

(Re)visiting (Non)decoupling

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based in part on our work in progress with J. Brehmer (ITP Heidelberg), A. Freitas (Pittsburg U.)
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CP3 - Université catholique de Louvain



Lunch seminars @ CP3 - May 27th 2015

Outline

- 1 Setting the ground
- 2 Theoretical aspects
 - The Appelquist–Carazzone theorem
 - Generalizations
 - Decoupling breakdown
 - The role of symmetries
 - (Non)decoupling & EFTs
- 3 Non-decoupling in Higgs physics BSM
- 4 Closing remarks

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Motivation (I) - Historical Background



TIMEFRAME ['80s]

CONTEXT GUT model building

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Kazama & Yao Phys. Rev. D 25 (1982) 1605

♠ *A central idea behind the unification of forces of vastly different strength is that such an apparent hierarchy arises not from the difference of the fundamental coupling constants of the theory but rather from that of the masses of the exchanged particles.*

♠ *In constructing a viable unified theory, these heavy particles must be incorporated into the structure with due caution. Among the most important requirements are (i) that superheavy particles must effectively decouple at low energies, (ii) that correct effective light-particle theory must emerge at low energies, and (iii) that the mass hierarchy, arranged at the tree level, should be stable against radiative corrections. As we shall see, these requirements are deeply interrelated. None of them are trivial to satisfy.*

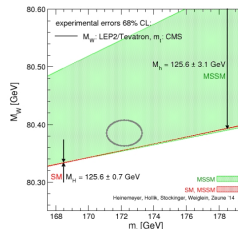
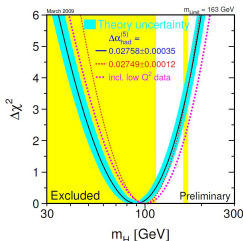
Motivation (II) - Phenomenological interest

- ♠ **DECOUPLING** – a somehow implicit notion
- Collider physics itself
 - Effective field theory & simplified model approaches to new physics
 - Model-independent new physics parametrization (S, T, U parameters)

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- ♠ **NONDECOUPLING** – – a somehow implicit notion (as well)
 - Precision EW and heavy flavor physics as probes of missing SM building blocks
 - $m_t - m_W$ correlations, m_H blueband
 - Precision EW & heavy flavor physics constraints on new physics



Keywords

High scale Λ_{UV} Heavy fields Φ Large masses M $\hat{\mathcal{O}}(\Lambda_{\text{UV}})$ $\mathcal{L}_{\text{UV}}(\Phi, \varphi; g, g^*, m, M)$



for $M \rightarrow \infty$

Low scale Λ_{IR} Light fields φ low masses m $\hat{\mathcal{O}}(\Lambda_{\text{IR}})$ $\mathcal{L}_{\text{IR}}(\varphi; g, m)$

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DECOUPLING



$\hat{\mathcal{O}}(\Lambda_{IR})$ UV-insensitive for $M \rightarrow \infty$



heavy field effects undetectable for $M \rightarrow \infty$

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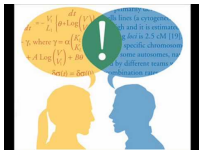
heavy field effects undetectable for $M \rightarrow \infty$

A more formal definition

The property by which field theory amplitudes computed from $\mathcal{L}_{\text{IR}}$ and $\mathcal{L}_{\text{UV}}$ are coincident in the asymptotic heavy mass limit – up to at most a redefinition of the renormalizable parameters in $\mathcal{L}_{\text{IR}}$

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NONDECOUPLING



$\mathcal{O}(\Lambda_{\text{IR}})$ are UV-sensitive for $M \rightarrow \infty$



heavy field effects remain for $M \rightarrow \infty$

A more formal definition

Remainder of the UV-scale dynamics in low-energy observables for asymptotically large heavy field masses

Questions ahead

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- What fundamental field theory mechanisms are responsible for decoupling?
- What field theory mechanisms lead to the breakdown of decoupling conditions?
- What are the implications of non-decoupling for BSM Higgs physics?
- What is the interplay between non-decoupling effects and effective field theories?

