Conformal Bootstrap with Four Supercharges

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with Sheer El-Showk, Dalimil Mazáč, Miguel F. Paulos

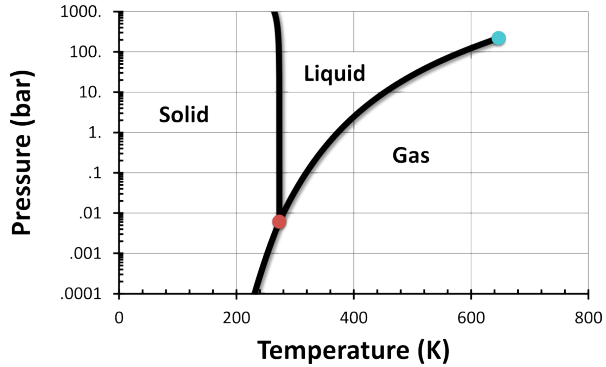
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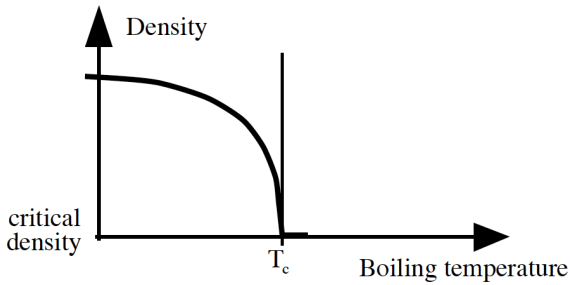
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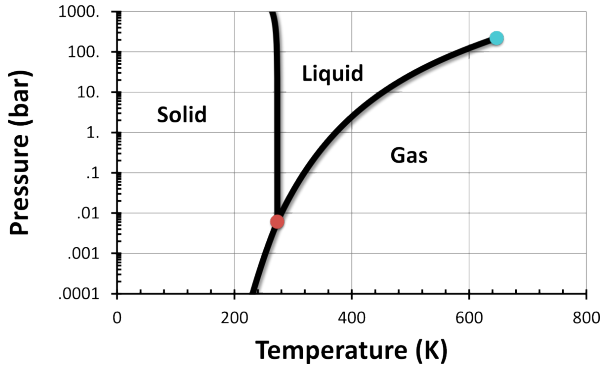


