Bound states in dark-matter phenomenology

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Long-range interactions
mediated by
a light or massless particle

Bound states

- **Iconoclastic-ism:**
  Most DM research has focused on contact interactions.
  Prototypical WIMP scenario: $m_{DM} \sim m_{mediators} \sim 100$ GeV.
  What happens when $m_{DM} >> m_{mediators}$?

- **Long-range interactions appear in a variety of DM theories:**
  - Self-interacting DM
  - Asymmetric DM
  - Dissipative DM
  - DM explanations of galactic positrons
  - DM explanations of IceCube PeV neutrinos

  } Hidden sector DM

  - WIMP DM with m_{DM} > few TeV ! [Hisano et al. 2002]

    - Minimal DM. [Cirelli et al.]
    - LHC implications for SUSY.
    - Direct/Indirect detection bounds.

- **Large logarithmic corrections:**

$$\delta\sigma/\sigma \sim \alpha \ln (m_{DM} / m_{mediator})$$

→ resummation techniques etc. [In WIMP pheno: Baumgart et al. (2014)]

- **Non-perturbative effects:**

Sommerfeld enhancement in the non-relativistic regime.

Usually invoked for DM annihilation into radiation, but in fact affects *all* processes with same initial state.

- **More processes:**

Radiative formation of bound states [Sommerfeld enhanced]

Bound states

Phenomenological implications

- **Asymmetric DM → stable bound states**
  - Kinetic decoupling of DM from radiation, in the early universe
  - DM self-scattering in haloes (screening)
  - Indirect detection signals (radiative level transitions)
  - Direct detection signals (screening, inelastic scattering)

- **Symmetric DM → unstable bound states**
  formation + decay = extra annihilation channel
  - Relic abundance [von Harling, KP (2014); Ellis et al. (2015)]
  - Indirect detection

Bound states

Varieties

A. Confining theories

Hadronic bound states (“non-perturbative non-perturbative bound states”).

Cosmologically, they definitely form.
May leave a remnant weakly coupled long(-ish)-range interaction.

B. Weakly coupled theories

“Perturbative non-perturbative bound states”, e.g. atoms.

Formation efficiency depends on the details:

(i) bound-state formation cross-section, and

(ii) thermodynamic environment (early universe, DM haloes)

C. Theories with contact interactions

Solitonic bound states, e.g. Q-balls. [Coleman (1985)]
Exist in SUSY models. [Kusenko (1997)]

Highly ordered systems. Typically, very special conditions of formation (low entropy), e.g. Q-ball formation after inflation but before reheating, via fragmentation of Affleck-Dine condensate. [Kusenko, Shaposhnikov (1998)]

- **Particle physics: Cross-sections**
  - Cosmology / Astrophysics: DM becomes / is non-relativistic

    → Sommerfeld enhancement at low velocities
  - High-energy colliders: Incoming species are typically relativistic.

- **Thermodynamics: efficiency / impact of bound-state processes**

  Balance between bound-state formation and disassociation processes depends on thermodynamic environment.

Outline

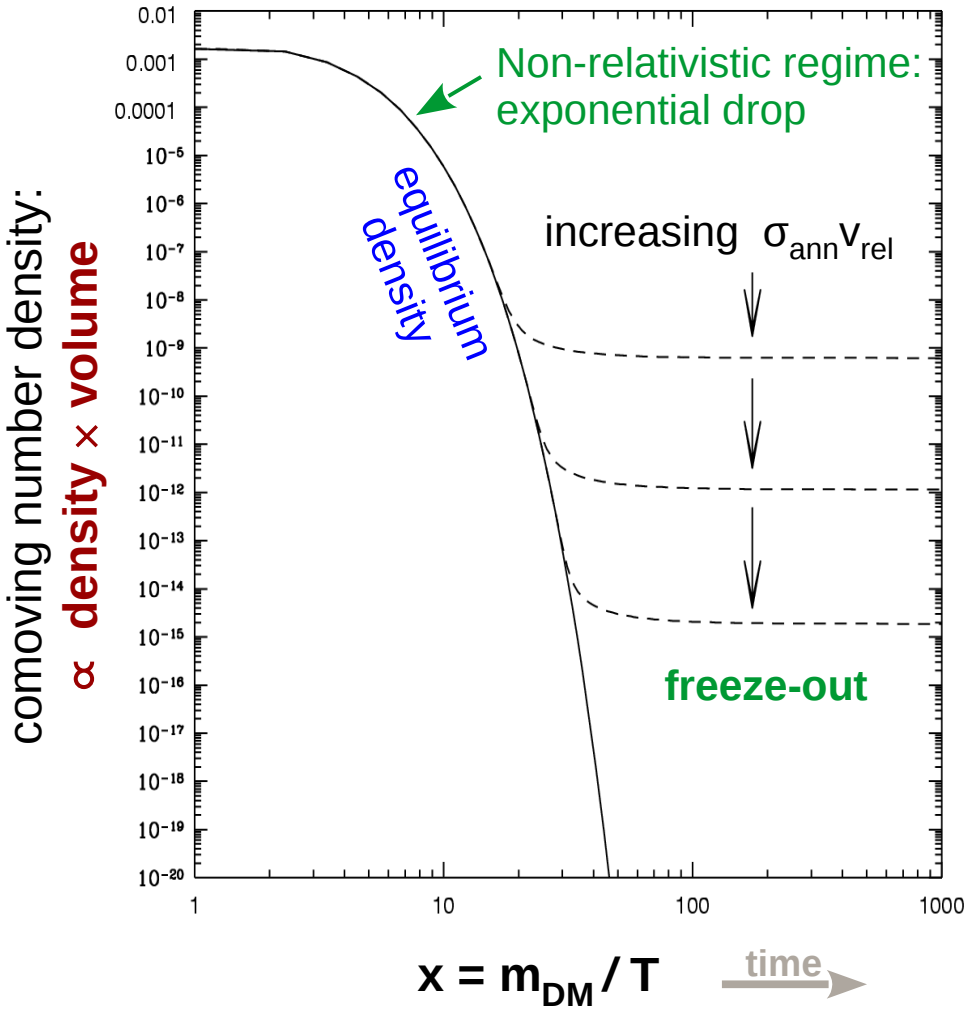
- [Pheno]
  Effect of bound-state formation and decay on freeze-out of symmetric thermal-relic DM. Indirect detection signals.
- [Formal]
  How (and why) to calculate bound-state formation cross-sections in weakly coupled theories, in a QFT framework.
- [Discussion]
  Relevance to WIMPs, and other DM theories.

Relic density of symmetric DM with contact interactions

- Early universe: Hot thermal bath of elementary particles, $n_X = n_X(T)$, kept in chemical equilibrium via annihilations, $X + \bar{X} \leftrightarrow f + \bar{f}$

- As universe expands and cools
  - → Density decreases
  - → Annihilations become inefficient
  - → Exponential decrease of $n_X(T)$ stalls: **freeze-out**
  - → Relic density

$$\Omega_X \simeq 0.26 \times \left[\frac{3 \times 10^{-26}\ cm^3/s}{\langle \sigma_{ann} v_{rel} \rangle} \right]$$



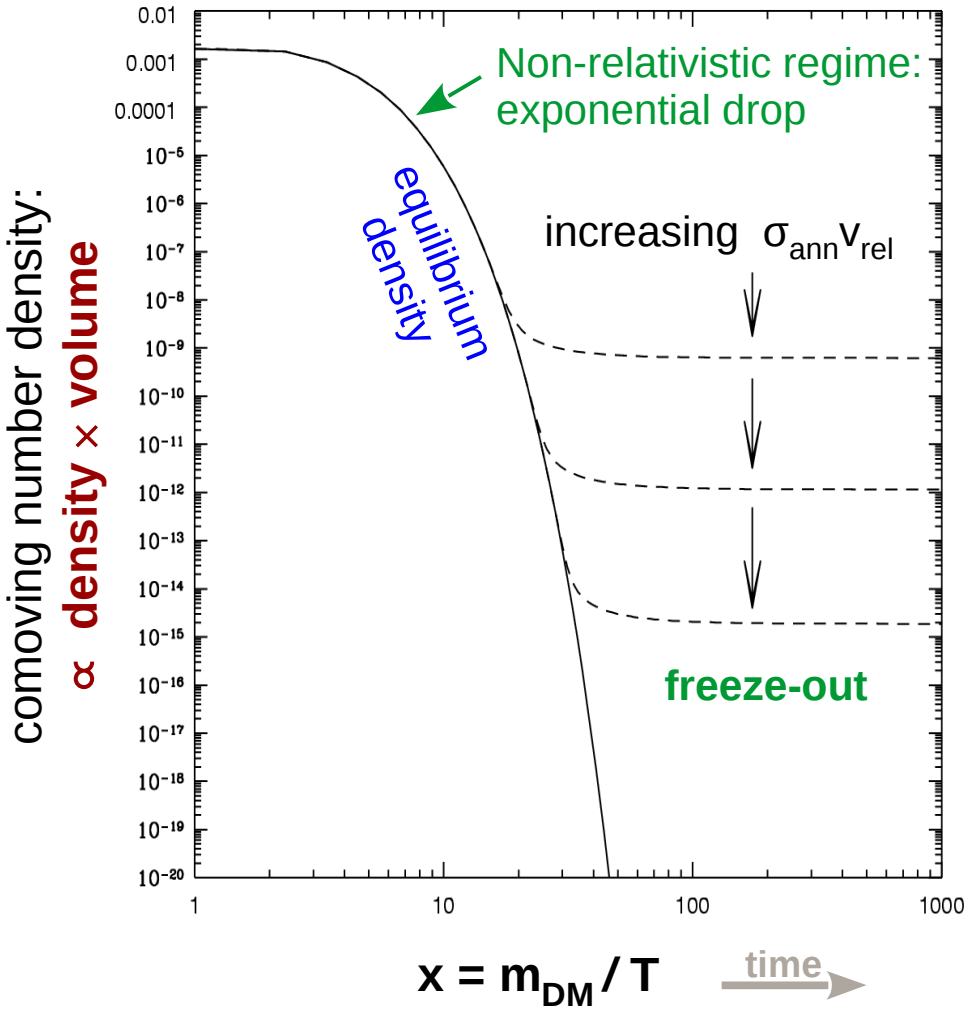
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Assumption:
$\sigma_{ann} v_{rel}$ **is velocity-independent.**
Valid for contact interactions.



