

Neutrino Physics and Astrophysics

Outline

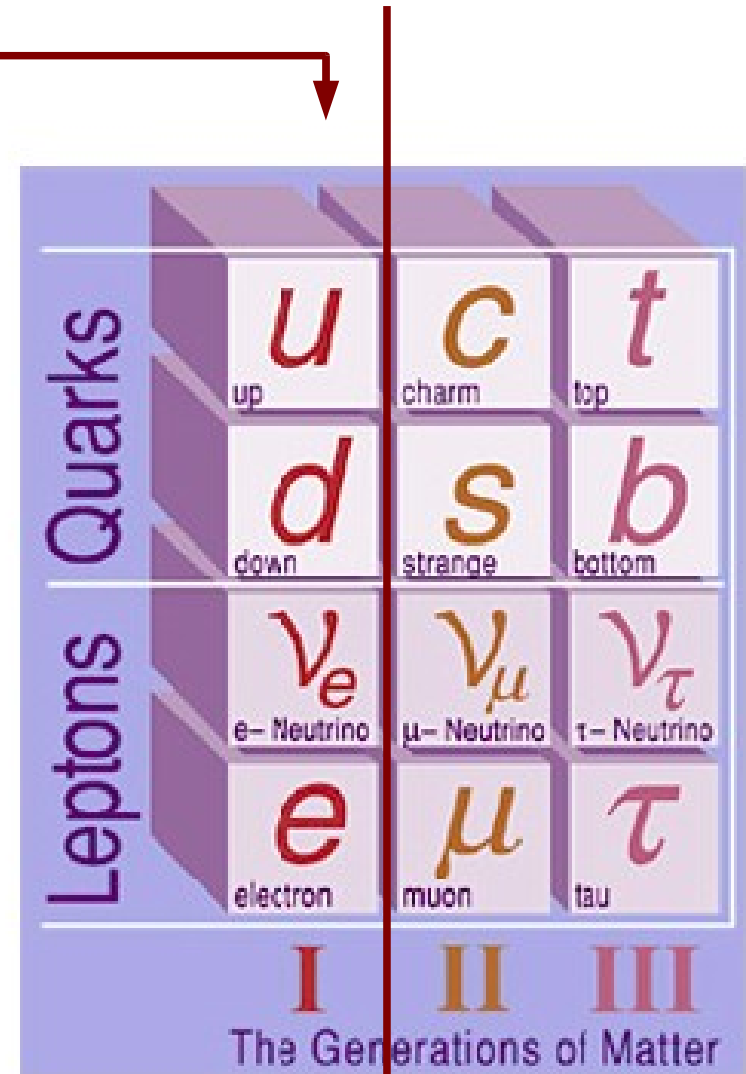
- Introduction:
 - The β decay problem or how the neutrino saved energy conservation
 - Discovery of the neutrinos
 - The neutrino and the Standard Model
 - Measured neutrino properties
 - What if the neutrino was massive ?

Outline

- Neutrino oscillations
 - The solar neutrino problem
 - The atmospheric neutrino anomaly
 - Reactor experiments
 - Accelerator experiments
- The neutrino as a tool
 - Geophysics with neutrinos
 - Astrophysics with neutrinos

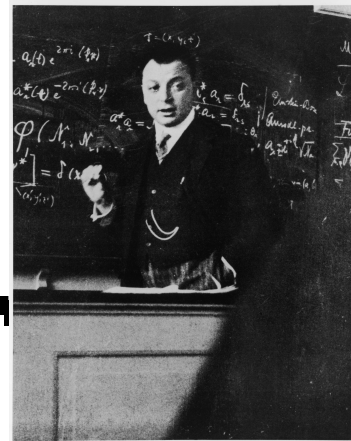
Matter as we know it today

- Ordinary matter is composed of
 - Quark u $Q=2/3$
 - Quark d $Q=-1/3$
 - Electron $Q=-1$
 - ... and a 4th fundamental particle, the neutrino



How the neutrino was born

Original: *Physikalische Zeitschrift* 1930, 31, 373
Abschrift/15.12.58



Offener Brief an die Gruppe der Radioaktiven bei der
Gesellschafts-Tagung zu Tübingen.

Abschrift

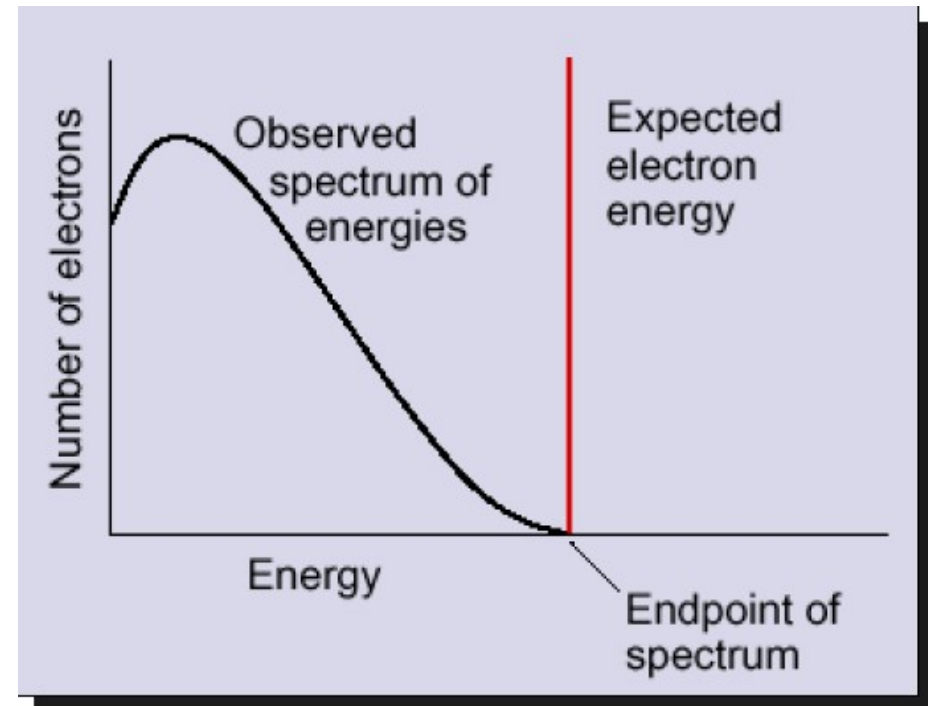
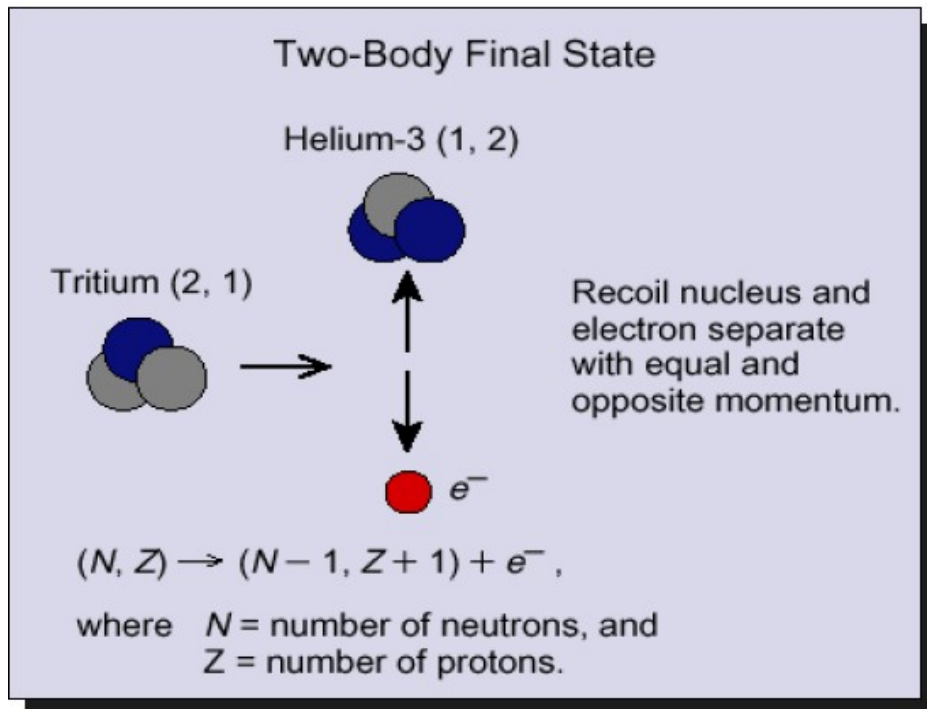
Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Oleriastrasse

Liebe Radioaktive Damen und Herren,

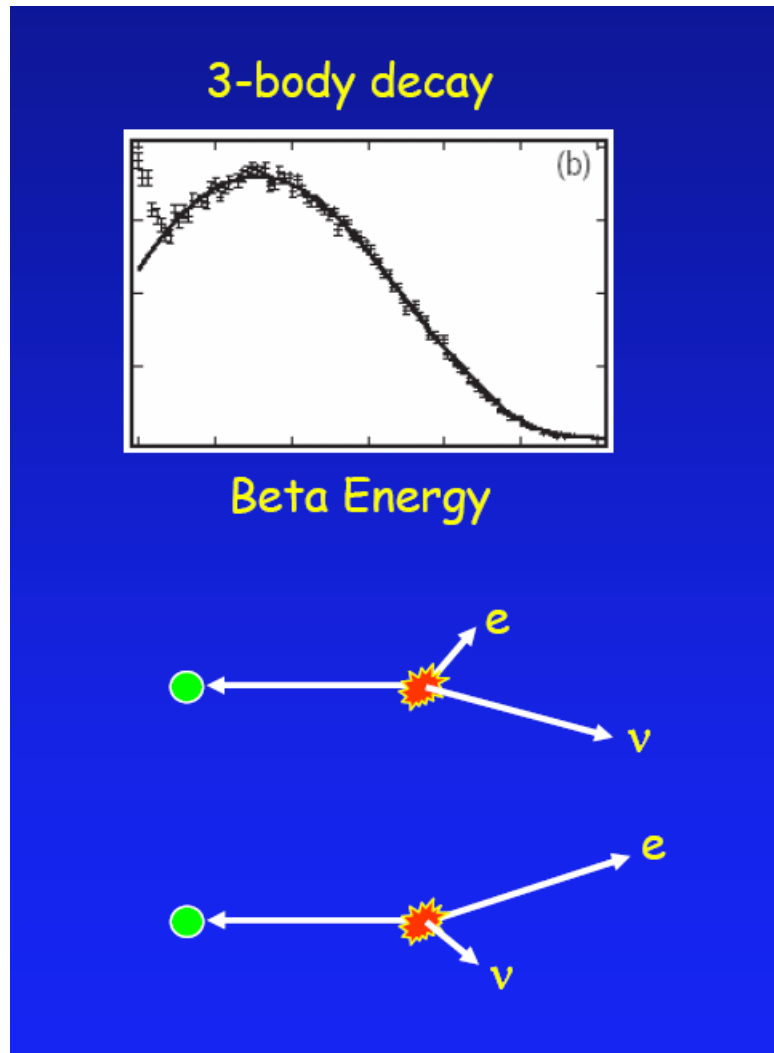
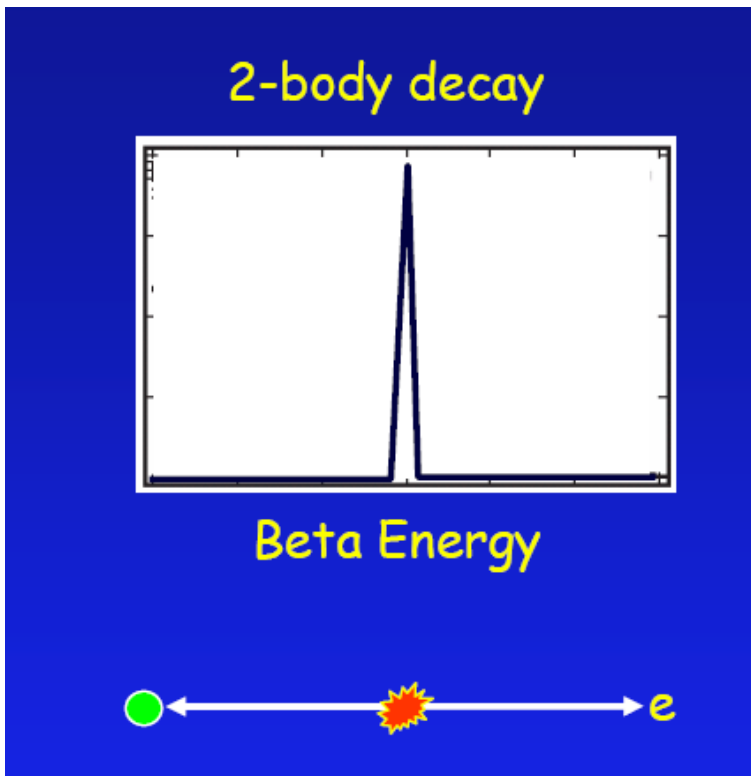
Wie der Überbringer dieser Zeilen, den ich kuldvollst
anzuhören bitte, Ihnen das näherem auseinandersetzen wird, bin ich
angesichts der "falschen" Statistik der N - und $Li-6$ Kerne, sowie
des kontinuierlichen beta-Spektrums auf einen verzweifeltten Ausweg
verfallen um den "Wechselstanz" (1) der Statistik und den Energieerhalt
zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale
Teilchen, die ich Neutronen nennen will, in den Kernen existieren,
welche den Spin $1/2$ haben und das Ausschliessungsprinzip befolgen und
sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie
nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen
müsste von derselben Grössenordnung wie die Elektronenmasse sein und
jedenfalls nicht grösser als $0,01$ Protonenmasse. Das kontinuierliche
beta-Spektrum wäre dann verständlich unter der Annahme, dass beim
beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert
wird, derart, dass die Summe der Energien von Neutron und Elektron
konstant ist.

The β ray spectrum

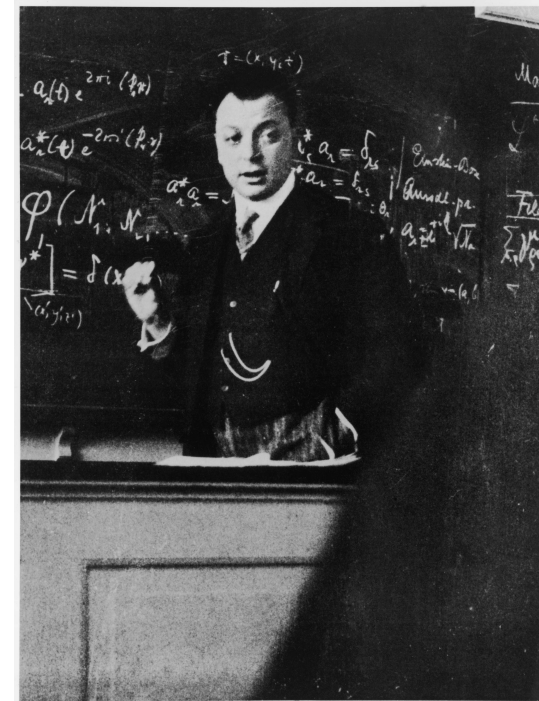
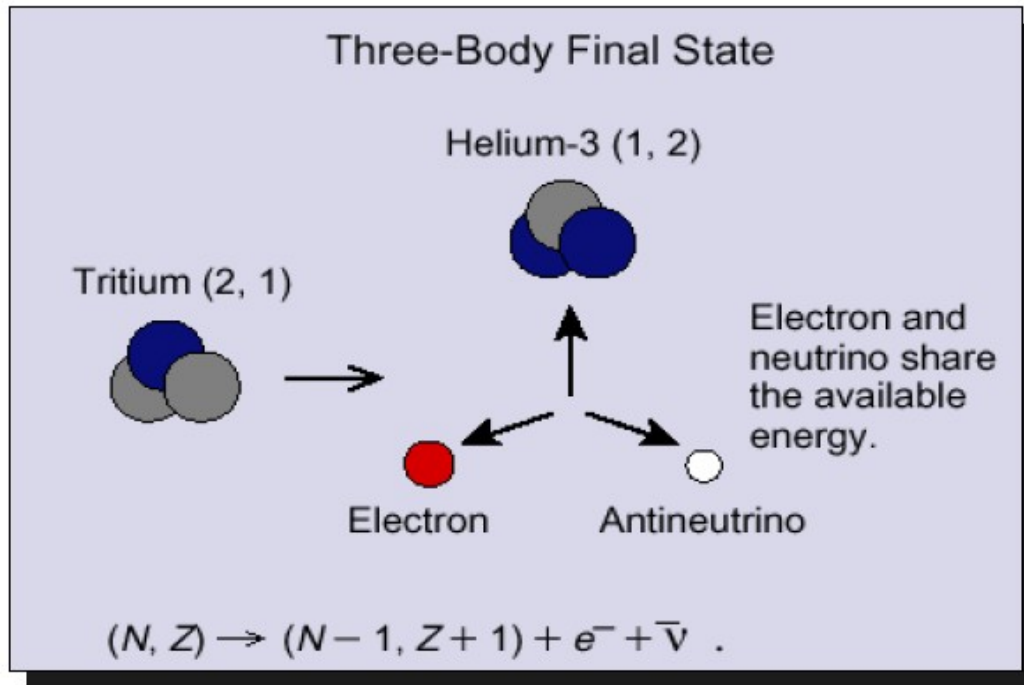


- Observed as a 2-body decay, the e^- was expected with a well defined energy, but...
- A spectrum is observed and energy conservation and causality are at stake

Two and three-body kinematics



I have done a terrible thing...



I have done a terrible thing. I have proposed a particle that cannot be detected. It is something no theorist should ever do.

In 1930, Pauli "invents" the neutrino to explain the β -decay spectrum

Liebe Radioaktive Damen und Herren

Zürich, Dec. 4, 1930

Dear Radioactive Ladies and Gentlemen,

...because of the “wrong” statistics of the N and ${}^6\text{Li}$ nuclei and the continuous β -spectrum, I have hit upon a desperate remedy to save the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin $\frac{1}{2}$ and obey the exclusion principle The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous β -spectrum would then become understandable by the assumption that in β -decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and electron is constant.

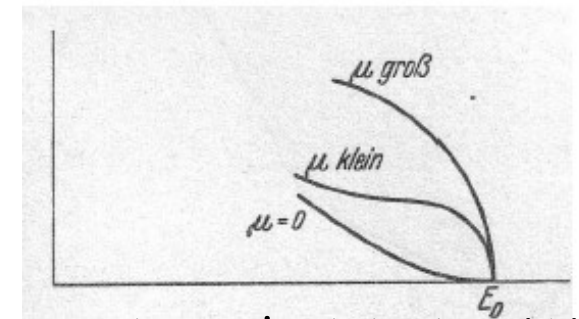
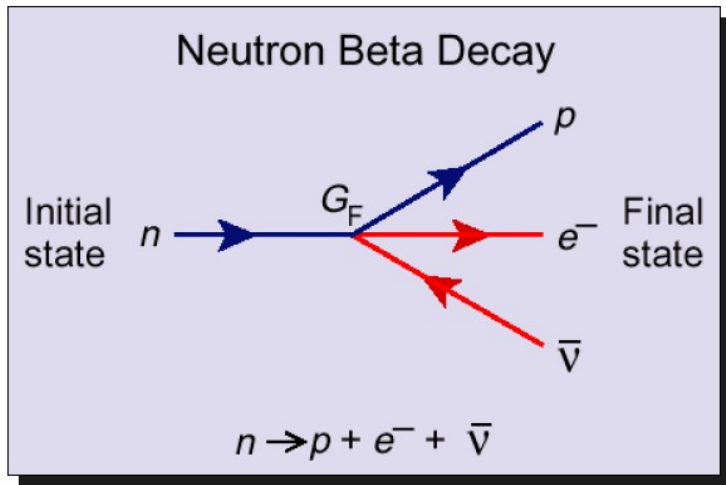
..... For the moment, however, I do not dare to publish anything on this idea

So, dear Radioactives, examine and judge it. Unfortunately I cannot appear in Tübingen personally, since I am indispensable here in Zürich because of a ball on the night of 6/7 December.

W. Pauli

β -ray theory

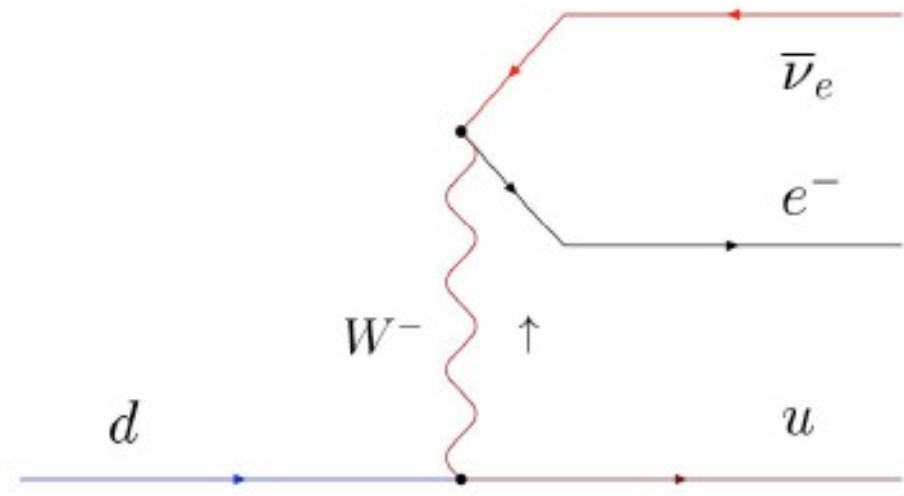
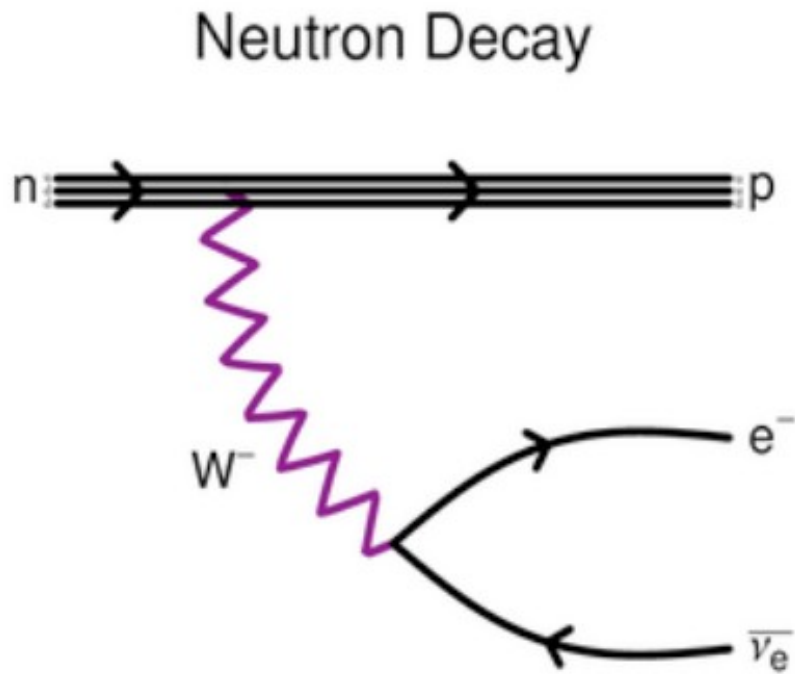
Fermi (1934), *Nuovo Cimento & Zeitschrift für Physik* (previously rejected by Nature)



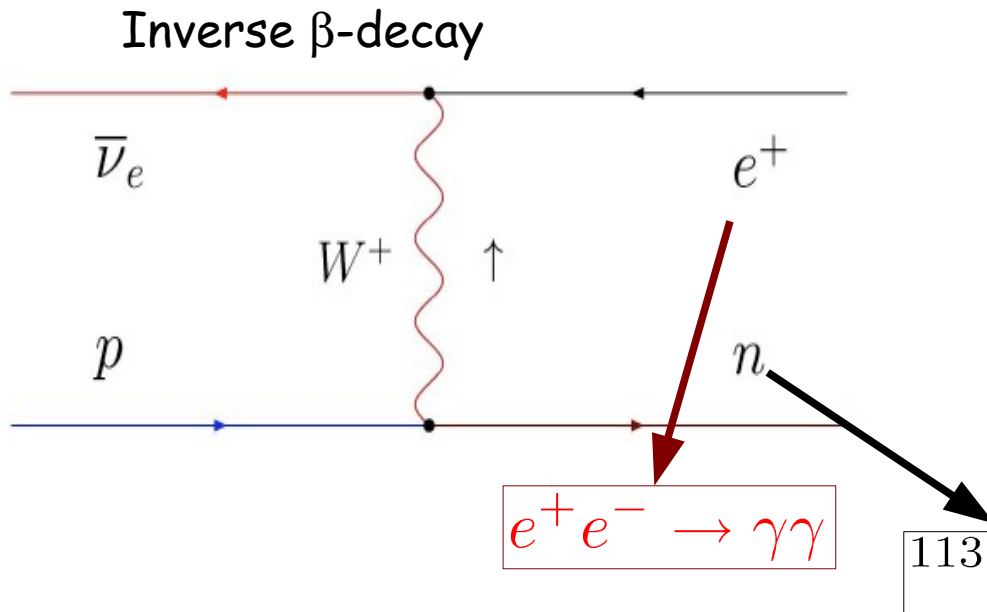
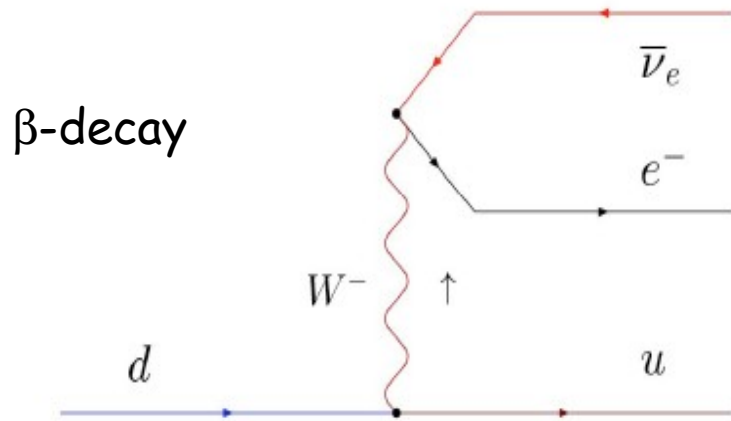
- Point interaction among 4 spin 1/2 particles. Theory is relativistic. Wave functions are spinors satisfying Dirac equation. Particles are created at the instant of decay.
- Prediction of β decay rates and electron energy spectra depends only of one constant, G_F , determined experimentally. Energy spectrum depends of neutrino mass μ . Measurable distortions near end-point of spectrum if $\mu > 0$.
- Not renormalizable, cross-sections diverge with s

β -decay today

- Today we know : at the quark level:



How do we detect ν ?



•If ν are produced by β decay, they can be detected using the inverse reaction. $E_n \approx 3\text{MeV}$

$$\sigma(\bar{\nu}p) \approx 10^{-43} \text{cm}^2$$

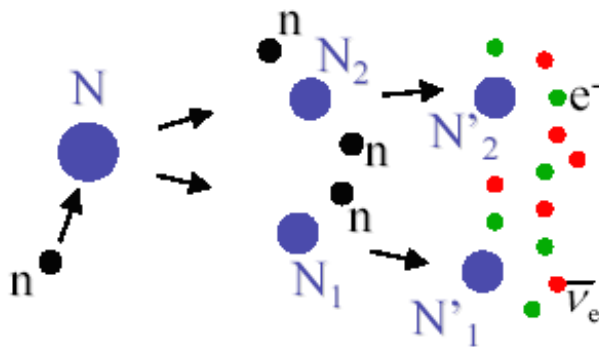
$$l = \frac{1}{N_A \rho \sigma}$$

$$l(\text{Pb}) \sim \frac{1}{610^{23} \cdot 7.9 \cdot 10^{-43}} \text{cm}$$

~ 4 light years !!!