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The Appelquist–Carazzone theorem

♠ The notion of heavy field decoupling - a first QFT formalization :

♠ K. Symanzik (1973), *Infrared singularities and small distance behavior analysis*

♠ T. Appelquist, J. Carazzone, (1975), *Infrared Singularities and Massive Fields*

Phys.Rev. **D11** (1975) 2856

We examine some problems associated with the low-momentum behavior of gauge theories and other renormalizable field theories. Our main interest is in the infrared structure of unbroken non-Abelian gauge theories and how this is affected by the presence of other heavy fields coupled to the massless gauge fields. It is shown in the context of a simple model of gauge mesons coupled to massive fermions that the heavy fields decouple at low momenta except for their contribution to renormalization effects.

The Appelquist–Carazzone theorem

♠, **Setup:** Simplified model of massless gauge bosons coupled to massive fermions:

$$\begin{aligned}
 \mathcal{L}_{\text{full}} &= \underbrace{-\frac{1}{4} F^{\mu\nu} F_{\mu\nu}}_{\text{light sector}} + \underbrace{i\bar{\psi}\not{D}\psi + M\bar{\psi}\psi}_{\text{heavy sector}} + \underbrace{\mathcal{L}_{\varphi_{\mu}-\psi}}_{\text{light-heavy cross-talk}} \\
 \mathcal{L}_{\text{full}} &= \mathcal{L}_{\text{IR}(\varphi, \{g\})} + \mathcal{L}_{\text{UV}(\varphi, \{g, g^*, M\})} + \mathcal{L}_{\varphi_{\mu}-\psi}
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$G^{(n)}(p_1, \dots, p_n; \mu)$ generic n-point light field Green's function computed from $\mathcal{L}_{\text{full}}$

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$$\underbrace{G^{(n)}(p_1, \dots, p_n; \mu)}_{\text{from } \mathcal{L}_{\text{full}}} = Z^{n/2} \underbrace{\tilde{G}^{(n)}(p_1, \dots, p_n; \mu)}_{\text{from } \mathcal{L}_{\text{eff}}} + \mathcal{O}(1/M),$$

$\mathcal{L}_{\text{eff}} \equiv \mathcal{L}_{\text{IR}}(\{\tilde{g}_i\})$, with $\tilde{g}_i \equiv \tilde{g}_i(\{g_i, M\})$ and $Z \equiv Z(\{g_i, M\})$.

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Proof keypoints:



SCALING



RENORMALIZABILITY

Generalizing Appelquist–Carazzone

♠ **Puzzling!!** – while GUT fields decouple, heavy SM fields do not

Examples

- Large charm mass effect in the $\mathcal{O}(\alpha_s)$ corrections to the axial isoscalar weak current: [Collins, Wilczek, Zee \[’78\]](#)
- Large top-mass correction to $\delta\rho$: [Marciano \[’75\]](#); [Toussaint \[’78\]](#)

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Generalized conditions for decoupling

- 1 A separably renormalizable light sector, which remains so at each possible stage of symmetry breaking
- 2 Asymptotically weak couplings

Decoupling breakdown: categories

A Nondecoupling taxonomy

- 1 Intrinsic , in situations beyond the validity range of the AC theorem
- 2 Accidental , in situations for which AC holds though is apparently violated

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Intrinsic

- One VEV + hierarchical Yukawas: c or t quarks in the SM
- Strong coupling: Higgsless SM



Accidental

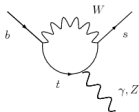
- Renormalization scheme – induced: On-shell Z' sector renormalization
- UV-scale induced: linked gauge couplings from GUTs
- Delayed decoupling: GUT-induced B -violating interactions

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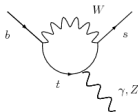


EW penguin contributing to the

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EW penguin contributing to the

$B \rightarrow X_S \gamma$ FCNC decay

$$\mathcal{M}(b \rightarrow s\gamma) = \sum_{q=u,c,t} \lambda_q F(x_q) \quad \text{where} \quad \lambda_q = V_{qb}^* V_{qs}$$

- ♠ CKM unitarity implies $\lambda_u + \lambda_c + \lambda_t = 0$ (tree-level GIM)

$$\text{♠ } F(x_q) \sim \frac{m_q^2}{m_W^2} \text{ resulting from a chirality flip} \Rightarrow \mathcal{O}\left(\frac{m_t^2}{m_W^2}\right)$$