Relic density of symmetric DM with *long-range* interactions

[von Harling, KP (2014)]

Toy model: **Dirac fermions (X, $\overline{\text{X}}$) of mass *m*,**
coupled to a massless dark photon γ, with dark fine-structure constant α.

Very important parameter: $\zeta = \alpha / v_{rel}$

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Annihilation

$$X + \bar{X} \rightarrow \gamma + \gamma$$

$$\sigma_{ann} v_{rel} = \sigma_0 S_{ann}(\zeta)$$

$$\sigma_0 = \pi \alpha^2 / m^2$$

$$S_{ann}(\zeta) = \frac{2\pi\zeta}{1 - e^{-2\pi\zeta}}$$

$$S_{ann}(\zeta \ll 1) \simeq 1$$

$$S_{ann}(\zeta \gtrsim 1) \simeq 2\pi\zeta$$

[Feng, Kaplinghat, Yu (2010)]

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Processes

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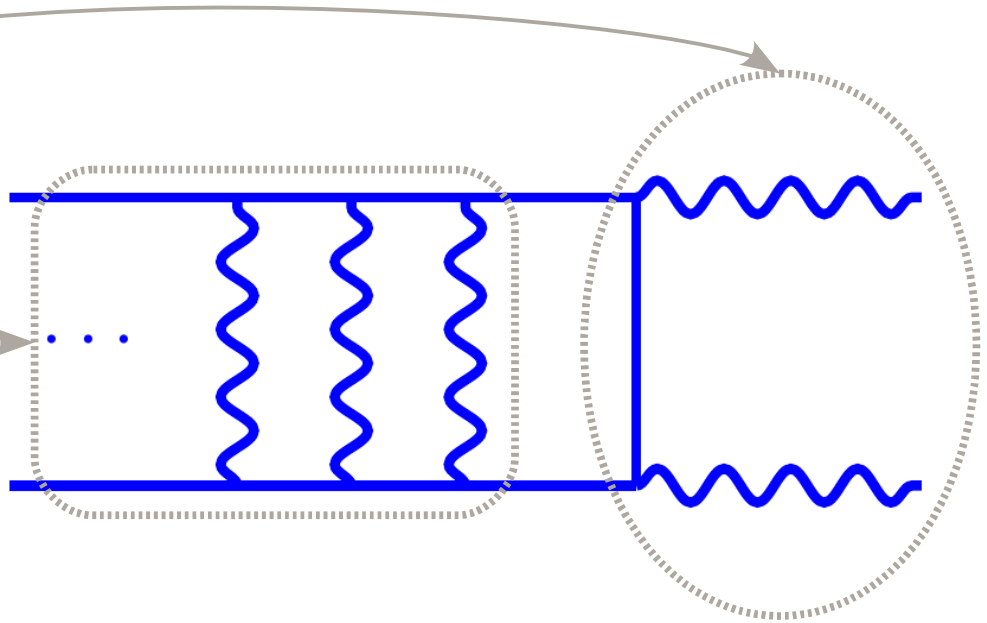
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Bound state formation αnd decay

$$X + \bar{X} \rightarrow (X\bar{X})_{bound} + \gamma$$

$$(X\bar{X})_{bound} \rightarrow 2\gamma \quad or \quad 3\gamma$$

$$\sigma_{BSF} v_{rel} = \sigma_0 S_{BSF}(\zeta)$$

$$\sigma_0 = \pi \alpha^2 / m^2$$

$$S_{BSF}(\zeta) = \left[\frac{2^9}{3 e^{4\zeta \operatorname{arccot}(\zeta)}} \frac{\zeta^4}{(1+\zeta^2)^2} \right] \frac{2\pi\zeta}{1 - e^{-2\pi\zeta}}$$

$$S_{BSF}(\zeta \ll 1) \simeq \frac{2^9 \zeta^4}{3} \ll 1$$

$$S_{BSF}(\zeta \gtrsim 1) \simeq \frac{2^9}{3 e^4} \times 2\pi\zeta \simeq \mathbf{3.13 \times S_{ann}}$$

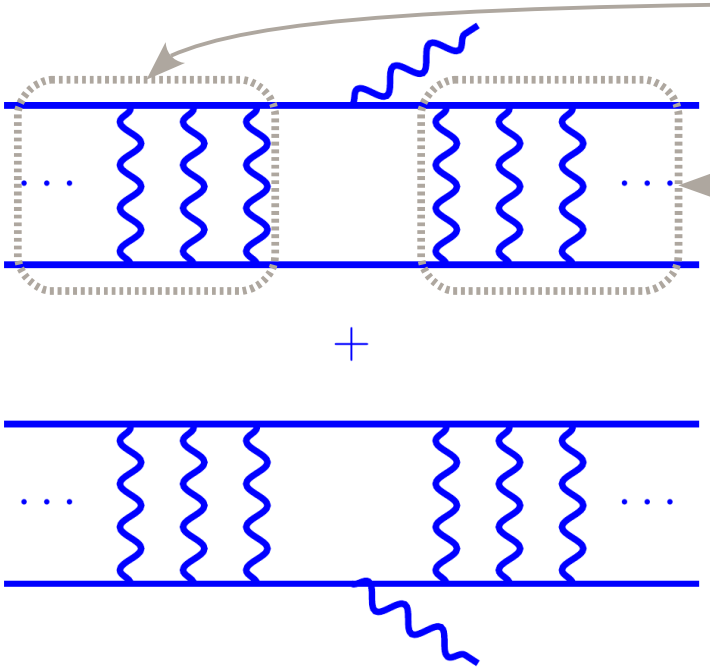
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[von Harling, KP (2014)]

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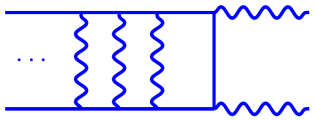
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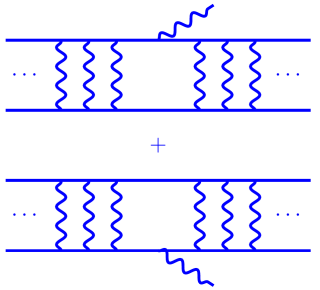
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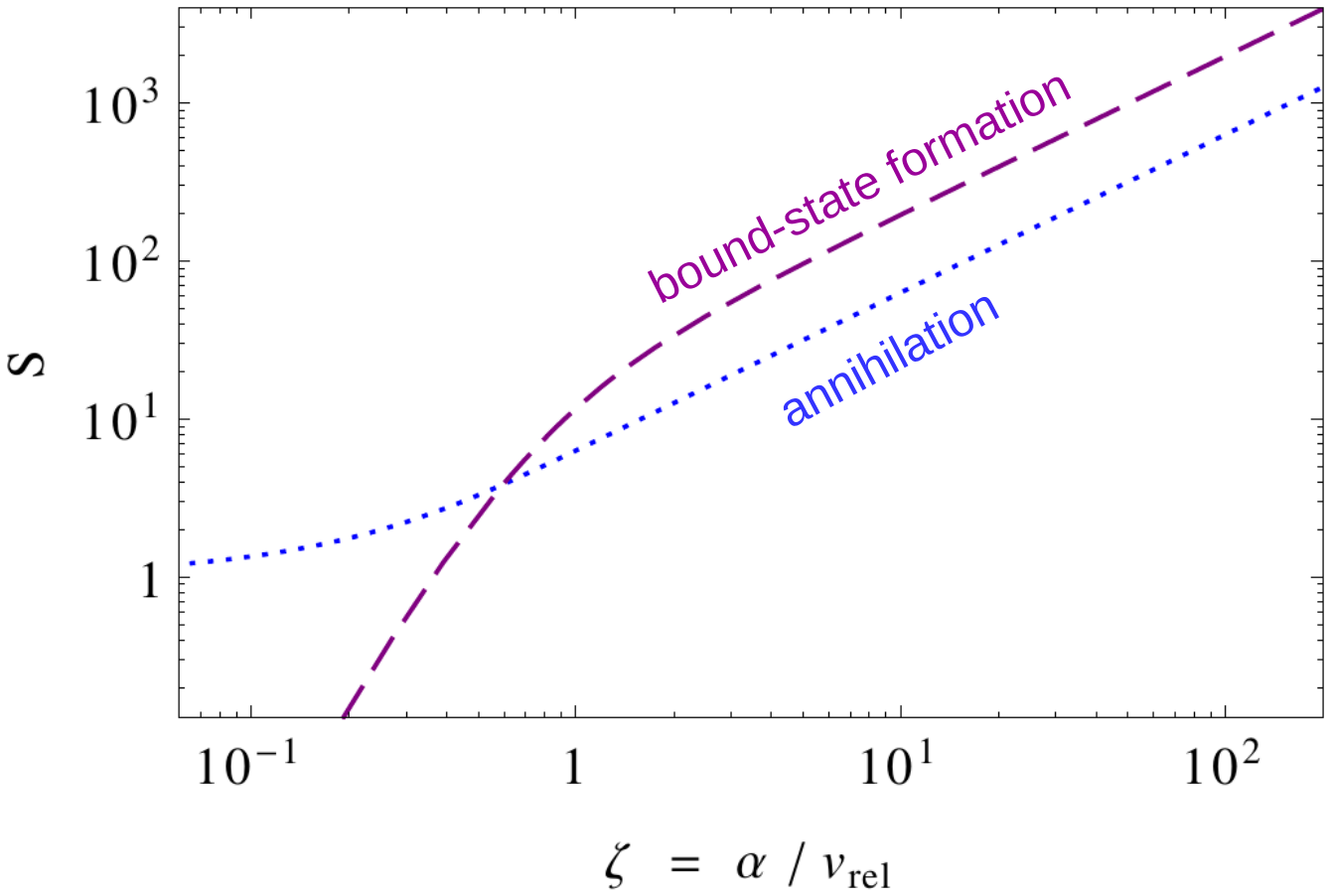
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Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]



BSF dominates over annihilation everywhere the Sommerfeld effect is important (ζ > 1) !