Experiments with reactor neutrinos

Nuclear reactors are very intense sources of $\bar{\nu}_e$ deriving from beta-decay of the neutron-rich fission fragments



Yield :
 200 MeV / fission
 6 $\bar{\nu}_e$ / fission

$$\bar{\nu} \text{ production rate} = \frac{6P_t}{200 \text{ MeV} \times \underbrace{1.6 \times 10^{-13}}_{\substack{\text{conversion factor} \\ \text{MeV} \rightarrow \text{J}}}} = 1.87 \times 10^{11} P_t \bar{\nu}/s$$

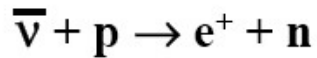
P_t : reactor thermal power [W]

For a typical reactor: $P_t = 3 \times 10^9 \text{ W} \Rightarrow 5.6 \times 10^{20} \bar{\nu} / s$ (isotropic)
Continuous $\bar{\nu}$ energy spectrum – average energy $\sim 3 \text{ MeV}$

First neutrino observation

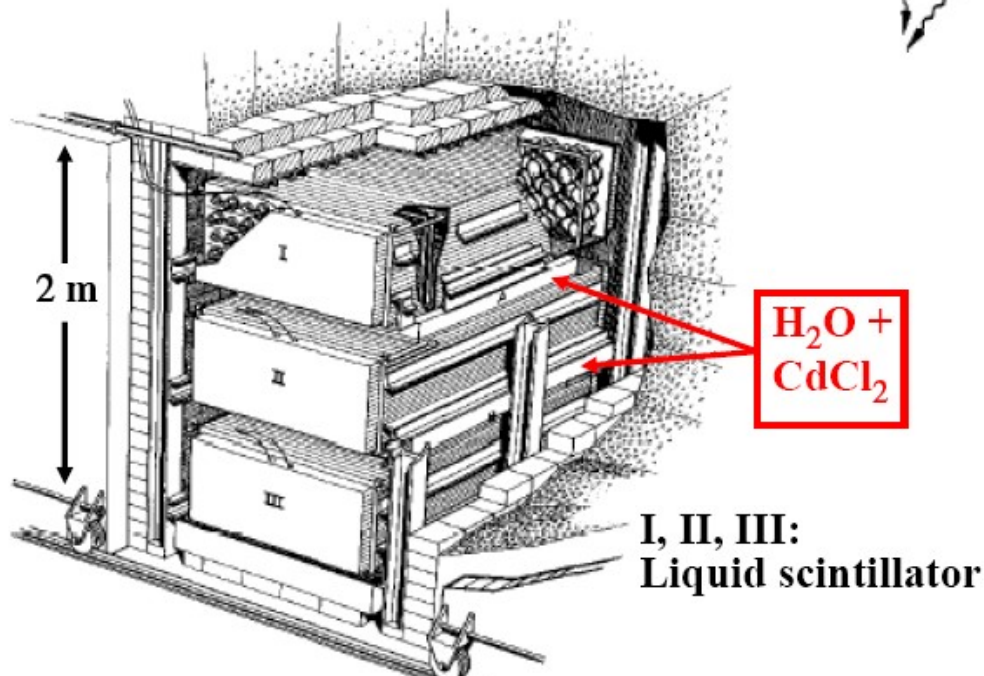
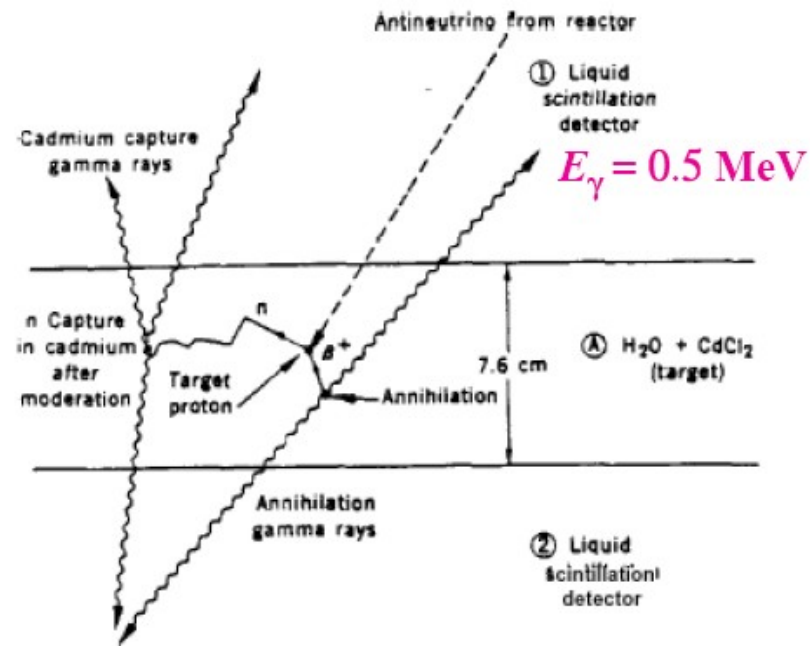
First neutrino detection

(Reines, Cowan 1953)



detect 0.5 MeV γ -rays from $e^+e^- \rightarrow \gamma\gamma$
($t = 0$)

neutron “thermalization” followed
by capture in Cd nuclei \Rightarrow emission
of delayed γ -rays (average delay $\sim 30 \mu\text{s}$)



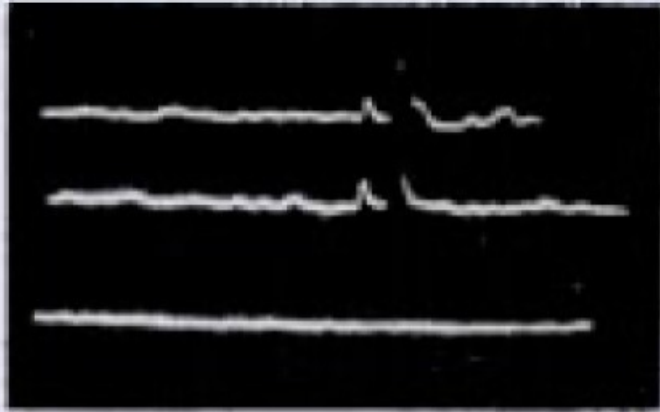
Event rate at the Savannah River
nuclear power plant:

3.0 ± 0.2 events / hour

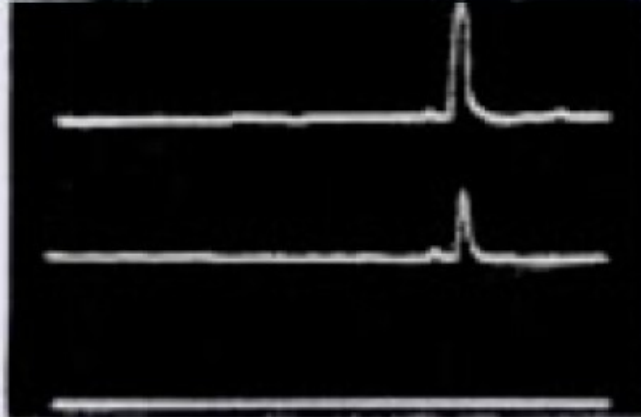
(after subtracting event rate measured
with reactor OFF)

in agreement with expectations

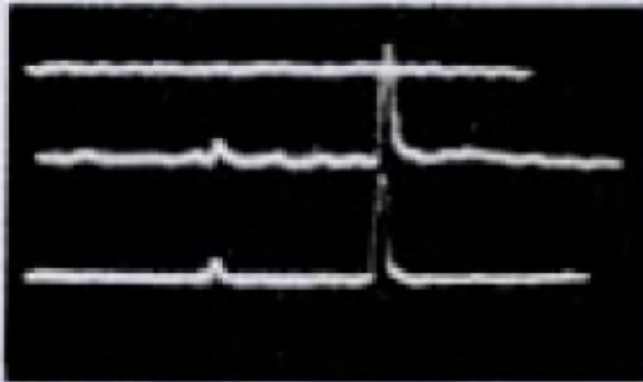
Experimental signature



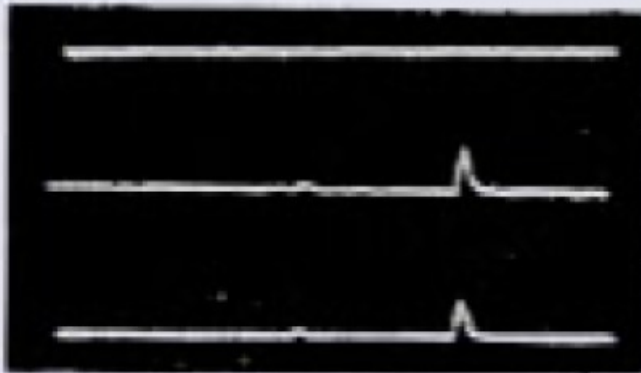
(a) Positron scope



Neutron scope



(b) Positron scope

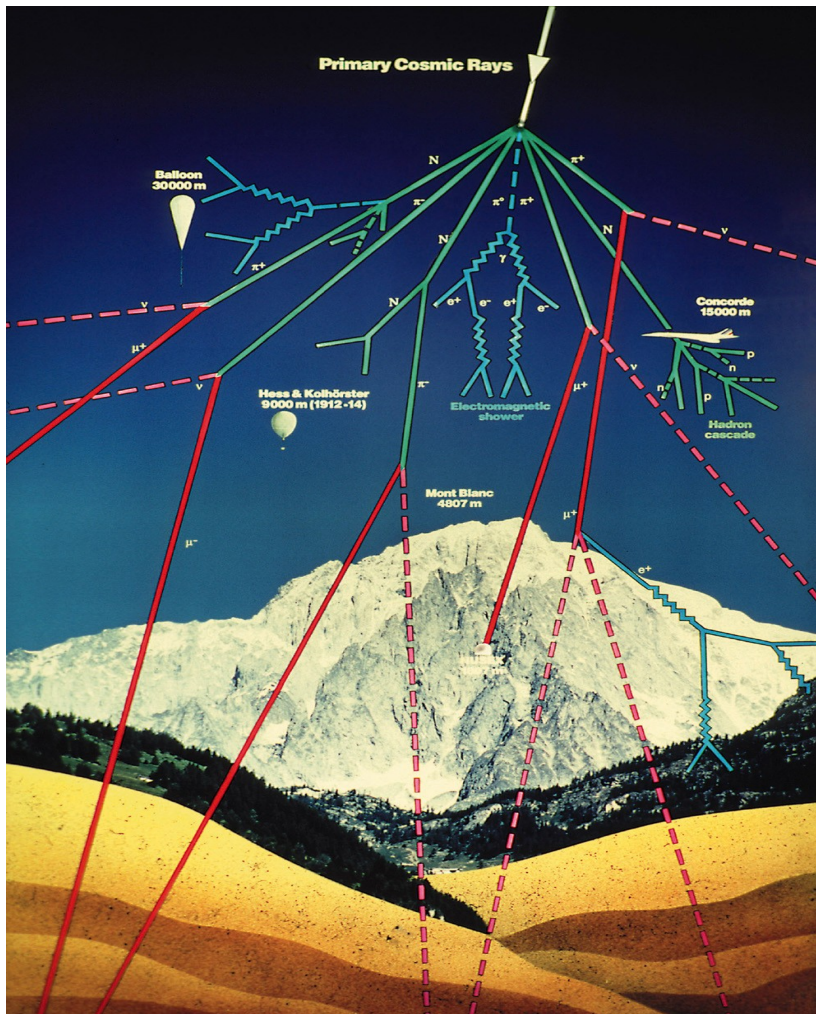


Neutron scope



Nobel prize 1995

Cosmic Rays

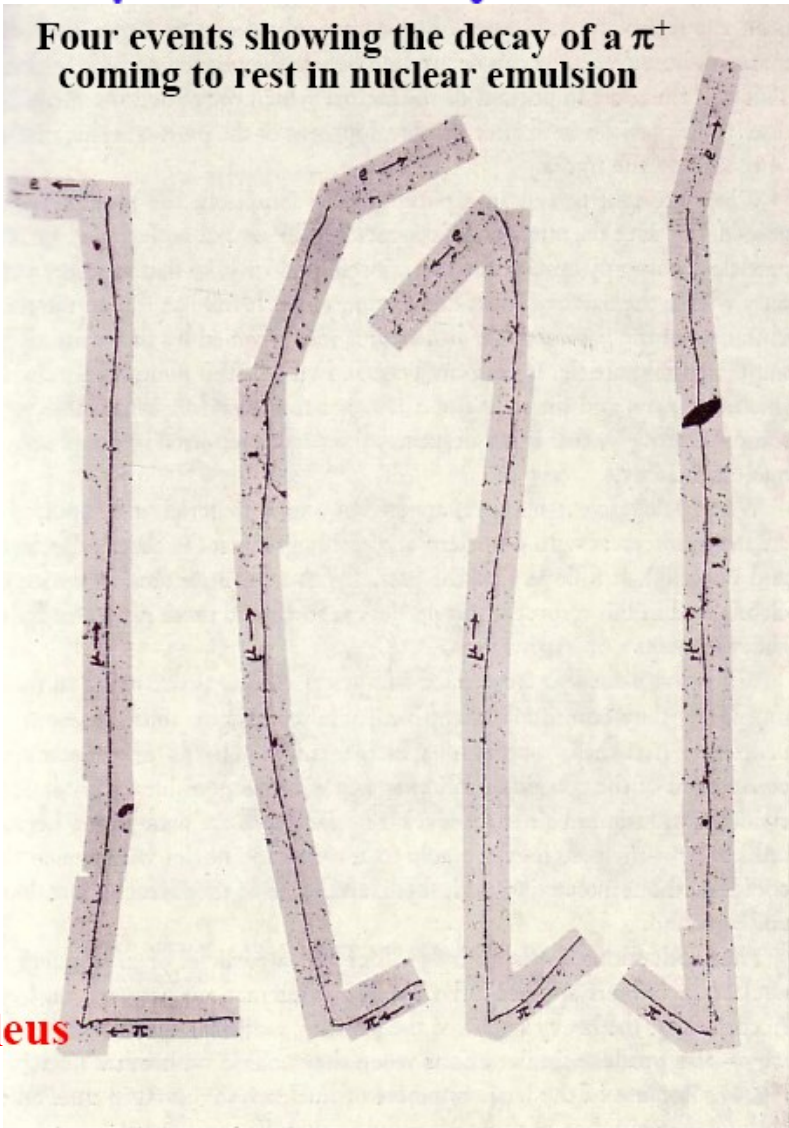


- The Earth is continuously bombarded by high-energy particles from outer space. Mostly protons.
- When a high energy proton hits a nuclei in the upper shells of the atmosphere it produces a shower of light hadrons (π , K) which, on decay, result in μ , e , ν .

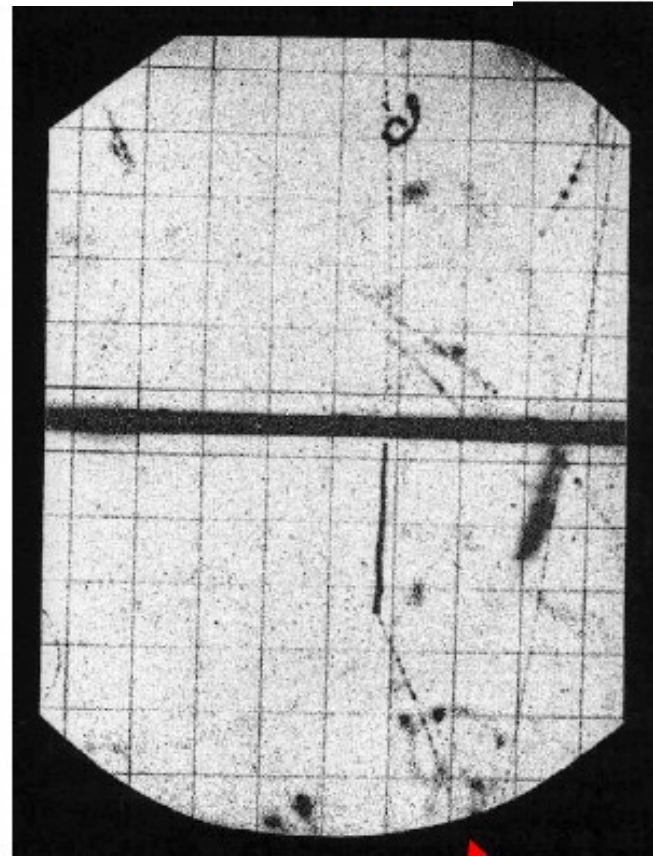
Who ordered that?

$\pi^+ \rightarrow \mu^+ \rightarrow e^+$ decay chain

Four events showing the decay of a π^+ coming to rest in nuclear emulsion



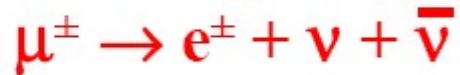
Cosmic ray muon stopping
in a cloud chamber and
decaying to an electron



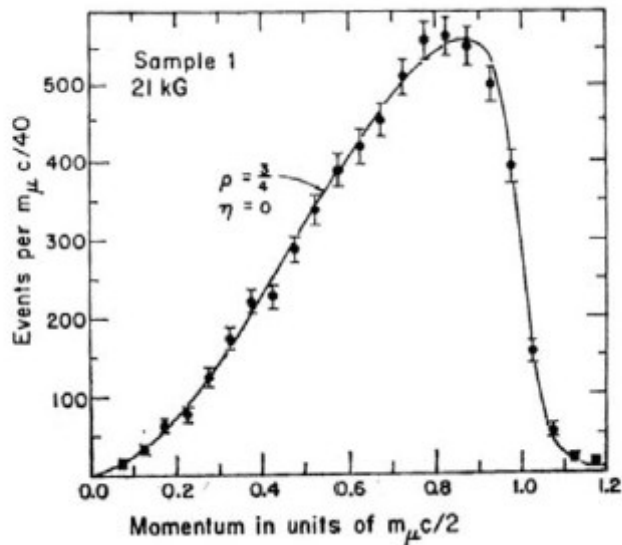
decay electron track

A second neutrino?

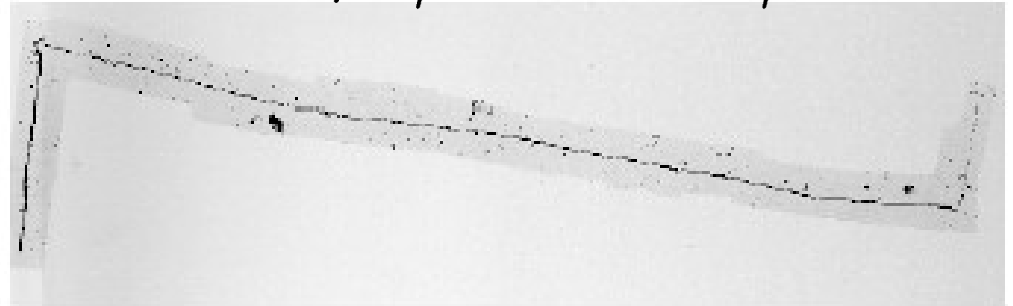
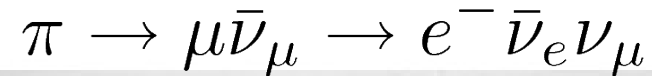
Muon decay



Decay electron
momentum distribution



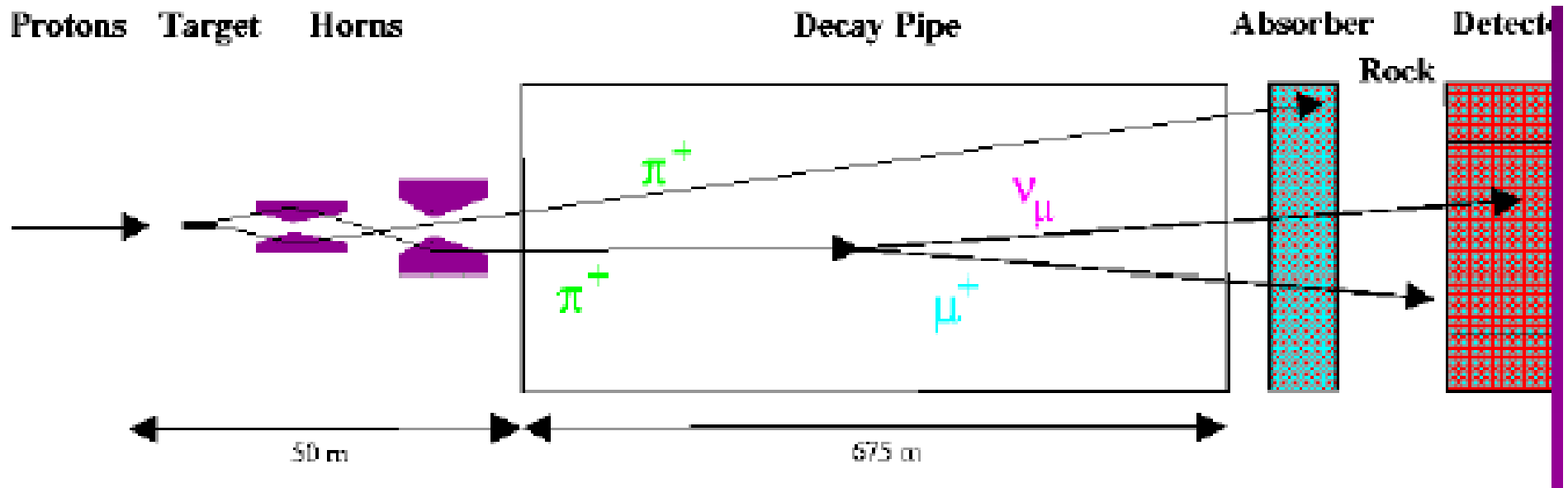
μ decay is a three body process (observed electron has a continuous spectrum). One needs two neutrinos



π decay is a two body process (muon has always the same energy when pion decays at rest). One undetected particle (kink in emulsion) signals the presence of a neutrino

Are all those neutrinos the same than the one emitted in β decay?

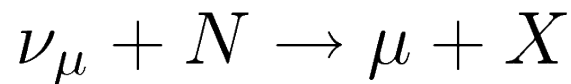
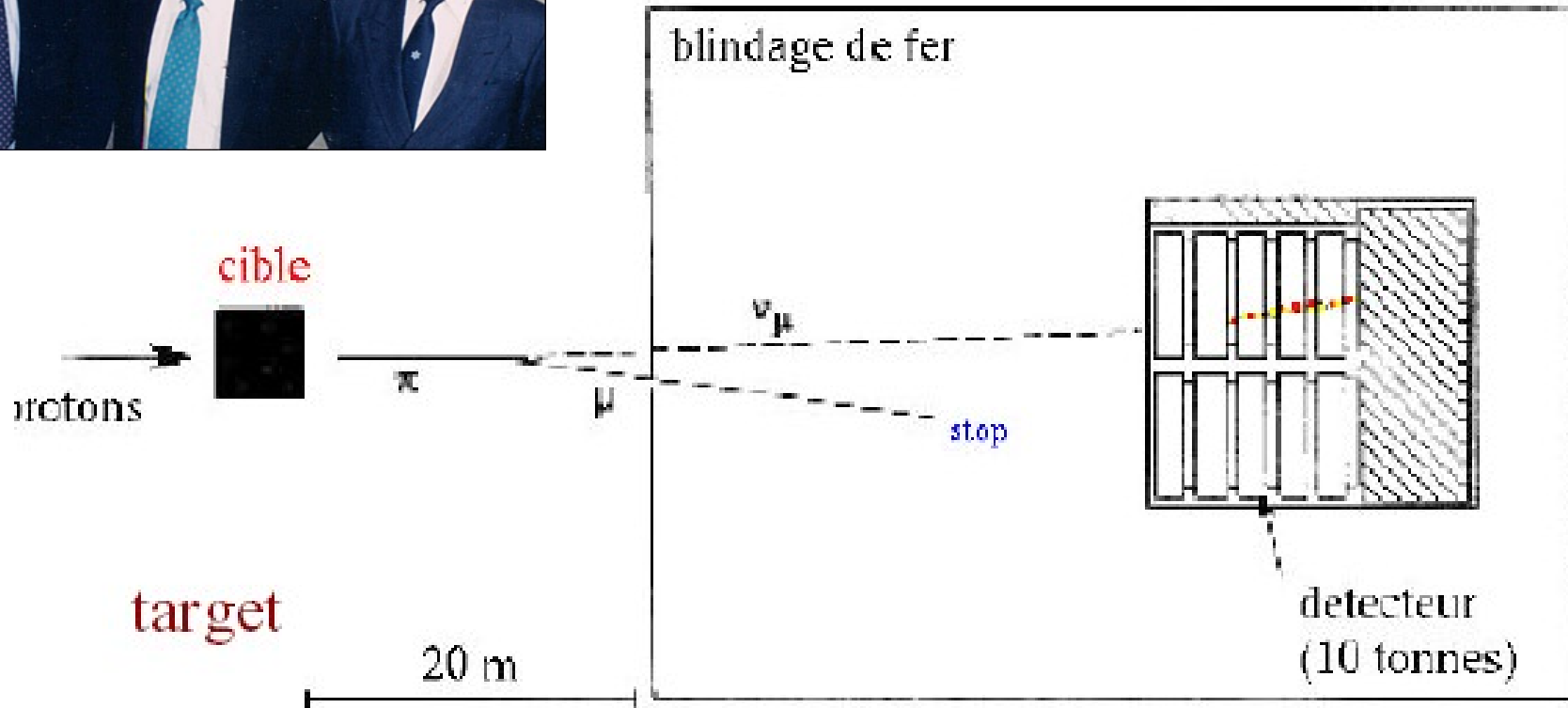
How to build a ν_μ beam



The discovery of ν_μ (1962)