- If all quarks were close-by in mass and hence separably renormalizable, GIM suppression would be stronger – non-decoupling would be tamed
- **Example:** FCNC top-quark decays – non-decoupling is at most $\mathcal{O}(m_b^2/m_W^2)$

The role of symmetries (I)

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- **renormalizability** protects Green's functions from receiving divergent contributions not absorbable into CTs
- While guaranteed, UV finiteness may rely on delicate cancellations between light and heavy virtual particle contributions
- Heavy mass dependence as the remainder of the would-be UV poles if the heavy fields were removed.

The role of symmetries (II)

Heavy fields decouple if:

♠ respect $SU(2)_L \otimes U(1)_Y$ and hence its breaking pattern

♠ Example: See-saw neutrino mass

$$\mathcal{L} \sim M(N_R^c)^T C^{-1} N_R + \bar{L}_L Y_N N_R + \text{h.c.} \quad \Rightarrow \quad M_{\nu, \text{eff}} \sim -\frac{Y_N^T Y_N}{M} v^2$$

♠ break $SU(2)_L \otimes U(1)_Y$ via a singlet VEV whilst remain separably renormalizable

♠ Example: Degenerate $SU(2)_L$ doublets – contributing to the T parameter as

$$T \sim G_F \frac{M_1^2 M_2^2}{M_1^2 - M_2^2} \log \left(\frac{M_1^2}{M_2^2} \right) - \frac{M_1^2 + M_2^2}{2} \sim \frac{(M_1^2 - M_2^2)}{M_1^2}$$

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Heavy fields do not decouple if:

♠ Strongly coupled – mass hierarchy = single scale(v^2) + coupling hierarchy)

♠ Not separably renormalizable

♠ Example: m_t -imprints on low-energy observables

$$\alpha T = \frac{3G_F}{8\sqrt{2}\pi^2} F_0(m_t^2, m_b^2) \quad \text{where} \quad F_0(m_t^2, m_b^2) = m_t^2 + m_b^2 - \frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \log \left(\frac{m_t^2}{m_b^2} \right)$$

The role of symmetries (III)

- ♠ Underlying/Accidental symmetries can temper non-decoupling effects

- ♠ Examples:

- Custodial symmetry leads to Veltman screening
- GIM mechanism keeps FCNC meson decay rates small
- Supersymmetry prevents mass splittings (viz. large T) from SUSY heavy Higgs bosons

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- delayed decoupling [B -violating interactions in GUTs]
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- Non-linearly realized symmetries are intrinsically linked to non-decoupling

(Non)decoupling and Effective Field Theories

🔹 Back to History: Kazama & Yao Phys. Rev. D 25 (1982) 1605

♠ *It is clear in the case of theories without spontaneously broken symmetry one cannot talk about decoupling without the existence of an effective light-particle theory, because in its absence we cannot absorb the large mass effects by redefinition of the parameters of the light theory. These two concepts are, therefore, two sides of one and the same subject.*

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$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{i_D} \sum_{D>4} \frac{C_{i_D}^{(D)}}{\Lambda^{D-2}} \mathcal{O}_{i_D}^{(D)}$$

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🔸 WEAK COUPLING:

Passarino ['12]

• Decoupling:

$$C_{i_D}^{(D)} \sim \mathcal{O}(1/\Lambda)$$

• Screening:

$$C_{i_D}^{(D)} \sim \log(\Lambda)$$

• Non-decoupling:

$$C_{i_D}^{(D)} \sim \mathcal{O}(\Lambda) \quad (\text{or higher})$$

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• Non-decoupling: $C_{i_D}^{(D)} \sim \mathcal{O}(\Lambda)$ (or higher)

♠ Non-decoupling **modifies** the naive correspondence between **mass dimension** and **perturbative** expansions

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♠ **STRONG COUPLING:** effective operators built from gauge-invariant operators for which the gauge symmetry is realized non-linearly

CCWZ formalism - EW_χ Lagrangian

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Loop-induced Higgs couplings: $H \rightarrow gg$

♠ We consider a Higgs scalar partner $\mathcal{L} \supset -\mu_S^2 |S|^2 - \lambda_s \Phi^\dagger \Phi |S|^2$,