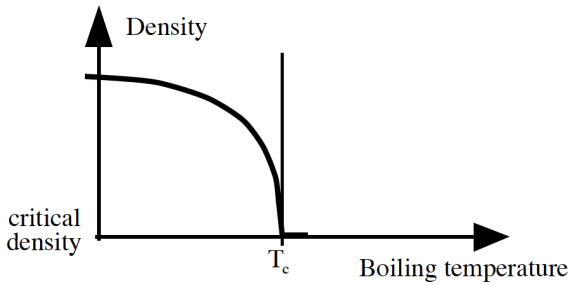


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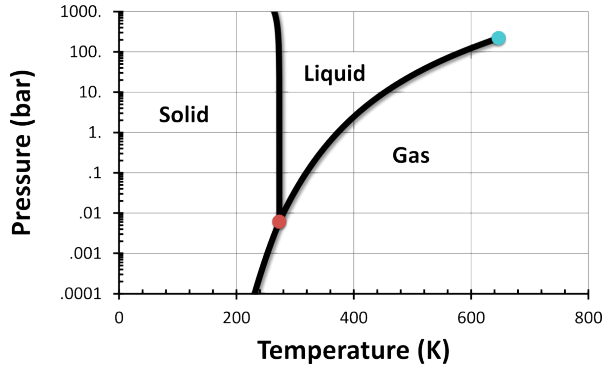
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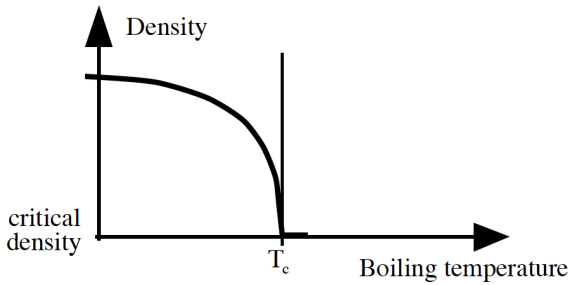
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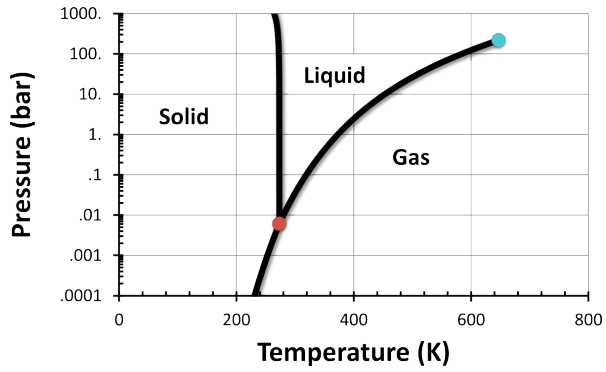
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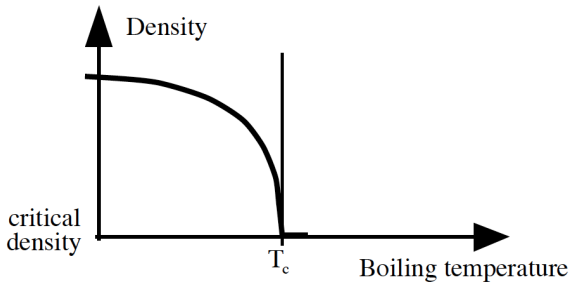
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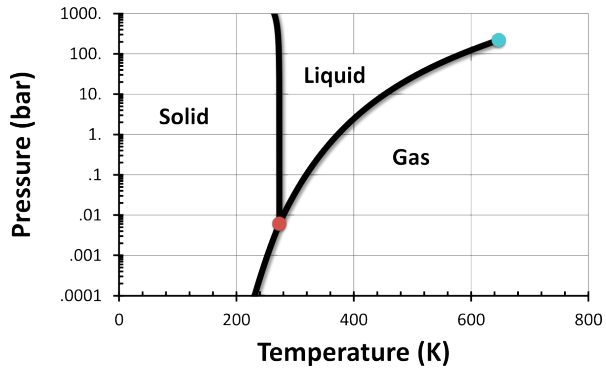
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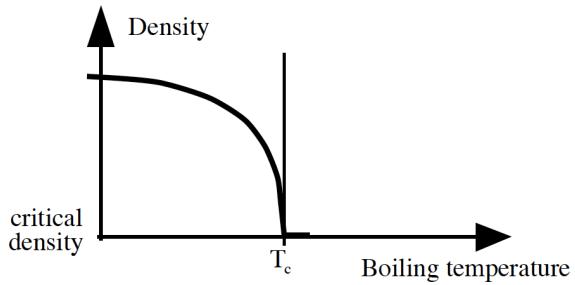
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Goal: Find β and ν using CFT

Motivation

Why Conformal Field Theory?

- Describe critical points in statistical mechanics.
- Quantum critical points in condensed matter systems.
- Universality - the same CFT describes multiple physical systems.
- Start/end points of Renormalization Group flows.
- Quantum gravity in AdS.

Motivation

Why supersymmetric CFTs?

- Useful calculable toy models.
- Teach us about dualities in field theory - Seiberg duality, S-duality, ...
- Relevance to string theory.
- Relevance to mathematics.
- Realized in Nature! Example: the 2d tri-critical Ising model ($c = 7/10$).

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All higher-point functions can be obtained from these by using the OPE [Wilson]

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Disclaimers:

1. In $d = 2$ I use only the finite part of the Virasoro algebra, i.e. $SO(2,2)$.
2. I discuss only local operators.

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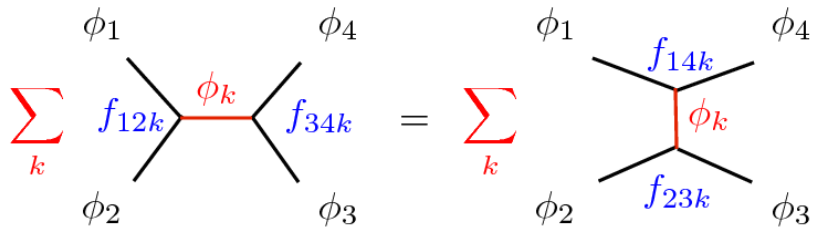
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can be expanded in inequivalent channels.