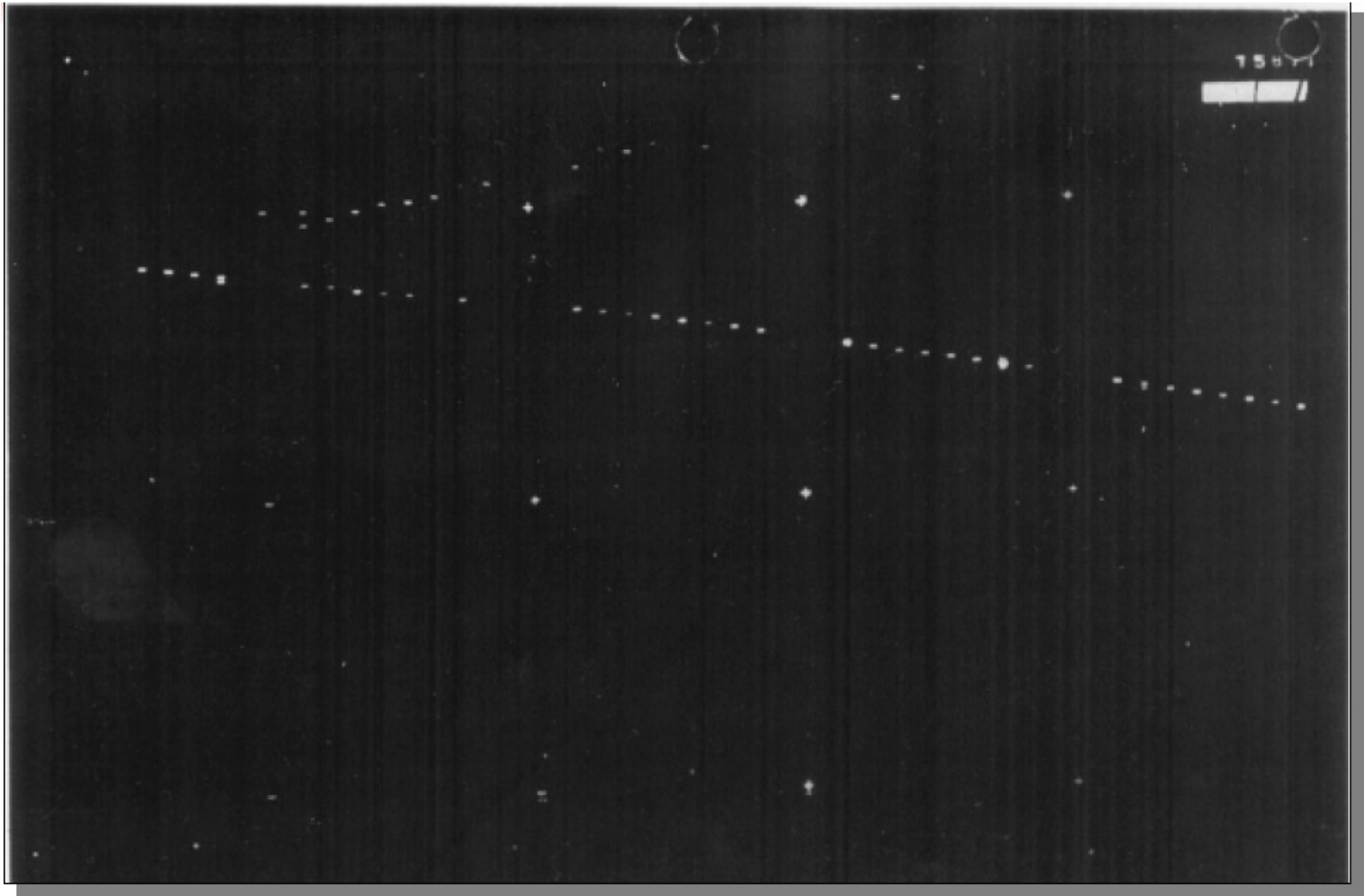


Swartz, Lederman & Steinberger
(Nobel 1989)



----- sparks along
a muon track

Experimental signature

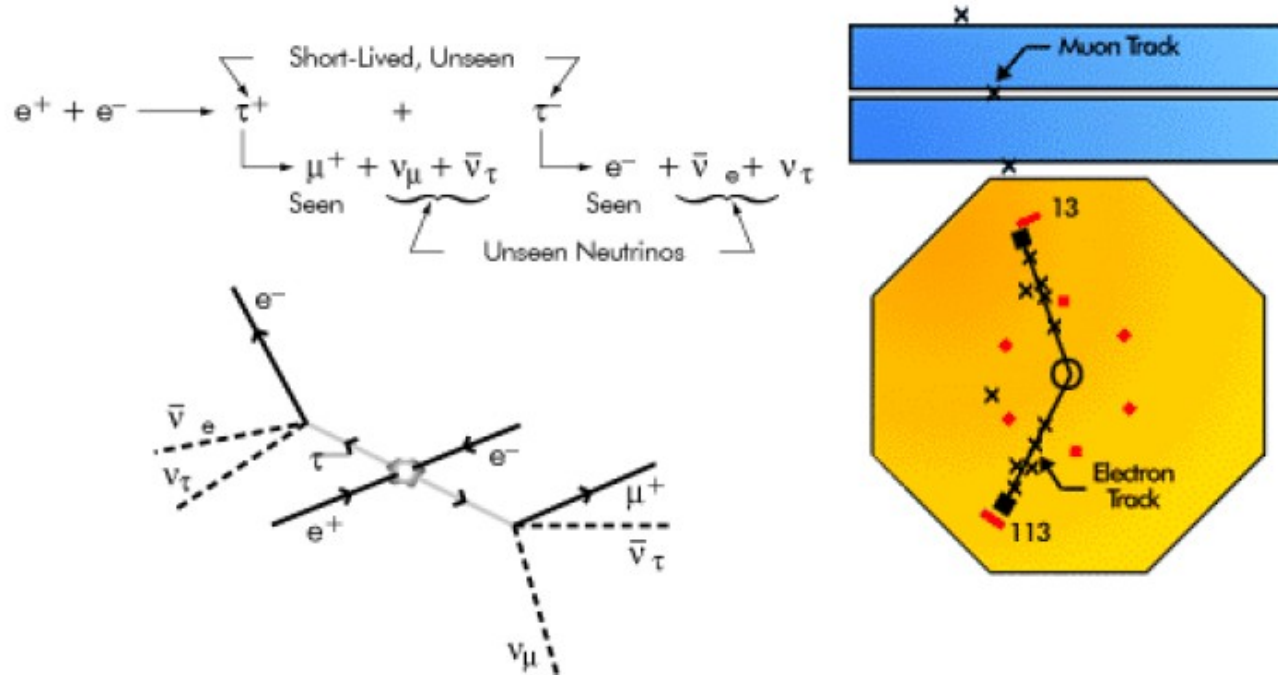


Is $\nu_\mu = \nu_e$?

- Muon decay $\mu \rightarrow e \nu \bar{\nu}$
- If $\nu_\mu = \nu_e$ expect
 - $\mu \rightarrow e \gamma$ BR $< 10^{-8}$...was expected $\sim 10^{-4}$
 - $\mu + p \rightarrow e + p$...not observed
- The non-observation of electron-like events in the same quantity as μ -like events in beams produced from ~~μ~~ decay rules it out (1962)

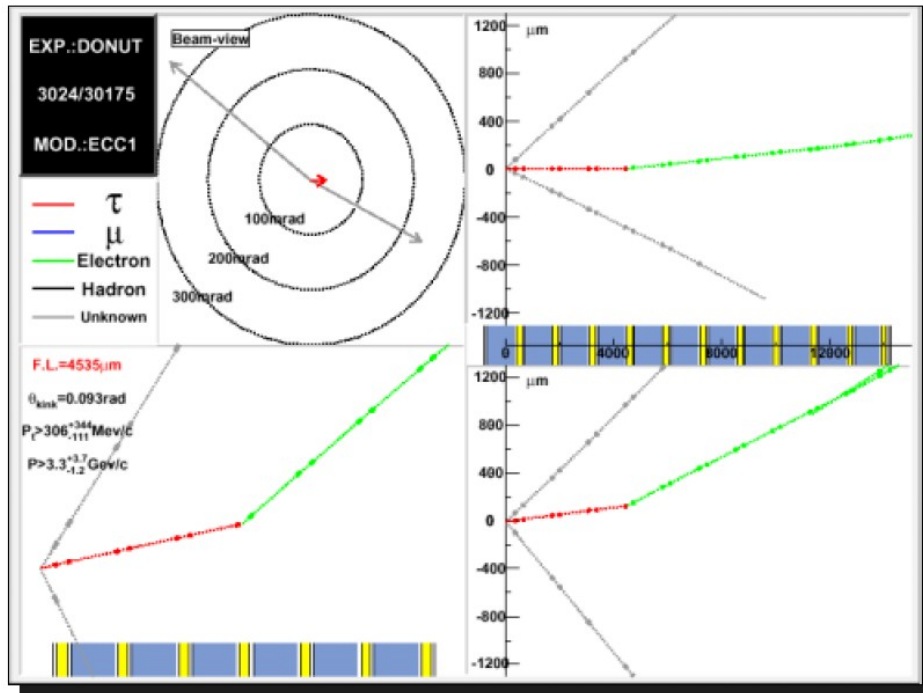
The third electron

The third heavy electron, the tau was discovered by M. Perl and collaborators in 1975 at SLAC

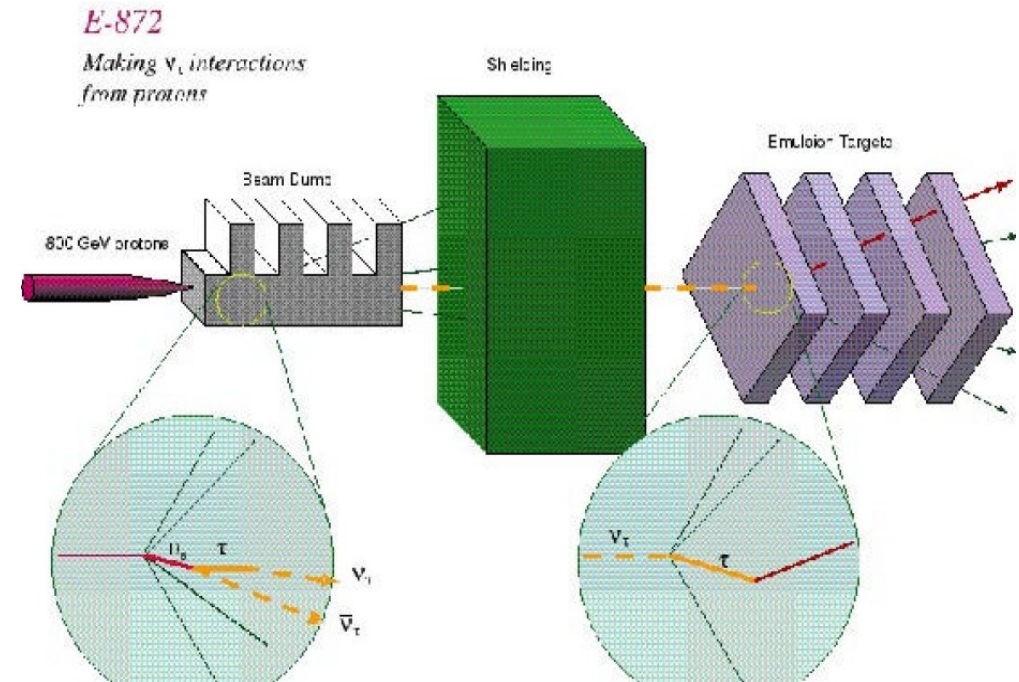


Direct observation of ν_τ

- DoNuT, 2000 : beam dump experiment



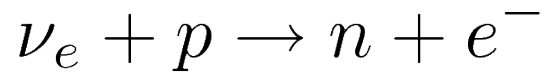
$$\nu_\tau + N \rightarrow \tau + X$$



$$D_s \rightarrow \tau \nu_\tau$$

$$\nu_e : \nu_\mu : \nu_\tau = 6 : 9 : 1$$

Neutrinos and anti-neutrinos



- If neutrino = anti-neutrino then we should observe $\bar{\nu}_e + p \rightarrow n + e^-$ but
- In 1955 R. Davis at a reactor tries unsuccessfully $\bar{\nu}_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$
- The reaction $\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$ would be used by his experiment at Homestake mine to detect solar neutrinos

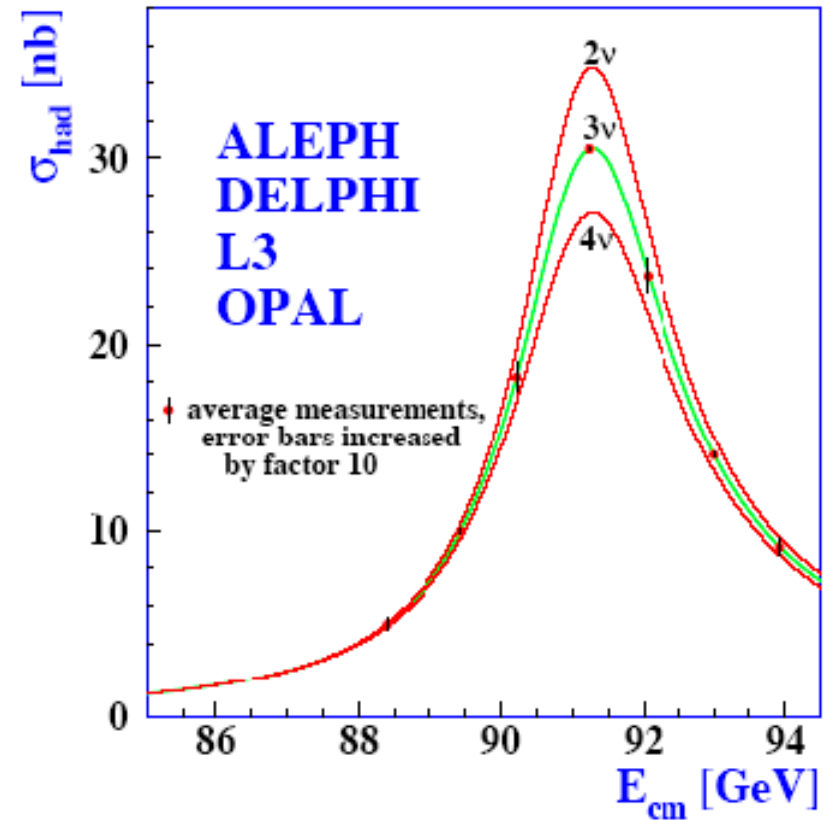
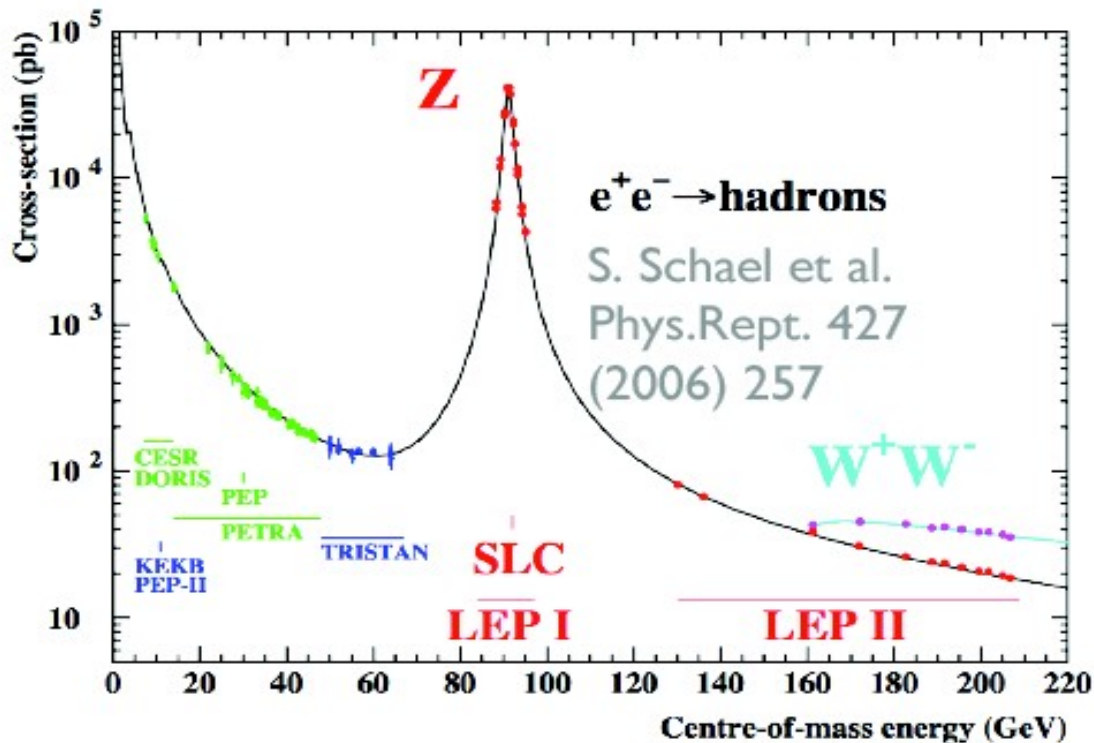
There are only three light neutrinos

$$N_\nu = \frac{\Gamma_{inv}}{\Gamma_l} \left(\frac{\Gamma_l}{\Gamma_\nu} \right)_{SM}$$

Measured Theory

$$\Gamma_{inv} = \Gamma_Z - \Gamma_{ee} - \Gamma_{\mu\mu} - \Gamma_{\tau\tau} - \Gamma_{had}$$

$$N_\nu = 2.984 \pm 0.008$$

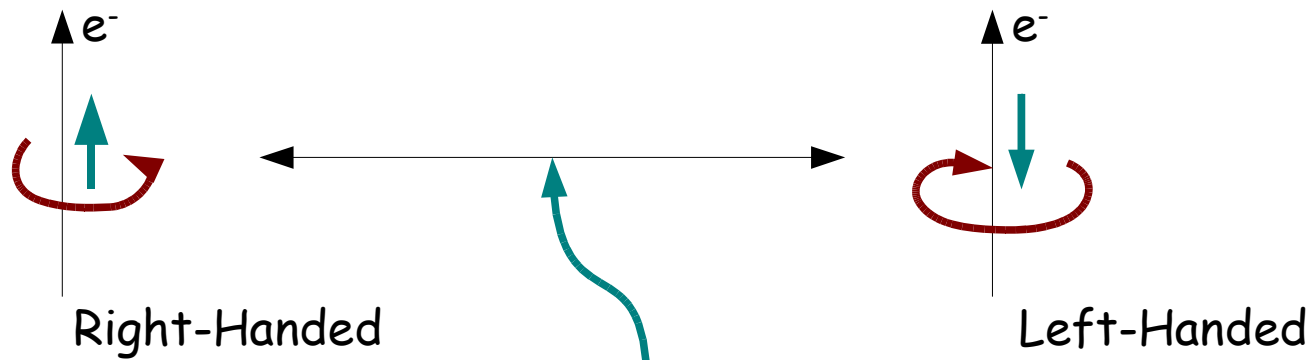
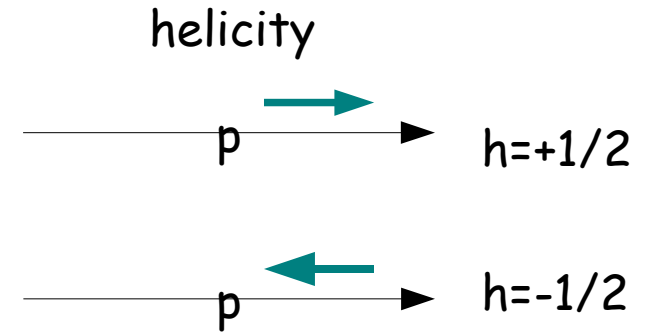


Chirality and Helicity

$$\psi_R = \frac{1}{2}(1 + \gamma_5)\psi$$

$$\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$$

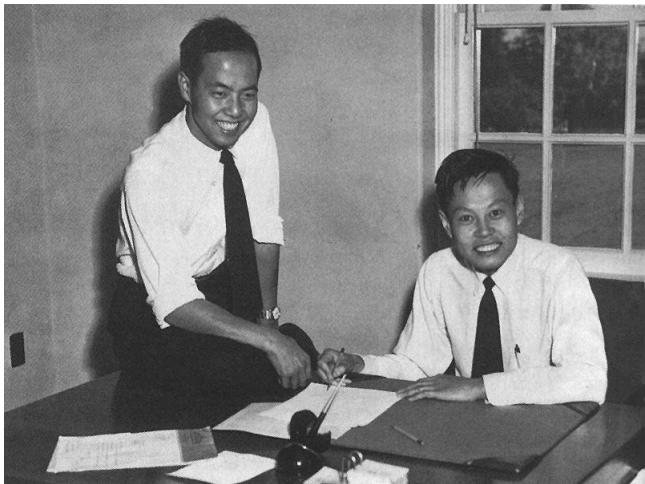
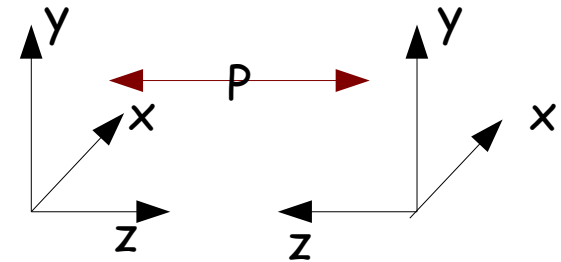
$$\xrightarrow{\frac{m}{E} \rightarrow 0}$$



"overtaking" only possible
for massive particles

Parity

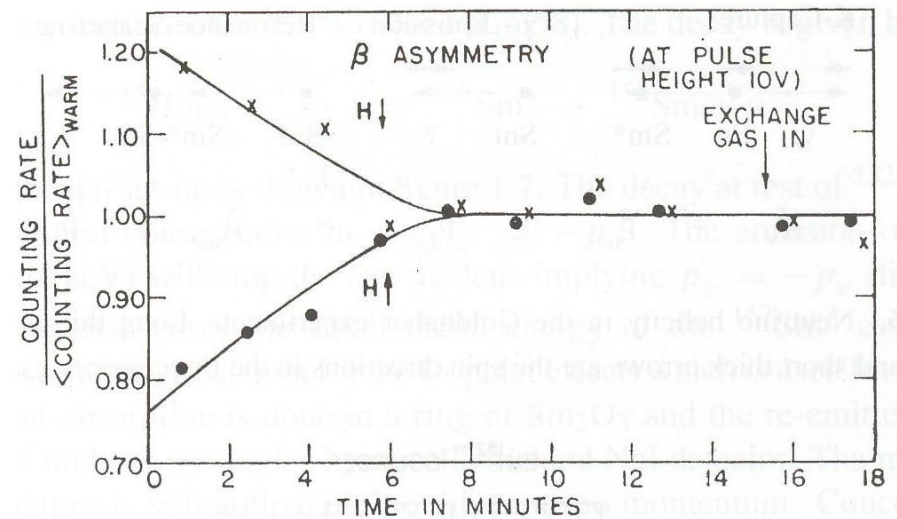
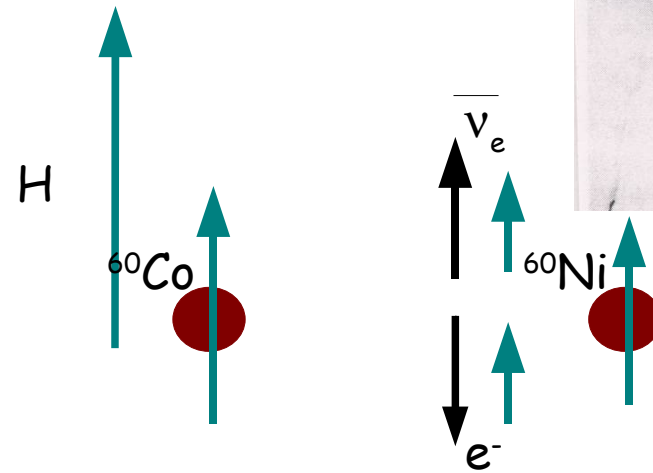
- Mirror image
- Parity invariance : For any physical particle system, its mirror image is equally probable. Nature does not "know" the difference between right and left



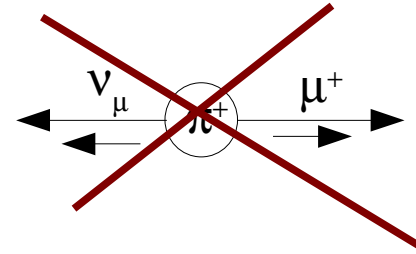
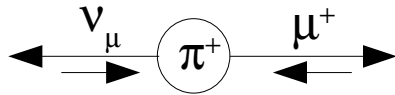
- Lee and Yang (1956)
To explain K^0 decays, they assume parity is violated in weak interactions...

Parity violation

- In 1957 Wu et al. Measure the rate of down-going e^- in polarized ^{60}Co β -decay.
- Parity maximally violated
- Only left-handed ν and right-handed $\bar{\nu}$



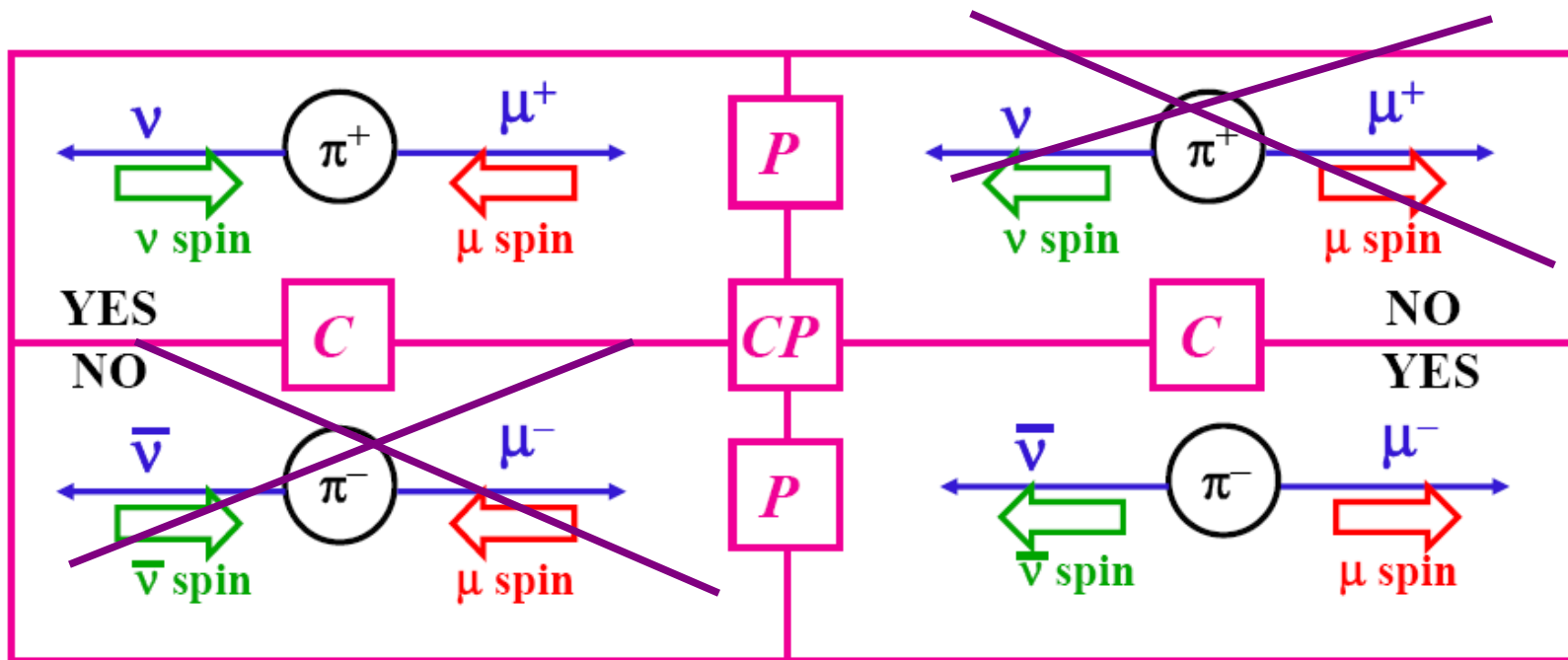
Pion Decay



- If parity was conserved : both should be observed with equal probabilities
- The μ^+ should not be polarised but is only observed left-handed \Rightarrow Parity is violated

