Probing the (an)-isotropy of expansion with weak gravitational lensing

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Outline

1) Overview of cosmology

2) Anisotropic cosmologies

3) Weak-lensing

4) Testing anisotropy with weak-lensing

Cosmological models

Their construction relies on 4 hypothesis

1) Theory of gravity [General relativity]

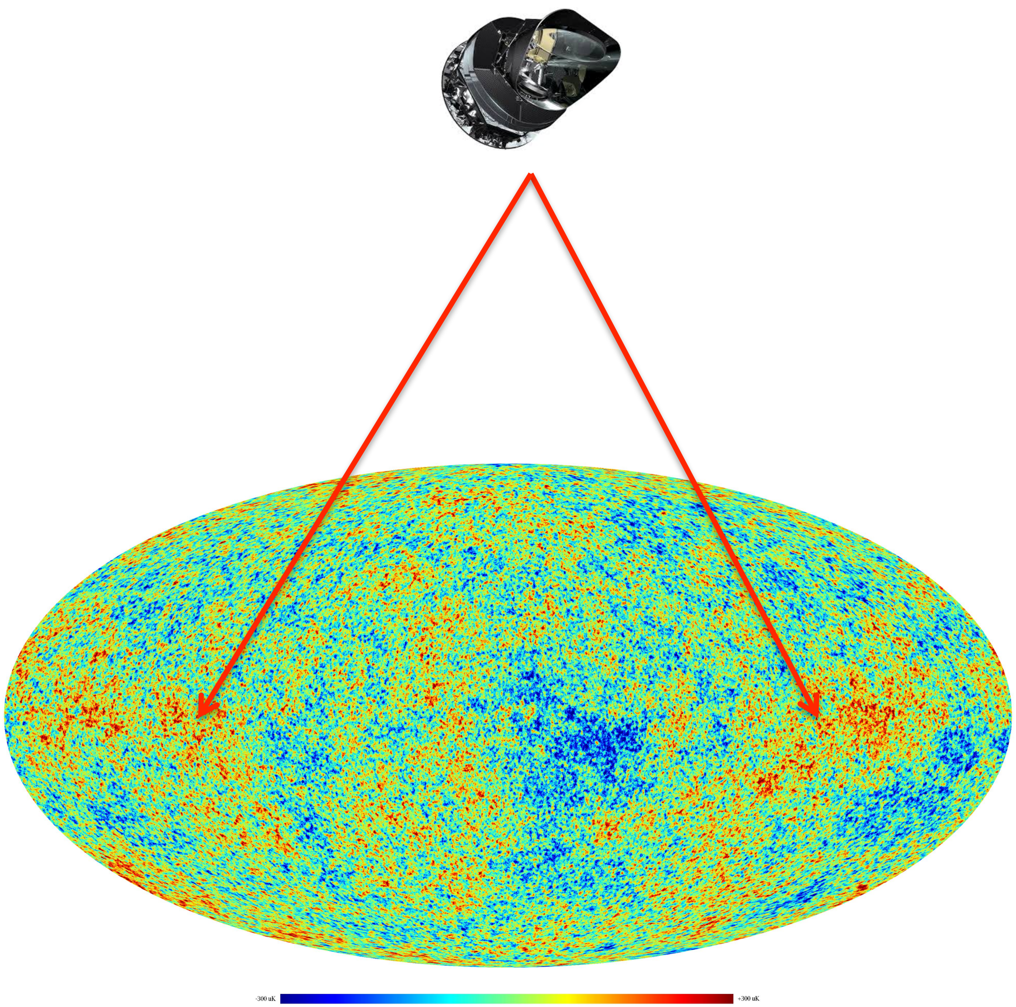
2) Matter [Standard model fields + CDM + Λ]

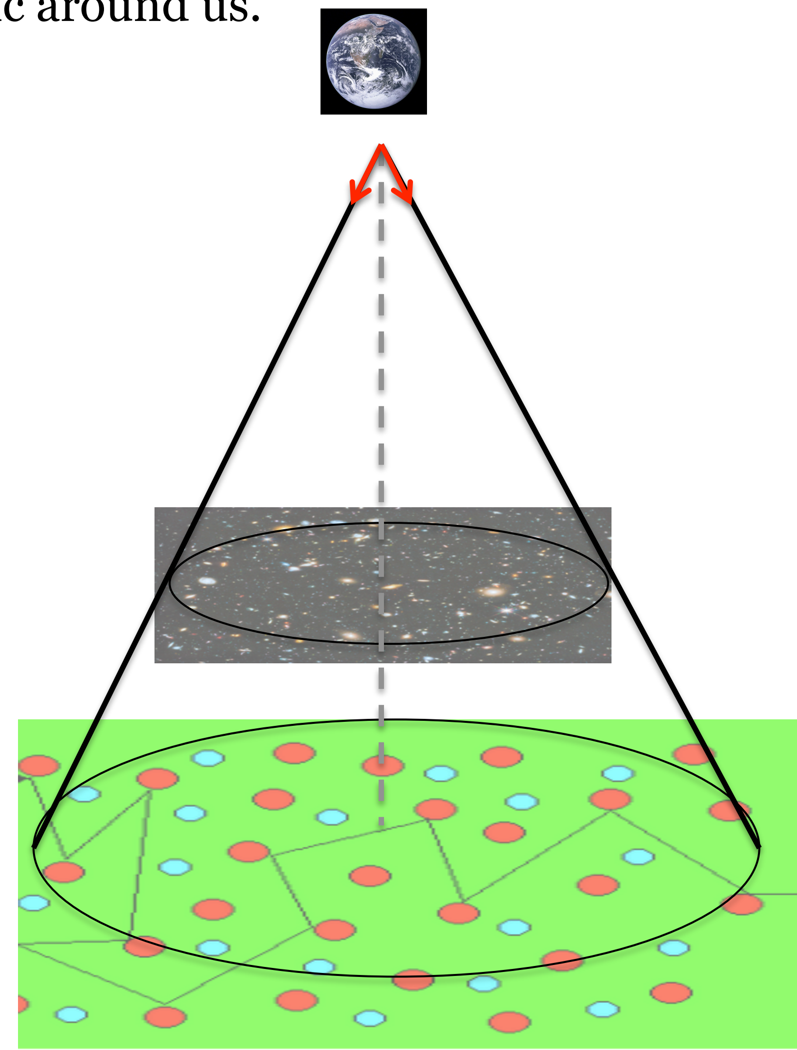
3) Symmetry hypothesis [Copernican Principle]

4) Global structure [Topology of space is trivial]

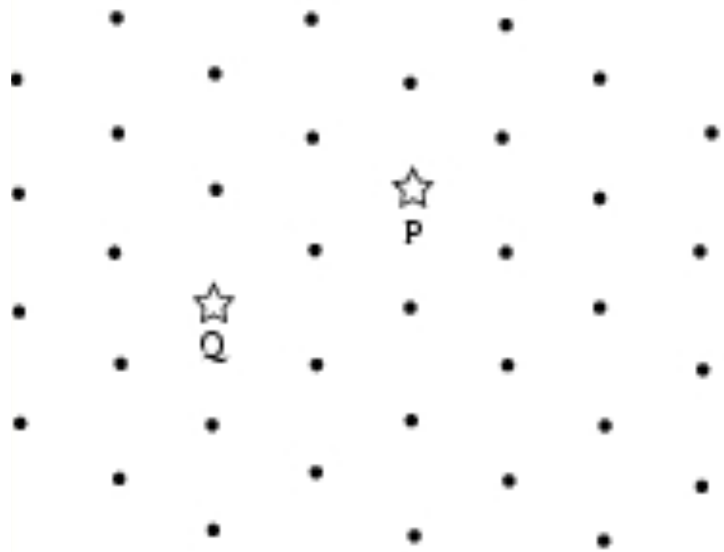
Isotropy: motivation

Observationally, the universe seems very isotropic around us.