Here I use the standard conformal invariant cross ratios

$$u \equiv \frac{x_{12}^2 \, x_{34}^2}{x_{13}^2 \, x_{24}^2} = z\bar{z} \, , \qquad v \equiv \frac{x_{14}^2 \, x_{23}^2}{x_{13}^2 \, x_{24}^2} = (1-z)(1-\bar{z}) \, .$$

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Hard to extend this to $d>2$ since the conformal group/algebra $SO(d,2)$ is finite.

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The conformal bootstrap is an old idea [Polyakov], [Ferrara-Gatto-Grillo], ...

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Previous work:

- 6d $\mathcal{N} = (2,0)$ [Beem-Rastelli-van Rees]
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- 4d $\mathcal{N} = 1$ [Polland-Simmons-Duffin], [Vichi], [Berkooz-Yacoby-Zait]
- 3d $\mathcal{N} = 8$ [Chester-Lee-Pufu-Yacoby]
- 3d $\mathcal{N} = 1$ [Bashkirov]

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Our goal: Implement the bootstrap for CFTs with 4+4 supercharges in $d \leq 4$, i.e. 4d $\mathcal{N} = 1$, 3d $\mathcal{N} = 2$, and 2d $\mathcal{N} = (2,2)$ CFTs.

Plan

- Motivation and overview ✓

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Superconformal algebras with 4 supercharges

Superconformal algebra

The generators are

$$D\ ,\quad P_i\ ,\quad K_i\ ,\quad M_{ij}\ ,\quad R\ ,\qquad i,j=1,\ldots,d$$
$$Q^+_\alpha\ ,\quad Q^-_{\dot\alpha}\ ,\quad S^{\alpha-}\ ,\quad S^{\dot\alpha+}\ ,\qquad \alpha,\dot\alpha=1,2\ .$$

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The structure of the non-trivial (anti)commutation relations is

$$[M_{ij},X]\sim X\ ,\qquad [D,Y]\sim Y\ ,$$
$$\{Q,Q\}\sim P\ ,\qquad \{S,S\}\sim K\ ,\qquad [P,S]\sim Q\ ,\qquad [K,Q]\sim S\ .$$

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Most importantly

$$\{S^{\alpha-},Q^+_\beta\}=\delta^\alpha{}_\beta\left(\mathrm{i}D-\frac{d-1}{2}R\right)+(m_{ij})_\beta{}^\alpha M_{ij}\,,$$
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The conjugation rules are (in Euclidean signature)

$$D^\dagger=-D\,,\qquad R^\dagger=R\,,\qquad M_{ij}^\dagger=M_{ij}\,,\qquad P_i^\dagger=K_i\,,$$
$$S^{\alpha-}=(Q^+_\alpha)^\dagger\,,\qquad S^{\dot\alpha+}=(Q^-_{\dot\alpha})^\dagger\,.$$

NB: One can formulate a superconformal algebra with 4+4 supercharges for general space dimension $d\leq 4$.

Superconformal algebra

Upshot:

Superconformal algebra

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- We found a way to define all "matrices" in the commutation relations such that the super-Jacobi identities are satisfied while keeping d a free parameter.
- When $d = 1, 2, 3, 4$ one finds the usual superconformal algebras in the respective dimension.
- Superconformal unitarity bounds as a function of d. They are stronger than the usual conformal unitarity bounds. For scalars the bound is

$$\Delta = \frac{d-1}{2}|q| \,, \qquad\qquad \Delta \geq \frac{d-1}{2}|q| + d - 2 \,.$$

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If $[K_i, \mathcal{O}] = [S, \mathcal{O}] = 0 \rightarrow$ superconformal primary. Operators saturating the bound obey in addition $[Q^{\pm}, \mathcal{O}] = 0$ and are called superconformal (anti-)chiral primary operators. They form short superconformal multiplets and their dimension is protected, i.e. not renormalized.

Superconformal blocks and bootstrap equations

Superconformal blocks

An elegant method for deriving conformal block is via the Casimir operator of the conformal algebra [Dolan-Osborn]. Apply this to our superconformal algebra

$$C = -D^2 - \frac{1}{2}(P_i K_i + K_i P_i) + \frac{1}{2} M_{ij} M_{ij} \\ - \frac{d-1}{4} R^2 + \frac{1}{2} \left(\left[S^{\dot{\alpha}+}, Q^-_{\dot{\alpha}} \right] + \left[S^{\alpha-}, Q^+_{\alpha} \right] \right) .$$