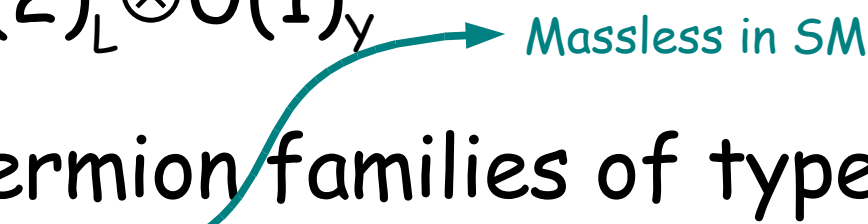
Pion decay and CP

- Pion decay conserves CP but violates C and P



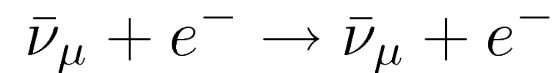
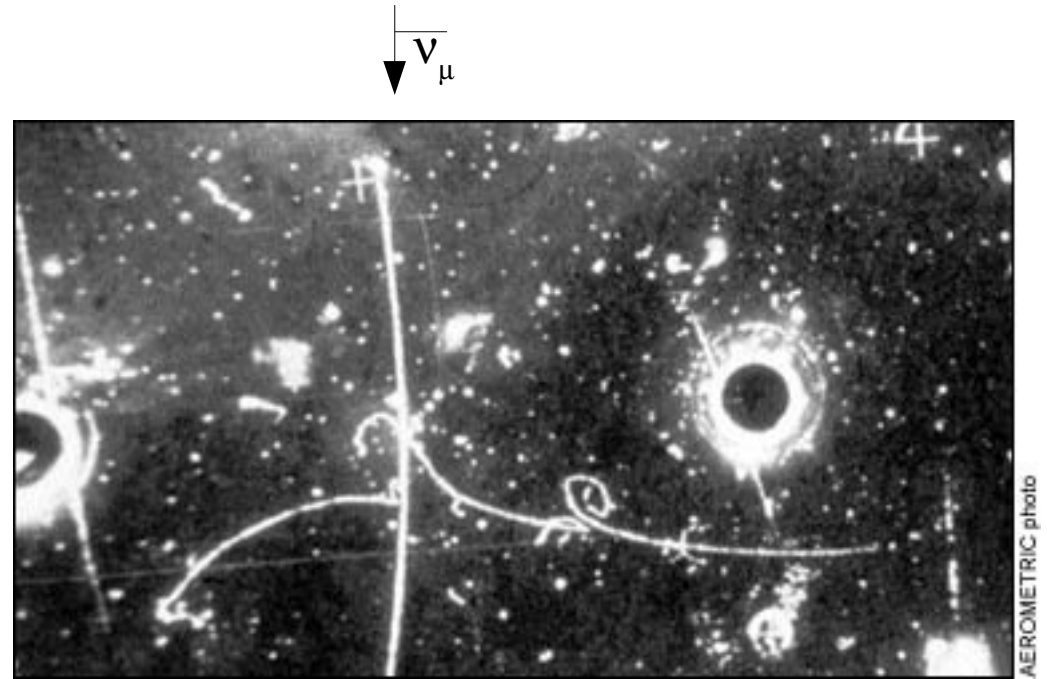
- Anti-neutrinos are right-handed

Standard model

- $SU(2)_L \otimes U(1)_Y$  Massless in SM
- 3 fermion families of type
 $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} e_R \quad \begin{pmatrix} u_L \\ d_L \end{pmatrix} u_R d_R$ + anti-particles
- Weak interaction W^\pm, Z^0 (spin 1)
- Electromagnetic interaction γ (spin 1)
- Higgs boson (spin 0)

Discovery of neutral currents

- Neutral currents predicted in SM
- Discovered at CERN in 1973 (Gargamelle)



Neutrino scattering on electrons

$$\sigma(\nu e^-) = \frac{2G_F^2 m_e E_\nu}{\pi} \left[c_L^2 + \frac{1}{3}c_R^2 - \frac{1}{2}c_R c_L \frac{m_e}{E_\nu} \right]$$

| | C_L | C_R | σ (cm ²) |
|---------------------|-------------------------|-------------------------|--|
| $\nu_e e^-$ | $1/2 + \sin^2\theta_W$ | $+\sin^2\theta_W$ | $0.952 \times 10^{-43} (E_\nu/10\text{MeV})$ |
| $\bar{\nu}_e e^-$ | $+\sin^2\theta_W$ | $1/2 + \sin^2\theta_W$ | $0.399 \times 10^{-43} (E_\nu/10\text{MeV})$ |
| $\nu_\mu e^-$ | $-1/2 + \sin^2\theta_W$ | $+\sin^2\theta_W$ | $0.155 \times 10^{-43} (E_\nu/10\text{MeV})$ |
| $\bar{\nu}_\mu e^-$ | $+\sin^2\theta_W$ | $-1/2 + \sin^2\theta_W$ | $0.134 \times 10^{-43} (E_\nu/10\text{MeV})$ |

$\sigma \sim 10^{-4} \text{fb} !!!$

• Where

• $C_L = 1/2 (g_V + g_A)$ $C_R = 1/2 (g_V - g_A)$ in Standard Model

• Measuring the ratio of ν^- cross-sections gives $\sin^2\theta_W$

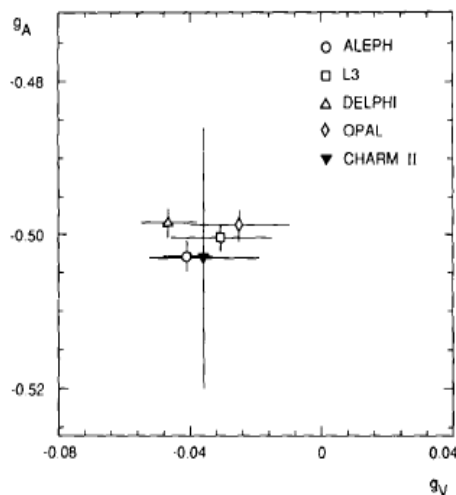
CHARM II results

- Expressing in terms of g_V, g_A

$$- s = E_\nu m_e \quad y = E_e / E_\nu$$

- For muons :

$$\frac{d\sigma_{\bar{\nu}}^{\nu}}{dy} = \frac{G_F^2 s}{4\pi} [(g_V \pm g_A)^2 + (g_V \mp g_A)^2 (1-y)^2]$$



$$g_V^{\nu e} = -0.035 \pm 0.012(\text{stat}) \pm 0.012(\text{syst}),$$

$$g_A^{\nu e} = -0.503 \pm 0.006(\text{stat}) \pm 0.016(\text{syst}).$$

$$\sin^2\theta_W = 0.2324 \pm 0.0058 \pm 0.0059$$

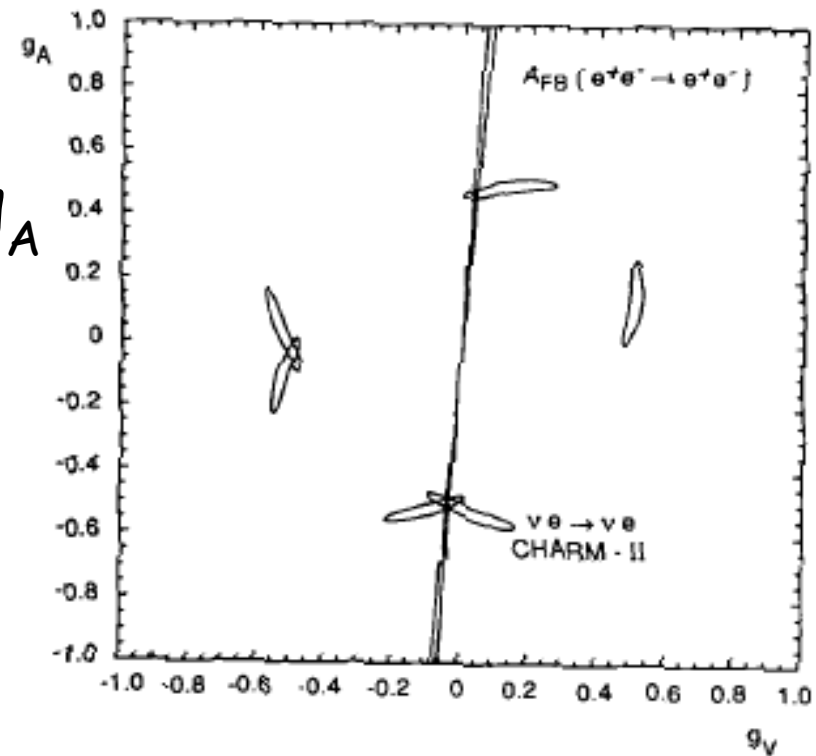
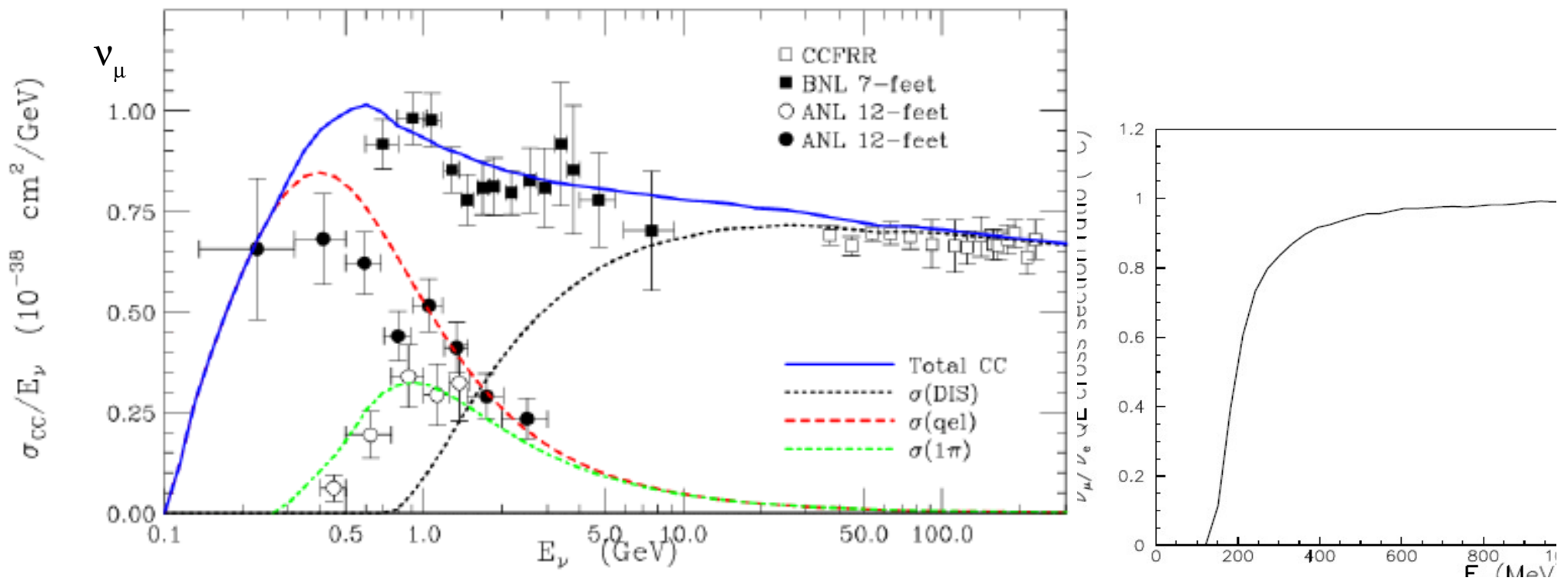


Fig. 3. 90% confidence level contours in the g_V - g_A plane, obtained from the fit to the data from the ν -beam, the $\bar{\nu}$ -beam and to other beams. Only statistical errors are considered. Results from experiments on the forward-backward asymmetry for $e^+e^- \rightarrow e^+e^-$ at LEP [13] are shown as well. Together they select a single solution in agreement with $g_A^e = -\frac{1}{2}$.

Neutrino-nucleon scattering

$$\sigma(\nu N) = (0.677 \pm 0.014) \cdot 10^{-38} \text{cm}^2 \frac{E_\nu}{\text{GeV}}$$

$$\sigma(\bar{\nu} N) = (0.334 \pm 0.008) \cdot 10^{-38} \text{cm}^2 \frac{E_\nu}{\text{GeV}}$$



Neutrinos probe the structure of the nucleon

$$\frac{d^2\sigma^i}{dx dy} = \frac{4\pi\alpha^2}{xyQ^2} \eta^i \left\{ \left(1 - y - \frac{x^2 y^2 M^2}{Q^2} \right) F_2^i + y^2 x F_1^i \mp \left(y - \frac{y^2}{2} \right) x F_3^i \right\},$$

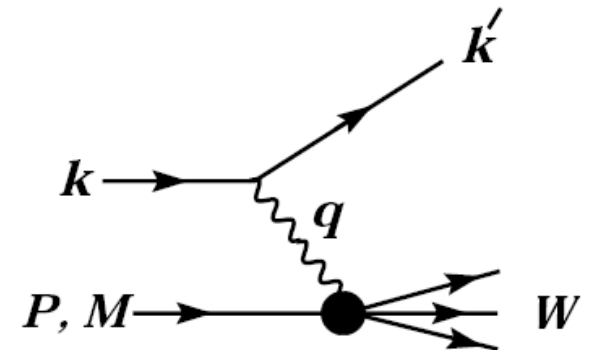
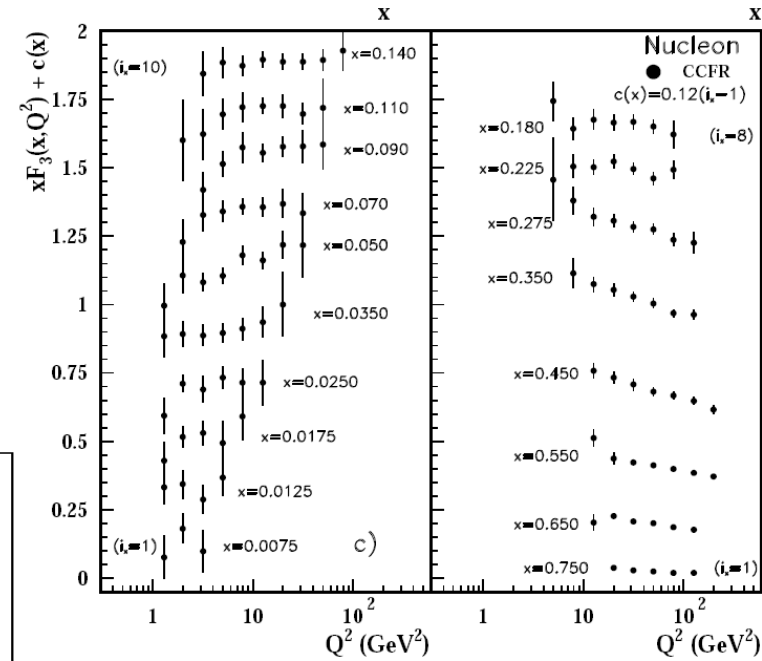
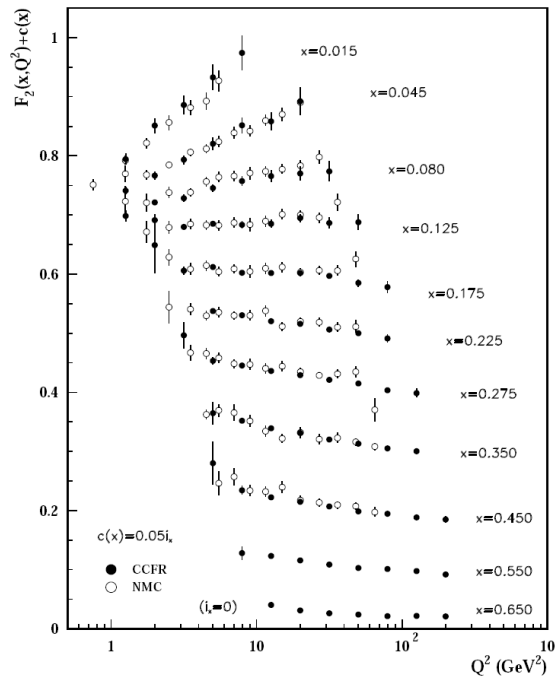
$$x = \frac{Q^2}{2M(E' - E)}$$

$$y = 1 - E'/E$$

$$\eta = 2 \left(\frac{G_F M_W^2}{4\pi\alpha} \frac{Q^2}{Q^2 + M_W^2} \right)$$

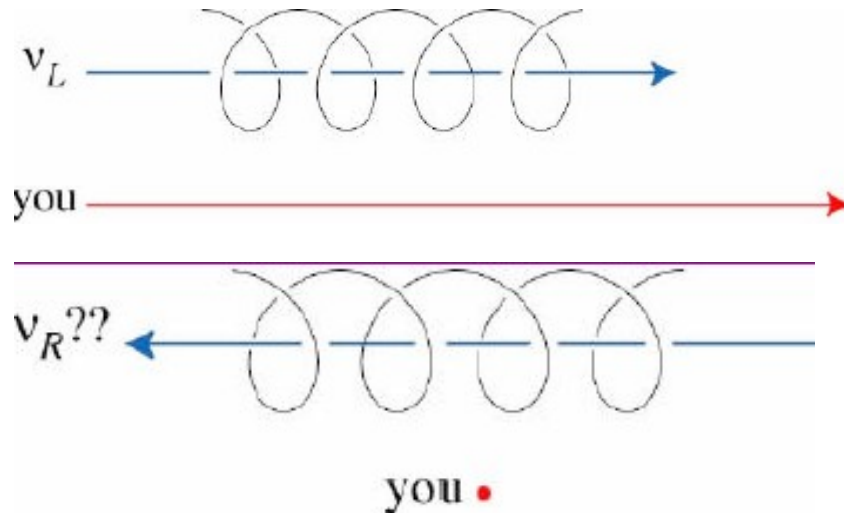
$$+ : \nu, - : \bar{\nu} \quad i = NC, CC$$

$$Q^2 = -q^2 \simeq EE' \sin^2\theta/2$$



What if neutrinos have mass ?

P. Dirac



E. Majorana



- There is a ν_R state

- Dirac spinor $\Psi_v = \nu_R + \nu_L$

- ν_R is the anti-particle of ν_L .

- $\Psi_v = (\nu_L)^c + \nu_L$

- Violates L

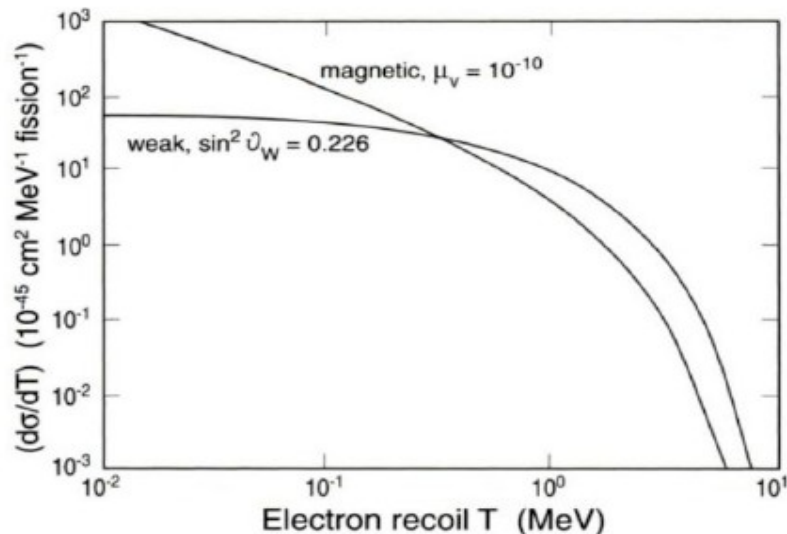
- Mass terms transform as SU(2) triplets: not gauge invariant, not renormalisable

Neutrino magnetic moment

- If neutrino have mass they can have a magnetic moment.
- Dirac neutrinos:
$$\mu_\nu = \frac{3G_F e}{8\sqrt{2}\pi^2} m_\nu = 3.2 \times 10^{-19} \left(\frac{m_\nu}{eV} \right) \mu_B$$

Too small to be measured, but some models predict larger magnetic moments

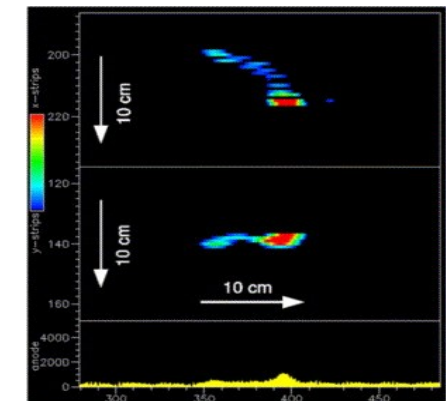
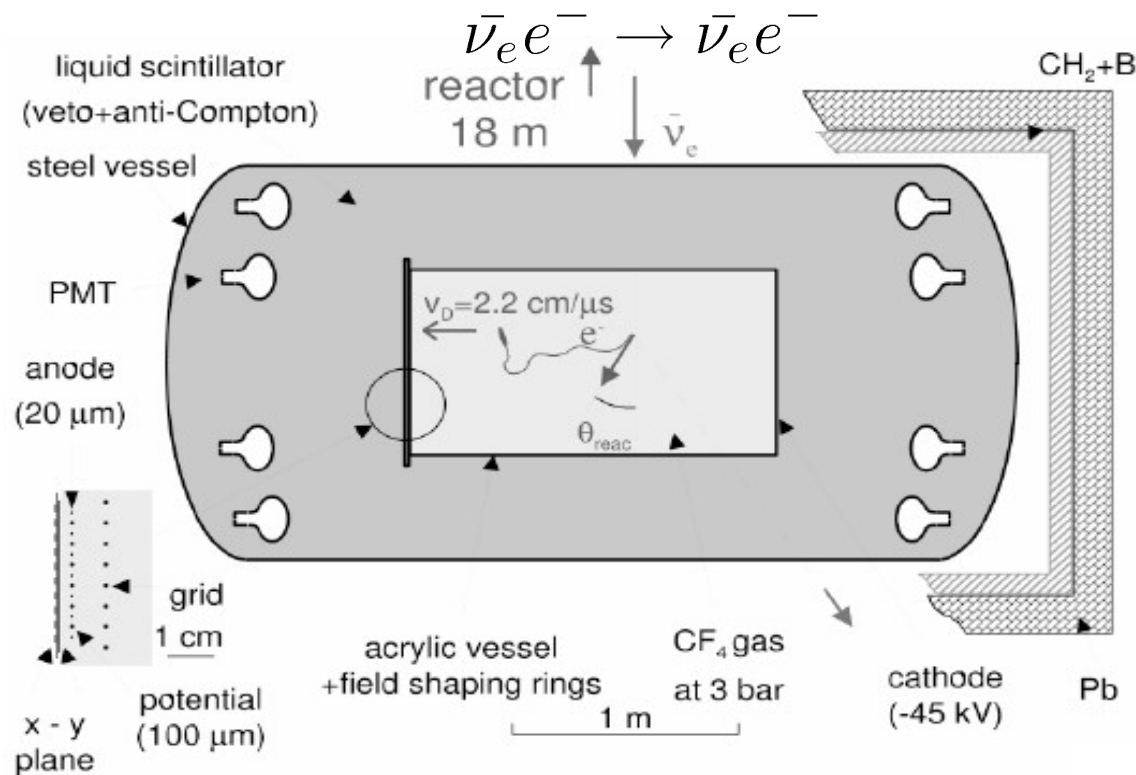
- Majorana neutrinos: CPT invariance requires $\mu_\nu=0$
- Measured via the differential cross-section for $\bar{\nu}_e$ scattering.



Effect of a magnetic moment on recoil electron kinetic energy distribution for $\bar{\nu}$ -e scattering

MUNU experiment

- Bugey reactor $1.5 < E_\nu < 8 \text{ MeV}$
- $\mu_{\bar{\nu}_e} < 0.9 \cdot 10^{-11} \mu_B$ @90% CL



190keV e^-

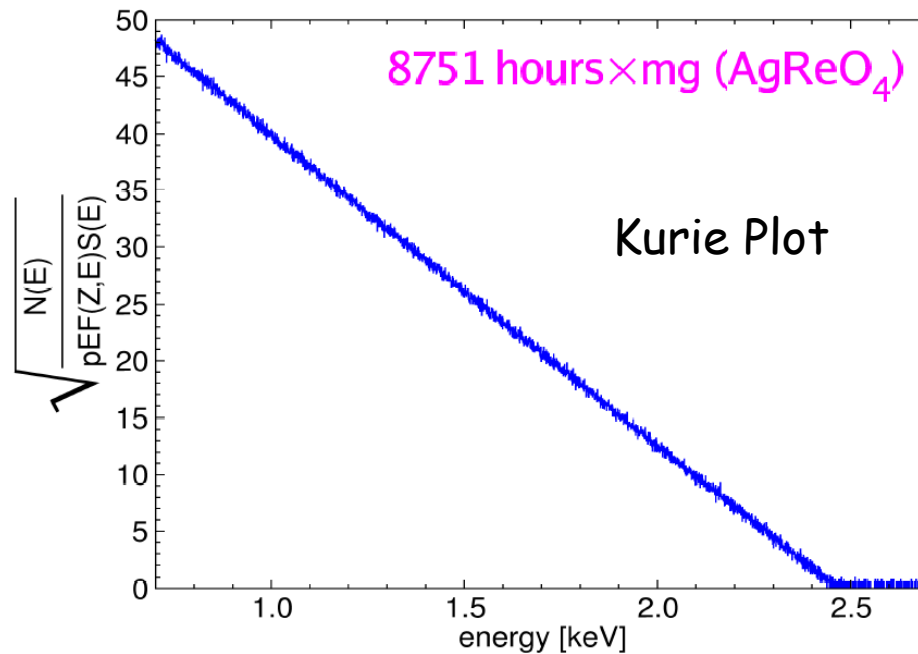
Handles on the neutrino mass

Electron neutrino mass

Direct measurement

Spectrum of electrons emitted in β decay for massless neutrinos

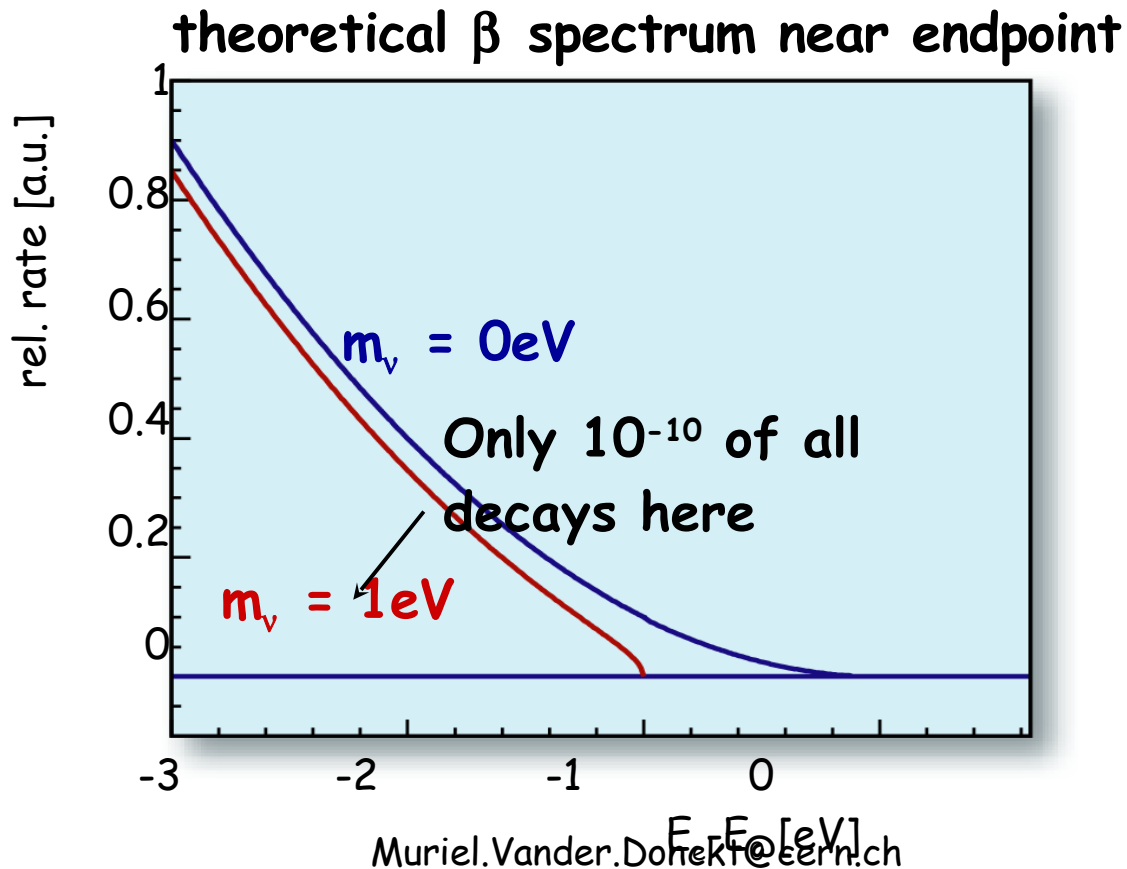
$$N_e(E) = E_e p F(Z, E) (E_0 - E_e)^2$$



