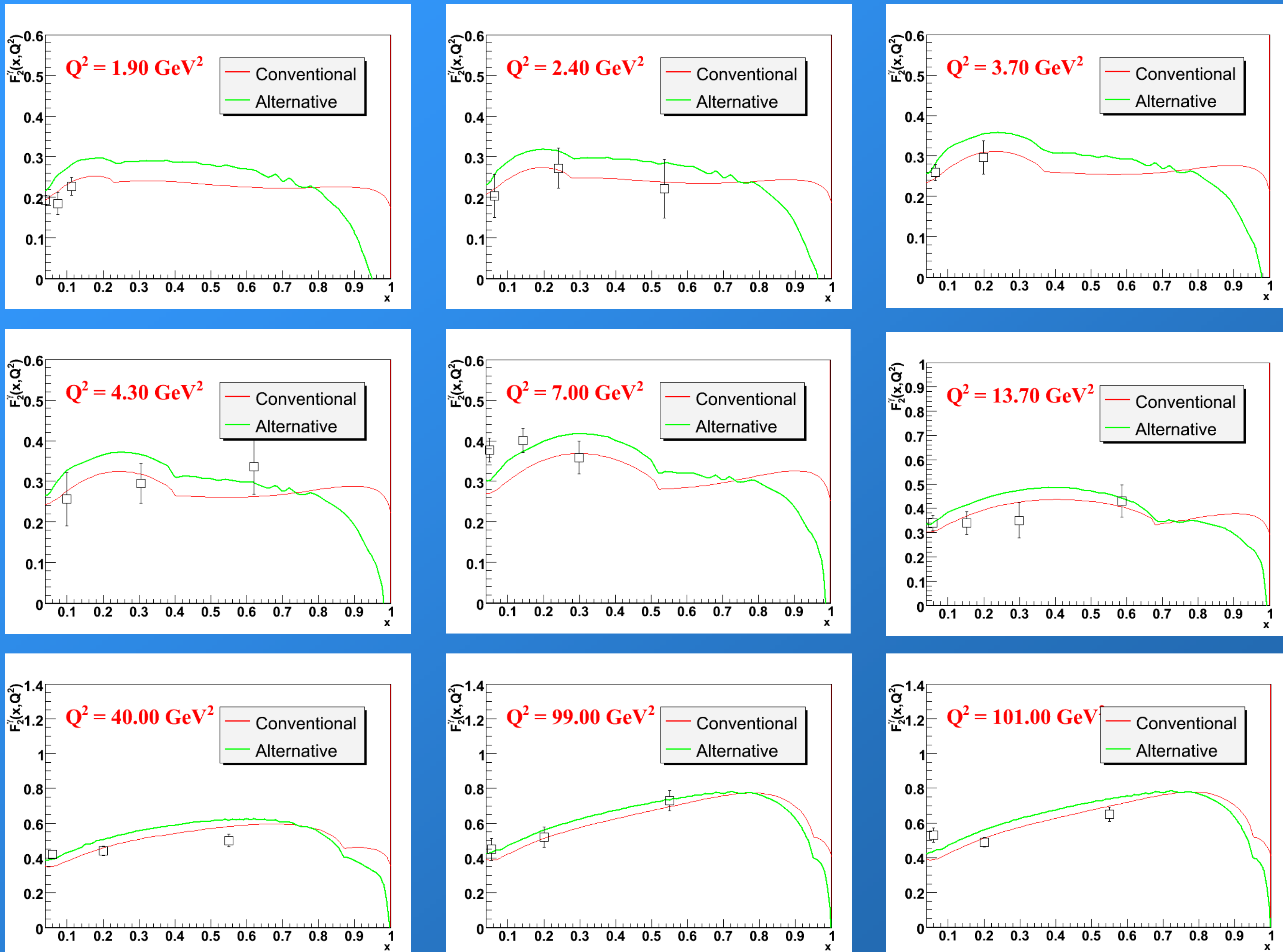




- The terms ~ $k_q^{(1)}$ represents an important positive contribution for x > 0.65
- $C_\gamma^{(0)}$ and $C_\gamma^{(1)}$ brings the numericicly important positive contribution up to x = 0.7
- In region x > 0.7 the negative contribution of $C_\gamma^{(0)}$ dominates.
- $C_q^{(1)}$, entering throught the convolution with q, compensates the negative contribution of $C_\gamma^{(1)}$