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Convert the Casimir operator into a differential operator when acting on

$$\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle = \frac{|x_{24}|^{\Delta_{12}} |x_{14}|^{\Delta_{34}-\Delta_{12}}}{|x_{12}|^{\Delta_1+\Delta_2} |x_{34}|^{\Delta_3+\Delta_4} |x_{13}|^{\Delta_{34}}} \sum_k C_{12k} C_{34k} \, \mathcal{G}^{\Delta_{12},\Delta_{34}}_{\Delta_k, s_k}(u, v) ,$$

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The result is a differential equation for the superconformal block $\mathcal{G}$

$$\mathcal{D}\mathcal{G}^{\Delta_{12},\Delta_{34}}_{\Delta,s}(u, v) = \lambda_C \mathcal{G}^{\Delta_{12},\Delta_{34}}_{\Delta,s}(u, v) ,$$

with

$$\lambda_C = \Delta(\Delta - d + 2) + s(s + d - 2) - \frac{d-1}{4} q^2 .$$

Superconformal blocks

The solution of the differential equation is

$$\mathcal{G}^{\Delta_{12},\Delta_{34}}_{\Delta,s}(u,v) = u^{-1/2} G^{\Delta_{12}-1,\Delta_{34}-1}_{\Delta+1,s}(u,v)\,,$$

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One can also explicitly derive these superblocks via OPE methods in $d = 2, 3, 4$.

For $d = 2, 4$ our superblocks reduce to previous results for $\Delta_{ij} = 0$

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In the $\Phi \times \Phi$ channel there is only one conformal primary (and its descendants) from a given superconformal multiplet contributing. Thus $\mathcal{G} = G$. The only way supersymmetry plays a role is through restricting the operators which can appear in the $\Phi \times \Phi$ OPE.

Bootstrap equations

$$\sum_{\mathcal{O}^+} |c^{\mathcal{O}^+}_{\Phi\bar{\Phi}}|^2 \begin{pmatrix} \mathcal{F}^{\Delta_\Phi}_{\Delta,s} \\ \tilde{\mathcal{F}}^{\Delta_\Phi}_{\Delta,s} \\ \tilde{\mathcal{H}}^{\Delta_\Phi}_{\Delta,s} \end{pmatrix} + \sum_{\mathcal{O}^-} |c^{\mathcal{O}^-}_{\Phi\bar{\Phi}}|^2 \begin{pmatrix} \mathcal{F}^{\Delta_\Phi}_{\Delta,s} \\ \tilde{\mathcal{F}}^{\Delta_\Phi}_{\Delta,s} \\ \tilde{\mathcal{H}}^{\Delta_\Phi}_{\Delta,s} \end{pmatrix} = -\sum_{\mathcal{P}} |c^{\mathcal{P}}_{\Phi\Phi}|^2 \begin{pmatrix} 0 \\ F^{\Delta_\Phi}_{\Delta,s} \\ -H^{\Delta_\Phi}_{\Delta,s} \end{pmatrix}$$

$\mathcal{O}^\pm$ are superconformal primaries with $q_{\mathcal{O}^\pm} = 0$ and even/odd spin.

$\mathcal{P}$ are conformal primaries with $q_{\mathcal{P}} = 2q_\Phi = \frac{4}{d-1}\Delta_\Phi$ and $[Q^+_\alpha, \mathcal{P}] = 0$.

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These constraints can be summarized as

$$\begin{aligned}
\mathcal{O}^+ &: \quad \Delta = 0\,,\ \Delta \geq s + d - 2\,, && s = 0, 2, 4, \dots\,, \\
\mathcal{O}^- &: \quad \Delta \geq s + d - 2\,, && s = 1, 3, 5, \dots\,, \\
\mathcal{P} &: \begin{cases} \Delta = 2\Delta_\Phi + s\,, & s = 0, 2, \dots\,, \\ \Delta = d - 2\Delta_\Phi\,, & s = 0, \quad \Delta_\Phi \leq \frac{d}{4}\,, \\ \Delta \geq |2\Delta_\Phi - (d-1)| + s + (d-1)\,, & s = 0, 2, \dots\,. \end{cases}
\end{aligned}$$

All these operators obey the superconformal unitarity bounds.

Important to include the operator $\mathcal{P} = Q^2\Psi$ with dimension $\Delta_\mathcal{P} = d - 2\Delta_\Phi$. Ψ is anti-chiral with dimension $\Delta_\Psi = d - 1 - 2\Delta_\Phi \rightarrow$ unitarity implies $\Delta_\Phi \leq \frac{d}{4}$.