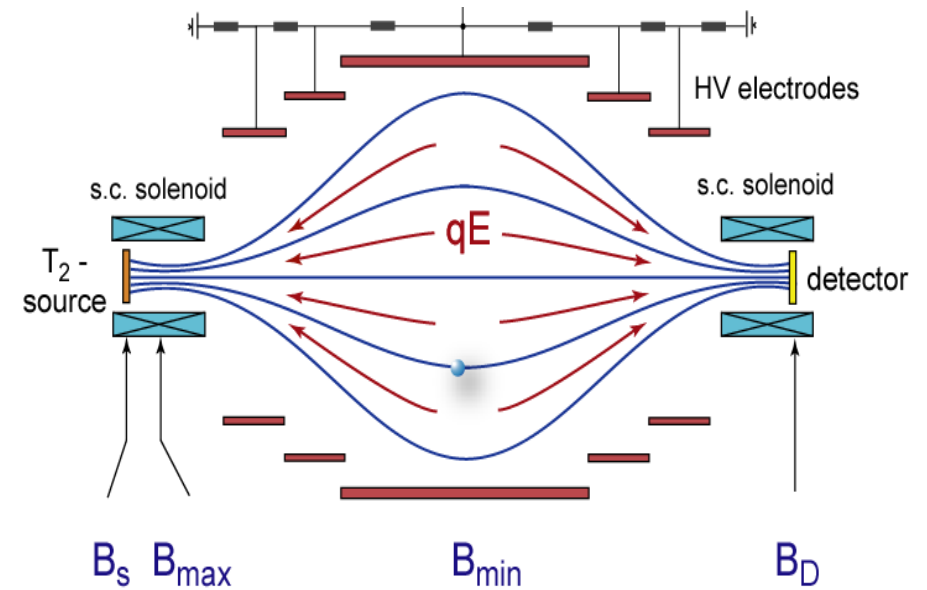
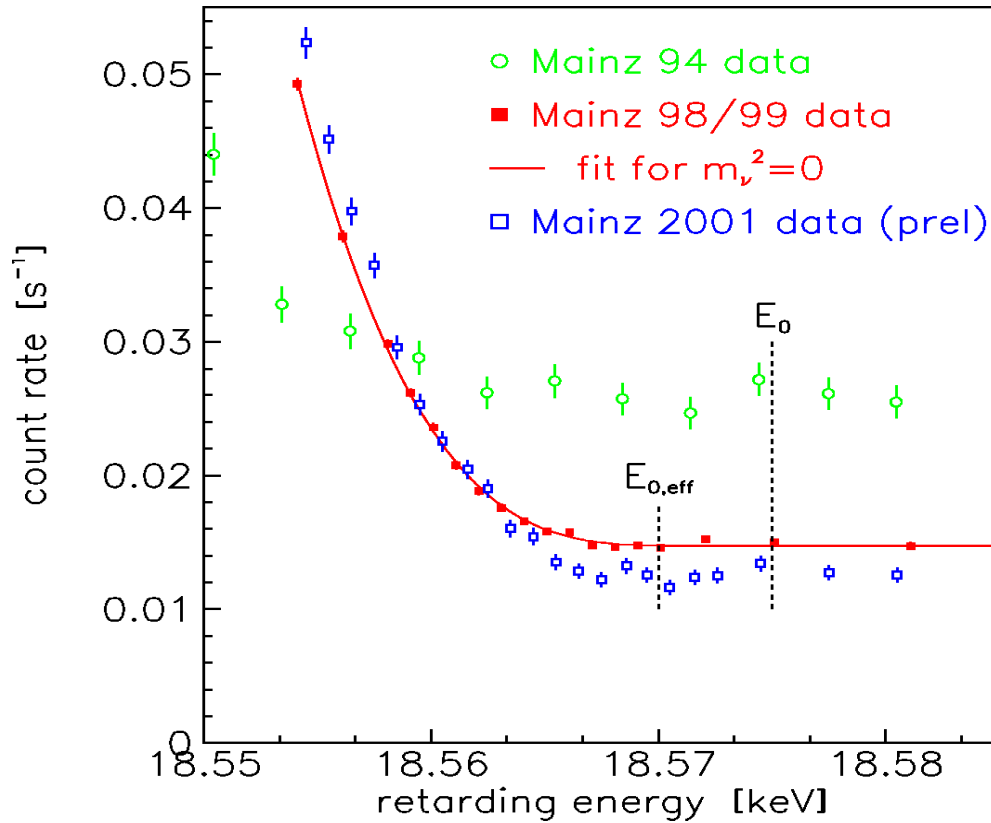
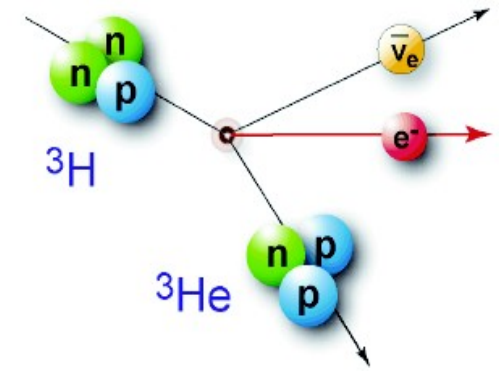
$$K(E) = \sqrt{\frac{N_e}{pEF(Z, E)}} \propto E_0 - E$$

Kurie Plot for a massive neutrino

$$K(E) \propto (E_0 - E) \cdot \left(1 - \frac{m_\nu^2}{(E - E_0)^2}\right)^{\frac{1}{4}}$$



Tritium β decay



Mainz : $m_\nu < 2.2\text{eV}$ (95%CL)

Sets threshold near the endpoint

Future: Katrin

expectation:

after 3 full run years

$$\sigma_{\text{syst}} \sim \sigma_{\text{stat}}$$



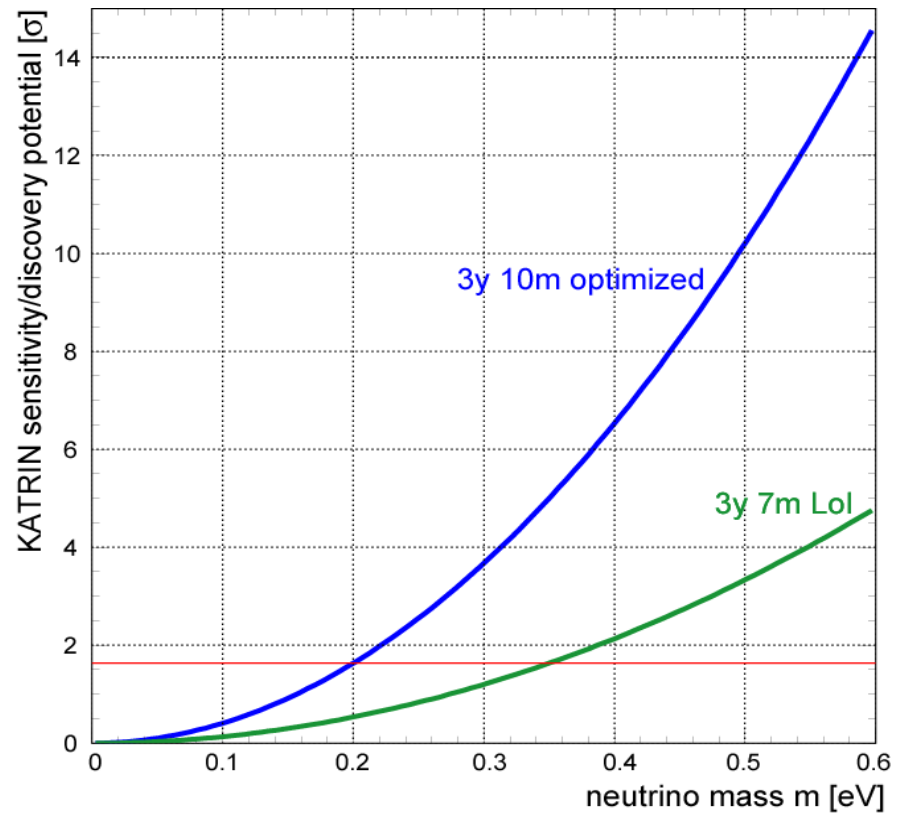
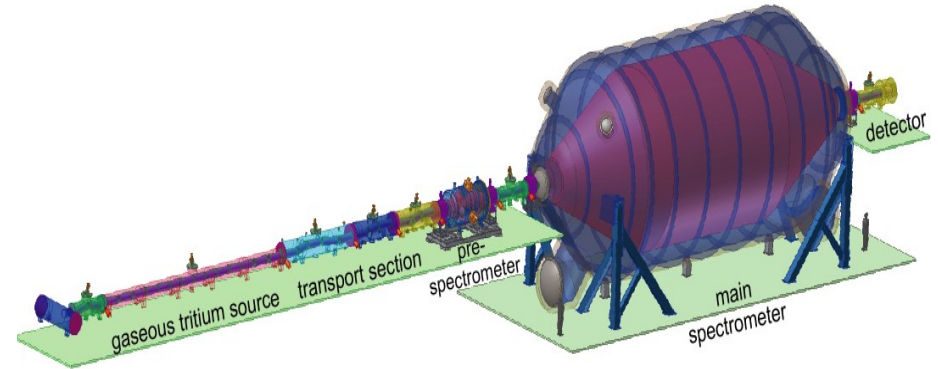
$$m_\nu = 0.35\text{eV} (5\sigma)$$

$$m_\nu = 0.3\text{eV} (3\sigma)$$

discovery potential

$$m_\nu < 0.2\text{eV} (90\%CL)$$

sensitivity



Pion and tau decay

- $\pi^+ \longrightarrow \mu^+ \nu_\mu$ $m_\nu^2 = m_\pi^2 + m_\mu^2 - 2\sqrt{m_\mu^2 + |p_\mu|^2}$

Measured at PSI :

$$|p_\mu| = 29.79200 \pm 0.00011 \text{ MeV}$$

$$m_\nu < 0.17 \text{ MeV}$$

- $\tau^- \longrightarrow 2\pi^- + \pi^+ + \nu_\tau$ and $\tau^- \longrightarrow 3\pi^- + 2\pi^+ (\pi^0) + \nu_\tau$

Measured in ALEPH: $m_\nu < 18.2 \text{ MeV}$

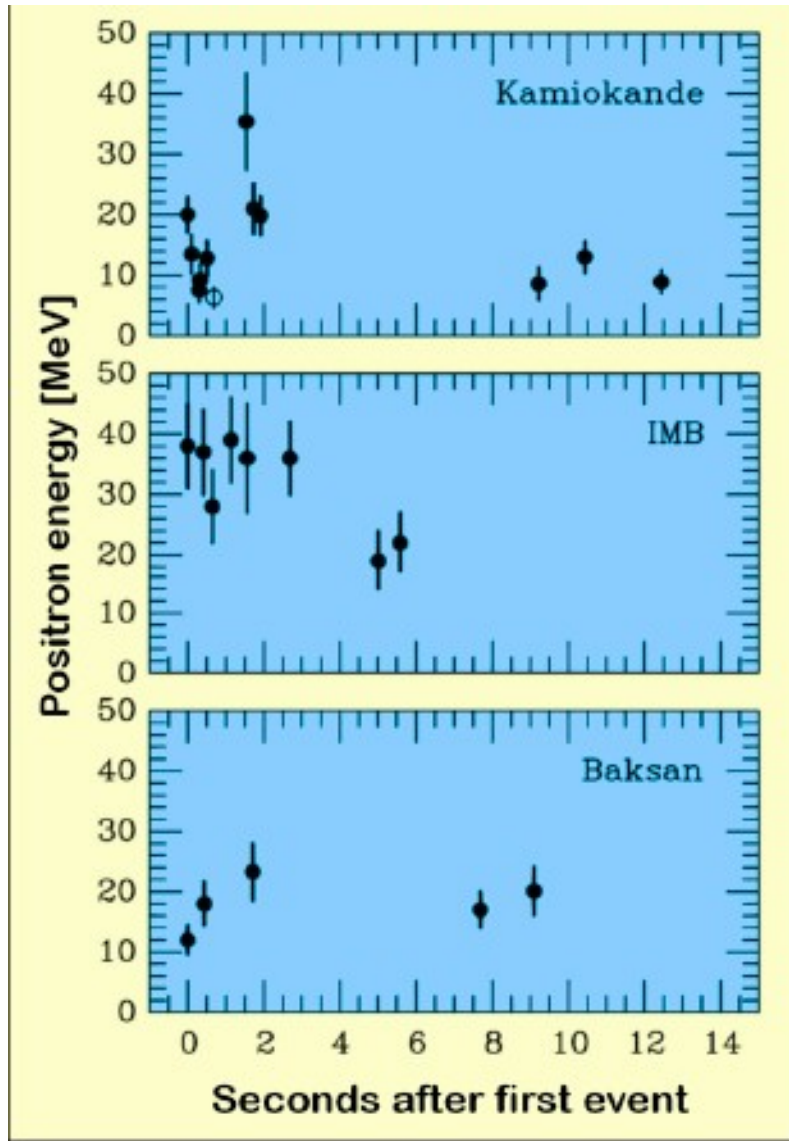
SuperNovae

- Type II supernova : Gravitational collapse of the Fe core of a $M \geq 8M_{\odot}$ star to form a neutron star or a black hole
- Nuclear binding energy released : $\sim 10^{53}$ erg carried by neutrinos with $\langle E \rangle = 10-20 \text{ MeV}$
- If neutrinos are massive Δt between detection of neutrinos with different E_{ν} gives a limit on the mass

$$t_F = \frac{L}{v} = \frac{L}{c} \frac{E_{\nu}}{\sqrt{E_{\nu}^2 - m_{\nu}^2 c^4}} \approx \frac{L}{c} \left(1 + \frac{m_{\nu}^2 c^4}{2E^2} \right)$$

$$\Delta t = \Delta t_0 + \frac{L m_{\nu}^2 c^3}{2} \left(\frac{1}{E_2^2} - \frac{1}{E_1^2} \right)$$

SN1987A



- Focused on proton decay, precise timing was not an issue for those detectors
- $m_{\nu_e} < 30\text{eV}$

Cosmological Neutrinos

- Nowadays relic neutrinos are non relativistic:

$$\rho_\nu = \sum_i m_{\nu_i} n_{\nu_i} = \Omega_\nu \rho_c \qquad \rho_c = \frac{3H^2}{8\pi G} \simeq 10.5 h^2 \text{keV cm}^{-3}$$

- WMAP, CBI, ACBAR: $\Omega_\nu h^2 < 0.0076$ (95%CL)

- $n_\nu = \frac{3}{4} \frac{\zeta(3)}{\pi^2} g_\nu T_\nu^3 = \frac{3}{4} \frac{\zeta(3)}{\pi^2} g_\nu \frac{4}{11} T_\gamma^3 = 113 \text{cm}^{-3} \quad (g_\nu = 2 \text{ dof})$

e+e- decoupling (0.511MeV)
reheats the photon CB

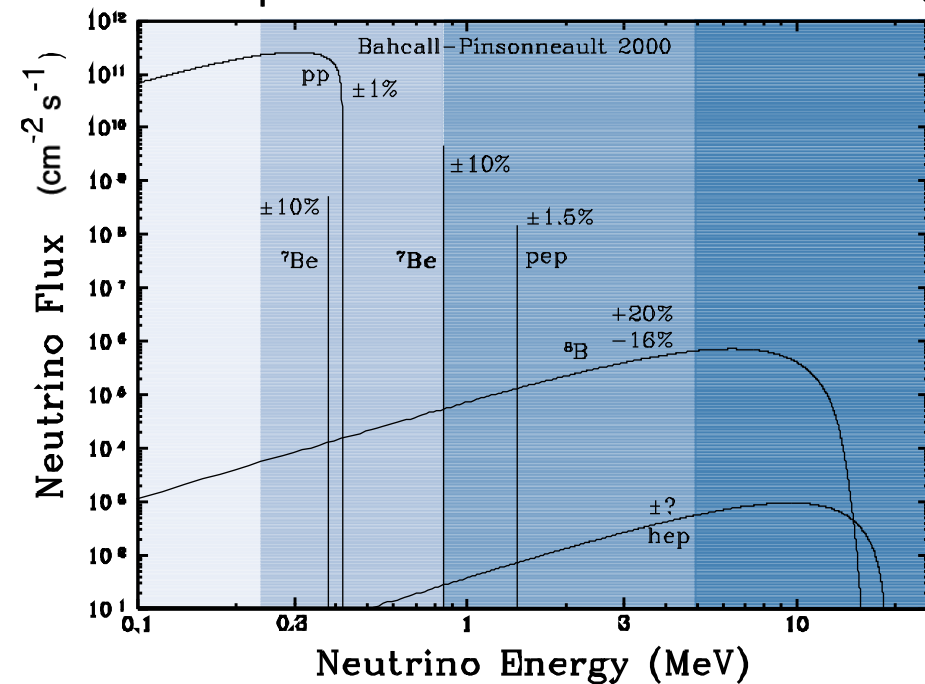
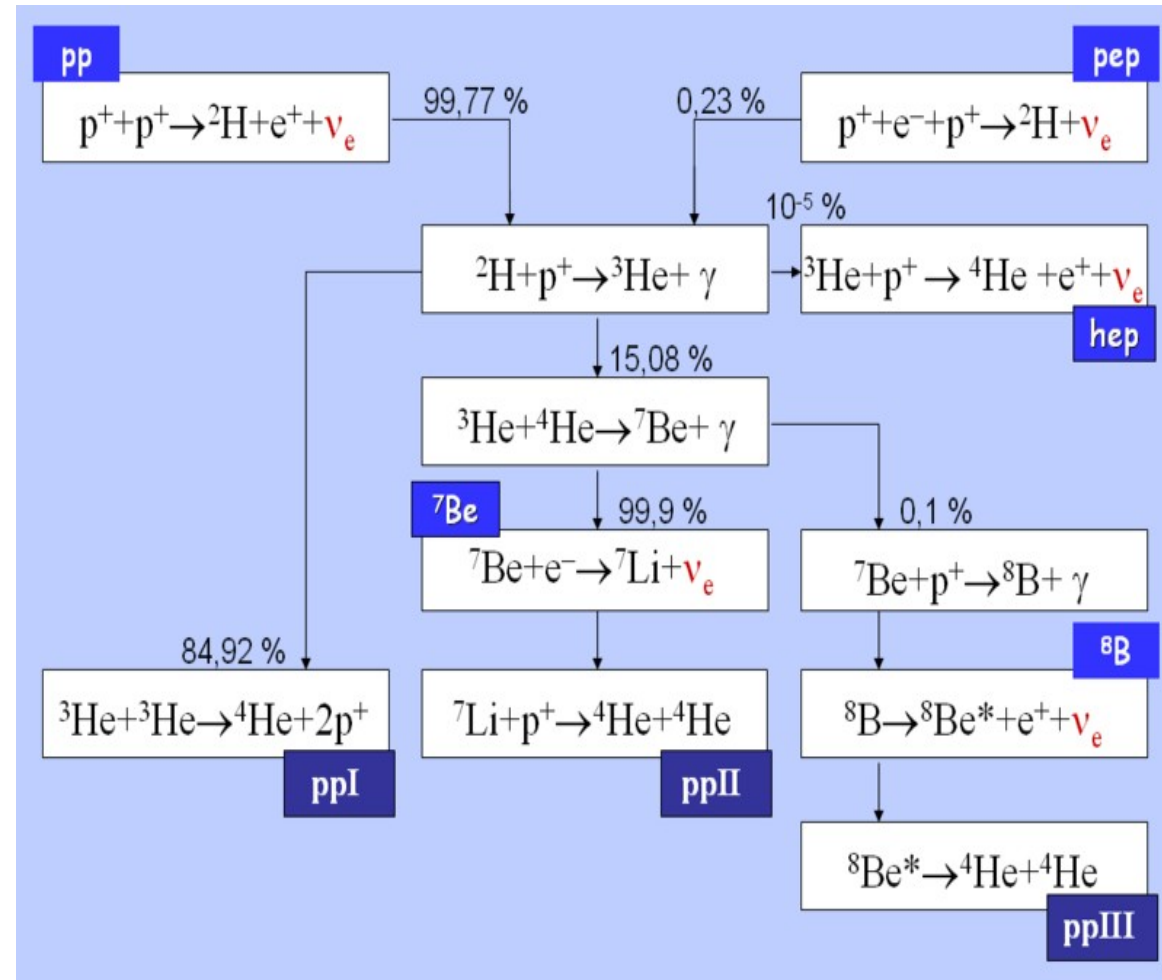
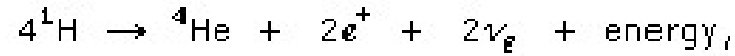
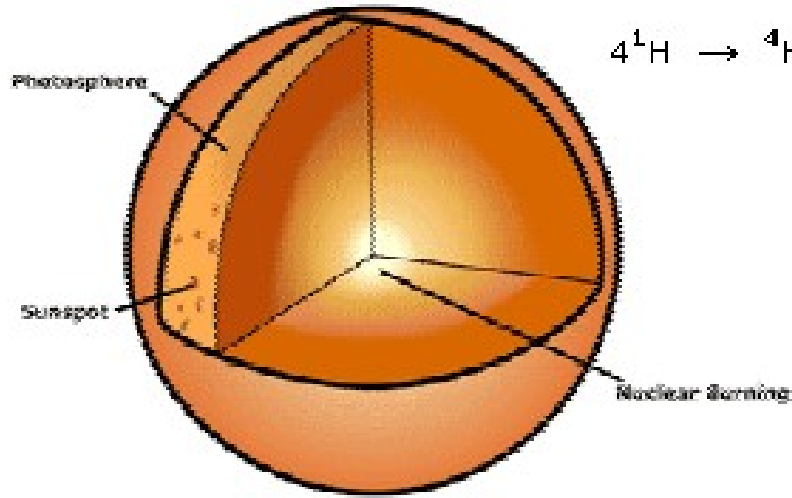
$$\sum_i m_{\nu_i} < 0.71 \text{eV}$$

To be taken with caution

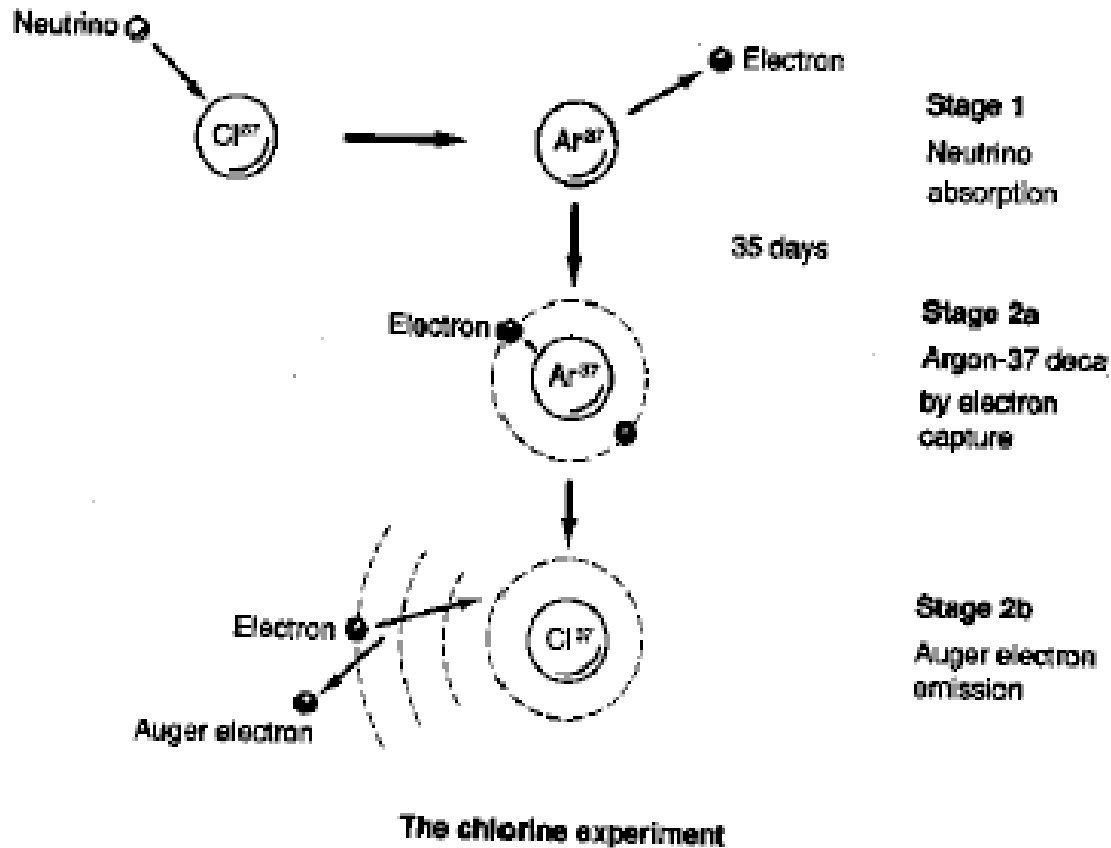
-Naturally available neutrino sources

-

Solar neutrinos



The chlorine experiment



$\text{Cl}^{37} \rightarrow 25\%$ of all natural chlorine

Inverse beta decay (0.86 MeV threshold)



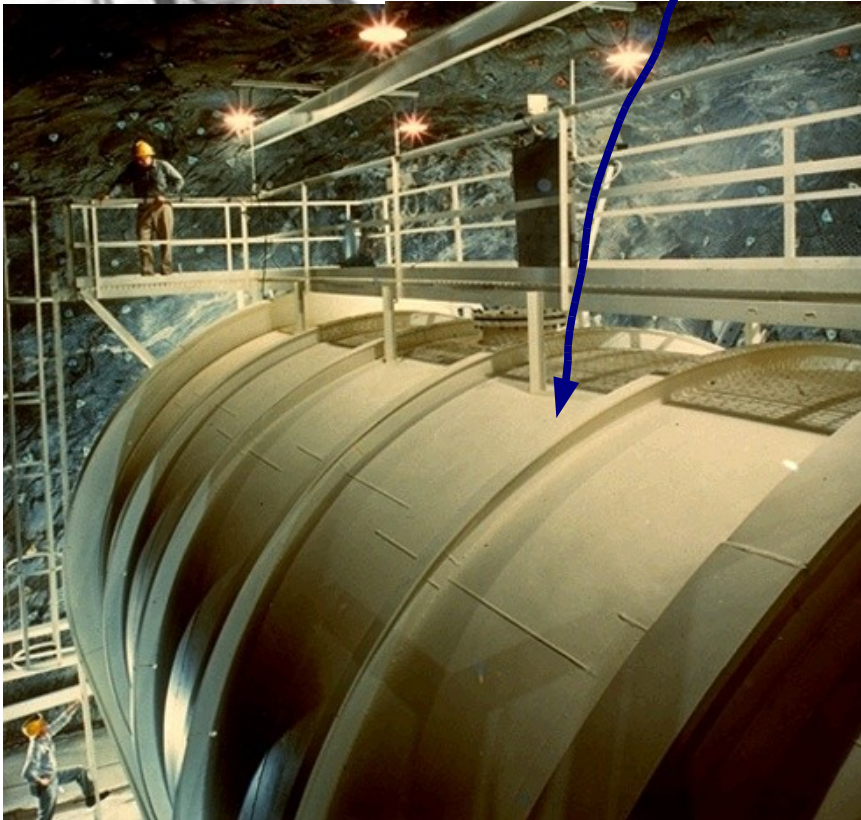
Ar is chemically very different from Chlorine. An inert gas that can be eventually removed from chlorine. It is radioactive and reverts to Cl^{37} emitting an Auger electron

R. Davis in the Homestake mine



Concept: Count the atoms of Ar^{37} produced by neutrinos

Only a few atoms of Ar^{37} per run!