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Boltzmann equations

$$\frac{dn_X}{dt} + 3Hn_X = -\left(n_X^2 - n_X^{eq\ 2}\right)\langle\sigma_{ann} v_{rel}\rangle - n_X^2\langle\sigma_{BSF} v_{rel}\rangle + \left(n_{\uparrow\downarrow} + n_{\uparrow\uparrow}\right)\Gamma_{ion}$$

$$\frac{dn_{\uparrow\downarrow}}{dt} + 3Hn_{\uparrow\downarrow} = + \frac{1}{4} n_X^2\langle\sigma_{BSF} v_{rel}\rangle - n_{\uparrow\downarrow}\left(\Gamma_{ion} + \Gamma_{decay,\uparrow\downarrow}\right)$$

$$\frac{dn_{\uparrow\uparrow}}{dt} + 3Hn_{\uparrow\uparrow} = + \frac{3}{4} n_X^2\langle\sigma_{BSF} v_{rel}\rangle - n_{\uparrow\uparrow}\left(\Gamma_{ion} + \Gamma_{decay,\uparrow\uparrow}\right)$$

$(X\bar{X})_{\uparrow\downarrow} \rightarrow 2\gamma$: $\quad \Gamma_{decay,\uparrow\downarrow} = \alpha^5 (m/2)$

$(X\bar{X})_{\uparrow\uparrow} \rightarrow 3\gamma$: $\quad \Gamma_{decay,\uparrow\uparrow} = \frac{4(\pi^2 - 9)}{9\pi} \alpha^6 (m/2)$

BSF important only when $\Gamma_{decay} > \Gamma_{ion}(T)$

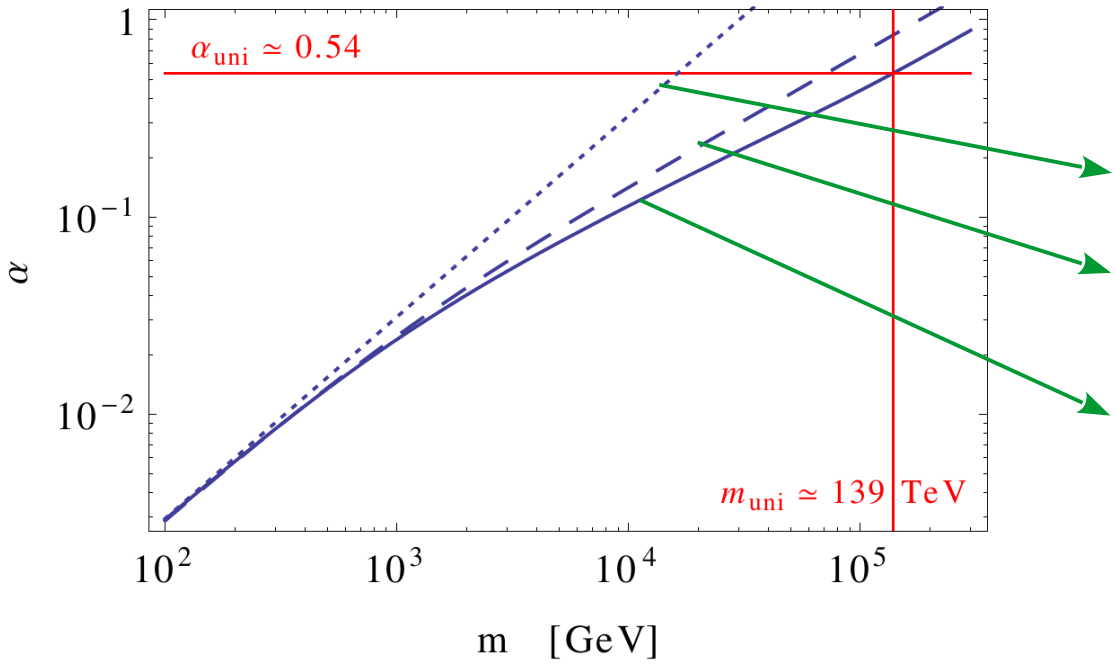
$(X\bar{X})_{\uparrow\downarrow \text{ or } \uparrow\uparrow} + \gamma \rightarrow X + \bar{X}$: $\quad \Gamma_{ion}(T) = \frac{2}{(2\pi)^3} 4\pi \int_0^\infty d\omega \, \frac{\omega^2}{e^{\omega/T} - 1} \sigma_{ion}(\omega)$

related to σ_{BSF}

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Results



Determination of α

Perturbative annihilation

Sommerfeld-enhanced annihilation

Sommerfeld-enhanced annihilation and BSF

Effect on the relic density

As large as a factor of 3 >> 1% !

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Partial-wave unitarity

$$\sigma_{inel,J}\, v_{rel} \leqslant \frac{(2J+1)\,4\pi}{m^2\, v_{rel}}$$

- **Implies upper limit on mass of thermal relic DM.** [Griest, Kamionkowski (1990)]

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feature of long-range inelastic processes

- **Implies upper limit on mass of thermal relic DM.** [Griest, Kamionkowski (1990)]
- **Can be realised only if DM possesses long-range interactions.**
  At $\alpha >> v_{rel}$: [von Harling, KP (2014)]

$$number \times \frac{\pi\,\alpha^2}{m^2}\,\frac{\alpha}{v_{rel}} \leqslant \frac{(2J+1)\,4\pi}{m^2\, v_{rel}} \quad \Rightarrow \quad \alpha \lesssim 0.5$$

Physical meaning: Saturation of interaction probability at large couplings.

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

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- For *s*-wave processes: $\mathbf{m_{UNI} < 83\ TeV \rightarrow 139\ TeV}$ (non-self-conjugate DM) [von Harling, KP (2014)]

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Partial-wave unitarity

$$\sigma_{inel,J}\, v_{rel} \leqslant \frac{(2J+1)\,4\pi}{m^2 v_{rel}}$$

feature of long-range inelastic processes

- **Unitarity realised (perturbatively) for α ~ 0.5,** [von Harling, KP (2014)]
  i.e. well below the perturbativity limit (α ~ π or 4π).

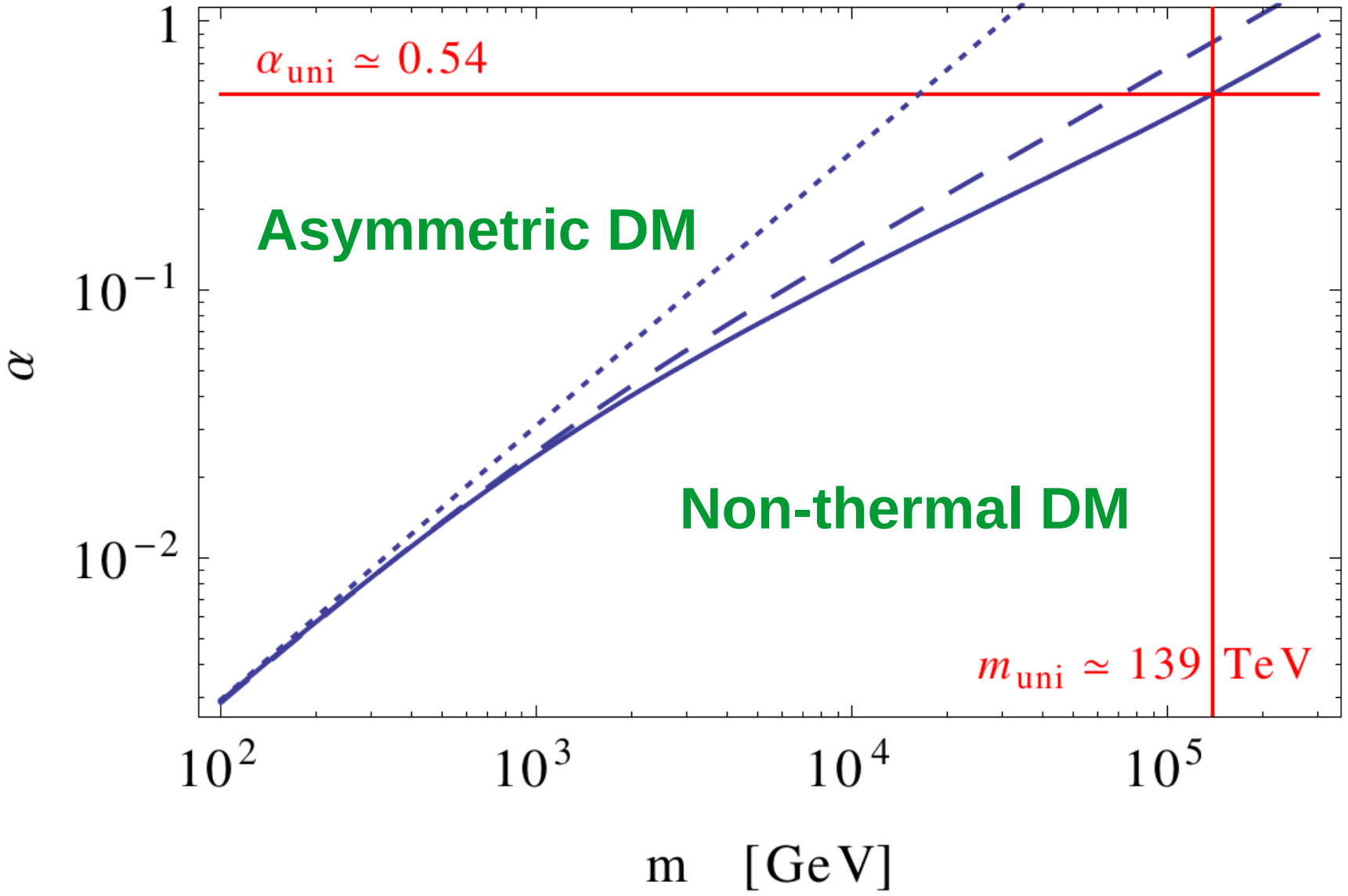
- **Close to the unitarity limit, all partial waves must have the same velocity dependence.**

  Confirmed by explicit calculations for long-range interactions. [Cassel (2009)]
  For annihilation, higher J → higher powers of α.