Solving the equations

Basic idea: Fix the dimension of the lightest chiral primary scalar in the theory. Assume that there is a gap in the spectrum and vary the dimension of the operator with lowest dimension above the gap until the crossing equation cannot be solved. This spectrum is then excluded. [Rattazi-Rychkov-Tonni-Vichi]

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Consider the space, $\mathcal{M}$, of N-th order differential operators of the type $\Lambda = \sum_{m,n}^{N} \alpha_{m,n} \partial_u^m \partial_v^n$.

If one finds a convex subspace, $\mathcal{M}_c$, such that $\Lambda[\, T_{\Delta_k, s_k}(u, v)] > 0$ then the spectrum $\{\Delta_k, s_k\}$ should be excluded since the crossing equation cannot be solved.

There are efficient numerical algorithms for searching for such convex subspaces. [Paulos], [Simmons-Duffin]

At the boundary between the allowed and disallowed region there is a unique solution of crossing. [El-Showk-Paulos]

The critical Wess-Zumino model in $d \leq 4$

The WZ model

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The Lagrangian in components is

$$\mathcal{L}_{\text{WZ}} = \partial_\mu\bar{\phi}\partial^\mu\phi + \mathrm{i}\bar{\psi}\gamma^\mu\partial_\mu\psi + |\lambda|^2|\phi|^4 + (\lambda\,\phi\psi_\alpha\epsilon^{\alpha\beta}\psi_\beta + c.c.)\,.$$

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We know little about the unprotected spectrum of this theory and its OPE coefficients in $d \neq 2, 4$.

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The ϵ-expansion ($d = 4 - \epsilon$) leads to [Thomas], [S.S. Lee]

$$\Delta_\Phi = \frac{d-2}{2} + \frac{\epsilon}{6}\,, \quad \rightarrow \quad \Delta_\Phi|_{d=3} = \frac{2}{3}\,,$$
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Consider the coefficients of the 2-pt function of the R-current, τ_{RR}, and the EM tensor, C_T. Superconformal symmetry leads to a Ward identity $\tau_{RR} \sim C_T$.

Localization leads to [Closset-Dumitrescu-Festuccia-Komargodski]

$$\tau_{RR} = \frac{2}{\pi^2}\mathrm{Re}\,\frac{\partial^2 F(b)}{\partial b^2}\bigg|_{b=1}\,,$$

This can be used to find [Nishioka-Yonekura]

$$\frac{C_T}{C_T^{(\mathrm{free})}} = \frac{\tau_{RR}}{\tau_{RR}^{(\mathrm{free})}} \simeq 0.7267845\,.$$