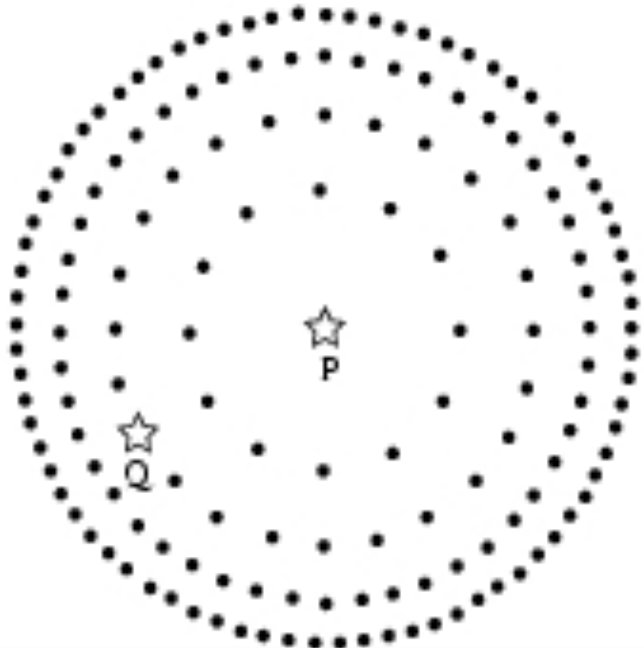




Two possibilities to achieve this:



Spatially homogeneous & isotropic



Spherically symmetric
Universe has a center

Copernican Principle: we do not occupy a particular spatial location in the universe

Geometrical implication

There exists a priviledged class of observers for which the spatial sections look isotropic and homogeneous.

$$ds^2 = -dt^2 + a^2(t)\left[d\chi^2 + f_K^2(\chi)d\Omega^2\right]$$

The spatial sections are constant curvature hypersurfaces.

The spacetime is of the Friedmann-Lemaître type

<u>Consequences</u>:

1- The dynamics of the universe reduces to the one of the scale factor

2- It is dictated by the Friedmann equations

$$\boxed{\begin{aligned} 3\left(H^2 + \frac{K}{a^2}\right) &= 8\pi G\rho \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3P) \end{aligned}}$$

Several ideas are currently developed to test the Copernican principle.

Motivations to test for isotropy

The Copernican principle has thus important implications for our understanding of the origin of the acceleration of the cosmic expansion.

Among other competing explanations, void models have been proposed.

At early time, the isotropisation of the universe is efficient only if inflation lasts long enough.

What is the dynamical effect of an early anisotropy?
Can it be constrained observationally?

Thiago Pereira, C. P., Jean.-Philippe. Uzan, JCAP ***09*** *(2007) 006*
C.P., Thiago Pereira, J.-P. Uzan., JCAP ***04*** *(2008) 004*

At late time:

- any model of dark energy involving vector fields and some models of modification of gravity have a non-vanishing anisotropic stress tensor;
- it has also been shown that in the approach in which the acceleration is explained by backreaction effect, the Hubble flow becomes anisotropic at late time

How can we constrain a late time anisotropy?

Anisotropic cosmologies

Anisotropic spaces in a one slide

Geometry: Spatial sections indexed by the time

$$g_{\mu\nu} = -\mathrm{d}t_\mu \otimes \mathrm{d}t_\nu + h_{ij}(t) e^i{}_\mu \otimes e^j{}_\nu$$

- $h_{ij}(t)$ tells how each spatial slice is glued to the previous one

- $[e_i, e_j] = C^k{}_{ij} e_k$ gives the structure of the spatial slices.

  Classification of anisotropic spaces from these constants.

$$^3\bar{R}_{ij}{}^{kl} = -\frac{1}{2} C^p{}_{ij} C_p{}^{kl} + \frac{1}{2} C_p{}^l{}_i C^{pk}{}_j + C_p{}^l{}_j C_i{}^{kp} + C_p{}^l{}_j C^k{}_i{}^p + C_{ijp}$$

Of course one slide is not enough… Shear

- $h_{ij}(t) = a(t)^2\delta_{ij}$

  Friedmann Lemaître (FL) case (for some cases).

  If $e^i = \mathrm{d}x^i,\ \ C^k{}_{ij} = 0$ Flat FL.

  The expansion describes everything $H = \dfrac{\dot{a}}{a}$

- $h_{ij} = a^2(t)[\mathrm{e}^{2\beta}]_{ij} = a^2(t)(\delta_{ij} + 2\beta_{ij} + \dots)$

  Matrix β_{ij} is traceless and encodes volume preserving deformations.

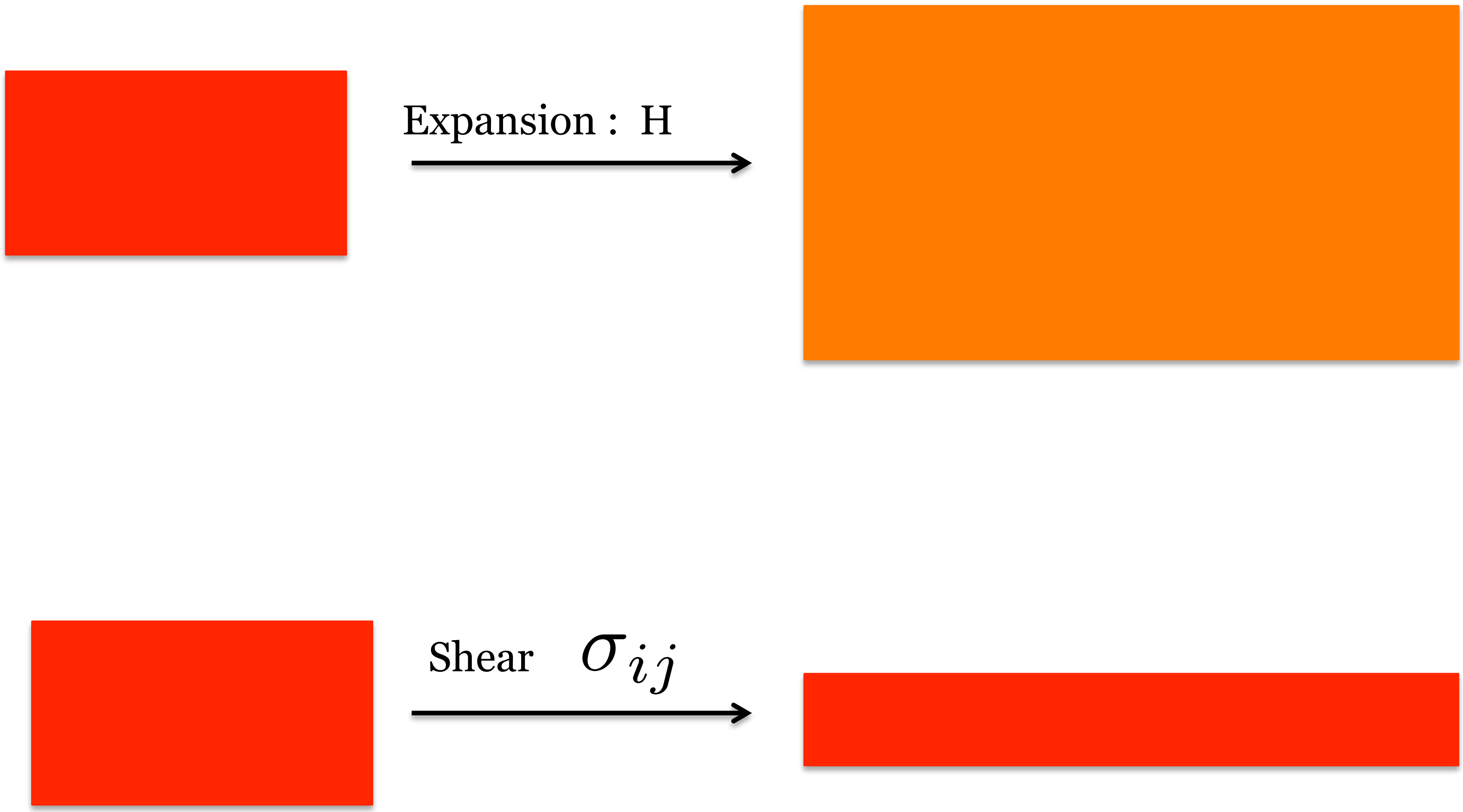
  Deformation rate is the shear $\boxed{\sigma_{ij} \equiv \dot{\beta}_{ij}}$