- **For BSF, higher partial waves give significant contribution.**
  E.g. BSF with vector boson emission: $\mathcal{M} \sim \sin\theta$ → J=0: 62%, J=2: 24% ...
  → Unitarity limit on m_{DM} even higher?

[KP, Postma, Wiechers (2015)]

Relic density of symmetric DM



Indirect detection of symmetric DM

BSF implies:

- Enhanced signal rate, $\sigma_{BSF} > \sigma_{ann}$.
- Features in the spectrum:

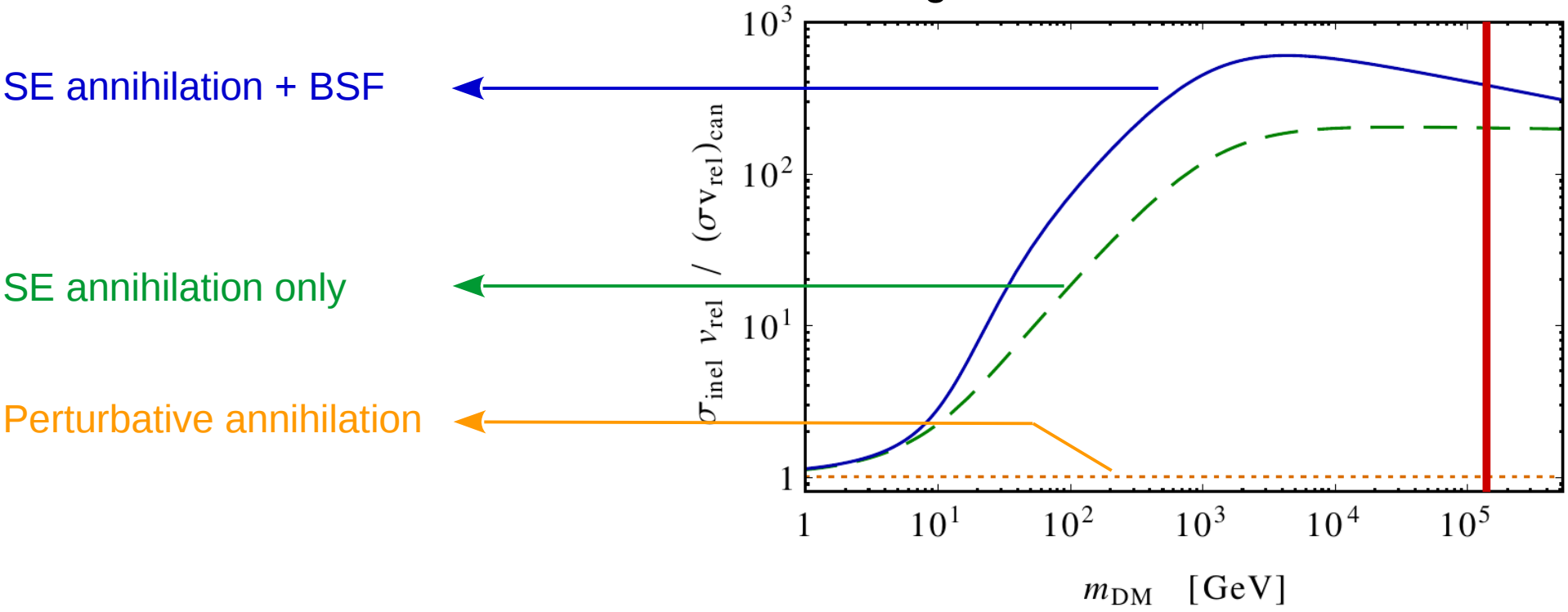
$X + \bar{X} \rightarrow (X\bar{X})_{bound} + \gamma$ **[$E = \alpha^2 m_x/4$]**

$(X\bar{X})_{bound,\uparrow\downarrow} \rightarrow 2\gamma$ **[$E = 2 \times m_x$]**

$(X\bar{X})_{bound,\uparrow\uparrow} \rightarrow 3\gamma$ **[E : extended spectrum]**

Milky Way: $v_{rel} \sim 10^{-3}$.

Large Sommerfeld enhancement



Bound states

Generalisations needed

- Massive mediator – Yukawa potential
- Different interactions (e.g. scalar mediator)
- Weakly coupled non-Abelian theories, e.g. electroweak interactions

Radiative capture in QFT

[KP, Postma, Wiechers (2015)]

Establish a QFT formalism (instead of QM),
for weakly-coupled theories with long-range interactions.
Take non-relativistic limit relevant for cosmo/astro DM applications.

- Can accommodate non-Abelian interactions,
  e.g. electroweak interactions.
- Allows systematic inclusion of higher-order corrections
  in the coupling strength and in the momentum transfer
  (particularly important when leading order terms cancel).

[Equivalent work for annihilation processes: Cassel (2009), Iengo (2009)]

Radiative capture in QFT

[KP, Postma, Wiechers (2015)]

In 3 easy steps

$$\mathcal{U}\{\chi_1 + \chi_2\} \rightarrow \mathcal{B}\{\chi_1\chi_2\} + \phi$$

[Radiation gets rid of the kinetic + binding energies.
Similar to internal bremsstrahlung, where kinetic energy only is radiated.]

1) Separate the initial / final asymptotic states from interaction part (which includes the radiative vertex).

2) Describe the asymptotic states.

Scattering state (initial): Long-range interaction between χ_1 & χ_2

→ Two-particle state ≠ Two plane waves [Sommerfeld (1931),

→ *Sommerfeld effect* (in the non-relativistic regime) Bethe & Salpeter (1957)]

Bound state (final): One-particle state (pole in scattering amplitude), with the quantum charges of χ_1 & χ_2. [Bethe & Salpeter (1957)]

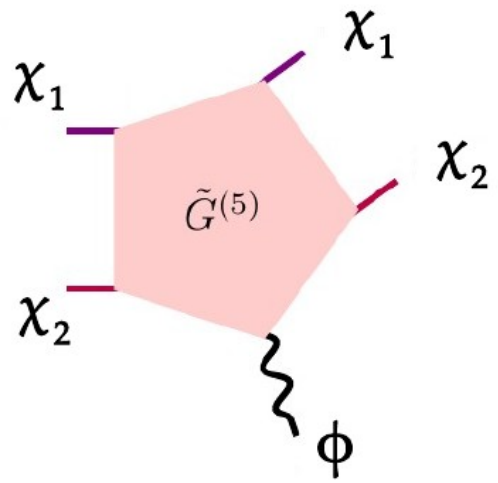
3) Put everything together to calculate the amplitude. LSZ reduction

Radiative capture in QFT

[KP, Postma, Wiechers (2015)]

Starting point

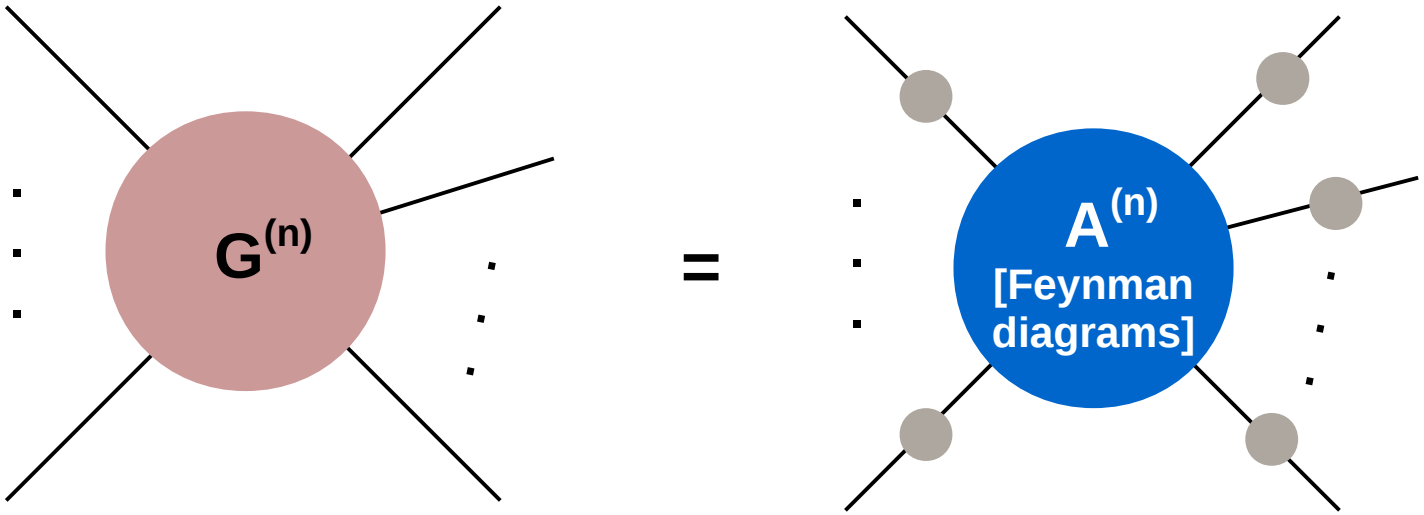
$$\mathcal{U}\{\chi_1 + \chi_2\} \rightarrow \mathcal{B}\{\chi_1\chi_2\} + \phi$$