Numerical difference of conventional and alternative approach in comparision with experimental data errors

Comparison F_2^γ in conventional and alternative approach



Alternative formulation

	Conventioanal	Alternative
LO QCD	$k^{(0)}, P^{(0)}$ ✓	$+ C_\gamma^{(0)}, k^{(1)}, C_\gamma^{(1)}, C_q^{(1)}$ ✓
NLO QCD	$k^{(0)}, P^{(0)}, k^{(1)}, C_\gamma^{(0)}, P^{(1)}, C_q^{(1)}$ ✓	$+ C_\gamma^{(1)}, k^{(2)}, C_q^{(2)}, C_\gamma^{(2)}$ ✗

All quantities except $C_\gamma^{(2)}$ are known ➔

- both LO and NLO analysis of F_2^γ in conventional approach is possible to perform
- In alternative approach only LO of F_2^γ analysis is possible to perform

Global analysis of photon structure function F_2^γ in LO

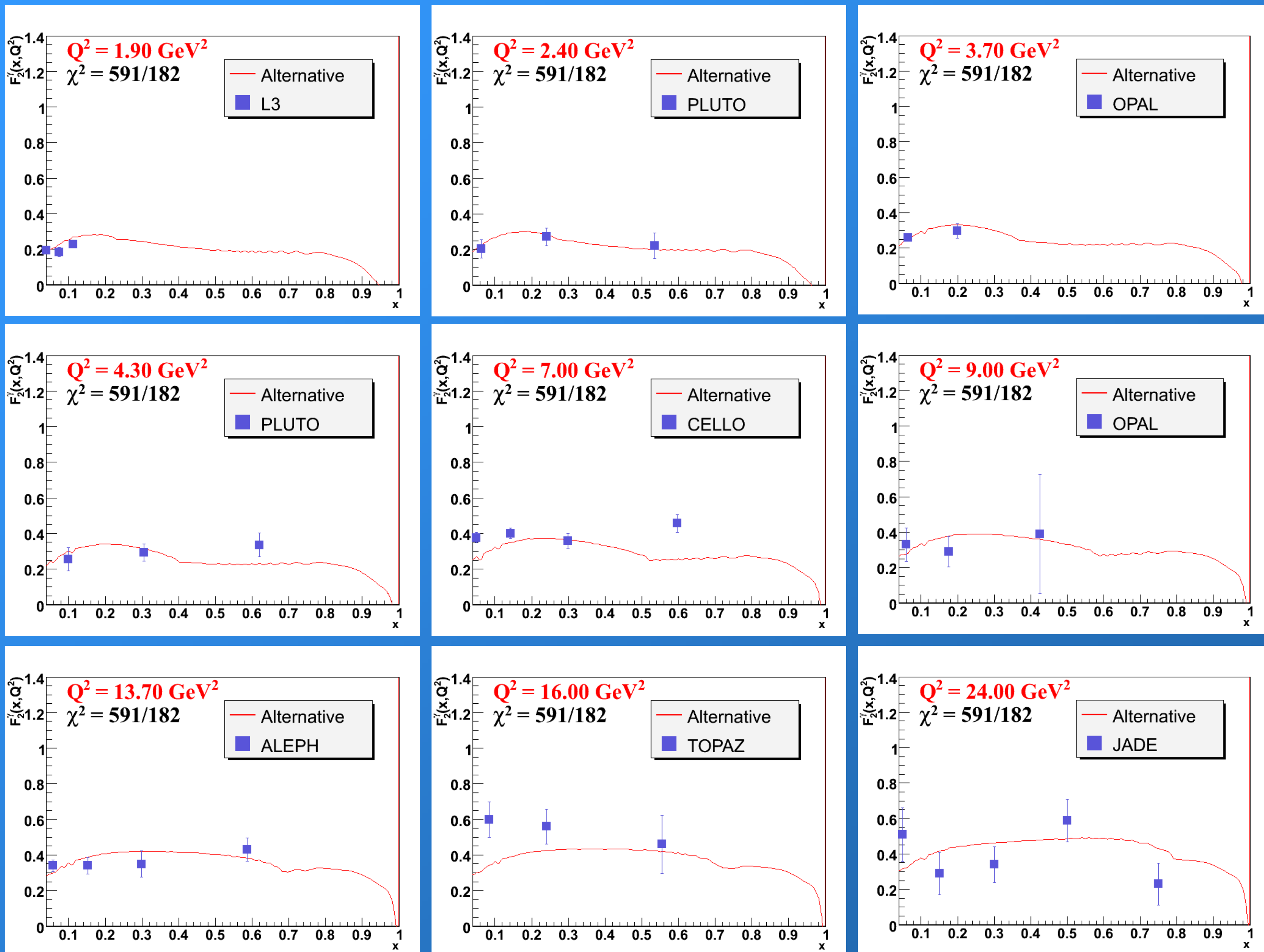
- follows $\text{FFNS}_{\text{CJKL}}$ model (3 active quarks, Λ=313 MeV)
- 182 data points used
- χ^2 = 357/182 reached