Illustration of shear and expansion

Expansion : H

Shear σ_{ij}

Illustration of shear and expansion



Friedmann equation

$$H^2 = \frac{8\pi G\rho}{3} + \frac{\sigma_{ij}\sigma^{ij}}{6} + \propto \frac{1}{a^2}\left[C^i{}_{jk}C_i{}^{jk} + \dots\right]$$

$\boxed{C^i_{jk}}$ Contributes as $1/a^2$

$\boxed{\sigma^2 = \sigma_{ij}\sigma^{ij}}$ Contributes as $1/a^6$

-Shear effects relevant at early times (BBN, CMB)

-Constants of structure effects relevant *at late time*
see e.g. Bianchi VIIh papers.

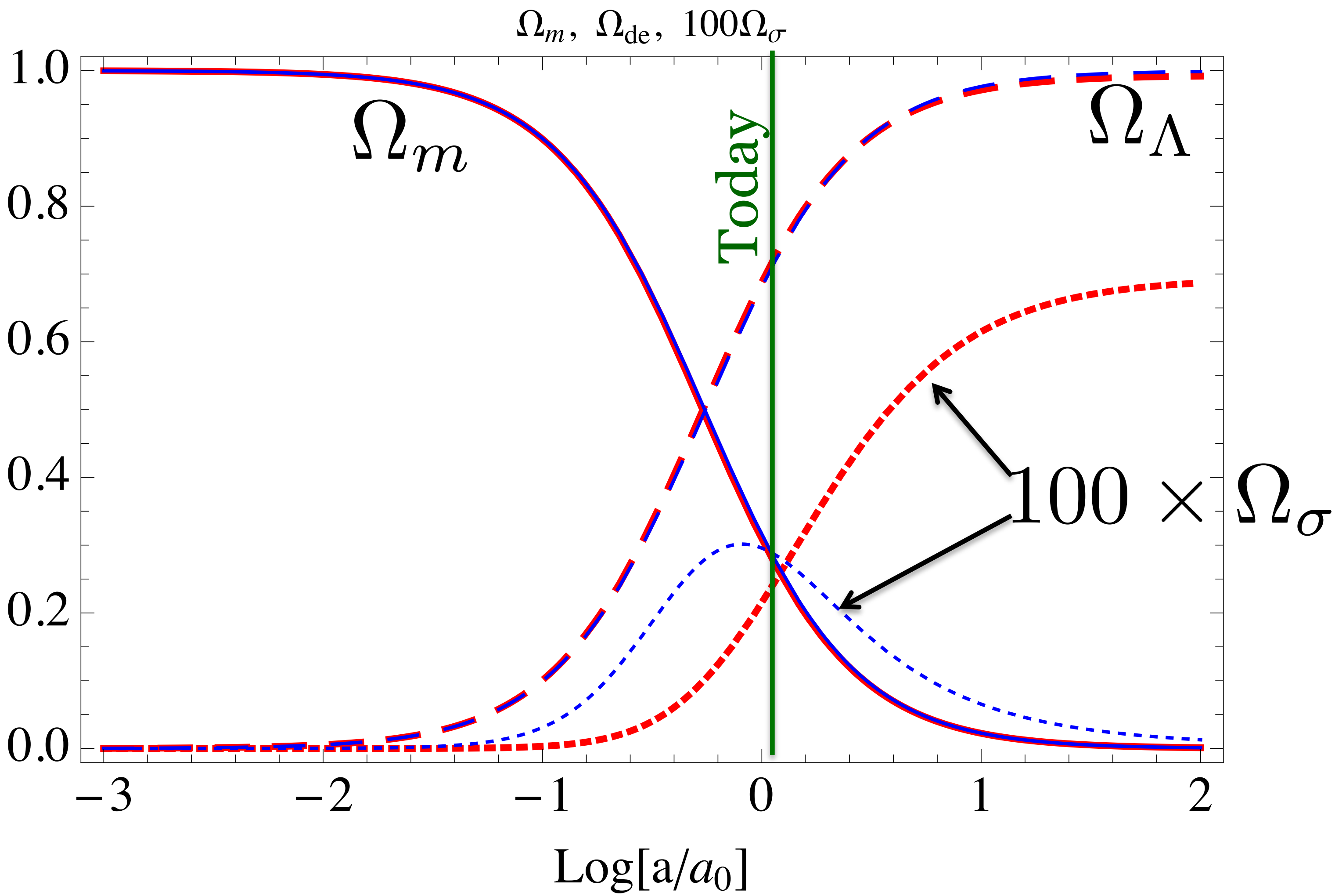
Bianchi I

$$\sigma_{ij} \neq 0$$

$$C^k{}_{ij} = 0$$

Unless there is anisotropic stress

$$\sigma'_{ij} + 3H\sigma_{ij} = \Pi_{ij} \neq 0$$



Weak-lensing

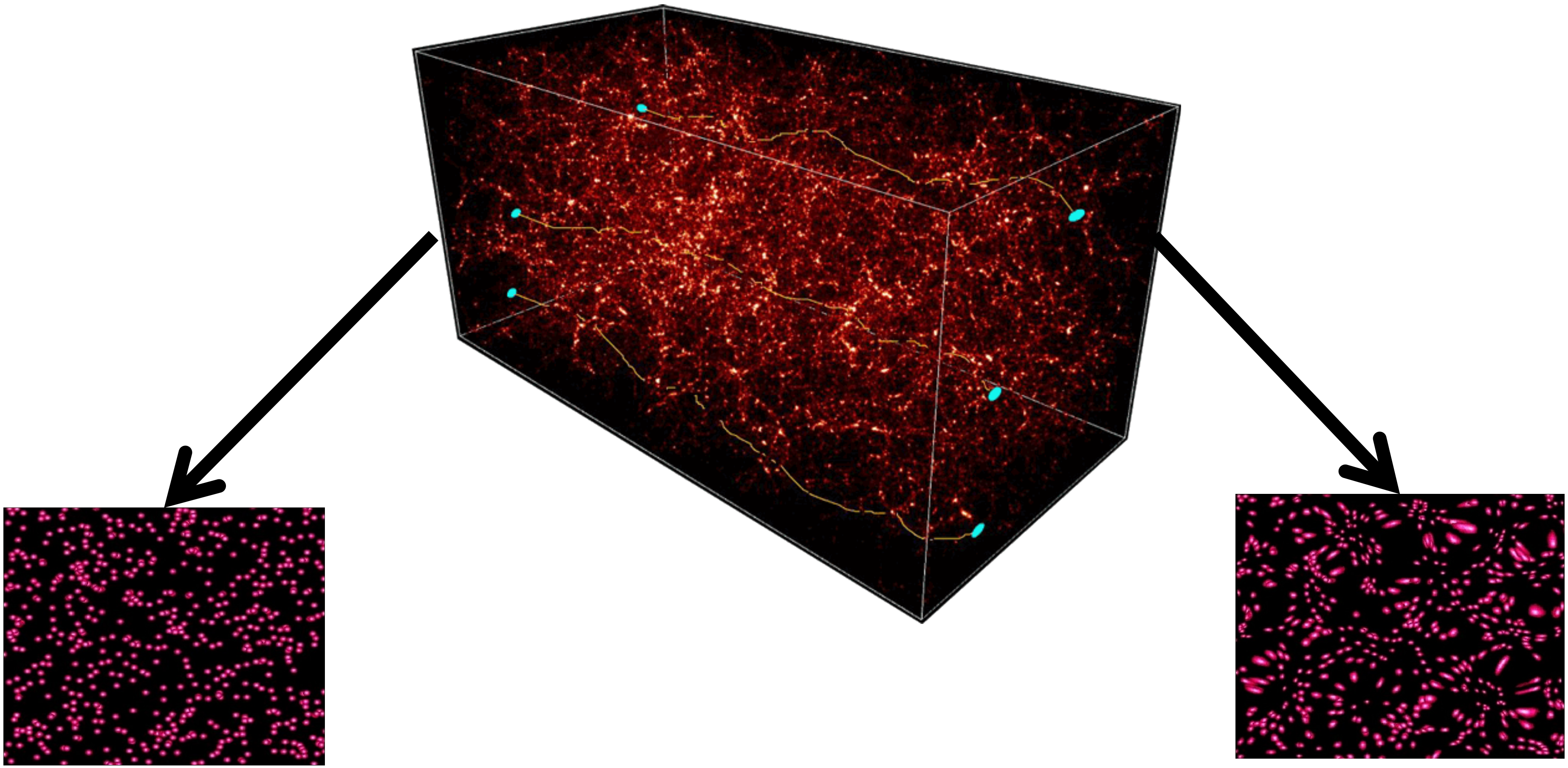
[CP, Jean-Philippe Uzan, Pereira, arXiv:1203.4069]