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$$g_{hGG} \sim \frac{\lambda v^2}{m_S^2} \left\{ \right.$$

$\rightarrow \text{constant}$ if $m_S \rightarrow \infty$ while $\mu_S \ll m_s$

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→ constant if $m_S \rightarrow \infty$ while $\mu_S \ll m_s$

♠ Non-decoupling contributions are connected to LETs

$$\mathcal{L} \supset \frac{\alpha_s}{12\pi v} \left[s C_2(r_C) \frac{\partial \log(m(v))}{\partial \log(v)} \right] H G^{\mu\nu A} G_{\mu\nu}^A.$$

⇒ Φ as a background field to which the (dynamical) heavy fields couple as $M_\Phi = M_\Phi(v)$

2HDM mass spectrum



Scalar potential

$$\begin{aligned} V(\Phi_1, \Phi_2) = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] \\ & + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 \\ & + \left[\frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1) (\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \text{h.c.} \right] , \end{aligned}$$

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 \end{aligned}$$



Mass eigenvalues

$$\begin{aligned}
 m_{\text{H}\pm}^2 = & \underbrace{\frac{m_{12}^2}{\sin \beta \cos \beta}}_{\text{Decoupling}} - \underbrace{\frac{v^2}{2} [\lambda_4 + \lambda_5 + \lambda_6 \cot \beta + \lambda_7 \tan \beta]}_{\text{non-decoupling}} \\
 m_{\text{A}0}^2 = & \frac{m_{12}^2}{\sin \beta \cos \beta} - \frac{v^2}{2} [2\lambda_5 + \lambda_6 \cot \beta + \lambda_7 \tan \beta]
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♠ Mass eigenvalues

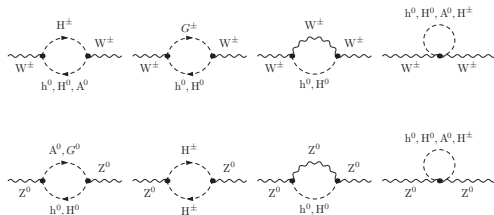
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 \end{aligned}$$

♠ Decoupling limit

$$\Phi_1 \supset \begin{pmatrix} H^+ \\ A^0 \\ H^- \end{pmatrix} \oplus H^0 \quad \text{or} \quad \Phi_1 \supset \begin{pmatrix} H^+ \\ H^0 \\ H^- \end{pmatrix} \oplus A^0 \quad \Phi_2 \supset \begin{pmatrix} G^+ \\ G^0 \\ G^- \end{pmatrix} \oplus \frac{v + h^0}{\sqrt{2}}.$$

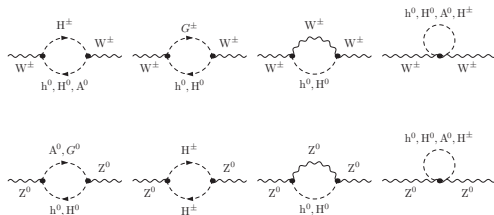
all Higgs states fall into the singlet and triplet representations of $SU(2)_{L+R}$

2HDM contributions to the T parameter



$$\begin{aligned}
 (\alpha_{em} T)_{\text{2HDM}} \supset & \frac{G_F}{8\sqrt{2}\pi^2} \left\{ m_{H^\pm}^2 \left[1 - \frac{m_{A^0}^2}{m_{H^\pm}^2 - m_{A^0}^2} \ln \frac{m_{H^\pm}^2}{m_{A^0}^2} \right] \right. \\
 & + \cos^2(\beta - \alpha) m_{h^0}^2 \left[\frac{m_{A^0}^2}{m_{A^0}^2 - m_{h^0}^2} \ln \frac{m_{A^0}^2}{m_{h^0}^2} - \frac{m_{H^\pm}^2}{m_{H^\pm}^2 - m_{h^0}^2} \ln \frac{m_{H^\pm}^2}{m_{h^0}^2} \right] \\
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 \end{aligned}$$