5-point Green's function

n-point Green's functions

Short-range interactions among initial/final state particles

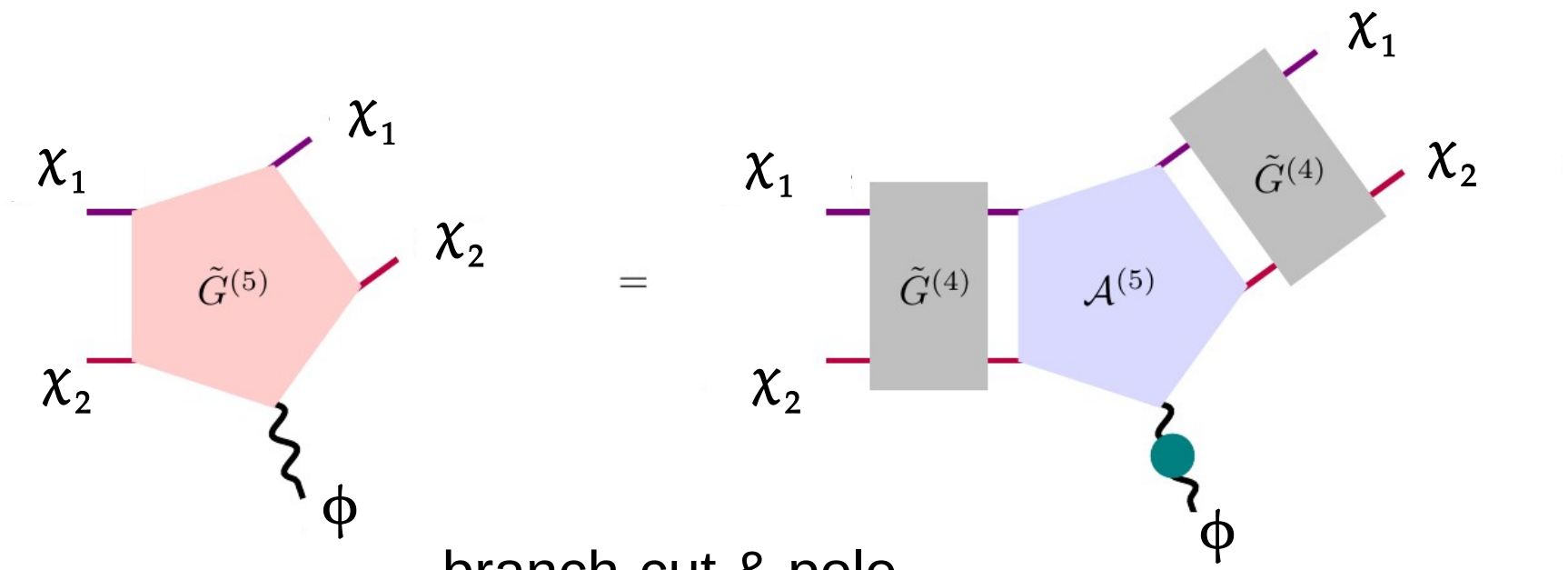


Radiative capture in QFT

[KP, Postma, Wiechers (2015)]

|1| Amputation

$$\mathcal{U}\{\chi_1 + \chi_2\} \to \mathcal{B}\{\chi_1\chi_2\} + \phi$$



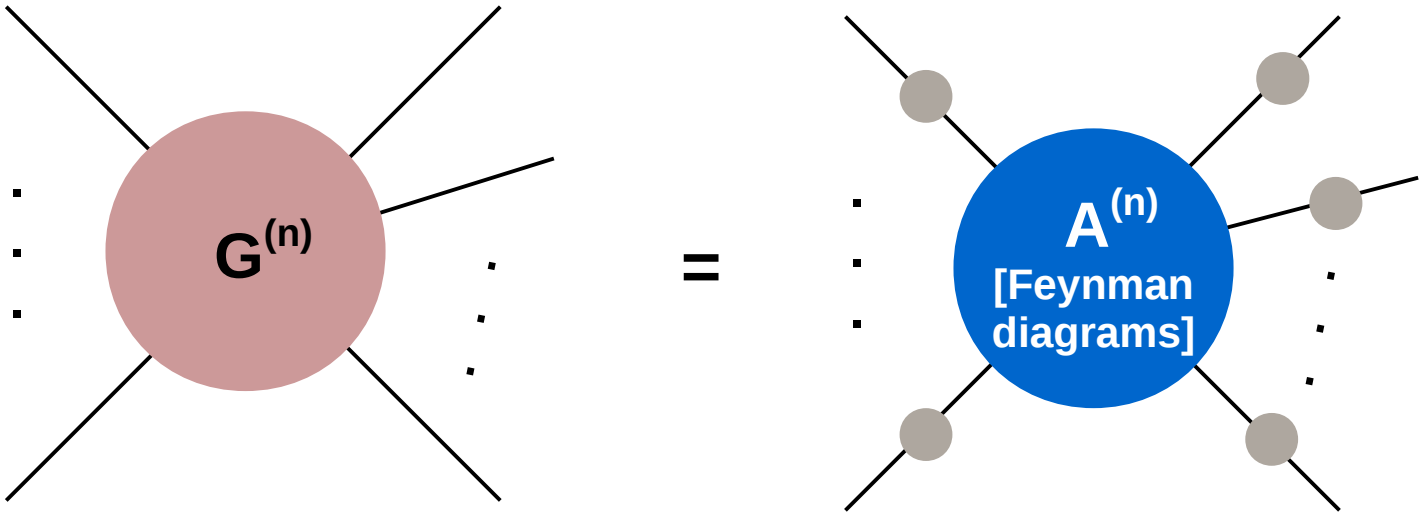
branch-cut & pole structure

5-point function ⟺ BSF amplitude

LSZ reduction

n-point Green's functions

Short-range interactions among initial/final state particles



$$G^{(2)} = \quad = \frac{iZ_{\vec{p}}}{p^2 - m^2 + i\epsilon}\,, \qquad Z_{\vec{p}} = |\langle\Omega|\phi|\vec{p}\rangle|^2$$

Radiative capture in QFT

[KP, Postma, Wiechers (2015)]

|1| Decomposition of $G^{(4)}$ into bound and scattering states

Insert unity in 4-point function

$$G^{(4)} = \langle \Omega | T \chi_1(x_1) \chi_2(x_2) \chi_1^\dagger(y_1) \chi_2^\dagger(y_2) | \Omega \rangle$$

$$1 \sim \sum_n \int d^3Q \, |\mathcal{B}_{\vec{Q},n}\rangle\langle\mathcal{B}_{\vec{Q},n}| + \int d^3q \, d^3Q \, |\mathcal{U}_{\vec{Q},\vec{q}}\rangle\langle\mathcal{U}_{\vec{Q},\vec{q}}|$$

$\sum_n \int d^3Q$: expectation value of relative momentum

$|\mathcal{B}_{\vec{Q},n}\rangle\langle\mathcal{B}_{\vec{Q},n}|$: bound states

$\int d^3q \, d^3Q$: scattering (unbound) states

$|\mathcal{U}_{\vec{Q},\vec{q}}\rangle\langle\mathcal{U}_{\vec{Q},\vec{q}}|$

Time-ordering operator magic

$$G^{(4)}(Q) \sim \sum_n \frac{i \Psi_{\vec{Q},n} \Psi^*_{\vec{Q},n}}{Q^0 - \omega_{\vec{Q},n} + i\epsilon} + \int d^3q \, \frac{i \Phi_{\vec{Q},\vec{q}} \Phi^*_{\vec{Q},\vec{q}}}{Q^0 - \omega_{\vec{Q},\vec{q}} + i\epsilon}$$

where

$$\Psi_{\vec{Q},n}(x_1,x_2) \equiv \langle \Omega | T \chi_1(x_1) \chi_2(x_2) | \mathcal{B}_{\vec{Q},n} \rangle$$

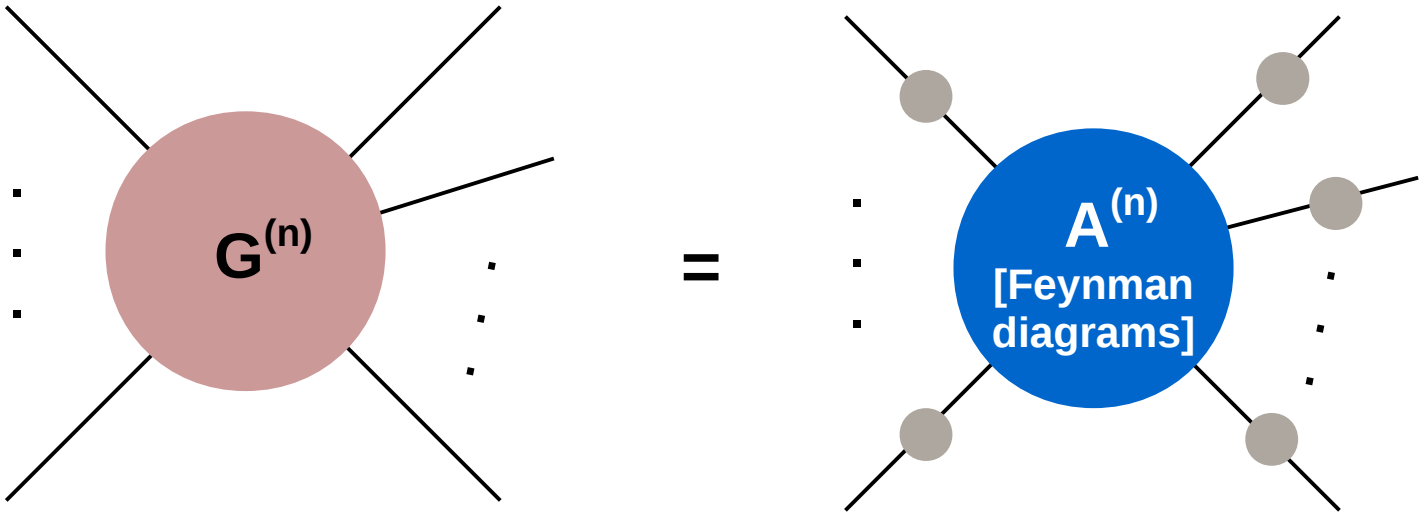
$$\Phi_{\vec{Q},\vec{q}}(x_1,x_2) \equiv \langle \Omega | T \chi_1(x_1) \chi_2(x_2) | \mathcal{U}_{\vec{Q},\vec{q}} \rangle$$