Galaxy Cluster Abell 2218 HST • WFPC2

NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

Lensing

Large scale structure



Distribution of matter

Gravitational lensing

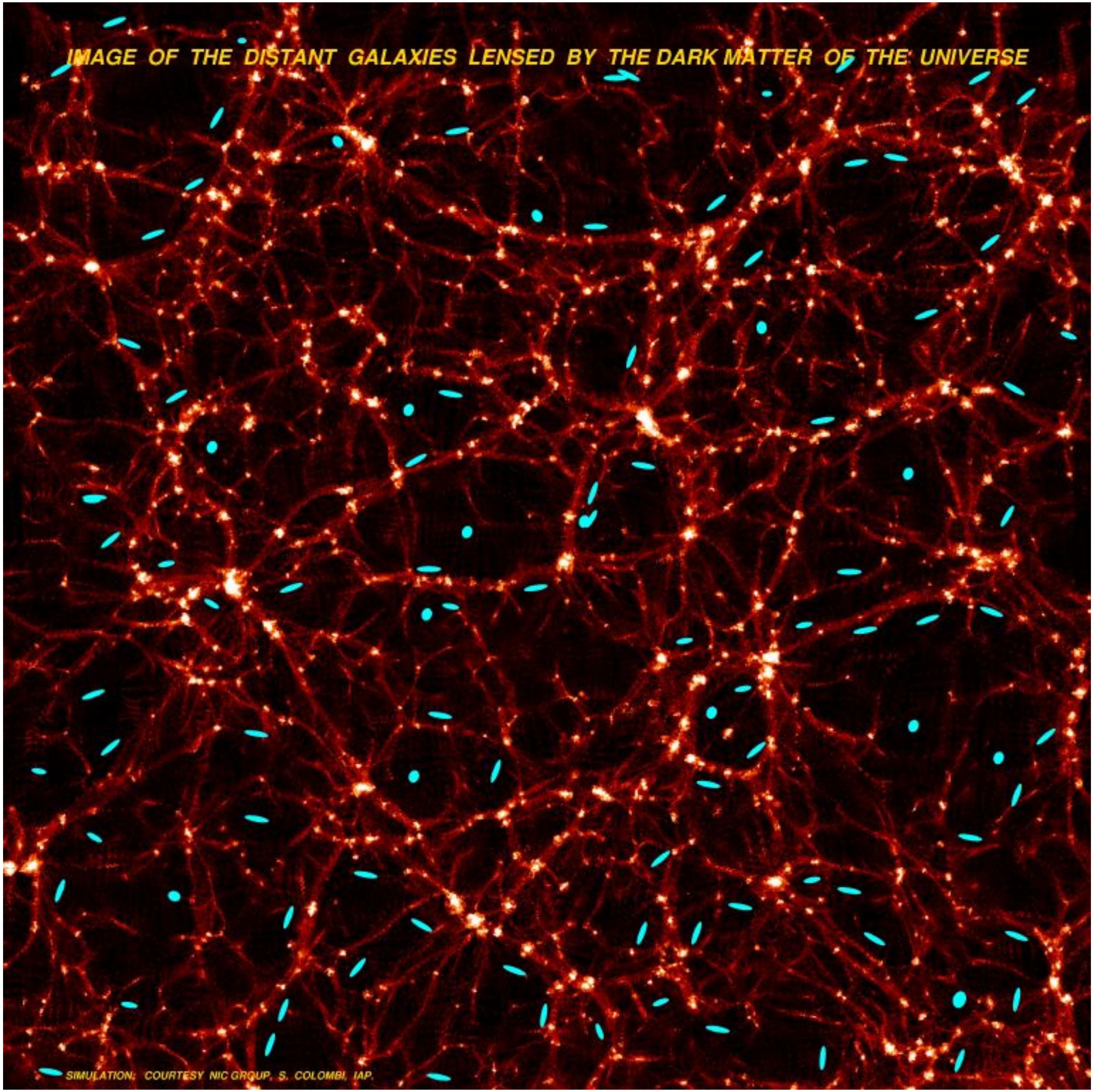


IMAGE OF THE DISTANT GALAXIES LENSED BY THE DARK MATTER OF THE UNIVERSE
SIMULATION: COURTESY NIC GROUP, S. COLOMBI, IAP.

Lensing (geodesic deviation)

-We define: k^μ as the tangent vector of the null-geodesic $$k^\mu \nabla_\mu k^\nu = 0$$

u^μ as the 4-velocity of the observer

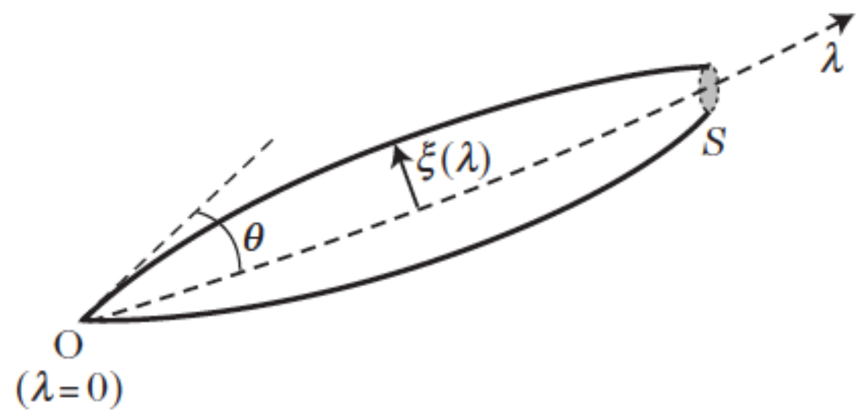
We then introduce n^μ, the spacelike unit vector pointing along the line of sight

$$\hat{k}^\mu \equiv U^{-1} k^\mu = -u^\mu + n^\mu \qquad u^\mu n_\mu = 0\,, \quad n_\mu n^\mu = 1$$