Non-decoupling beyond screening

as

power-like prefactors to Veltman logs



$$(\alpha_{em} T)_{[\mathbf{2HDM}]} \simeq - \frac{3 G_F m_Z^2 s_W^2}{8\sqrt{2}\pi^2} \xi^2 \log \left(\frac{m_\Phi^2}{m_W^2} \right) \quad \text{for degenerate } m_{\Phi=H^0, A^0, H^\pm}$$

Loop-corrected Higgs couplings in the 2HDM

♠ Coupling shift parametrization: $\hat{\kappa}_f \equiv \frac{\Gamma_{hff}^S}{\Gamma_{hff,\text{SM}}^S} \equiv (1 + \Delta_f)$

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♠ **Leading quantum effects on Higgs couplings** [Kanemura et. al. [arXiv:1502.07716](https://arxiv.org/abs/1502.07716)]

♠ **Loop Form factor:** $\Gamma_{hff}(p, k_1, k_2) = \Gamma_{hff}^S + \gamma_5 \Gamma_{hff}^P + \not{k}_1 \Gamma_{hff}^{V1} + \dots$

♠ **Coupling shift:** $1 + \Delta_t = \xi \cot \beta - \frac{1}{2} \xi^2 - \frac{1}{96\pi^2} \sum c_\Phi \frac{m_\phi^2}{v^2} \left(1 - \frac{M^2}{m_{\phi^2}^2}\right)^2$

$$- \frac{1}{12\pi^2} \cot^2 \beta \sum c_\Phi \left[\frac{m_t^4}{v^2 m_\phi^2} + \tan^2 \beta \frac{m_b^2 m_t^2}{v^2 m_{H^\pm}^2} \right]$$

with $\Phi = A^0, H^0, H^\pm$ and $\xi = \cos(\beta - \alpha) \simeq \frac{\tilde{\lambda} v^2}{M^2}$

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♠ **Non-decoupling contributions**

NLO EW in the full theory

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$\mathcal{O}^{\text{D}=8}$ in the EFT

$$\boxed{\frac{G^{\mu\nu A} G_{\mu\nu A} [\Phi^\dagger \Phi]^2}{\Lambda^4}}$$

Outline

- 1 Setting the ground
- 2 Theoretical aspects
 - The Appelquist–Carazzone theorem
 - Generalizations
 - Decoupling breakdown
 - The role of symmetries
 - (Non)decoupling & EFTs
- 3 Non-decoupling in Higgs physics BSM
- 4 Closing remarks

Closing remarks

Take-home ideas

♠ General conditions for decoupling

- A separably renormalizable heavy sector
- Asymptotically weak couplings
- Light and heavy sectors not linked by renormalization conditions

♠ Categories of non-decoupling:

- **Accidental** UV-scale induced; renormalization-induced; delayed decoupling
- **Intrinsic**: strong coupling; one single scale for light and heavy masses

♠ **Symmetries** (gauge, spontaneous breaking, accidental) are the **key QFT ingredient**

♠ **Non-decoupling effects** are **ubiquitous in Higgs physics**

♠ **EFT perspective**: screening (*polynomial*) non-decoupling contributions lead to Wilson coefficients with **logarithmic** (*power-like*) dependence in Λ_{UV} .

ADDITIONAL MATERIAL

Non-decoupling: a broaden perspective



Non-decoupling

cross-talk between distant scales

Non-decoupling: a broaden perspective



Non-decoupling

cross-talk between distant scales

Many phenomena can be adscribed into this category



Low-energy manifestation of relevant or marginal operators

- $\hat{\mathcal{O}}^{D=0}$: the Cosmological Constant
- $\hat{\mathcal{O}}^{D=2}$: the SM Hierarchy problem – m_H radiative instability



Long-range interactions

- Gravity
- Temperature



Non-local correlations

- In space – quantum entanglement
- In time – secular effects



Multi-scale & criticality

NLO EW corrections to $h \rightarrow \gamma\gamma$

Leading $\mathcal{O}(\alpha_{ew})$ corrections to $h \rightarrow \gamma\gamma$: [Degrassi et al., \['94\]](#)

$$\mathcal{L}_{ggH} \supset (\sqrt{2} G_F)^{1/2} \frac{\alpha_s}{12\pi} H G^{\mu\nu} G_{\mu\nu} \left[1 + \frac{G_F \sqrt{2}}{32 \pi^2} m_t^2 \right]$$