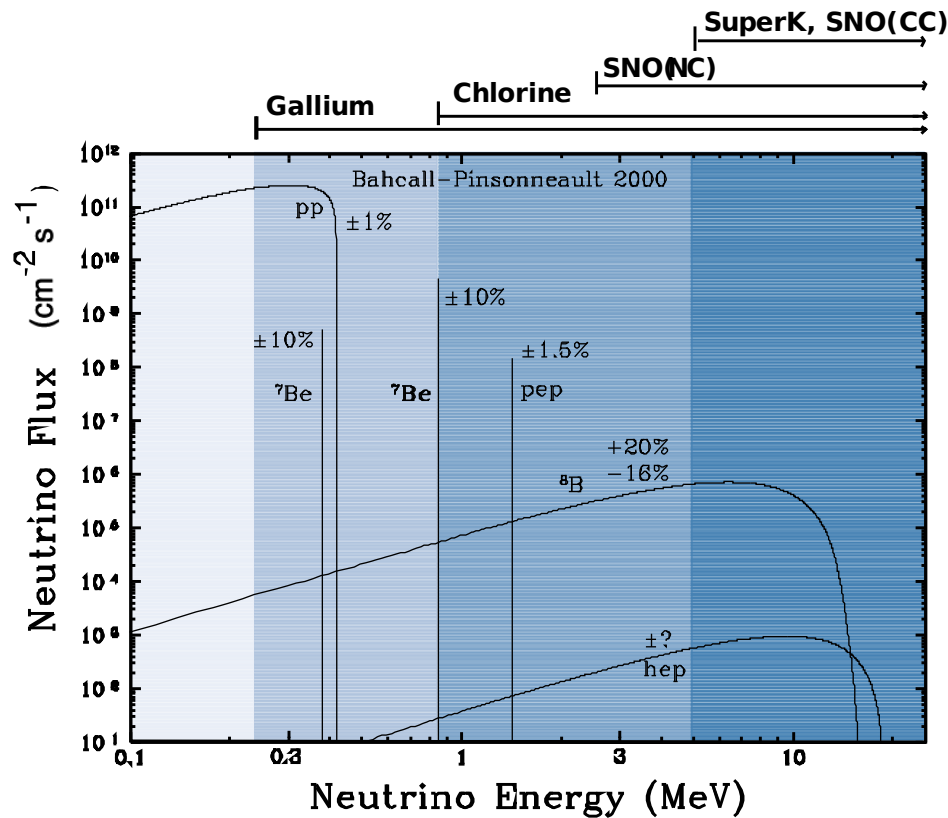


380 000 liters of C_2Cl_4 (a cleaning fluid) deep inside Homestake mine to shield from natural radiation (a Olympic swimming pool)

Let Argon-37 accumulate from 1 to 3 months. Flush with He gas to remove Ar from fluid. Let the Ar condensate in a 77 K charcoal trap. Collect and purify Ar.

Count the number of Auger electrons from Ar^{37}

Chlorine in numbers



Expected:

8.2 SNU ± 1.8

Observed

2.56 SNU ± 0.23

$$1\text{SNU} = 10^{-36} \nu \text{ capture s}^{-1} \text{ atom}^{-1}$$

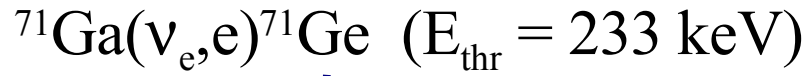
The solar neutrino problem

- Also called "paradox", "dilemma", "puzzle" and other nice words that showed that every body (secretly) believed that:
- Davis (Chlorine experiment) was wrong
- Bahcall (The solar model) was wrong

Or, more likely that:

Both were wrong!

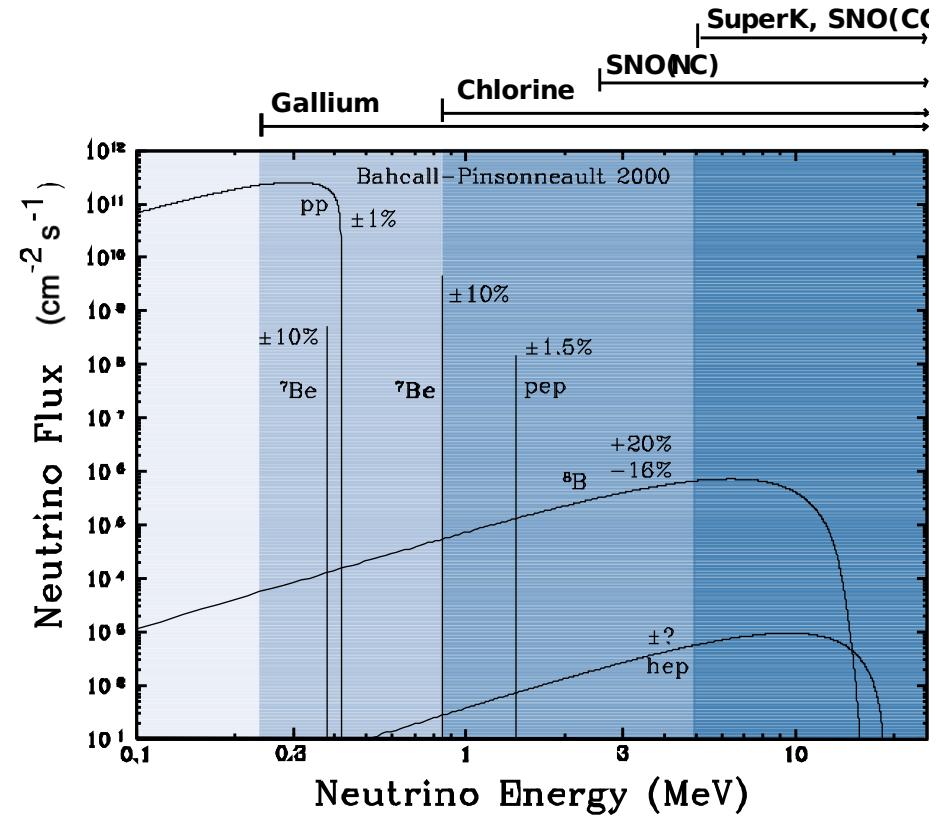
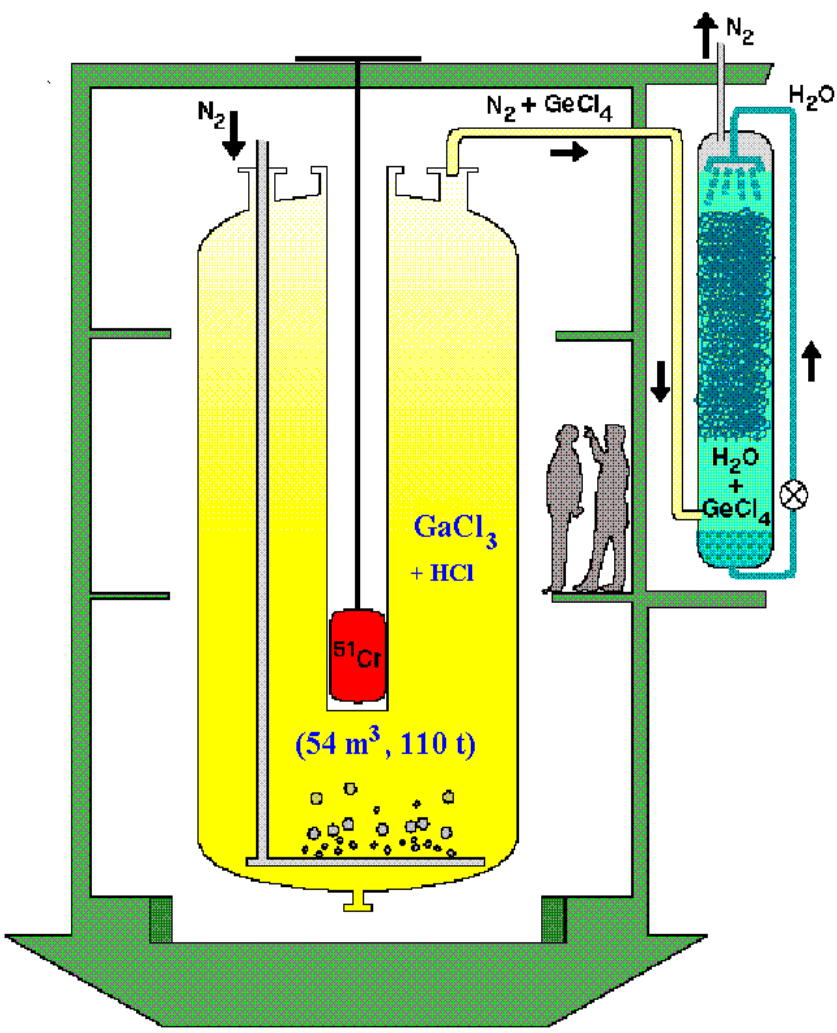
Gallium experiments



K,L shell EC

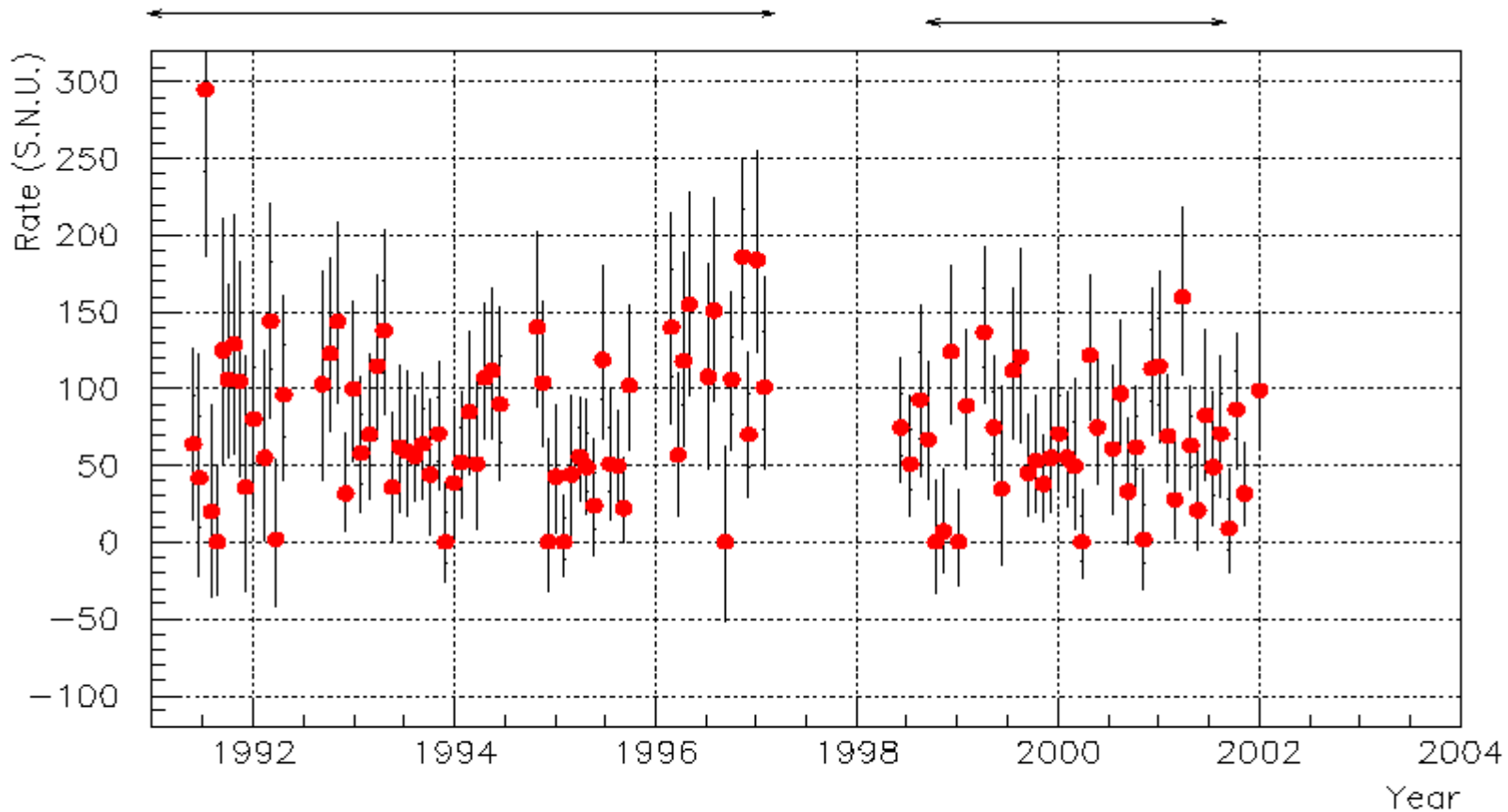
$\tau = 16.5 \text{ d}$

$^{71}\text{Ga} + 10 \text{ keV}, 1 \text{ keV}$
(Auger e^- , X)



Energy-RT analysis
GALLEX

Energy-PS/NN analysis
GNO



GALLEX

65 SR

77.5 ± 6.2 (stat) ± 4.5 (sys) SNU

GNO

43 SR

65.2 ± 6.4 (stat) ± 3.0 (sys) SNU

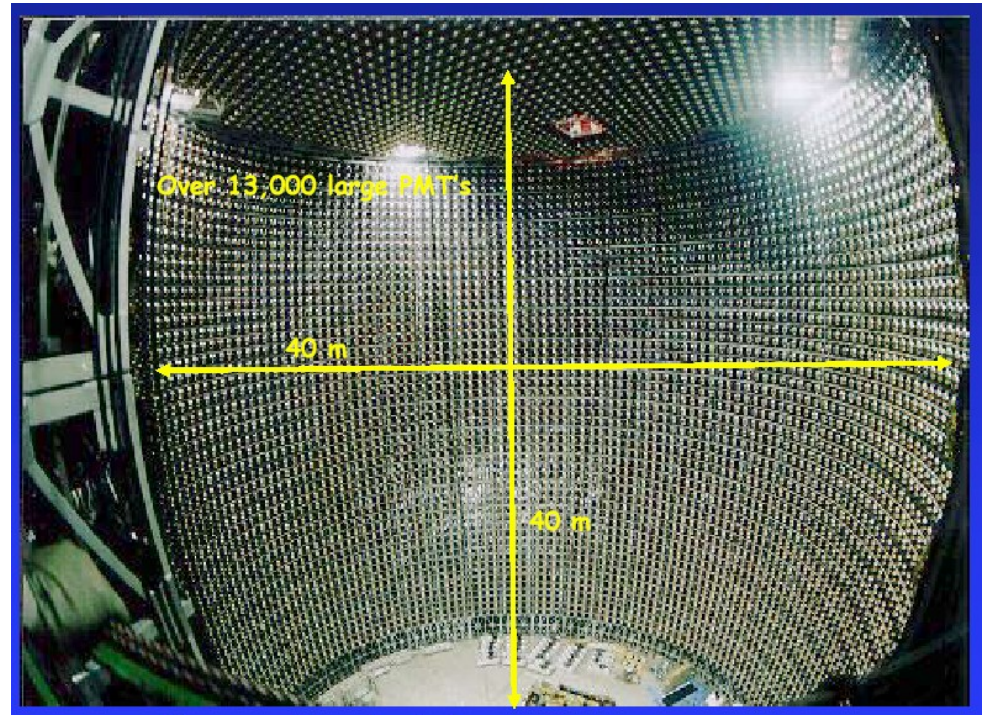
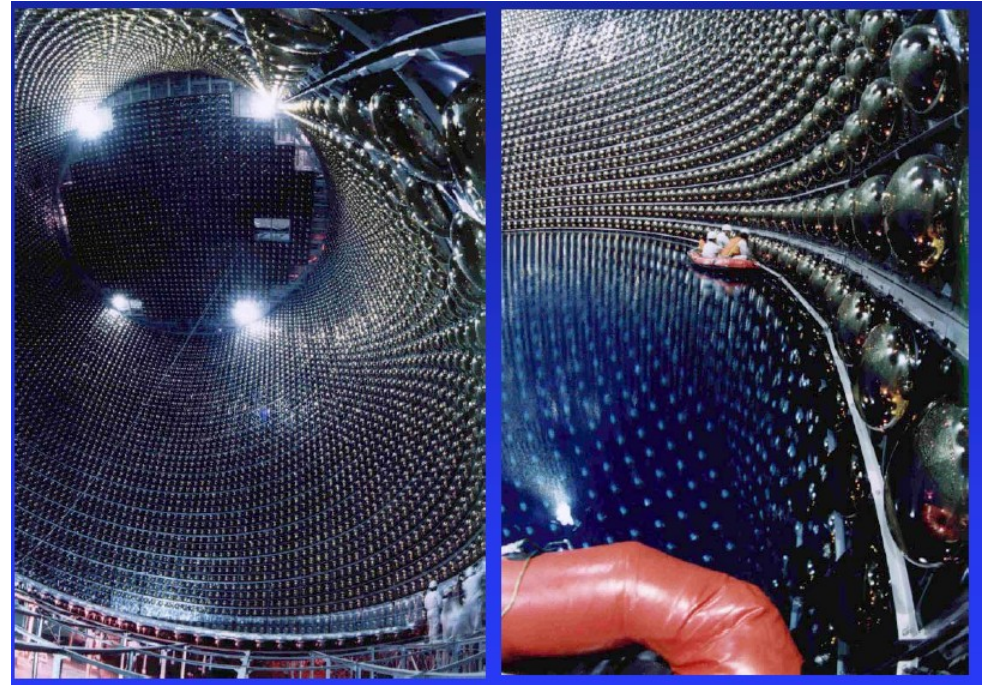
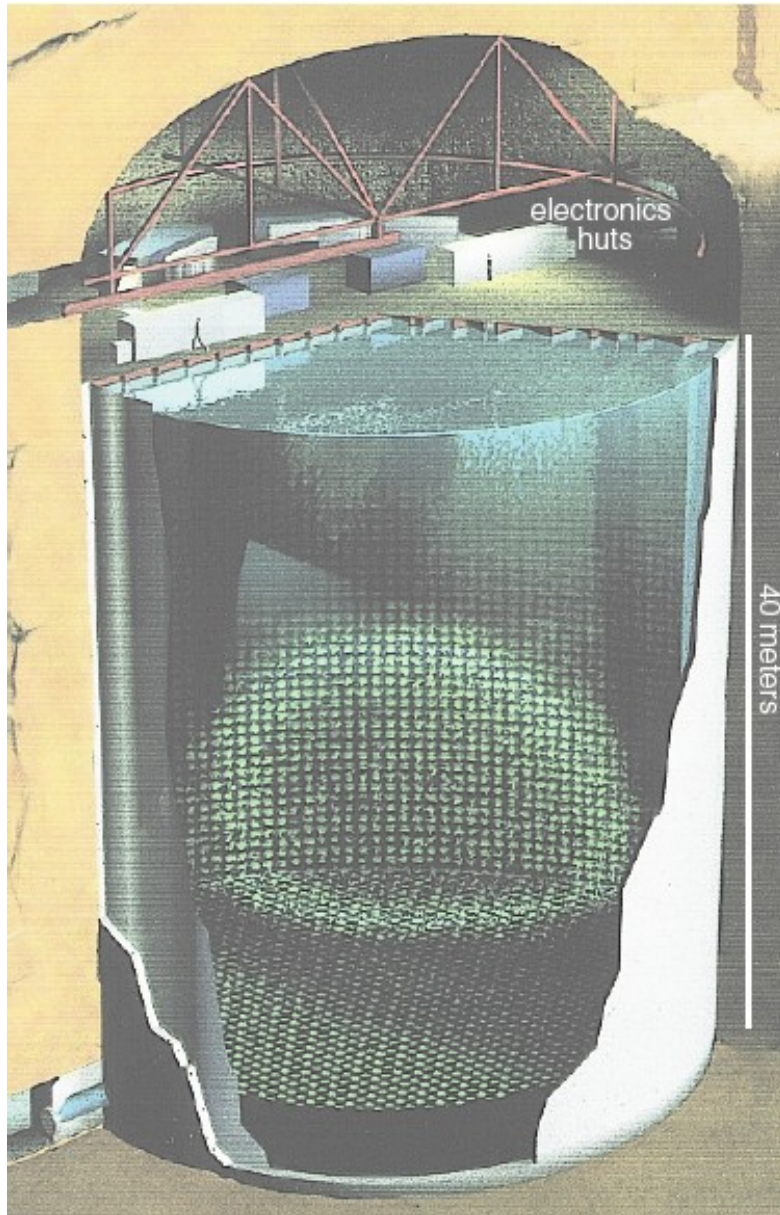
GNO+GALLEX