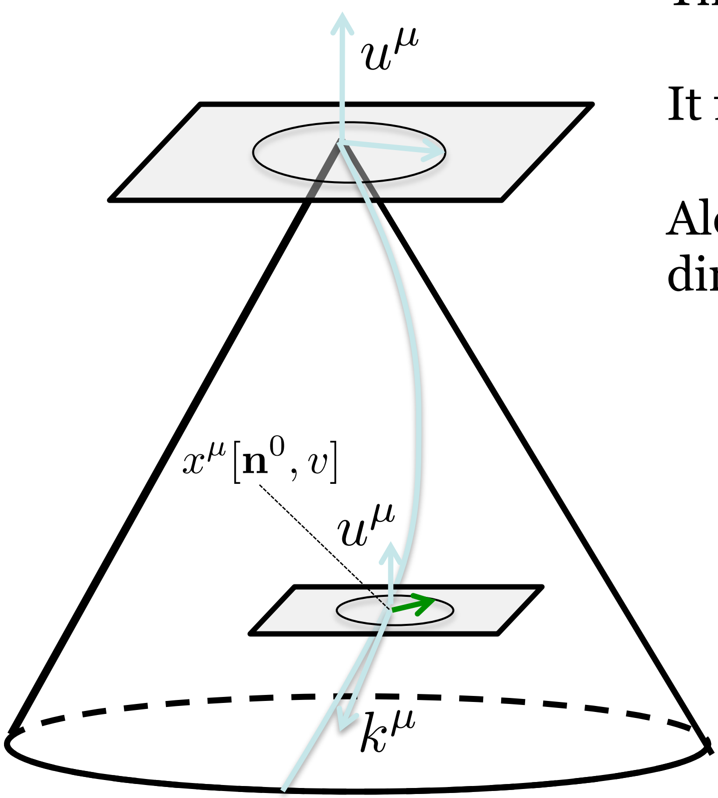
The deviation vector is the taken from a reference geodesic in the bundle

$$x^\mu(\lambda) = \bar{x}^\mu(\lambda) + \xi^\mu(\lambda),$$

$$\frac{D^2}{d\lambda^2}\xi^\mu = R^\mu{}_{\nu\alpha\beta} k^\nu k^\alpha \xi^\beta$$



Screen description



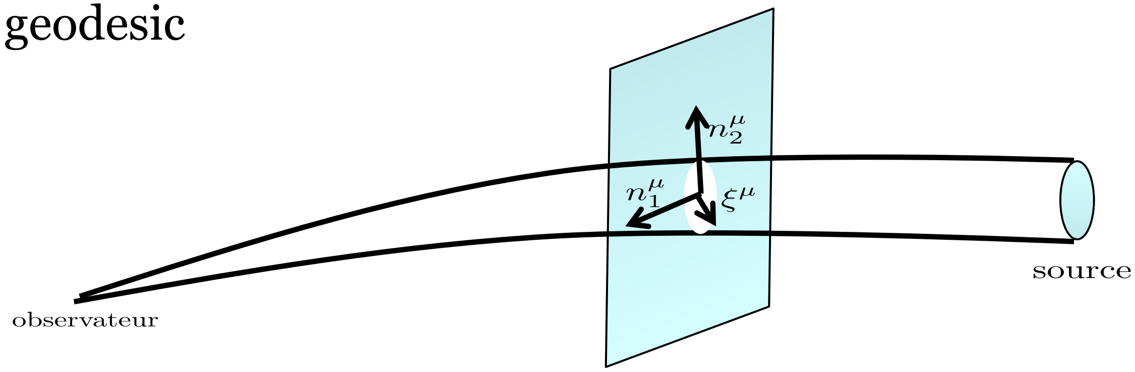
The integration of the geodesic equation gives $x^\mu[\boldsymbol{n}^o, v]$.

It follows that we get $\boldsymbol{n}[\boldsymbol{n}^o, v]$.

Along the geodesic, we can define a basis of the 2-dimensional space orthogonal to $\boldsymbol{n}$.

$$n_a^\mu n_{b\mu} = \delta_{ab}\,, \quad n_a^\mu u_\mu = n_a^\mu n_\mu = 0\,, \quad (a = 1, 2)$$

This basis can be parallely transported along the geodesic



From this basis, we can define the helicity basis by

$$\mathbf{n}_\pm = \frac{1}{\sqrt{2}}(\mathbf{n}_1 \pm i\mathbf{n}_2)$$

Sachs equation

We decompose the connecting vector on the basis of the screen to obtain

$$\frac{\mathrm{d}^2\eta_a}{\mathrm{d}v^2} = \mathcal{R}_{ab}\eta^b \qquad \text{with} \qquad \mathcal{R}_{ab} \equiv R_{\mu\nu\alpha\beta}k^\nu k^\alpha n_a^\mu n_b^\beta$$

The linearity of this equation implies that $\eta^a(v) = \mathcal{D}^a_b(v)\left(\mathrm{d}\eta^b/\mathrm{d}v\right)|_{v=0}$

Sachs equation	Initial condition
$\frac{\mathrm{d}^2}{\mathrm{d}v^2}\mathcal{D}^a_b = \mathcal{R}^a_c\mathcal{D}^c_b$	$\mathcal{D}^a_b(0) = 0\,, \quad \frac{\mathrm{d}\mathcal{D}^a_b}{\mathrm{d}v}(0) = \delta^a_b$

Program

Sachs equation

$$\frac{\mathrm{d}^2}{\mathrm{d}v^2}\mathcal{D}^a_b = \mathcal{R}^a_c\mathcal{D}^c_b$$

Initial condition

$$\mathcal{D}^a_b(0) = 0\,, \qquad \frac{\mathrm{d}\mathcal{D}^a_b}{\mathrm{d}v}(0) = \delta^a_b$$

Derivative on central geodesic

Source related to curvature

Deformation Matrix

1- Express the projected Riemann tensor
in terms of Ricci / E&B of the Weyl

2- Decompose the Jacobi matrix
in terms of a convergence/shear (E&B)/rotation

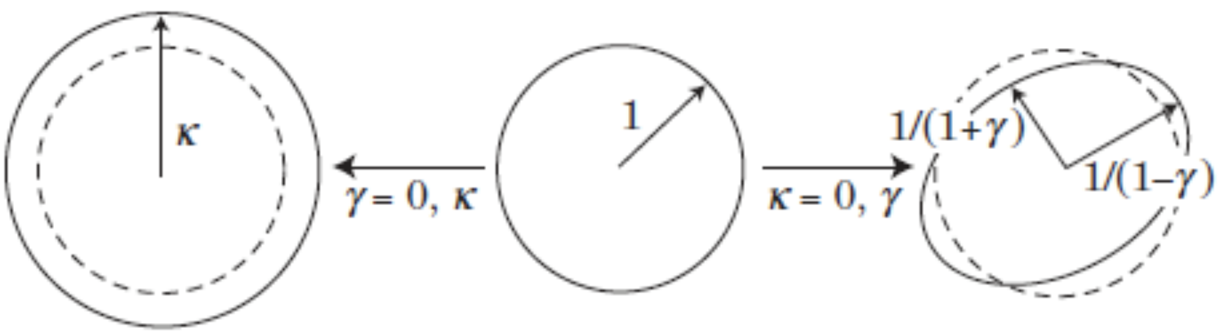
3- Derive an equation of propagation for these components

4- Perform an harmonic decomposition.