Parameterization:

$$f_{had}^{\gamma}(x,Q_0^2) = \kappa \frac{4\pi\alpha}{\hat{f}_\rho^2} f^\rho(x,Q_0^2,\alpha,\beta)$$

	χ^2	κ	α	β
conventional	357	1.726	0.465	0.127
alternative	work in progress			

Global analysis



Next-to-leading order – nonsinglet case

Coupling constant $\alpha_s(M)$ satisfies $$\frac{d\alpha_s(M)}{d\ln M} = -\beta_0 \frac{\alpha_s(M)}{4\pi} - \beta_1 \frac{\alpha_s^2(M)}{16\pi^2}$$

In conventional approach:

- qPL satisfy the evolution equation, where splitting functions $k^{(1)}$ and $P^{(1)}$ are included

$$\frac{dq_{NS}(x,M)}{d\ln M^2} = \frac{\alpha}{2\pi} k_q^{(0)} + \frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi} k_q^{(1)} + \left(\frac{\alpha_s}{2\pi} P_{NS}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{NS}^{(1)}\right) \otimes q_{NS}$$

In alternative approach:

- qPL satisfy the evolution equation, where, moreover, splitting functions $k^{(2)}$ is included

$$\frac{dq_{NS}(x,M)}{d\ln M^2} = \frac{\alpha}{2\pi} k_q^{(0)} + \frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi} k_q^{(1)} + \frac{\alpha}{2\pi}\left(\frac{\alpha_s}{2\pi}\right)^2 k_q^{(2)} + \left(\frac{\alpha_s}{2\pi} P_{NS}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{NS}^{(1)}\right) \otimes q_{NS}$$

These two approaches differs by

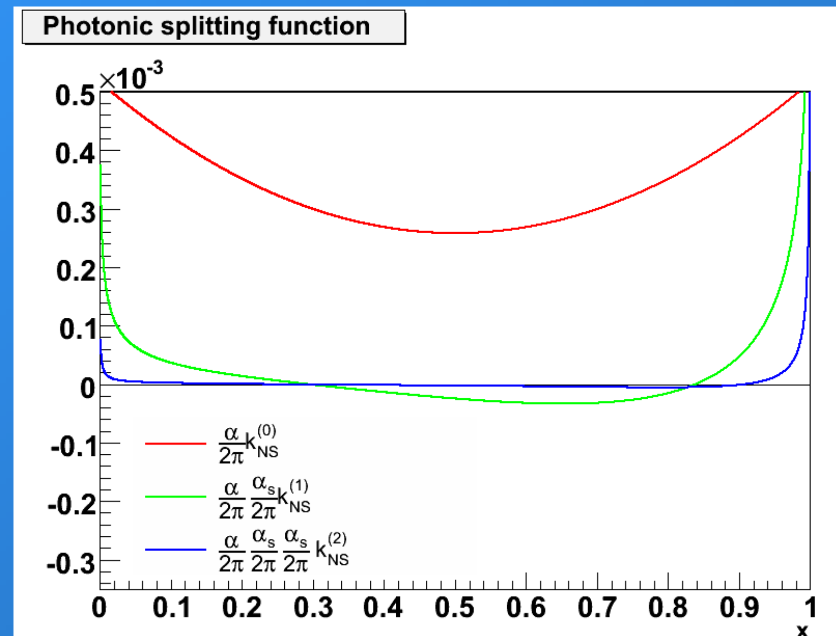
$$q_{NLO}^{PL,ALT}(n) - q_{NLO}^{PL,CON}(n) = \frac{\alpha_s(M)}{-1-\frac{2P^{(0)}(n)}{\beta_0}} \frac{\alpha k^{(2)}(n)}{2\pi^2\beta_0}\left(1-\left(\frac{\alpha_s(M)}{\alpha_s(M_0)}\right)^{-1-\frac{2P^{(0)}(n)}{\beta_0}}\right)$$