Bethe-Salpeter wavefunctions

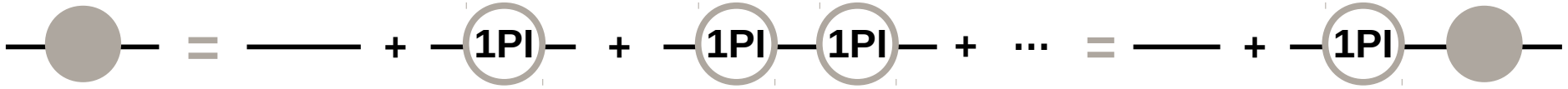
By fixing the energy Q^0, we pick out a state.

n-point Green's functions

Short-range interactions among initial/final state particles



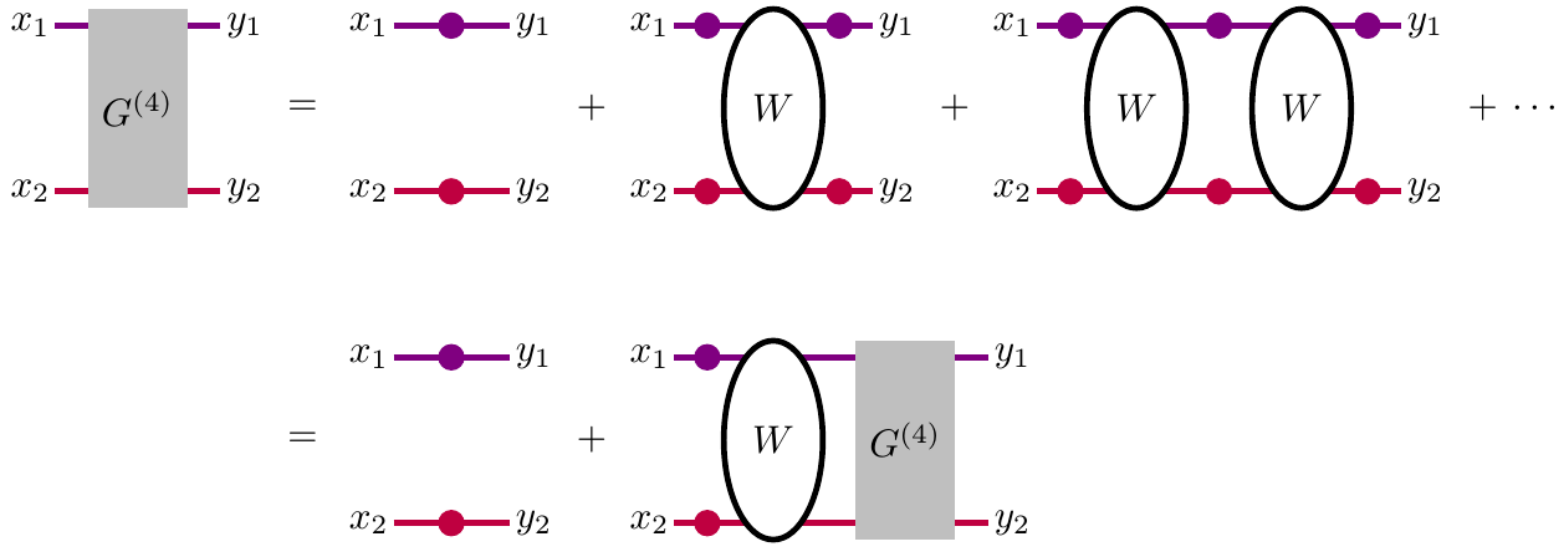
$$G^{(2)} = \quad = \frac{iZ_{\vec{p}}}{p^2 - m^2 + i\epsilon}, \qquad Z_{\vec{p}} = |\langle \Omega | \phi | \vec{p} \rangle|^2$$



Radiative capture in QFT

[KP, Postma, Wiechers (2015)]

|2| Dyson-Schwinger for $G^{(4)}$



+

$G^{(4)}$ decomposition into bound & scattering-state contributions

=

Bethe – Salpeter equations for bound & scattering state WFs

$$\tilde{\Psi}_{\vec{Q},n}(p) = S(p;Q) \int \frac{d^4k}{(2\pi)^4} \tilde{W}(p,k;\,Q)\tilde{\Psi}_{\vec{Q},n}(k)$$

$$\tilde{\Phi}_{\vec{Q},\vec{q}}(p) = S(p;Q) \int \frac{d^4k}{(2\pi)^4} \tilde{W}(p,k;\,Q)\tilde{\Phi}_{\vec{Q},\vec{q}}(k)$$

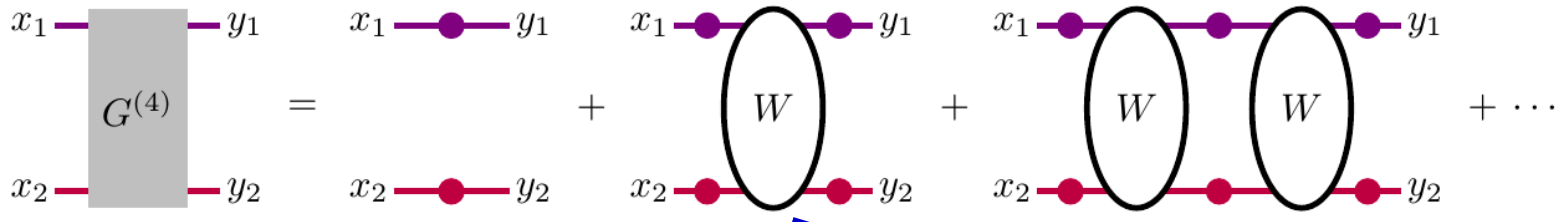
Sommerfeld effect!

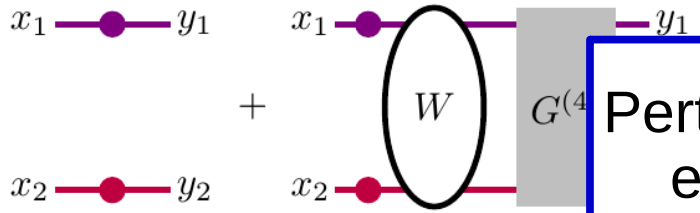
Reduce to Schroedinger eq in non-relativistic regime

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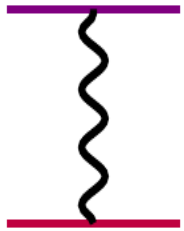


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Perturbative interaction kernel, e.g. one-boson exchange



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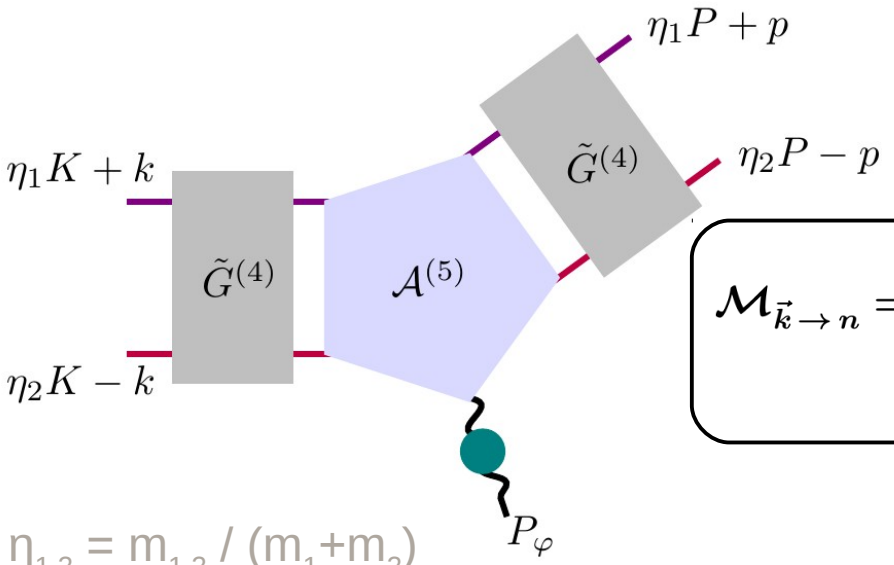
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[KP, Postma, Wiechers (2015)]

|3| LSZ reduction
Green's function → Amplitude



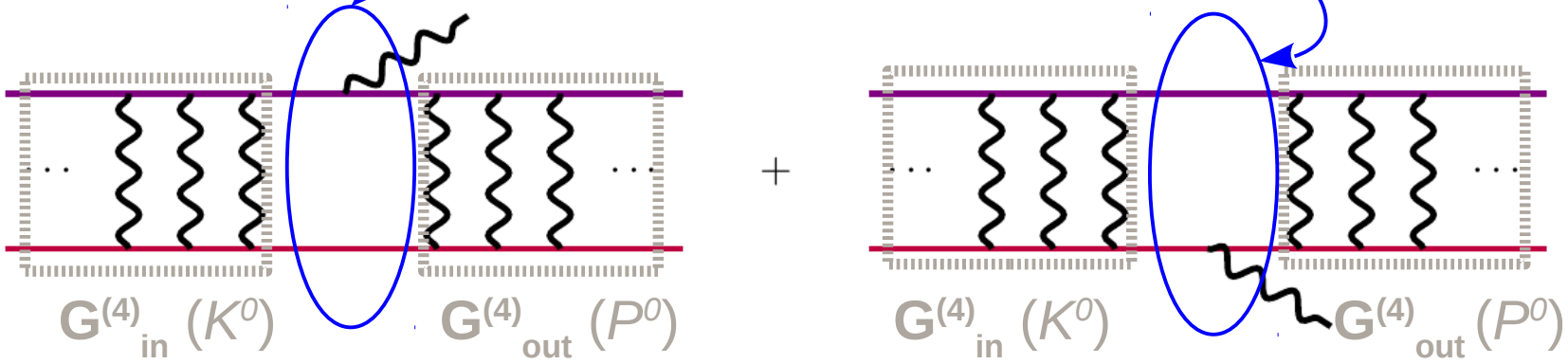
$\eta_{1,2} = m_{1,2} / (m_1+m_2)$

non-perturbative physics

$$\mathcal{M}_{\vec{k}\to n} = \int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\ \tilde{\Psi}^*_{\vec{P},n}(p)\ \tilde{\Phi}_{\vec{K},\vec{k}}(q)\times$$
$$\mathcal{A}^{(5)}(P_\varphi, \eta_1 P + p, \eta_2 P - p;\ \eta_1 K + q, \eta_2 K - q)$$