Decomposition of the Jacobi matrix

$$\mathcal{D}_{ab} \equiv \kappa I_{ab} + V\epsilon_{ab} + \gamma_{ab}$$

$$\epsilon_{ab} = 2\mathrm{i} n^{-}_{[a} n^{+}_{b]}$$



$$\gamma_{ab}(\boldsymbol{n}^o, v) \equiv \sum_{\lambda=\pm} \gamma^{\lambda}(\boldsymbol{n}^o, v) n^{\lambda}_a n^{\lambda}_b$$

Expression of the projected Riemann tensor

The projected Riemann tensor takes the form

$$\mathcal{R}_{ab} = U^2 \left(\mathcal{R} I_{ab} + \mathcal{W}_{ab} \right)$$

Ricci

Weyl

$$\mathcal{R} \equiv -\frac{1}{2} R_{\mu\nu} \hat{k}^\mu \hat{k}^\nu \,, \quad \mathcal{W}_{ab} \equiv C_{\mu\rho\sigma\nu} \hat{k}^\rho \hat{k}^\sigma n_a^\mu n_b^\nu$$

The Ricci term is a scalar.

The Weyl term is a spin-2.

$$\mathcal{W}_{ab}(\boldsymbol{n}^o, v) \equiv -2 \sum_{\lambda=\pm} \mathcal{W}^\lambda(\boldsymbol{n}^o, v) n_a^\lambda n_b^\lambda.$$

It can be expressed as

$$\mathcal{W}_{ab}(\boldsymbol{n}^o, v) = -2 n^\mu_{\langle a} n^\nu_{b\rangle} \left[\mathcal{E}_{\mu\nu} + \mathcal{B}_\mu{}^\sigma \epsilon_{\sigma\nu}(\boldsymbol{n}) \right] \Big|_{\substack{x^\mu(\boldsymbol{n}^o, v) \\ \boldsymbol{n}(\boldsymbol{n}^o, v)}},$$

with $\mathcal{E}_{\mu\nu} \equiv C_{\mu\rho\nu\sigma} u^\rho u^\sigma \,, \quad \mathcal{B}_{\mu\nu} \equiv \dfrac{1}{2} \varepsilon_{\mu\alpha\beta\sigma} u^\sigma C_{\nu\rho}{}^{\alpha\beta} u^\rho$

Propagation of the degrees of freedom

The Sachs equation gives the equation of propagation for the components of the Jacobi matrix

$$\boxed{\left(\frac{\mathrm{d}^2}{\mathrm{d}\hat{v}^2} + H_{\|}\frac{\mathrm{d}}{\mathrm{d}\hat{v}} - \mathcal{R}\right)\begin{pmatrix}\kappa \\ \mathrm{i}V \\ \gamma^{\pm}\end{pmatrix} = -2\begin{pmatrix}\mathcal{W}^{(-\gamma^{+})} \\ \mathcal{W}^{[-\gamma^{+}]} \\ \mathcal{W}^{\pm}(\kappa \pm \mathrm{i}V)\end{pmatrix}}$$

The integration of this system requires:

- to determine the light-cone structure

  $\boldsymbol{n}^a(\boldsymbol{n}^o, v)$, $H_{//}(\boldsymbol{n}^o, v)$,....

  in this sense, this equation is non-local.

  in general $\mathbf{n}(\boldsymbol{n}^o, v) \neq \boldsymbol{n}^o$

- the extraction of the component +/- also depends on $\boldsymbol{n}^a(\boldsymbol{n}^o, v)$

  $\boldsymbol{n}^o$ is the direction of observation.

Harmonic space

$$\boxed{\mathcal{D}_{ab} \equiv \kappa I_{ab} + V\epsilon_{ab} + \gamma_{ab}}$$

$$\gamma_{ab} = D_a D_b E + \epsilon_{ac} D^c D_b B$$

Depends on the observing direction $\mathbf{n}^o$

$$E(\mathbf{n}^o) = \sum_{\ell\, m} E_{\ell m} Y_{\ell m}(\mathbf{n}^o)$$

$$\kappa(\mathbf{n}^o) = \sum_{\ell\, m} \kappa_{\ell m} Y_{\ell m}(\mathbf{n}^o)$$

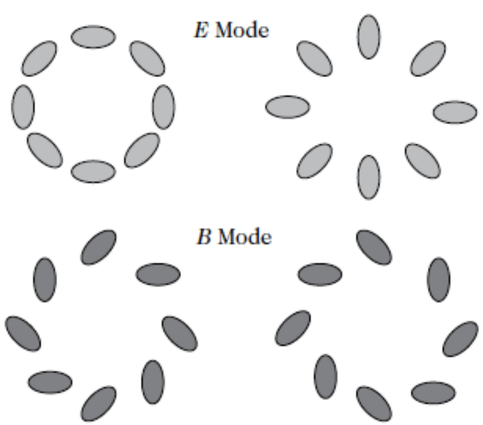
$$B(\mathbf{n}^o) = \sum_{\ell\, m} B_{\ell m} Y_{\ell m}(\mathbf{n}^o)$$

Harmonic dictionary

$$\boxed{D_a \to \ell}$$

Summary of multipolar decomposition

	Scalars	Spin-2
Jacobi matrix *[shape of the bundle]*	$\kappa(\boldsymbol{n}^o,\hat{v}) = \sum_{\ell,m} \kappa_{\ell m}(\hat{v}) Y_{\ell m}(\boldsymbol{n}^o)$ $V(\boldsymbol{n}^o,\hat{v}) = \sum_{\ell,m} V_{\ell m}(\hat{v}) Y_{\ell m}(\boldsymbol{n}^o)$	$\gamma^{\pm}(\boldsymbol{n}^o,\hat{v})$
Source terms *[Properties of the spacetime]*	$\mathcal{R}(\boldsymbol{n}^o,\hat{v}) = \sum_{\ell,m} \mathcal{R}_{\ell m}(\hat{v}) Y_{\ell m}(\boldsymbol{n}^o)$ $H_{\parallel}(\boldsymbol{n}^o,\hat{v}) = \sum_{\ell,m} h_{\ell m}(\hat{v}) Y_{\ell m}(\boldsymbol{n}^o)$	$\mathcal{W}^{\pm}(\boldsymbol{n}^o,\hat{v})$



$$\mathcal{W}^{\pm}(\boldsymbol{n}^o,\hat{v}) = \sum_{\ell,m} \left[\mathcal{E}_{\ell m}(\hat{v}) \pm \mathrm{i}\mathcal{B}_{\ell m}(\hat{v})\right] Y^{\pm 2}_{\ell m}(\boldsymbol{n}^o)$$

$$\gamma^{\pm}(\boldsymbol{n}^o,\hat{v}) = \sum_{\ell,m} \left[\underbrace{E_{\ell m}(\hat{v})}_{(-1)^{\ell}} \pm \mathrm{i}\underbrace{B_{\ell m}(\hat{v})}_{(-1)^{\ell+1}}\right] Y^{\pm 2}_{\ell m}(\boldsymbol{n}^o)$$

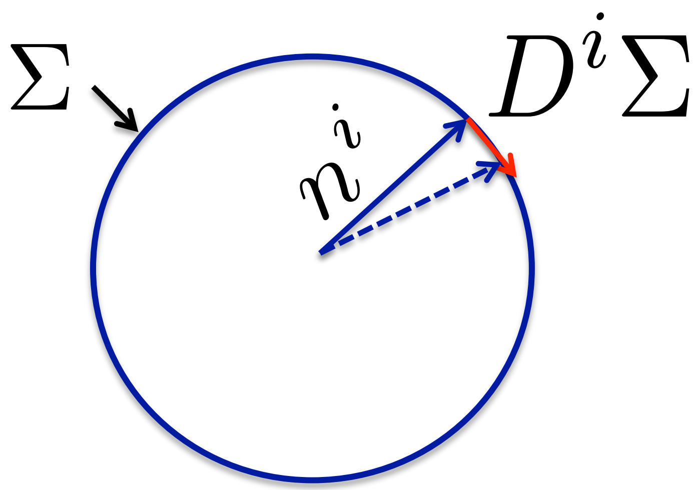
Testing (late-)anisotropy with weak-lensing