108 SR

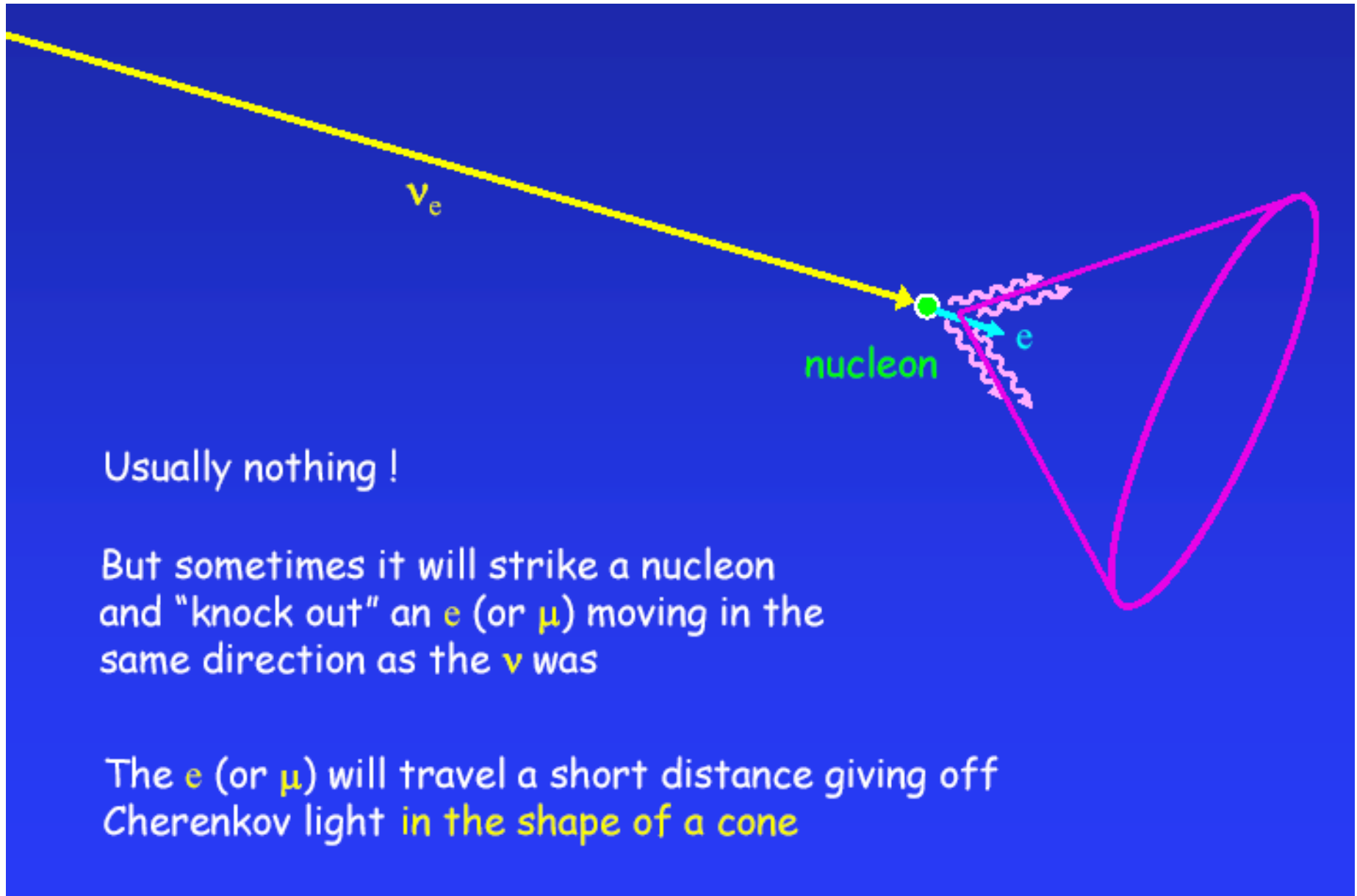
70.8 ± 4.5 (stat) ± 3.8 (sys) SNU

Expected 129^{+9}_{-8} SNU

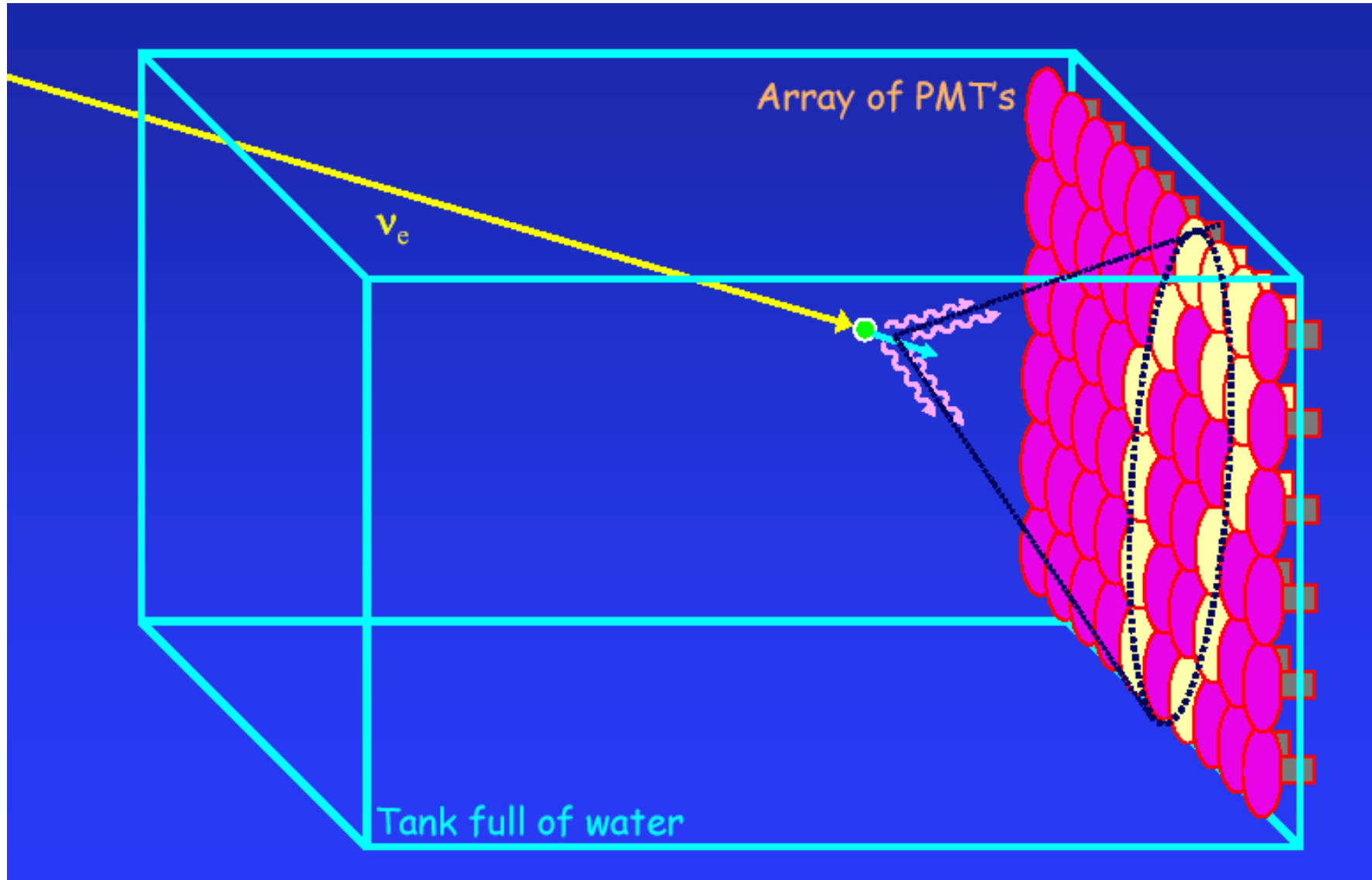
Super Kamiokande



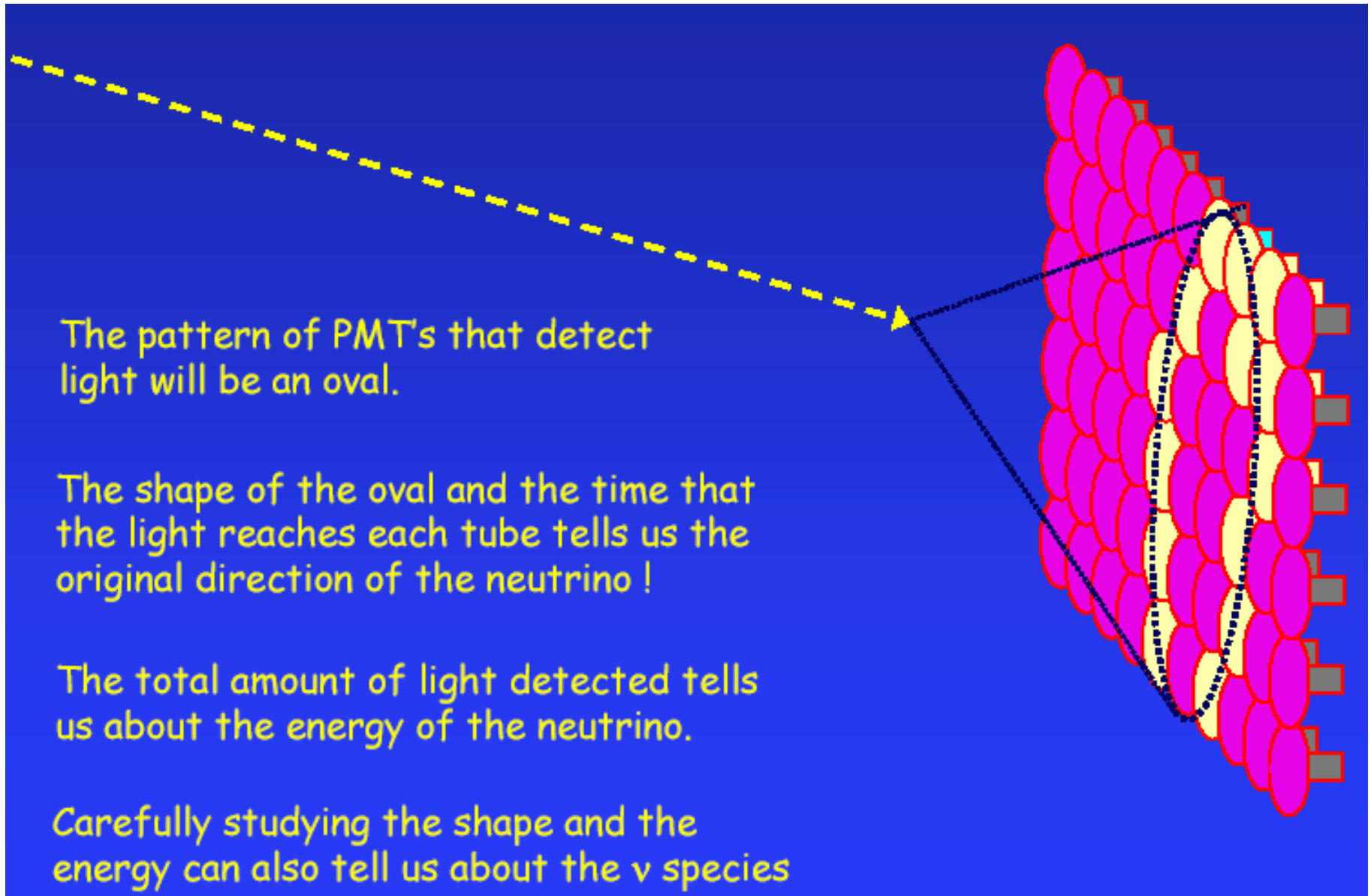
What does a neutrino do in water?



Water detectors: Concept

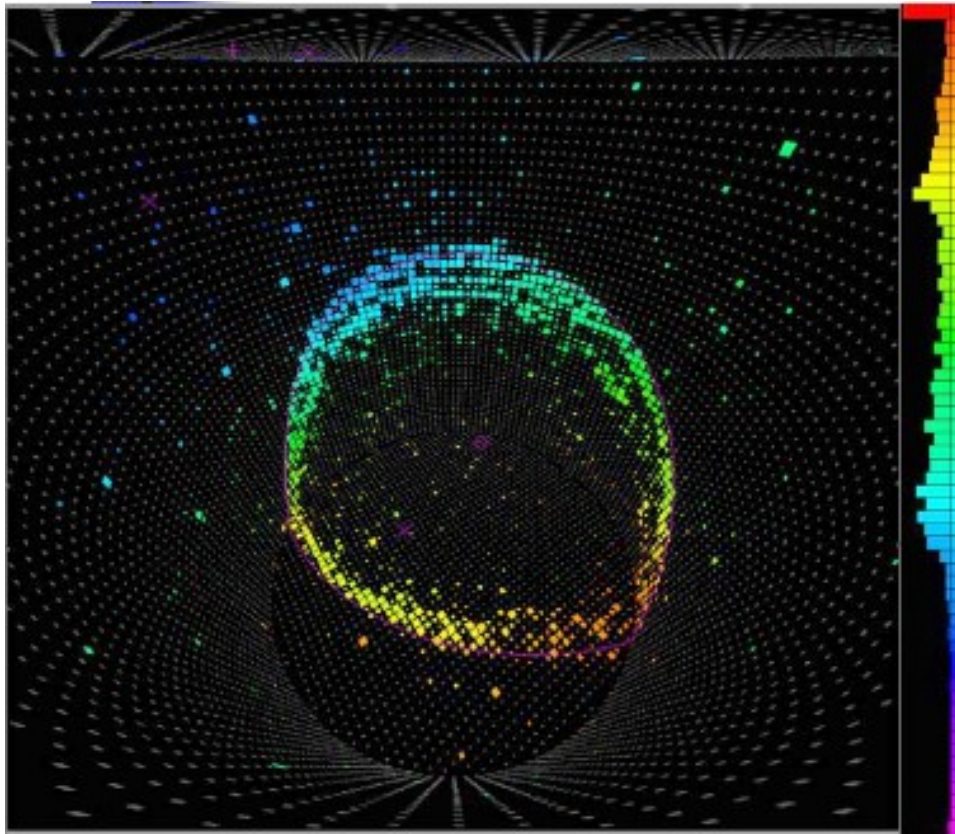


Water detectors: Concept

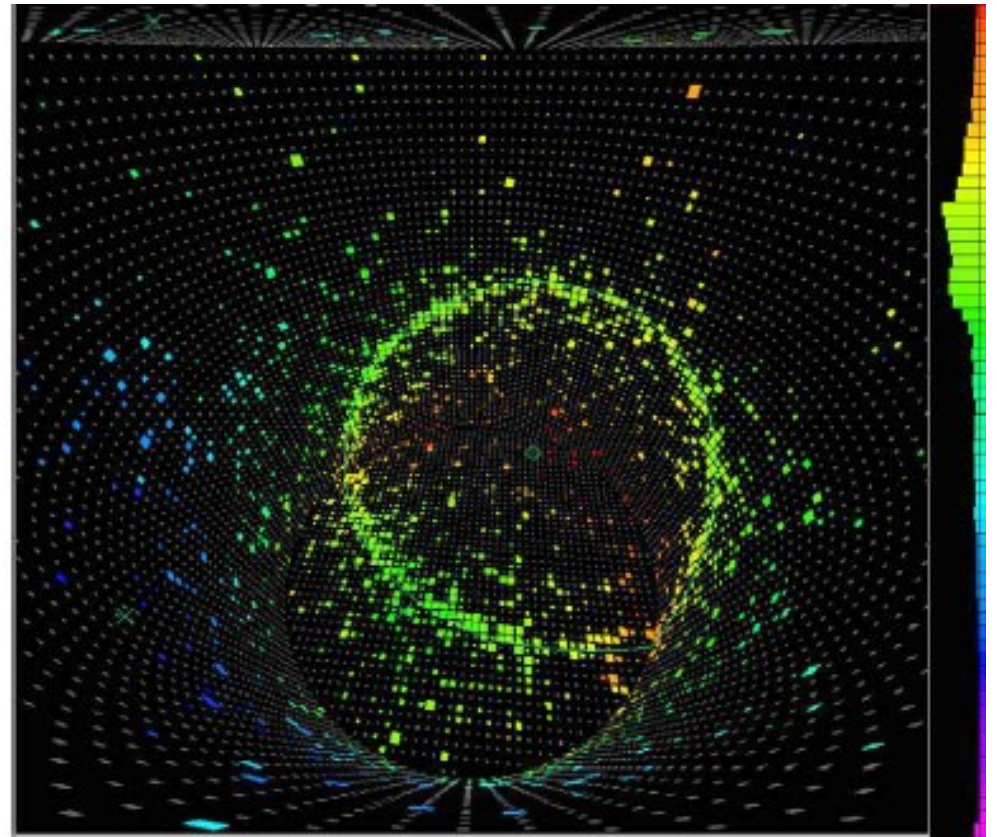


The eyes of Super-Kamiokande

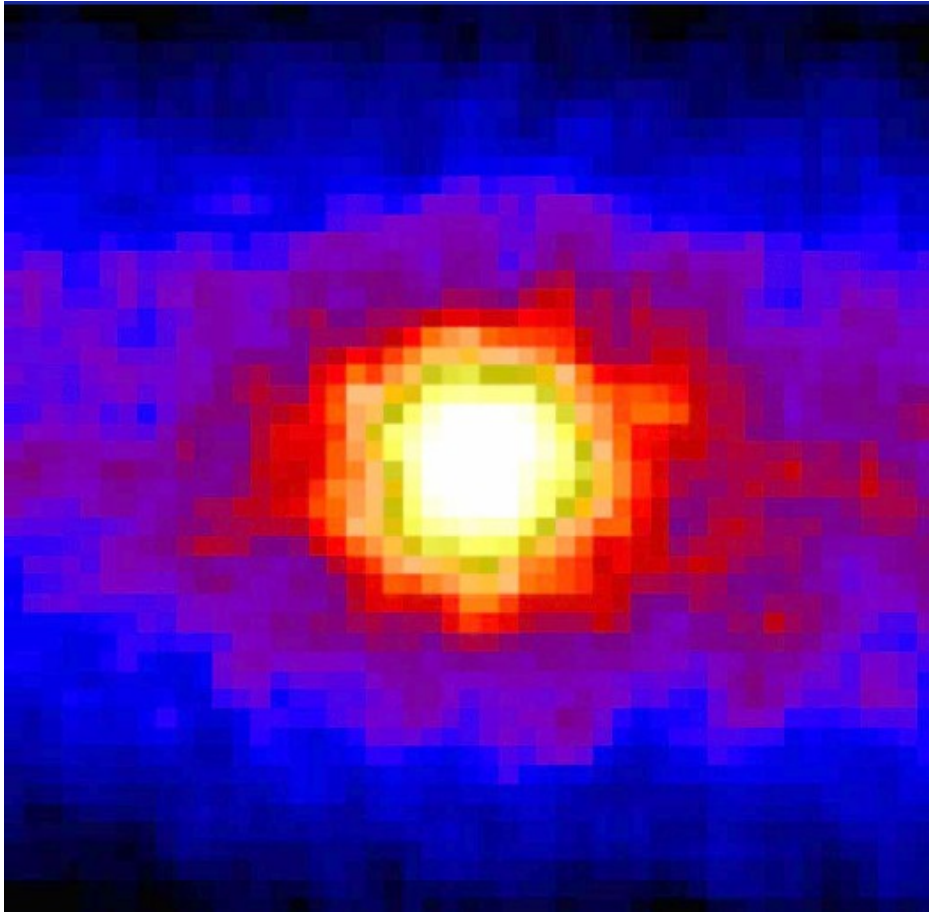
Muon ring



Electron ring (fuzzy)



A neutrino picture of the sun



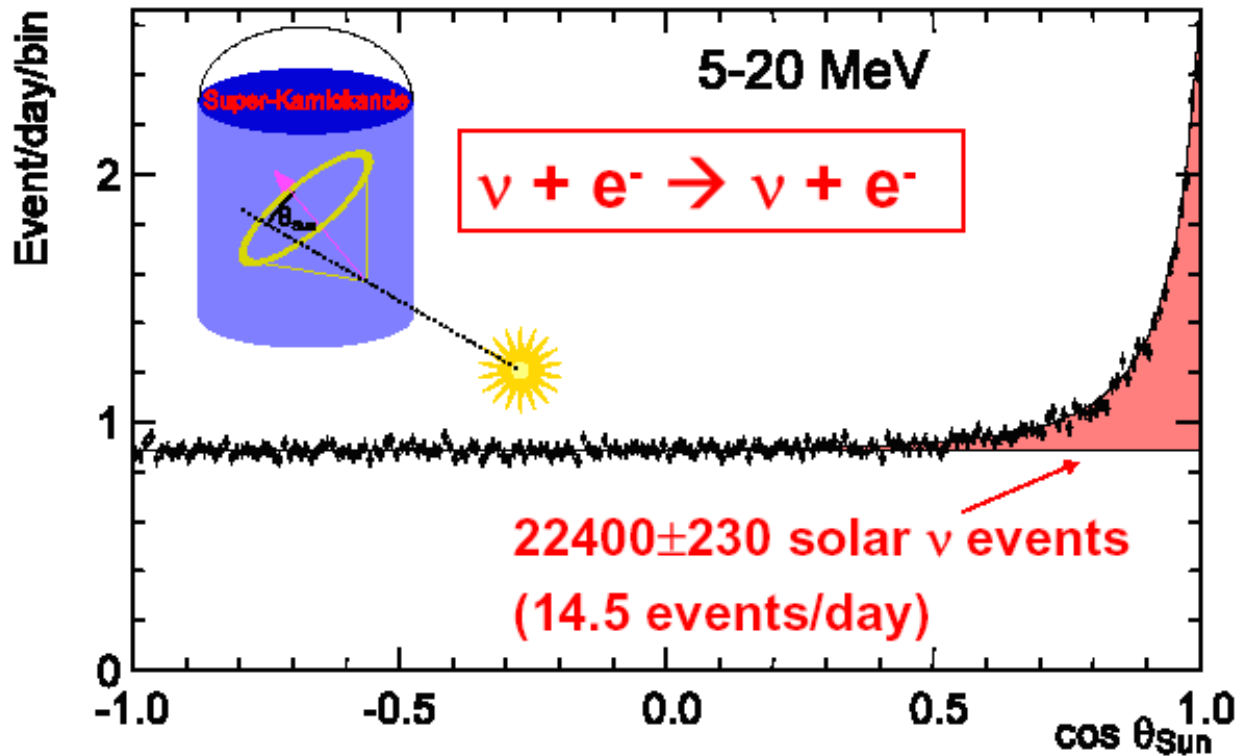
Super-K observation

Neutrinos come from
the sun indeed!

Super-Kamiokande solar results

Super-Kamiokande-I solar neutrino data

May 31, 1996 – July 13, 2001 (1496 days)

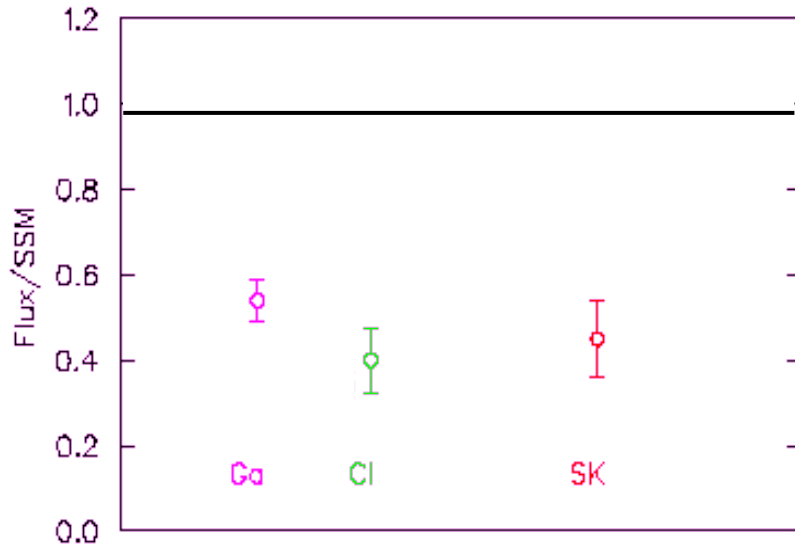


^8B flux : $2.35 \pm 0.02 \pm 0.08$ [$\times 10^6$ /cm²/sec]

$$\frac{\text{Data}}{\text{SSM(BP2004)}} = 0.406 \pm 0.004 \begin{matrix} +0.014 \\ -0.013 \end{matrix}$$

$$(\text{Data/SSM(BP2000)} = 0.465 \pm 0.005 \begin{matrix} +0.016 \\ -0.015 \end{matrix})$$

Solar Neutrino Problem summary



$$\text{GALLEX : } \frac{\phi_{Ga}(\nu_e)}{\phi_{SSM}(\nu_e)} = 0.58 \pm 0.05$$

$$\text{SAGE : } \frac{\phi_{Ga}(\nu_e)}{\phi_{SSM}(\nu_e)} = 0.60 \pm 0.05$$

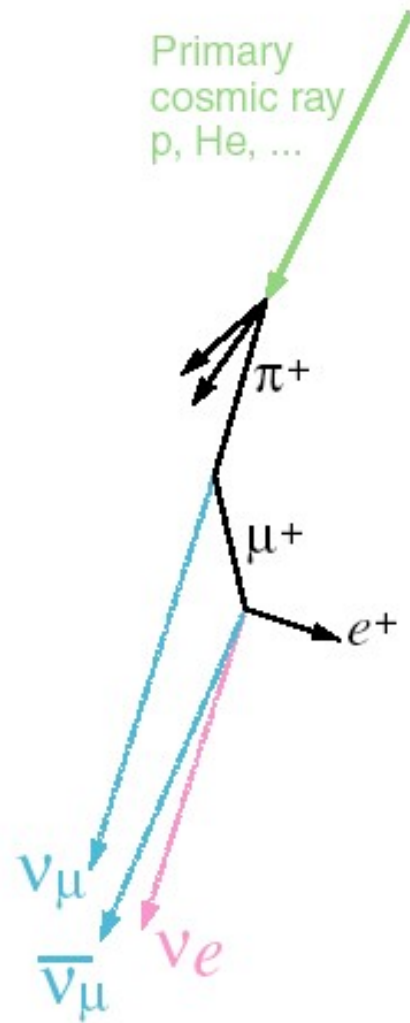
$$\text{Homestake : } \frac{\phi_{Cl}(\nu_e)}{\phi_{SSM}(\nu_e)} = 0.34 \pm 0.03$$

$$\text{Super - K : } \frac{\phi_{SK}(\nu_x)}{\phi_{SSM}(\nu_e)} = 0.451^{+0.017}_{-0.015}$$

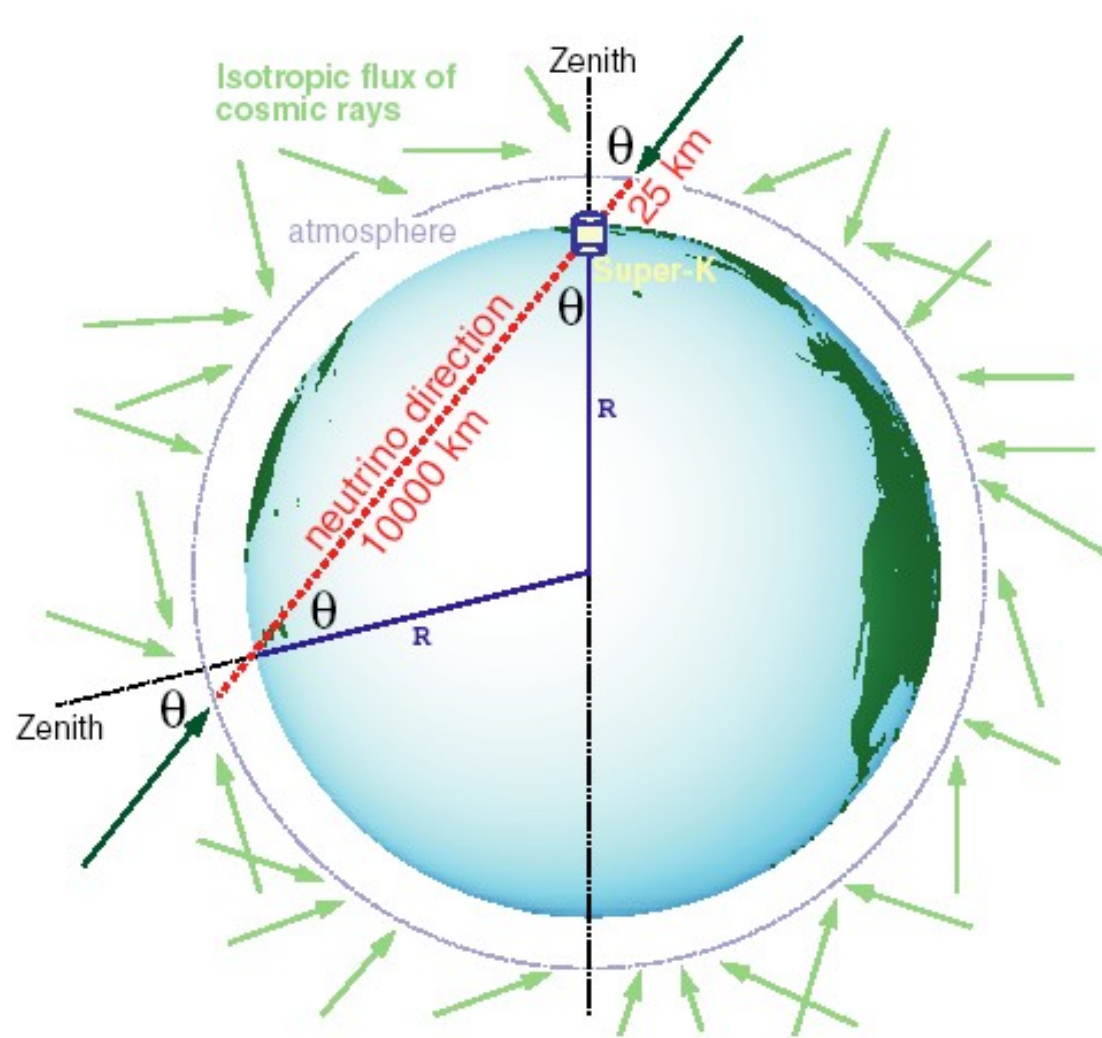
| <i>Experiment</i> | <i>Reaction</i> |
|--|---|
| Homestake | $\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e$ |
| SAGE | $\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e$ |
| Gallex + GNO | $\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e$ |
| Kamiokande + Super-Kamiokande | $\nu_x + e \rightarrow \nu_x + e$ |

R. Davis was NOT wrong !!!

Atmospheric neutrinos

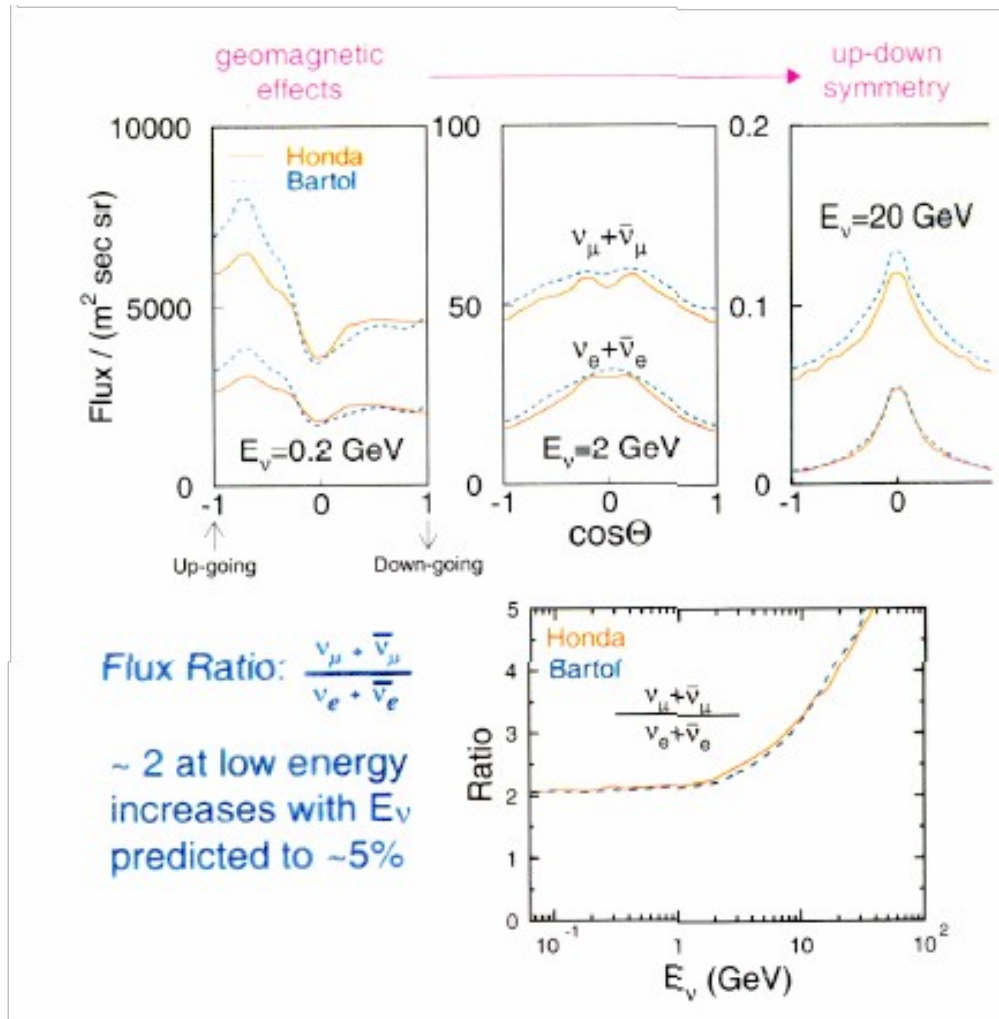


Ratio of $\nu_\mu/\nu_e \sim 2$
(for $E_\nu < \text{few GeV}$)



Up/Down Symmetric Flux
(for $E_\nu > \text{few GeV}$)