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♠ Non-decoupling **non-trivially relates** coupling constant & operator mass dimension expansions

SUSY D-term contributions to the BMSSM Higgs potential



Toy model:

MSSM



$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U_x(1)$

[Batra, Delgado, Kaplan, Tait hep-ph/0309149]

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♠ Additional (super)field content:

• $U(1)_X$ -breaking complex scalars φ, φ^C

$$\langle \varphi \rangle^2 \sim m_\varphi^2$$

• Heavy Z' boson

$$M_{Z'} = 2q_X g_X \langle \varphi \rangle$$

♠ Extended Lagrangian:

♠ **D-terms**: $\mathcal{L}_{MSSM \otimes U(1)_X}^{\text{D-term}} \supset \mathcal{L}_{MSSM}^{\text{D-term}} + \frac{g_X^2}{2} \left[\frac{1}{2} |\tilde{\Phi}_u|^2 + \frac{1}{2} |\tilde{\Phi}_d|^2 + q_X |\varphi|^2 - q_X |\varphi^C|^2 \right]$

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♠ If $m_\varphi^2 > v^2$ SUSY breaks down for φ, Z' before EWSB

$\Rightarrow$ Naturalness remains protected

$\Rightarrow U(1)_X$ D-terms do not decouple and contribute to the light Higgs potential

$$V_H \supset \frac{g_X^2}{2} \left[\frac{1}{2} |\Phi_u|^2 + |\tilde{\Phi}_d|^2 \right] \times \left(1 + \frac{m_{Z'}^2}{2m_\Phi^2} \right)^{-1}$$

Higgs triplet contributions to the T parameter

Heavy scalar triplet

$$T \equiv T^a \frac{\sigma^a}{2} = \begin{pmatrix} \frac{T^0 + v_T}{2} & -\frac{T^+}{\sqrt{2}} \\ \frac{T^-}{\sqrt{2}} & -\frac{T^0 + v_T}{2} \end{pmatrix} \quad [\text{Chen, Dawson '05}]$$

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♠ Tree-level analysis:

$$m_Z^2 = \frac{1}{4}(g^2 + g'^2) v_D^2$$

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$$\alpha T = \frac{1}{1 - 4\sqrt{2} G_F v_T^2} - 1$$

♠ Tree-level $SU(2)_V$ breaking $\Rightarrow \delta v_T$ **must be added at one-loop**

♠ It is yet possible to obtain $\alpha T^{\text{tree}} \rightarrow 0$ for $v_T \rightarrow 0$ and $m_\Phi^2 \rightarrow 0$

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One-loop analysis

$$T^{\text{1-loop}} \sim -T^{\text{tree}} \left[\frac{8\sqrt{2} G_F v_T}{32\pi^2 (1 - 4\sqrt{2} G_F v_T)} \times \frac{3\mu v_T (\kappa v_D^2 + \mu v_T)}{2 m_\Phi^2} \left(\log \frac{m_\Phi^2}{Q^2} - 1 \right) \right]$$

Once $v_D \neq 0$ breaks $SU(2)_L \otimes U(1)_Y$, gauge symmetry can no longer keep $v_T = 0$ radiatively stable (barring fine-tuned Q^2) $\Rightarrow$ **Nondecoupling appears for $\mu \sim m_\Phi$**

Sample case (II): QCD corrections to J_5^μ

Collins, Wilczek & Zee [PRD 18 (1978)] *Low-energy manifestations of heavy particles*

- ♠ consider a sequential four-quark Weinberg–Salam model of weak interactions.
 - The states (u, d, s) are treated as **light** and c as **massive** separately
 - The full theory, as well as the **light** (u, d) and **heavy** (c, s) subsectors are **separably gauge invariant** and hence **renormalizable**

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Screening-like non-decoupling contribution induced by the charm mass

- Gauge invariant theory $\Rightarrow$ all quarks (regardless of their masses) contribute
- Gauge symmetry spontaneously broken $\Rightarrow$ quarks become massive
- One single mass scale & hierarchical Yukawas $\Rightarrow m_{u,d,s} \ll m_c$
- Heavy quark sector alone **Not separably renormalizable**