Observation? Effects on geodesic

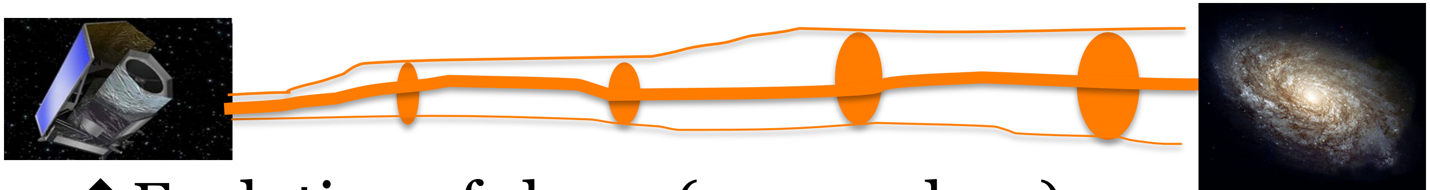
- Evolution of Energy $$\dot{E} + HE + 2\Sigma E = 0$$

- Evolution of direction $$\dot{n}^i = -D^i\Sigma$$

Lensing Potential $$\boxed{\Sigma = \frac{1}{2}\sigma_{ij}n^i n^j}$$



Effect on geodesic deviation (weak lensing)



◆ Evolution of shape (e.g. a galaxy)

$$\frac{\mathrm{d}^2\mathcal{D}^a_b}{\mathrm{d}\lambda^2} = \mathcal{R}^a_c\mathcal{D}^c_b$$

Deformation Matrix

Derivative on central geodesic

Source related to curvature

◆ FL case : $\mathcal{R}_{ab} = 2D_aD_b\Phi$

Gravitational potential
(metric perturbation)

Gradient orthogonal to line of sight

O Born Trajectory S

$$D_aD_b\Phi$$

Effect of anisotropic background

◆ Anisotropy as a perturbation

$$\text{Physical Space} \xrightarrow{\Phi} \text{Anisotropic Background} \xrightarrow{\frac{\sigma}{H},\ \beta_{ij}} \text{FL Background}$$

◆ Double expansion scheme.

◆ Effect of anisotropy appears at order $\dfrac{\sigma}{H}\Phi$

Dominant effect

Standard FL lensing

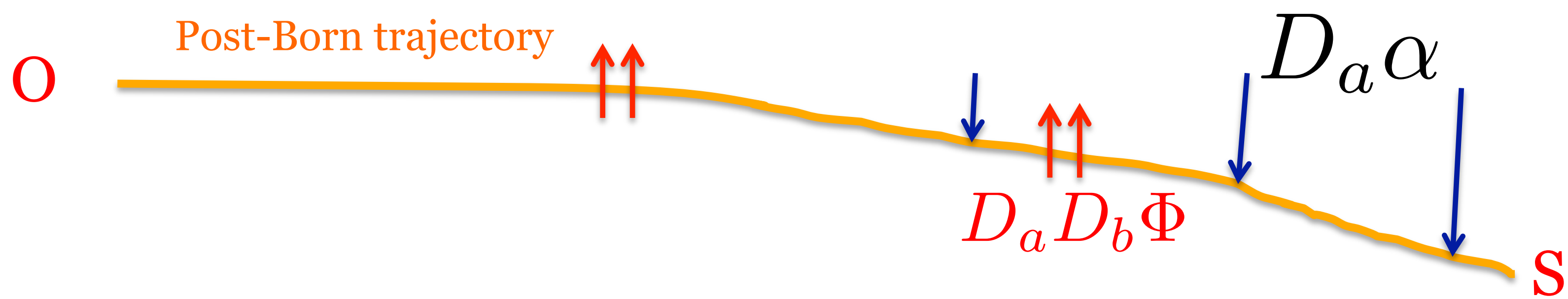
Only E modes, diagonal correlations

$$\gamma_{ab}^{\mathrm{FL}} \propto 2 D_a D_b \Phi \Rightarrow \langle E_{\ell m} E_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell^E$$

Anisotropic case. Dominant effect is

$$\gamma_{ab}^{\mathrm{Anis}} \propto D_c \alpha D^c \left(2 D_a D_b \Phi \right)$$

α is related to $\beta_{ij} n^i n^j$



Anisotropy acts like a deflecting potential on the central geodesic

E-B mode structure

- **Analogy with CMB lensing:**
  B modes generated by lensing.

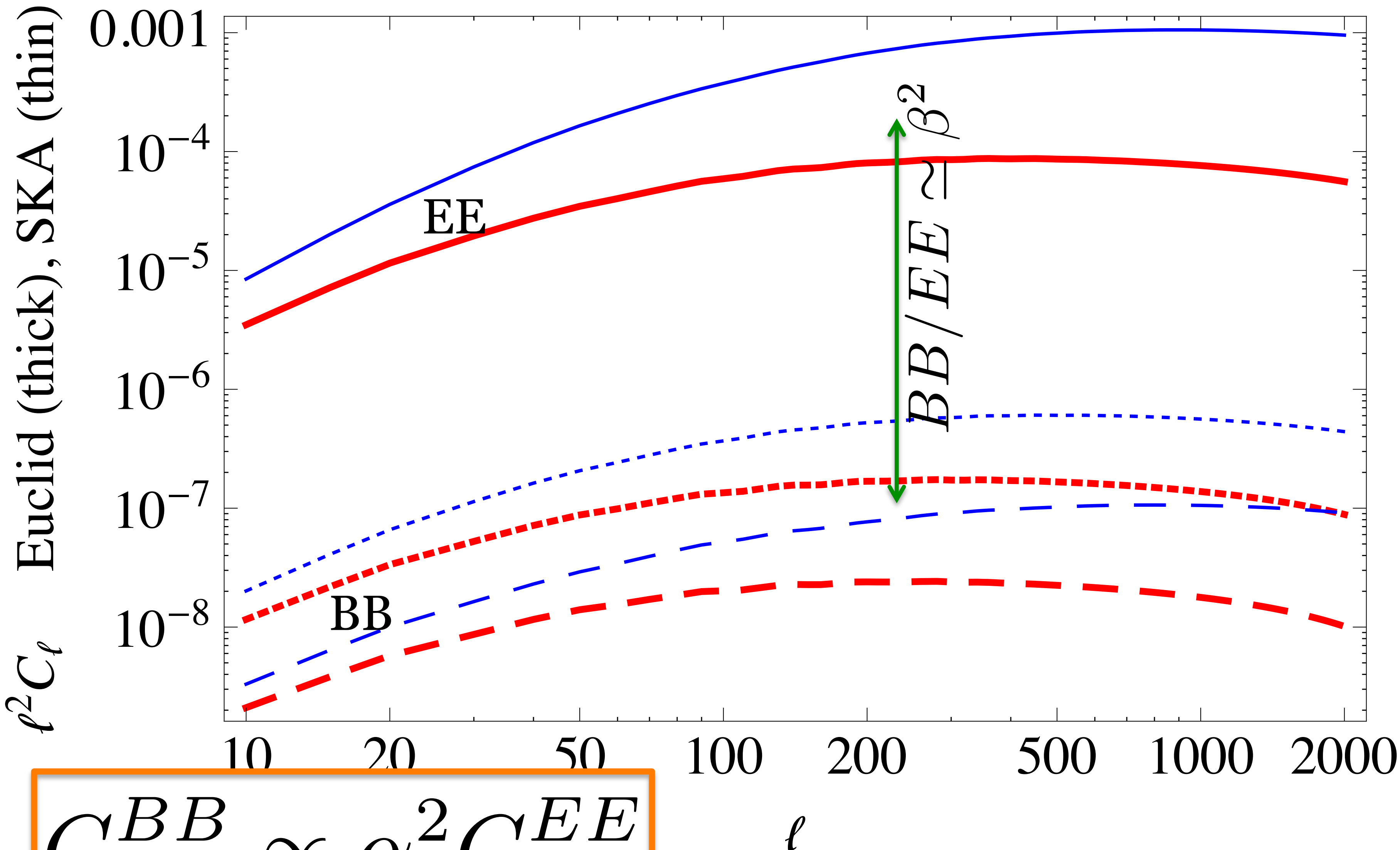
- **Difference with CMB lensing:**
  The lensing potential is a pure quadrupole.
  The 5 degrees of freedom are those of β_{ij}