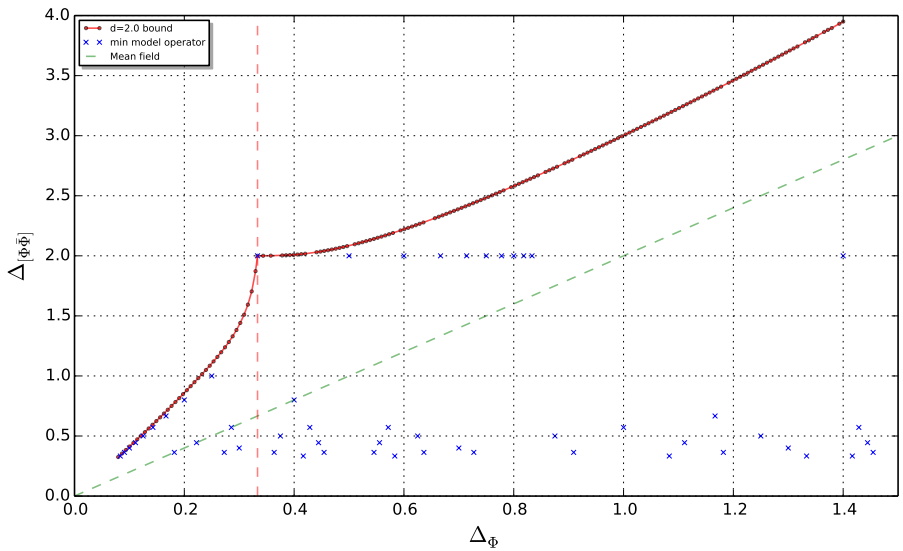
Numerical results

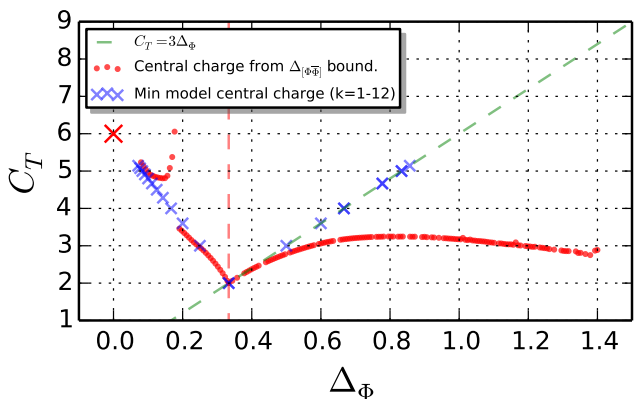
$\mathcal{N}=2$ minimal models

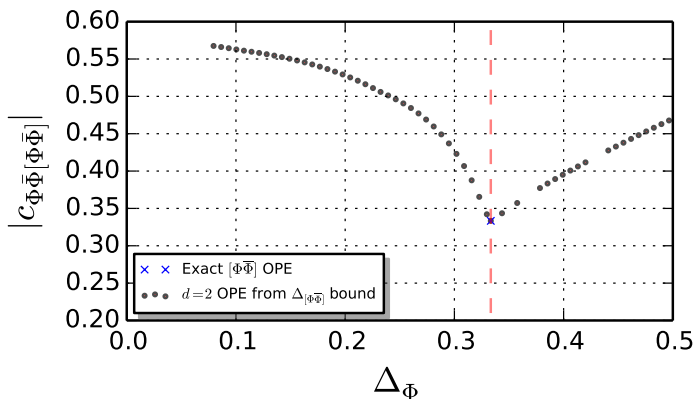
2d $\mathcal{N}=2$ minimal models have $c=\frac{3k}{k+2}$ for $k=1,2,3\ldots$.

$$h_{n,m}=\frac{n\left(n+2\right)-m^2}{4\left(k+2\right)}\,,\qquad \Omega=\frac{m}{\left(k+2\right)}\,.\quad n=0,\ldots,k\,,m=-n,-n+2,\ldots,n$$

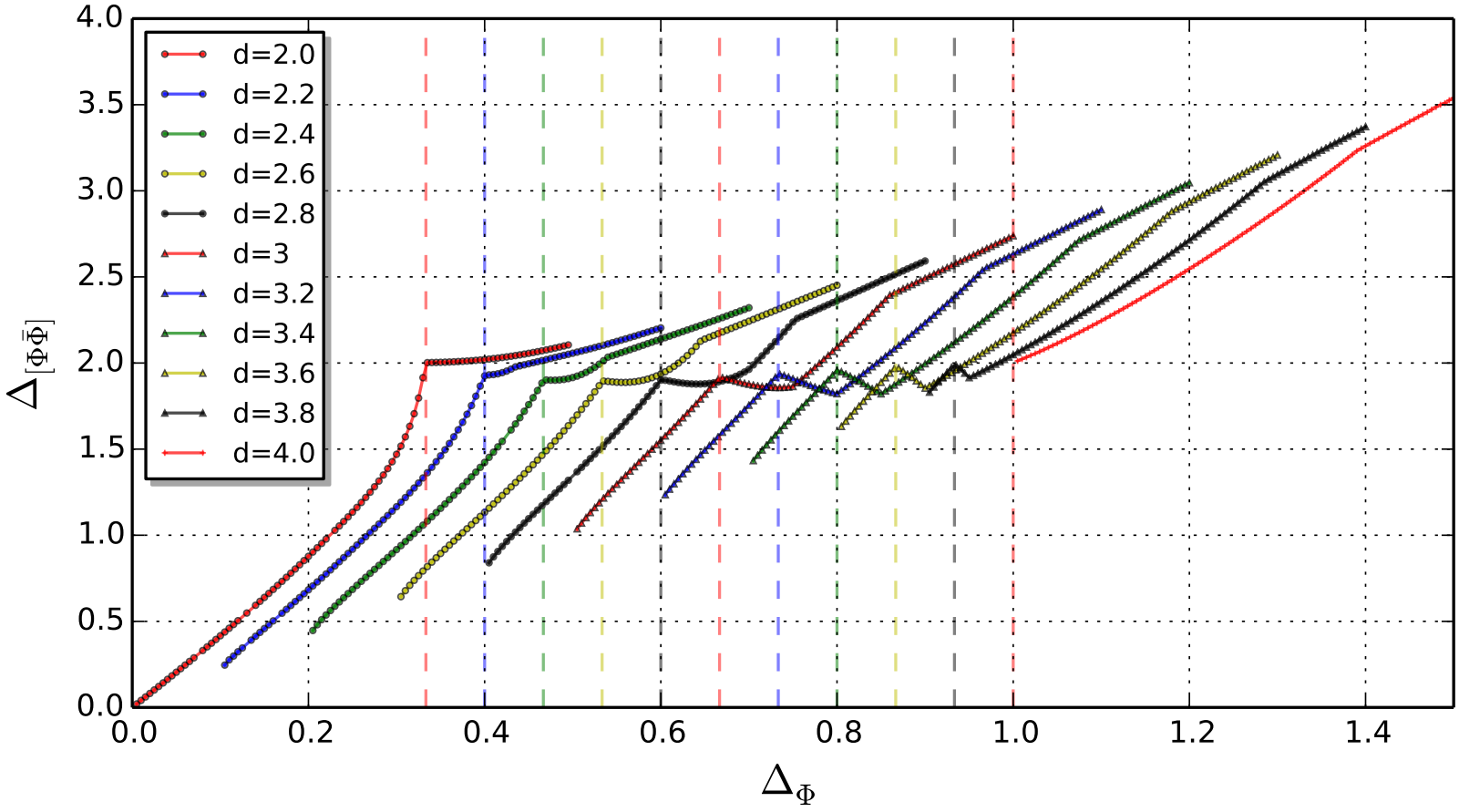
The chiral (antichiral) primaries have $m=\pm n$, respectively.