- this contribution is significantly smaller than the corresponding contribution in leading order ($\sim k^{(1)}$)

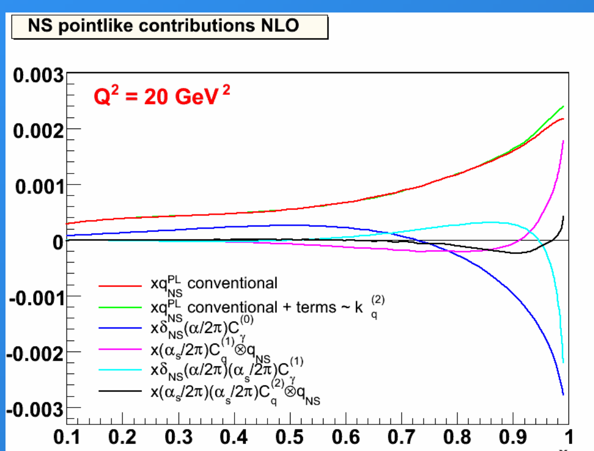
Next-to-leading order – nonsinglet case

Photonic splitting function:

$$k(x) = \frac{\alpha}{2\pi} k^{(0)}(x) + \frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi} k^{(1)}(x) + \frac{\alpha}{2\pi}\left(\frac{\alpha_s}{2\pi}\right)^2 k^{(2)}(x)$$