$$\alpha(\mathbf{n}^o) = \sum_m \alpha_{2m} Y_{2m}(\mathbf{n}^o)$$

$$\boxed{B_{\ell m} \propto \ldots \alpha_{2M} E^{\mathrm{FL}}_{\ell+1\, m-M} + \ldots \alpha_{2M} E^{\mathrm{FL}}_{\ell-1\, m-M}}$$

BB correlations

Cont: EE, Dashed and Dots: BB



$$\boxed{C_\ell^{BB} \propto \alpha^2 C_\ell^{EE}}$$

Off diagonal correlations. EE and EB.

The anisotropy generates also off-diagonal correlations

$$\langle E_{\ell m} E^{\star}_{\ell+2 m+M} \rangle \qquad \propto \alpha_{2M} C_{\ell}^{EE}$$

$$\langle B_{\ell m} E^{\star}_{\ell+1 m+M} \rangle$$

$$M = -2, -1, 0, 1, 2$$

Interesting for two reasons

- Of order β compared to EE correlations.
- Enable to get the components α_{2M} of the deflecting potential and thus the β_{ij} .

Extension: Direction dependent modulation

Let us consider a direction dependent modulation of a spin s observable

$$\tilde{X}^s(\mathbf{n}) = \alpha(\mathbf{n}) X^s(\mathbf{n})$$

X^s

- CMB temperature (s=0)
- CMB polarization (s=2)
- Weak lensing convergence (s=0)
- Weak Lensing shear (s=2)

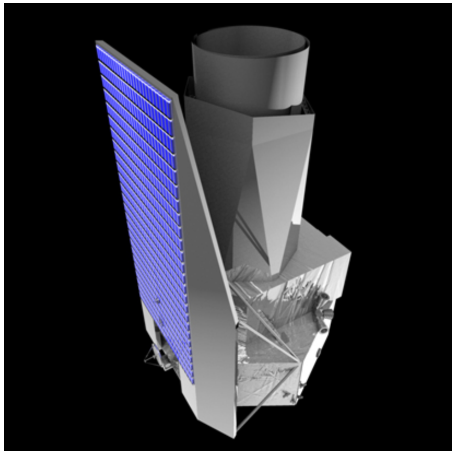
If $\alpha(\mathbf{n}) = \sum_{m=-L}^{L} \alpha_{Lm} Y_{Lm}(\mathbf{n})$

The correlations of the modulated field are

$$\langle B_{\ell m} E_{\ell' \, m+M} \rangle \;\; \ell' = \ell - L + 1, \ell - L + 3, \dots, \ell + L - 1$$

$$\langle E_{\ell m} E_{\ell' \, m+M} \rangle \quad \ell' = \ell - L, \ell - L + 2 \dots, \ell + L$$

What to expect with Euclid



(i) Visible imaging (ii) NIR photometry (iii) NIR spectroscopy.
15,000 square degrees
100 million redshifts, 2 billion images
Median z~1

I will provide:
- P(k,z) on 15,000 square degrees, 70,000,000 galaxy redshift with 0.5<z<2.
- weak lensing on 15,000 square degrees, 40 galaxy images per square arcmin with 0.5<z<3.

The error bars on the B-modes should be divided by (10-40) compared to CFHTLS.

Linear regime typically for scales larger than 1 deg. And Euclid will probe scales up to ~40 deg.

It will probe large scales for which astrophysical effects leading to B-modes are important.

Conclusion.

- BB correlations in weak lensing can probe a late anisotropic era
- The sensitivity in β^2 is given by the upper bounds on C_ℓ^{BB}

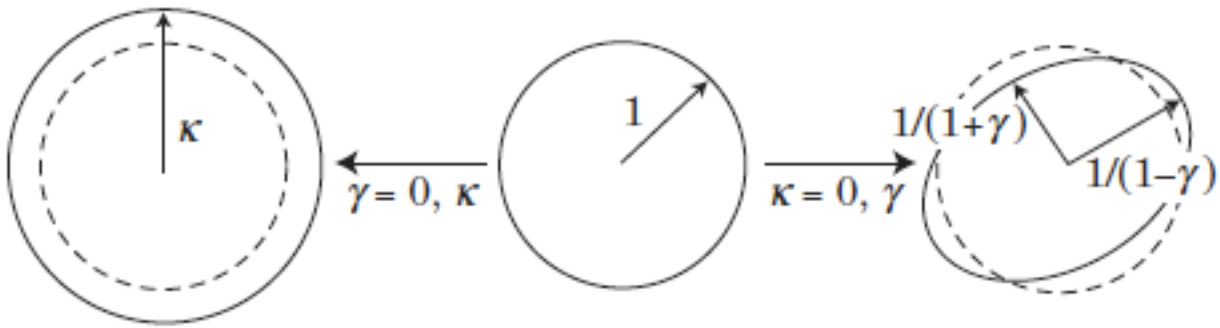
- Off diagonal correlations are more powerful (first order in β)
- Off diagonal correlations allow for consistency checks.

- Euclid

Thanks

Decomposition of the Jacobi matrix

$$\mathcal{D}_{ab} \equiv \kappa I_{ab} + V\epsilon_{ab} + \gamma_{ab} \qquad \epsilon_{ab} = 2in^{-}_{[a}n^{+}_{b]}$$



The shear can be shown to be a spin-2 and can thus be decomposed as

$$\gamma_{ab}(\boldsymbol{n}^o, v) \equiv \sum_{\lambda=\pm} \gamma^{\lambda}(\boldsymbol{n}^o, v) n^{\lambda}_a n^{\lambda}_b$$

Linear perturbations in the FL case

We work at linear order in perturbation

3 types of modes: S, V, T. Only S are important at low redshift.

$$\mathcal{R}^{(1)}_{ab} = -D_a D_b(\Phi + \Psi)$$

This implies that the Weyl has no magnetic part: $\mathcal{B}^{(1)}_{ab} = 0$

We can work in the Born approximation: $\boldsymbol{n}(\boldsymbol{n}^o, \hat{v}) = \boldsymbol{n}^o$ so that only $h_{oo} \neq 0$

Shear: E and B modes

$$E^{(1)}_{\ell m} \to \left(\mathcal{R}^{(0)}_{00} - h^{(0)}_{00}\frac{d}{d\hat{v}}\right) E^{(1)}_{\ell m} - 2\kappa^{(0)}_{00}\,\mathcal{E}^{(1)}_{\ell m}$$

$$B^{(1)}_{\ell m} \to \left(\mathcal{R}^{(0)}_{00} - h^{(0)}_{00}\frac{d}{d\hat{v}}\right) B^{(1)}_{\ell m}$$

Only E-modes are sourced.

Why investigating (an)-isotropy in expansion?

- To test the cosmological principle
  - We keep the homogeneity assumption
  - We release the isotropy assumption and want to constrain the anisotropy.

- To constrain the nature of Dark Energy

  We investigate a possible homogeneous anisotropic stress.