Perturbative *off-shell* transition amplitude

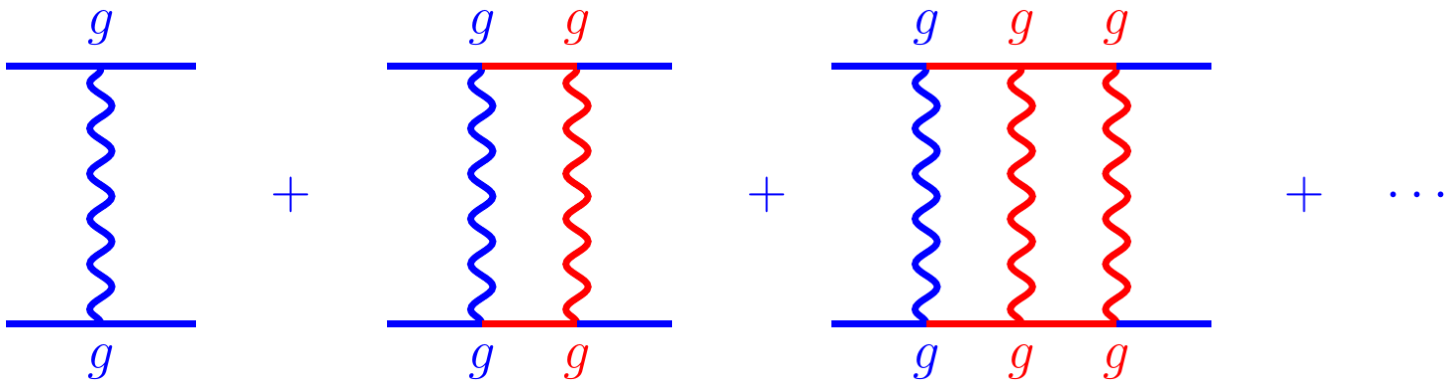
One-boson-exchange kernel:



Setup generalisable to multiple interacting quarteta $\chi_i + \chi_j \to \chi_k + \chi_l$ with interaction kernels W_{ijkl}, described by $G^{(4)}_{ijkl}$.

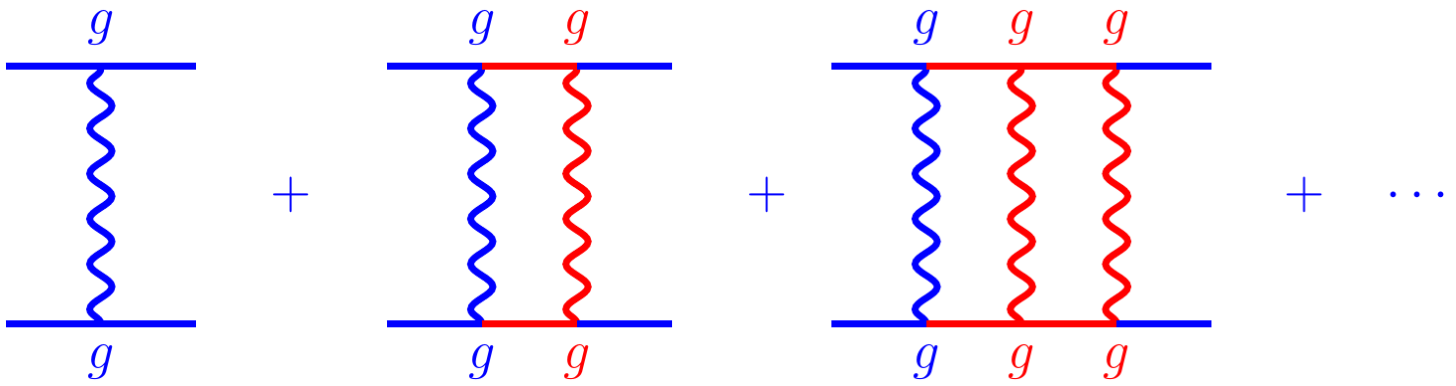
Making sense of the ladder diagrams

Every force mediator exchange introduces an additional $\alpha = g^2/4\pi$ suppression. How do we get an enhancement?



Making sense of the ladder diagrams

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Energy and momentum transfer scale with α !

Momentum transfer: $\bar{q} \sim \mu\,\alpha$. Energy transfer: $q^0 \sim \bar{q}^2/\mu \sim \mu\,\alpha^2$.

$$\text{one boson exchange} \sim \alpha\,\frac{1}{k_\gamma^2} \sim \alpha\,\frac{1}{(\mu\alpha)^2} \propto \frac{1}{\alpha}$$

$$\text{each added loop} \sim \alpha \times \int dq^0\, d^3q\, \frac{1}{q_\gamma^2}\,\frac{1}{q_1 - m_1}\,\frac{1}{q_2 - m_2}$$

$$\sim \alpha \times (\mu\alpha^2)(\mu\alpha)^3 \times \frac{1}{(\mu\alpha)^2}\left(\frac{1}{\mu\alpha^2}\right)^2$$

$$\sim 1$$

1/α scaling responsible for non-perturbative effects (not the largeness of the coupling)

Bound states
in WIMP phenomenology

Are bound states relevant to WIMP DM?

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Sommerfeld effect in annihilation processes is important → BSF is likely too.
In toy model (massless mediator), BSF affects relic density for m_{DM} > TeV.

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WIMP-onium formation is potentially important if $m_{WIMP} \gtrsim 6$ TeV !

Radiative BSF cross-sections $m_{mediator} \lesssim (m_X/2)\,\alpha^2/2$.
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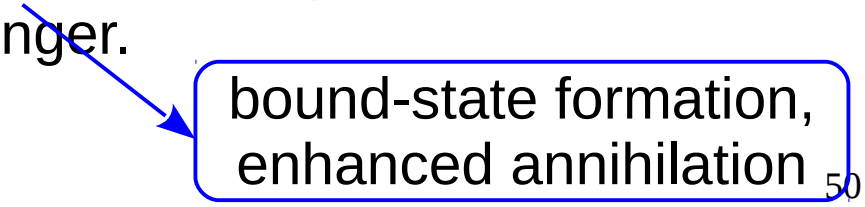
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What about sub-TeV WIMPs?

- MSSM contains parameter space where charged/coloured sparticles (e.g. stops, staus etc) are nearly degenerate to the neutralino LSP. Generically, this occurs even for light (few x 100 GeV) NLSP/LSP.
- NLSP – LSP co-annihilations significant for LSP relic density.
- Because of mass degeneracy, NLSP decay very slow, possibly *after* LSP freeze-out. Then, the NLSP abundance itself is important!

  → We need to accurately compute NLSP total annihilation rate.

  [QCD corrections (DM@NLO): Klasen, Harz, Herrmann, Kovarik, Meinecke, Steppeler]
- For charged/coloured NLSPs, there is no threshold for bound-state existence and formation.
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Bound states potentially important even for sub-TeV WIMPs!

bound-state formation, enhanced annihilation

Conclusion

- **The early universe regulates the DM manifestations today.**

  Familiar example — Symmetric thermal relics with contact interactions:

  $$\Omega_{DM} \propto 1/(\sigma_{ann}\, v_{rel}) \Rightarrow d\Gamma_{ann}/dV = \rho_{DM}^2\,(\sigma_{ann}\, v_{rel}) \propto 1/(\sigma_{ann}\, v_{rel})$$

  For long-range interactions, the regulator is bound-state formation:
  - Symmetric thermal relics: Reduced abundance.
  - Asymmetric thermal relics: Neutralises / screens the interaction.

- **BSF is important for important DM theories,**
  e.g. WIMPs, self-interacting DM, asymmetric DM.

- **Need tools to calculate bound-state related processes.**

  Non-relativistic regime relevant for DM phenomenology.
  Nevertheless, QFT formalism preferable or even necessary.

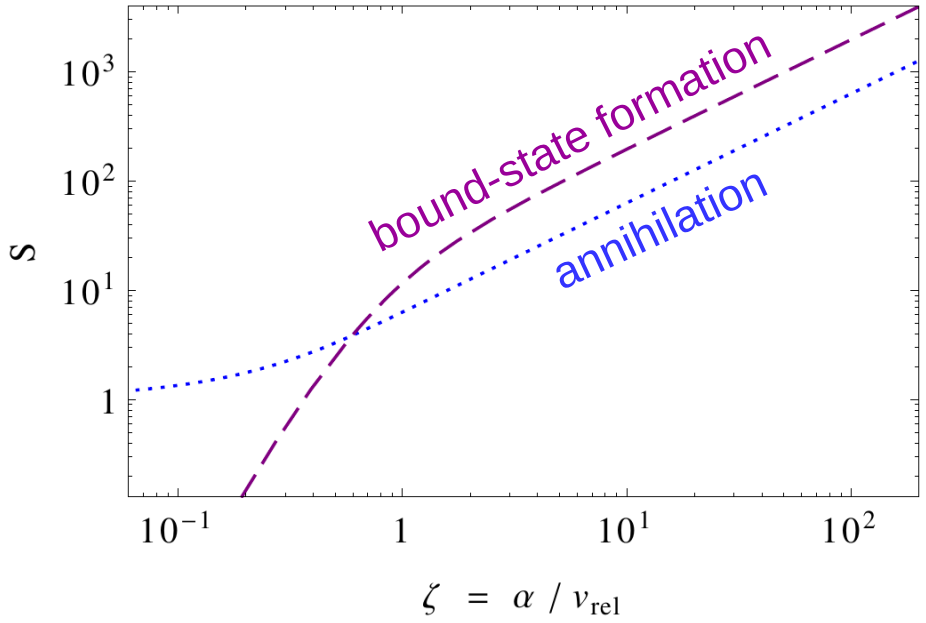
  Can accommodate DM residing in multiplets, and multiple interaction kernels.
  Can reliably yield higher-order corrections.