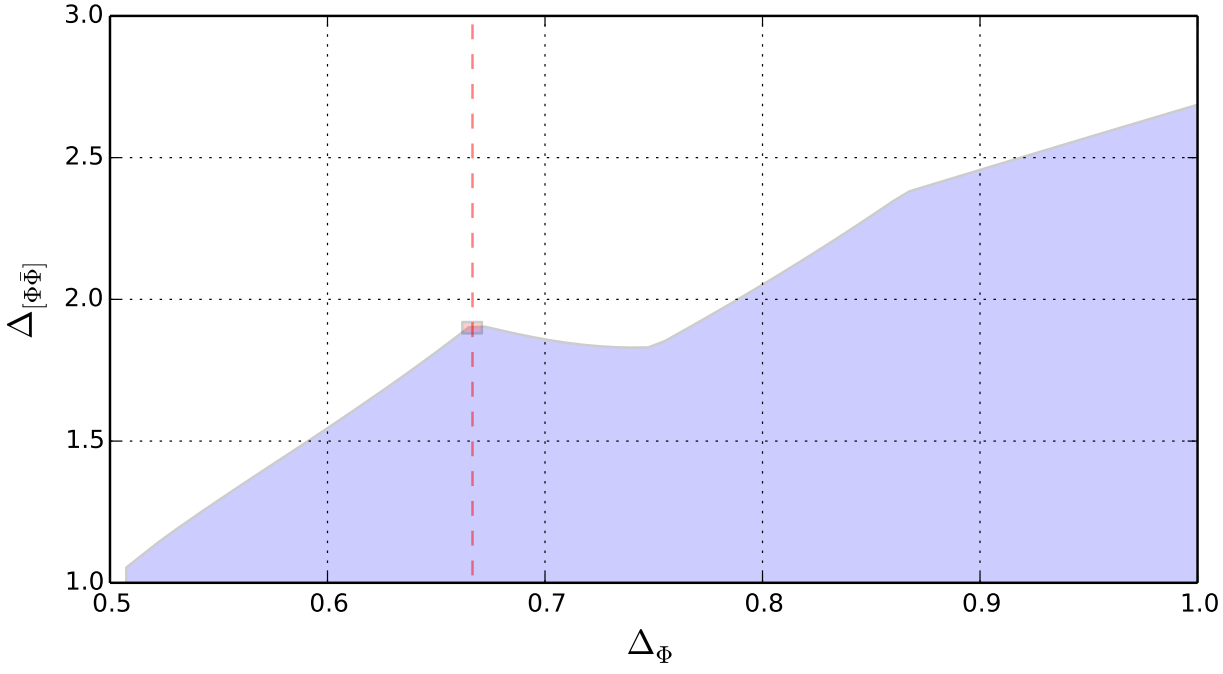


Bounds for $2 \leq d \leq 4$



Bound on the dimension of the first scalar superconformal primary, $[\Phi\bar{\Phi}]$, in the $\Phi \times \bar{\Phi}$ OPE. The range is $\frac{d-2}{2} \leq \Delta_\Phi \leq \frac{d-1}{2}$.