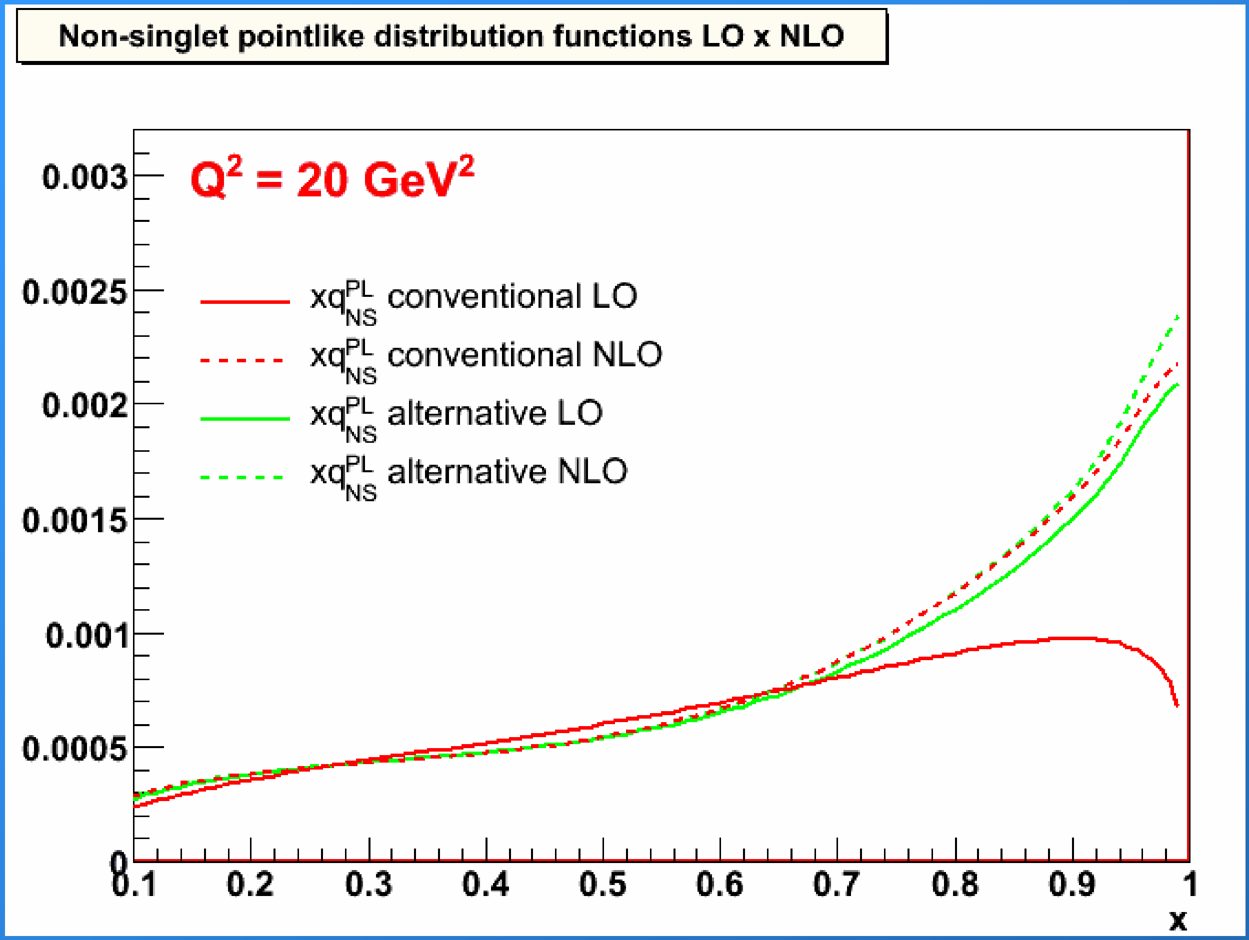


Including the terms ~ $k^{(2)}$ to the evolution equation for PL NS distribution function has only small numerical effect for its solution



- The terms ~ $k^{(2)}$ represents very small positive correction for x close to 1
- $C_\gamma^{(0)}$ and $C_\gamma^{(1)}$ brings the numericicly important positive contribution up to x = 0.7
- In region x > 0.7 the negative contribution of $C_\gamma^{(0)}$ dominates.
- $C_q^{(1)}$, entering throught the convolution with q, compensates the negative contribution of $C_\gamma^{(1)}$
- $C_q^{(2)}$ has only small numerical effect in region x close to 1

Next-to-leading order – nonsinglet case – pointlike part



- the difference between LO and NLO in conventional approach is much bigger than in the alternative one
- this difference is comparable with effect of including $k^{(1)}$ to standard LO → LO in alternative approach

! • PL distribution function in NLO in conventional approach is comparable with the PL distribution function in LO in alternative one

Conclusion

- The alternative approach to QCD analysis of F_2^γ was proposed

  In LO it differs

  - by including terms ~ $k^{(1)}$ to the evolution equations for photonic distribution functions
  - by including terms ~ $C_\gamma^{(0)}$, $C_\gamma^{(1)}$ and $C_q^{(1)}$ to the formula for F_2^γ

  In NLO it differs

  - by including terms ~ $k^{(2)}$ to the evolution equations for photonic distribution functions
  - by including terms ~ $C_\gamma^{(2)}$ and $C_q^{(2)}$ to the formula for F_2^γ

- The numerical difference with the conventional approach was shown

  In LO in the alternative approach

  - the pointlike distribution functions are significantly bigger and they are numerically comparable with the pointlike distribution functions in NLO in conventional approach
  - numerical difference of F_2^γ is significant in comparison to the errors of experimental data

  In NLO in the alternative approach

  - the pointlike distribution functions are numerically comparable to those of conventional approach
  - in order to compare F_2^γ, the photonic coefficient function $C_\gamma^{(2)}$ is need

- Global analysis of the structure function F_2^γ in alternative approach in LO was performed

  In $FFNS_{CJKL}$ model the quality of the fit is unsatisfactory ➡ work in progress