extra slides

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

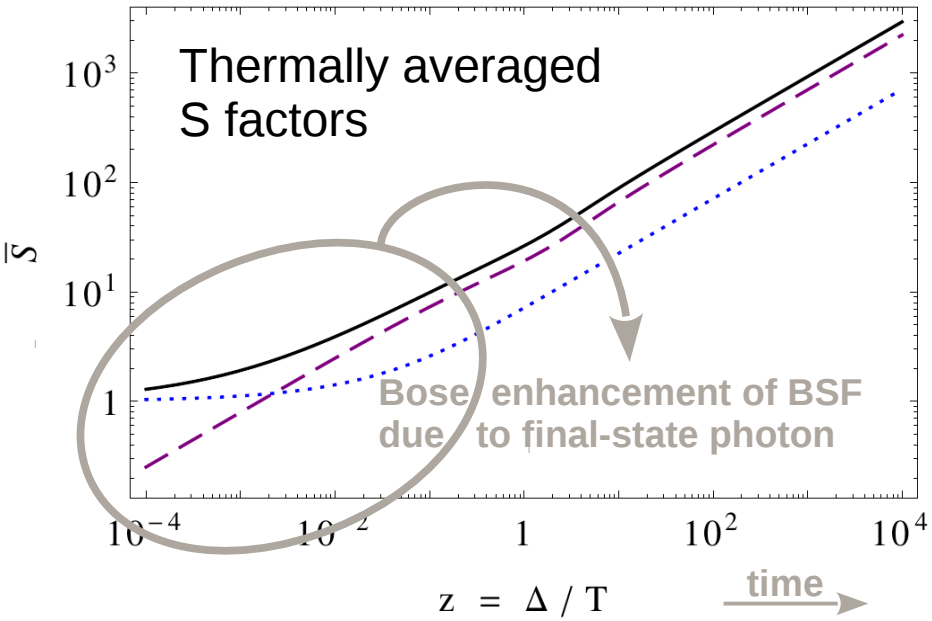
Rates



$$\text{reduced mass} \quad \mu = m/2$$

$$\zeta \equiv \frac{\text{Bohr momentum}}{\text{relative momentum}} = \frac{\mu\,\alpha}{\mu\, v_{rel}}$$

BSF dominates over annihilation everywhere the Sommerfeld effect is important (ζ > 1) !



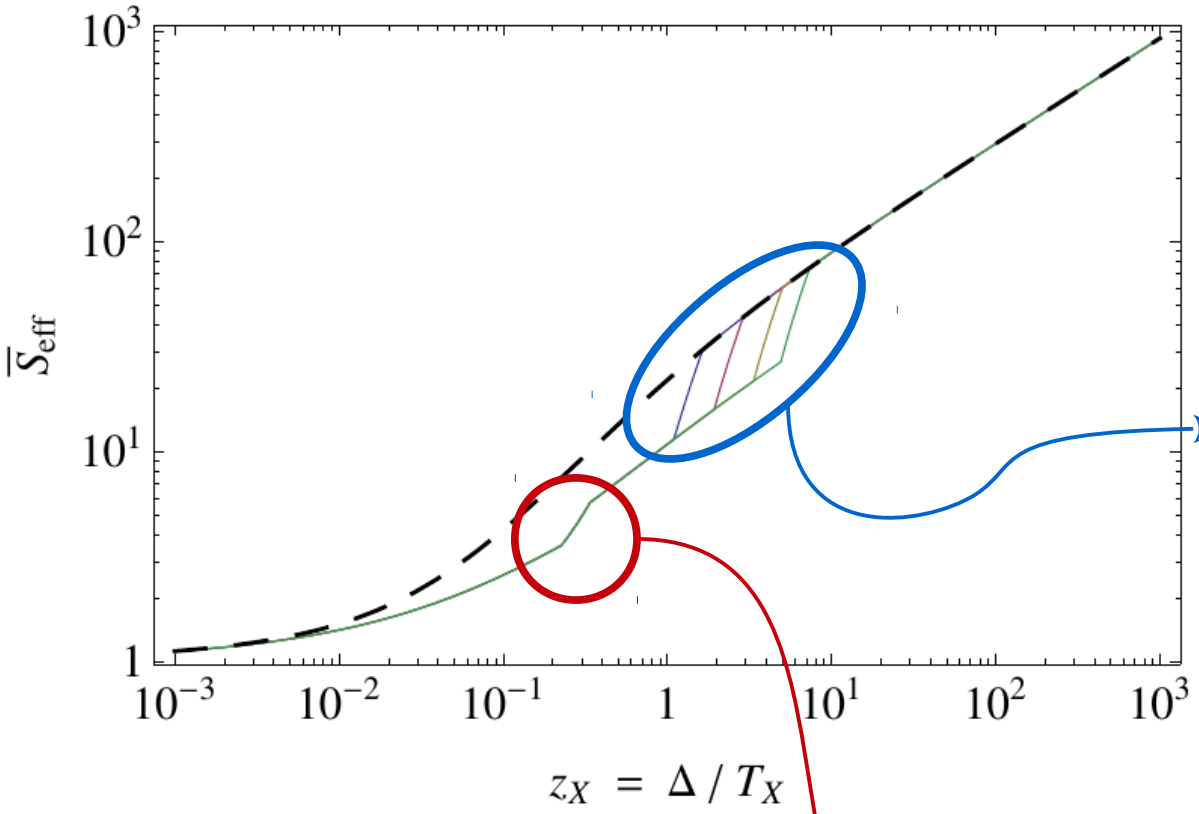
$$z = \frac{\text{binding energy } [\Delta]}{T} \sim \frac{(1/2)\,\mu\,\alpha^2}{(1/6)\,\mu\,\langle v_{rel}^2 \rangle} \sim \langle \zeta^2 \rangle$$

BSF can deplete DM at z >1, when disassociation of bound states becomes unimportant.

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

"Effective" enhancement



Decay of ↑↑ bound state faster than ionisation

(depends on α; from left to right: α = 0.5, 0.1, 0.01, 0.001)

$S_{eff} = S_{ann} + (1/4)S_{BSF} + (3/4)S_{BSF}$

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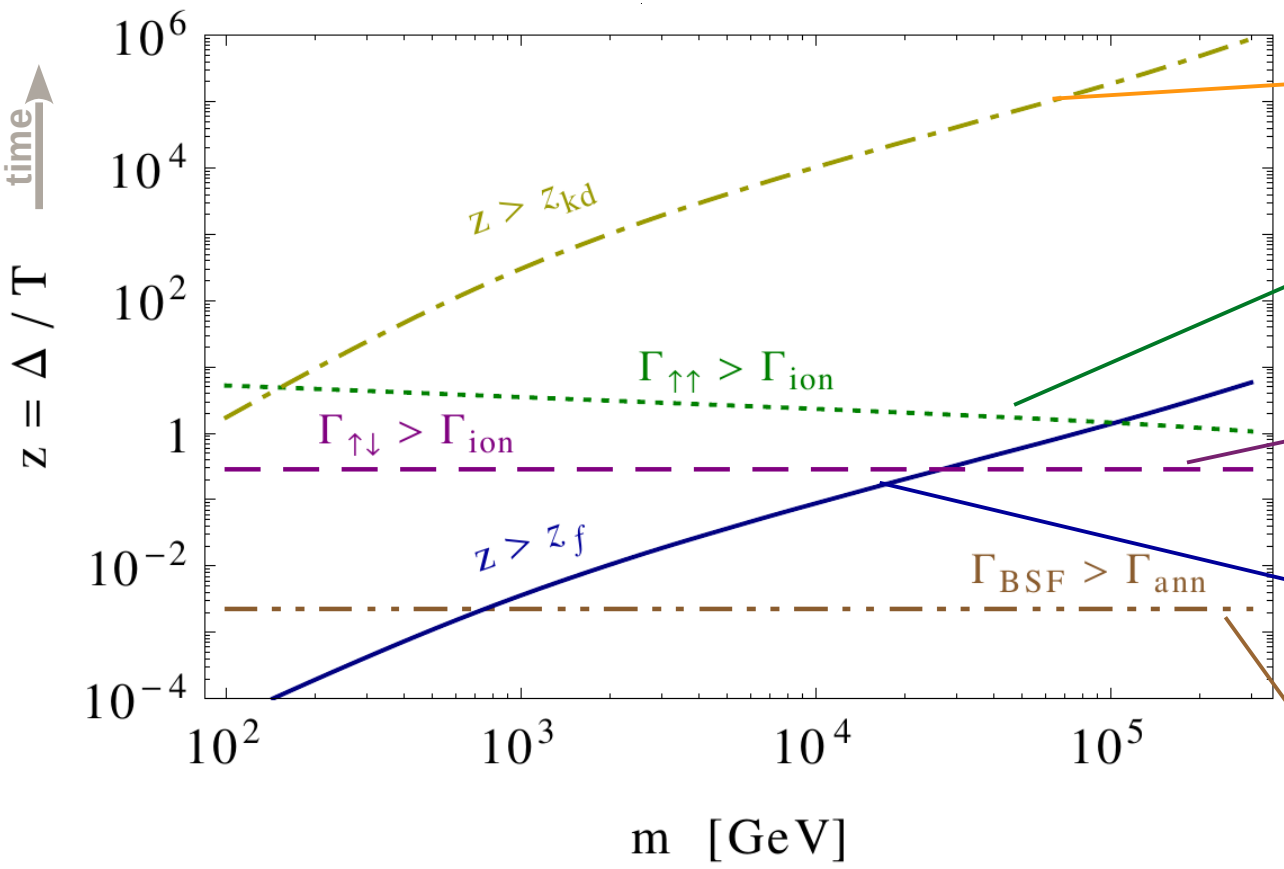
(independent of α)

$S_{eff} = S_{ann} + (1/4)S_{BSF}$

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

$\alpha = \alpha(m)$ fixed from relic abundance (see results)



Kinetic decoupling

Ortho-positronium decay faster than ionisation.

Para-positronium decay faster than ionisation.

Freeze-out: DM density differs from equilibrium value by O(1) factor [standard definition]

BSF becomes faster than annihilation