Critical WZ model in $d = 3$

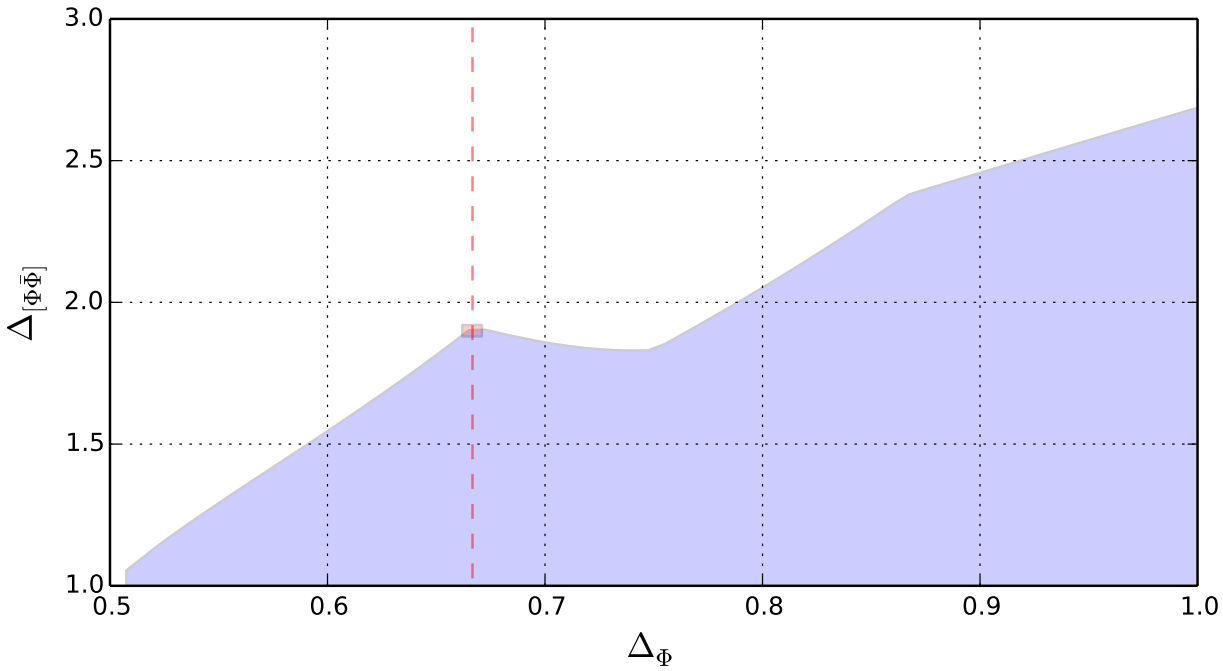


Precision spectroscopy of the critical WZ model in 3d.

$$\Delta_\Phi = \frac{1}{2} + \frac{\eta}{2} = 2/3\,, \qquad \Delta_{[\Phi\bar{\Phi}]} = 3 - \frac{1}{\nu} \simeq 1.9098(20)\,,$$

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The "central charge" is

$$\frac{C_T}{C_T^{(\text{free})}} \simeq 0.7265(3)\,.$$

To be compared with 0.7267845 from localization.

Summary

- Derivation of superconformal blocks for theories with 4+4 supercharges in $d \leq 4$.
- Universal bounds on the spectrum of unprotected operators in such theories.
- The critical WZ model sits at a kink on the bounding curve. The most accurate estimates to date of the unprotected spectrum of this theory.
- Two other kinks that remain mysterious.

Outlook

Extensions of our work

- Corroborate the bootstrap results for the WZ model using the ϵ expansion. [work in progress with E. Lauria]
- Precision numerical spectroscopy of the WZ model. Include other crossing constraints.
- Do the other kinks correspond to physically interesting SCFTs?
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Future goals

- Large c limit and connection to holography.
- Can we find bootstrap constraints on the string landscape?
- Analytic handle on the bootstrap - large spin, large c, extended supersymmetry, approximate higher-spin symmetry, ...

MERCI!