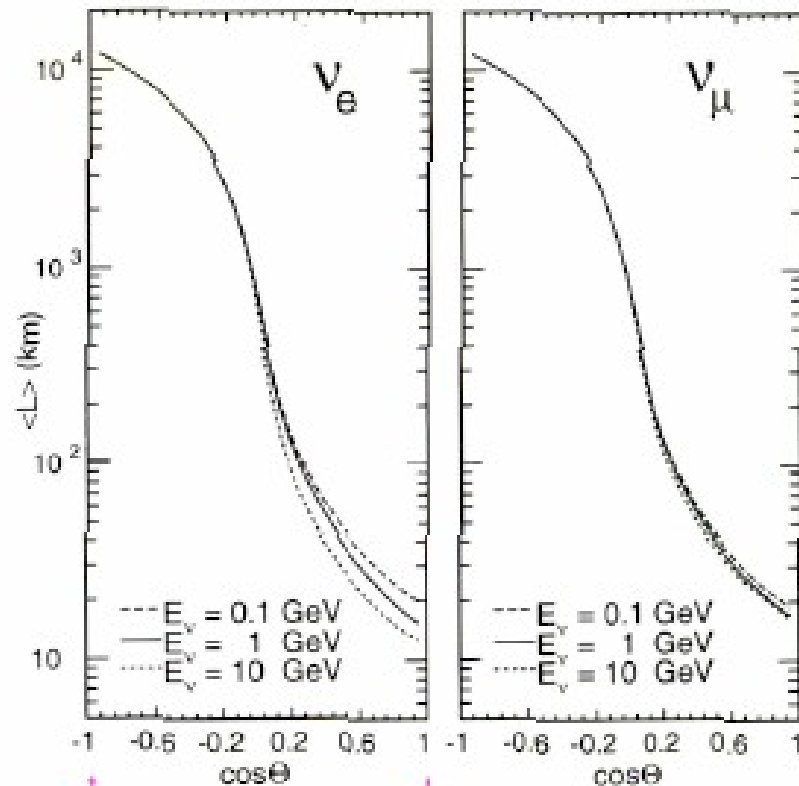
Atmospheric flux prediction



Ingredients:

- Primary proton flux
- Hadronic production models
- Hadron propagation (geomagnetic effects)

Zenith angle measures ν path length



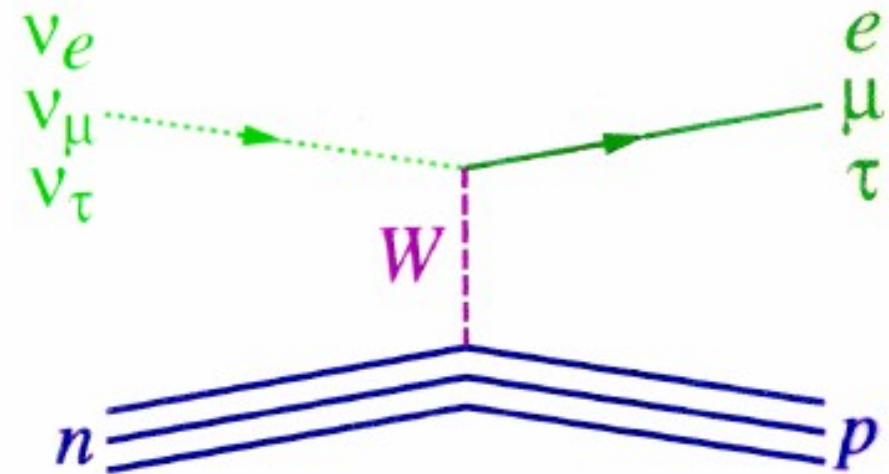
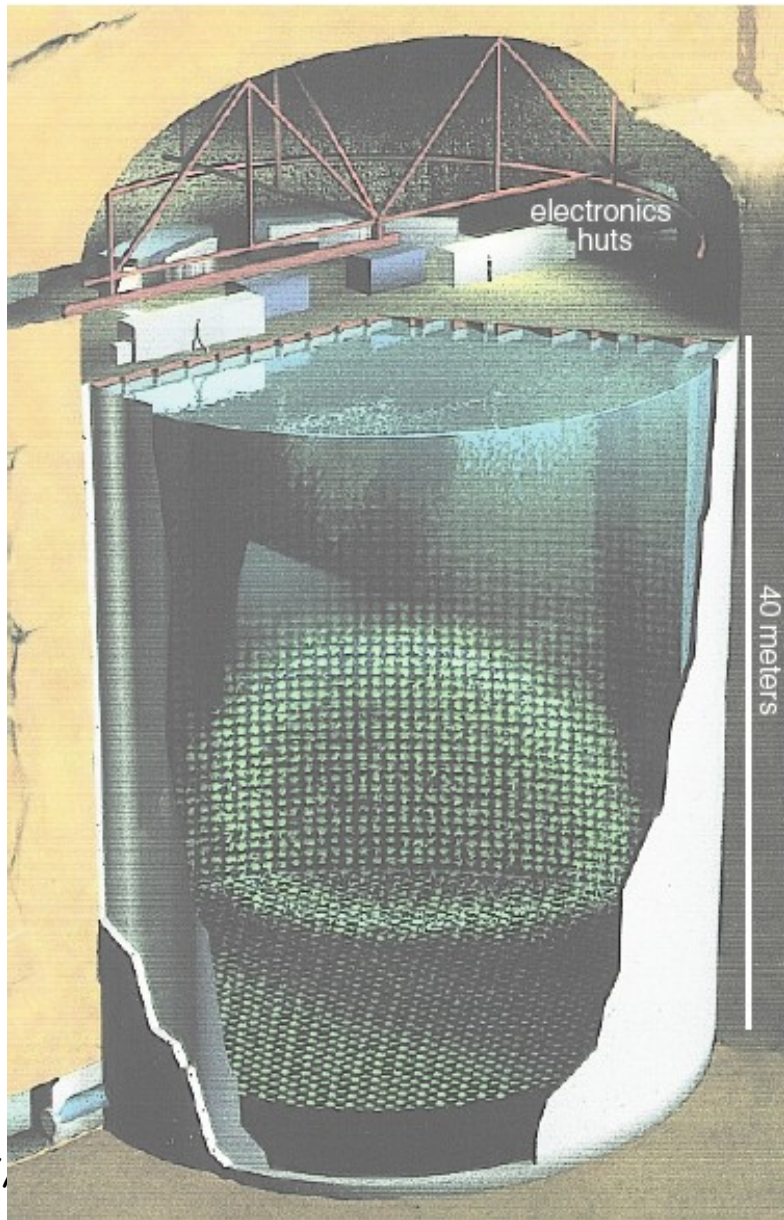
Three decades of path length from $O(10^4)$ to $O(10)$ km.

upgoing

downgoing

note rapid transition through horizon

Super-K sees also atmospheric neutrinos



$$\sigma \sim G_F^2 M E$$

$$\sim 10^{-38} \text{ cm}^2 \text{ at } 1 \text{ GeV}$$

Reaction Threshold

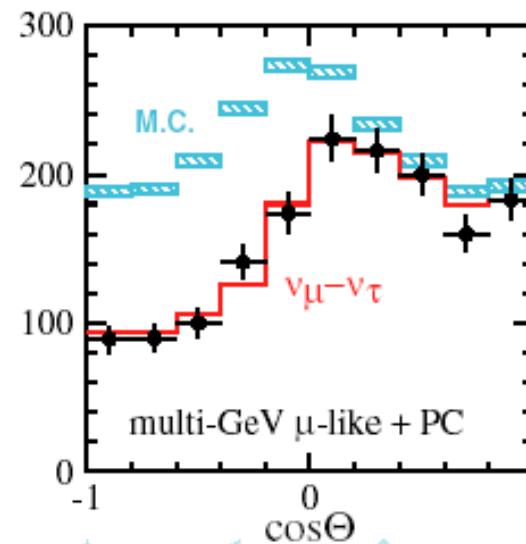
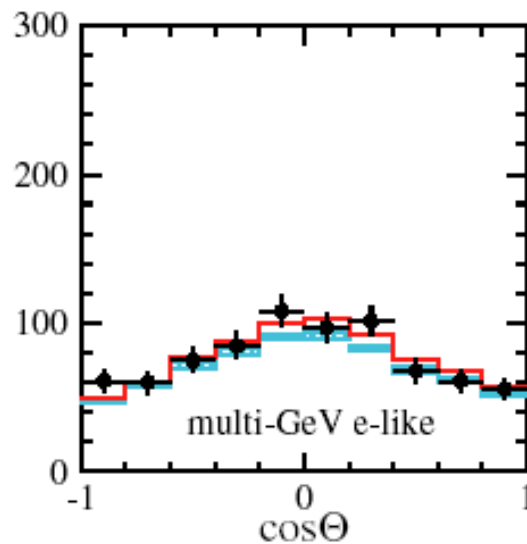
| | |
|--------|----------------------------|
| e | $E_\nu > 1.5 \text{ MeV}$ |
| μ | $E_\nu > 110 \text{ MeV}$ |
| τ | $E_\nu > 3500 \text{ MeV}$ |

The atmospheric neutrino problem

sub-GeV: $\frac{(N_{\mu}/N_e)_{\text{DATA}}}{(N_{\mu}/N_e)_{\text{M.C.}}} = 0.688 \pm 0.016 \pm 0.050$
 stat. sys.

multi-GeV: $\left(\frac{N_{\text{UP}} - N_{\text{DOWN}}}{N_{\text{UP}} + N_{\text{DOWN}}} \right)_{\mu\text{-like}} = -0.303 \pm 0.030 \pm 0.004$
 stat. sys.

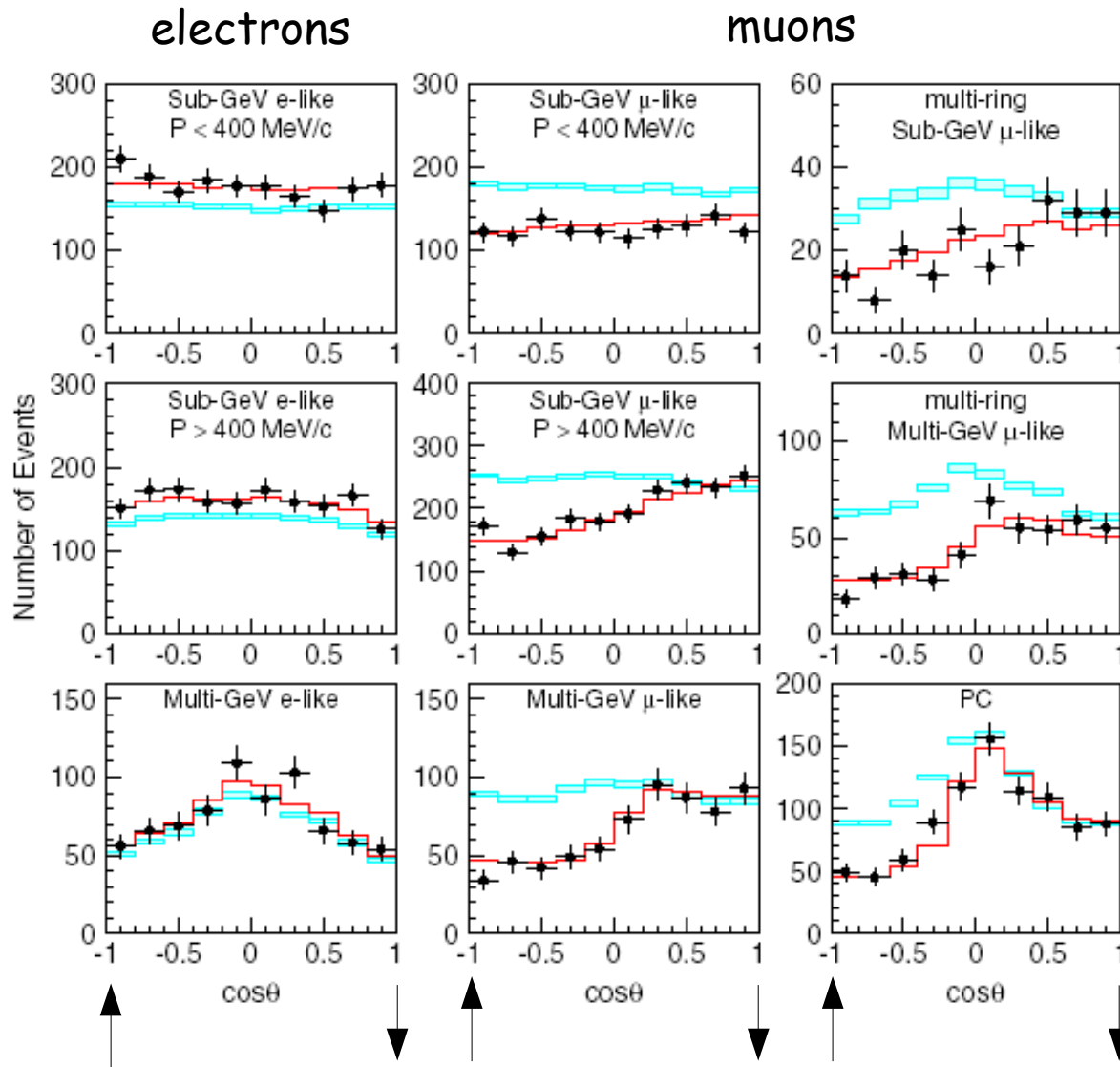
> 10 σ deviation!



1489 day
Super-K
preliminary

Neutrino travel distance: 12800 6200 700 40 15 km

Energy dependence



Effect depends not only on the path length but also on the energy

Solar & Atmospheric neutrino problems

- The combination of all solar neutrino experiments implied that solar neutrinos were disappearing between production (in the sun core) and detection in the earth.
- The zenith dependence observed by Super-Kamiokande experiment showed that ν_μ (but not ν_e) produced in the atmosphere were also disappearing. The effect depends on the zenith angle, that is on the neutrino path length, between production and detection and on the energy

That's it for today

Neutrino mixing

- EW CC:
$$j_\alpha^W = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma_\alpha l_L + 2 \sum_{q=u,c,t} \sum_{q'=d',s',b'} \bar{q}_L \gamma_\alpha q'_L$$

where ν_{lL} , l_L , q_L are left-handed neutrino, lepton

and quark fields and $q'_L = \sum_{q=d,s,b} U_{q'q} q_L$

- In analogy to quark mixing in the lepton sector: $\nu_{lL} = \sum_k U_{lk} \nu_{kL}$ where U is unitary, ν_k is the ν field with mass m_k .

- Since $Q_\nu = 0$ neutrinos can be Dirac or Majorana particles...

Dirac mass term

$$L^D = - \sum_{l'} \bar{\nu}_{l'R} M_{l'l} \nu_{lL} + h.c. = - \bar{\nu}'_R M \nu'_L + h.c.$$

$$\nu'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \quad \nu'_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}$$

- Diagonalisation $M = V m U^+ \implies \mathcal{L}$ in terms of mass eigenstates :

$$L^D = - \bar{\nu}_R m \nu_L - \bar{\nu}_L m \nu_R = - \bar{\nu} m \nu = - \sum_{k=1}^3 m_k \bar{\nu}_k \nu_k$$

$$\begin{aligned} \nu_R &= V^+ \nu'_R \\ \nu_L &= U^+ \nu'_L \end{aligned} \quad \nu = \nu_R + \nu_L = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Majorana mass term

- Using ν_{lL} and $(\nu_{lL})^c = C\bar{\nu}_{lL}^T$ (Charge conjugation matrix)

- $$L^M = -\frac{1}{2} \sum_{l',l} (\bar{\nu}_{l'L})^c M_{l'l} \nu_{lL} + h.c. = -\frac{1}{2} (\bar{\nu}'_L)^c M \nu'_L + h.c.$$

- M is symmetric $M = (U^+)^T m U^+$

$$L^M = \frac{1}{2} (\nu'_L)^T C^{-1} (U^+)^T m U^+ \nu'_L + h.c. = -\bar{\chi} m \chi = \frac{1}{2} \sum_{k=1}^3 m_k \bar{\chi}_k \chi_k$$

$$\chi = U^+ \nu'_L + (U^+ \nu'_L)^c = \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}$$

↙ Field with majorana mass m_k

Violates L

General term

- Dirac +Majorana mass terms with L&R-handed fields

$$L^{D-M} = -\frac{1}{2} \left[(\bar{\nu}'_L)^c M_L \nu'_L + (\bar{\nu}'_R M_R (\nu'_R)^c + \bar{\nu}'_R M_D \nu'_L + (\bar{\nu}'_L)^c M_D^T (\nu'_R)^c \right] + h.c.$$

$$L^{D-M} = -\frac{1}{2} (\bar{n}_L)^c M n_L + h.c.$$

$$n_L = \begin{pmatrix} \bar{\nu}'_L \\ (\nu'_R)^c \end{pmatrix}$$

$$M = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}$$

- Diagonalising M we get :

$$L^{D-M} = -\frac{1}{2} \bar{\chi} m \chi = -\frac{1}{2} \sum_{k=1}^6 m_k \bar{\chi}_k \chi_k$$

$$\chi = U^+ n_L + (U^+ n_L)^c = \begin{pmatrix} \chi_1 \\ \vdots \\ \chi_6 \end{pmatrix}$$

↑
Majorana type

More on the general mass term

$$n_L = U \chi_L$$

$$\nu_{lL} = \sum_{k=1}^6 U_{lk} \chi_{kL}$$

1st 3 rows

$$(\nu_{lR})^c = \sum_{k=1}^6 U_{\bar{l}k} \chi_{kL}$$

Last 3 rows

Right handed neutrinos (sterile)

- $\nu_i \rightarrow \nu_j$ flavour transitions are possible
- Active to sterile neutrinos are also possible !

For one family

- $M = O m' O^T$ (no CP violation) $O = \begin{pmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{pmatrix}$

$$\nu_L = \sin\theta \chi_{1L} - \cos\theta \chi_{2L}$$

$$(\nu_R)^c = \cos\theta \chi_{1L} + \sin\theta \chi_{2L}$$

$$m_L = \sin^2\theta m'_1 + \cos^2\theta m'_2$$

$$m_R = \cos^2\theta m'_1 + \sin^2\theta m'_2$$

$$2m_D = \sin 2\theta (m'_1 - m'_2)$$

- M eigenvalues :

$$m'_{1,2} = \frac{1}{2} (m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2})$$

$$\sin 2\theta = \frac{2m_D}{\sqrt{(m_L - m_R)^2 + 4m_D^2}}$$

Seesaw Mechanism

• Limiting case $m_L \approx 0$, $m_R \gg m_D$

$$m_1 \approx m_R, m_2 \approx m_D^2 / m_R$$

$$\theta \approx m_D / m_R$$

$$V_L \approx -\chi_{2L} \quad (V_R)^c \approx \chi_{1L}$$

If $m_D \approx m_{l,q}$ then $m_\nu \approx m_q^2 / m_R$ or m_l^2 / m_R

m_R is assumed GUT scale $\sim 10^{14} \text{ GeV}$

The heavy right-handed Majorana mass generates the small active neutrino mass

$$m'_{1,2} = \frac{1}{2}(m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2})$$

$$\sin 2\theta = \frac{2m_D}{\sqrt{(m_L - m_R)^2 + 4m_D^2}}$$

Oscillations in vacuum

$$|v_l\rangle = \sum_k U_{lk}^* |v_k\rangle$$

$$|v_k(t)\rangle = e^{-iE_k t} |v_k(0)\rangle$$

$$|v_l(t)\rangle = \sum_{l'} \sum_k U_{l'k} e^{-iE_k t} U_{lk}^* |v_{l'}\rangle$$

Transition amplitude: $A_{ll'}(t) = \langle v_{l'} | v_l(t) \rangle = \sum_k U_{l'k} U_{lk}^* e^{-iE_k t}$

$$P(v_l \rightarrow v_{l'}) = |A_{ll'}(t)|^2 = \sum_{kj} U_{lk}^* U_{l'k} U_{lj} U_{l'j}^* e^{-i(E_k - E_j)t}$$

relativistic v : $m \ll E : E_k - E_j \approx \Delta m_{kj}^2 / 2E$

$$P(v_l \rightarrow v_{l'}) = |A_{ll'}(t)|^2 = \sum_{kj} U_{lk}^* U_{l'k} U_{lj} U_{l'j}^* e^{-i(\Delta m_{kj}^2 L / 2E)}$$

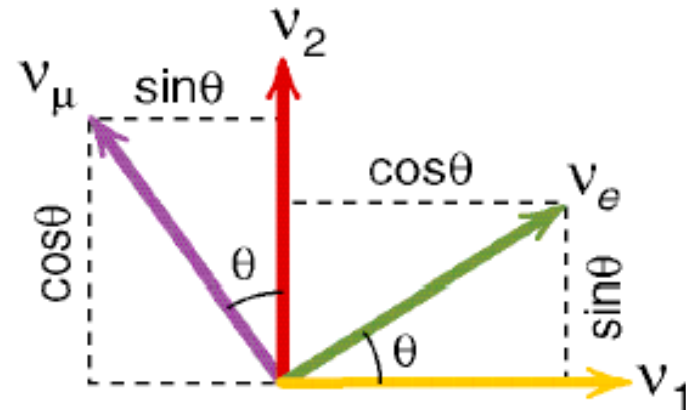
Oscillations cont'd

- Measuring oscillations will give us information on the mass differences
- Measurement of symmetry conservation:
 - CP violated : $P(\nu_\alpha \longrightarrow \nu_\beta) \neq P(\bar{\nu}_\alpha \longrightarrow \bar{\nu}_\beta)$
 - T violated : $P(\nu_\alpha \longrightarrow \nu_\beta) \neq P(\nu_\beta \longrightarrow \nu_\alpha)$
 - CPT conserved : $P(\nu_\alpha \longrightarrow \nu_\beta) \neq P(\bar{\nu}_\beta \longrightarrow \bar{\nu}_\alpha)$

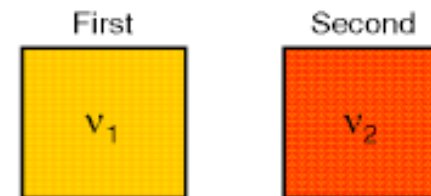
Mixing in two families

- Assume instead that weak and mass states are connected by a simple two-dimensional rotation (assume, for simplicity two families)
- Then mass and weak states are connected by means of an unitary transformation, the PMNS mixing matrix, which depends of a single parameter, the mixing angle θ

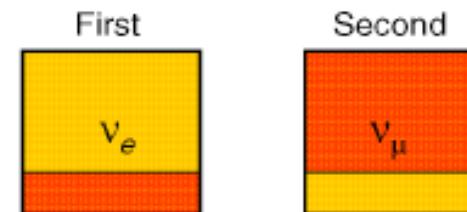
$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$



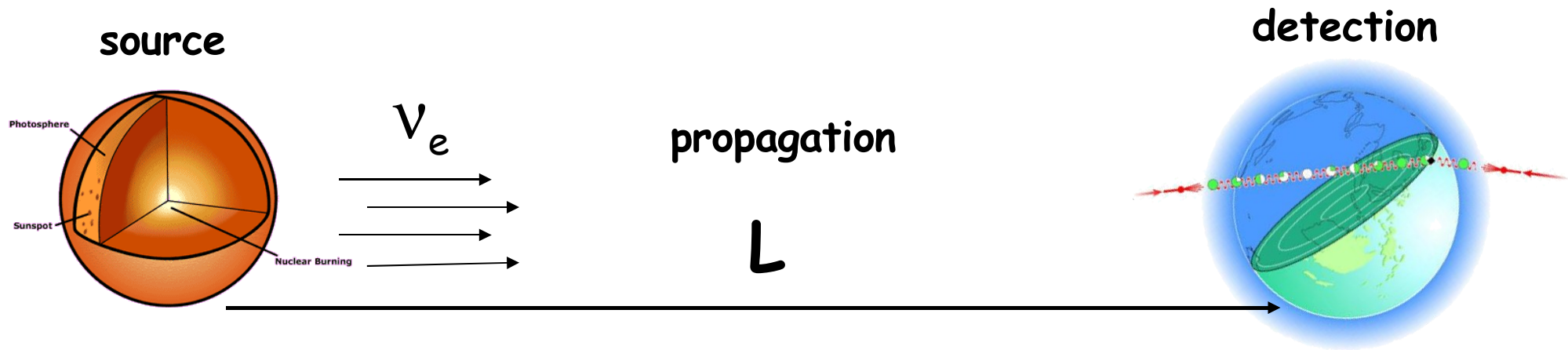
Mass states



Weak states



Neutrino oscillations



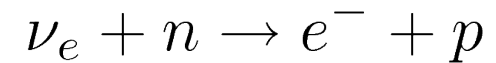
The weak interaction produces neutrinos of a given flavour

The mass eigenstates propagate at different velocities

Detection again via weak interaction

$$|\nu(x_0)\rangle = |\nu_e\rangle = c|\nu_1\rangle + s|\nu_2\rangle$$

$$|\nu(x)\rangle = c|\nu_1\rangle e^{i(Et - \vec{k}_1 \vec{x})} + s|\nu_2\rangle e^{i(Et - \vec{k}_2 \vec{x})}$$

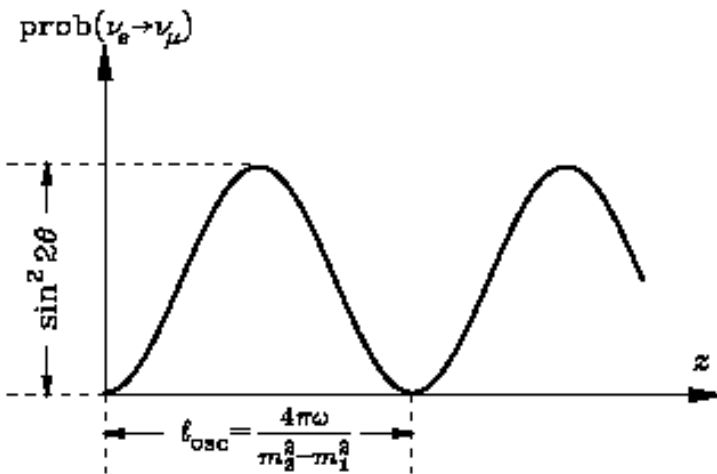


$$P(\nu_e \rightarrow \nu_\mu) = |\langle \nu_\mu | \nu(x) \rangle|^2$$

Oscillation Probability

$$P_{(\nu_e \rightarrow \nu_\mu)}(L) = \sin^2(2\theta) \sin^2 \left(1.27 \frac{\Delta m^2 (eV^2)}{E (GeV)} L (km) \right)$$

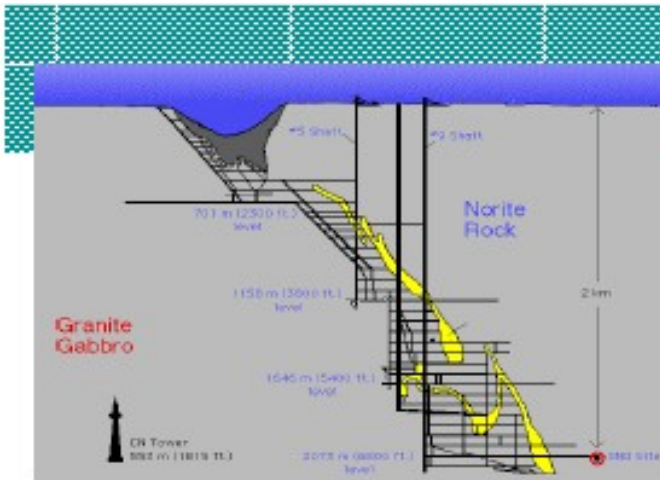
$$P_{\nu_e \rightarrow \nu_e}(L) = 1 - P_{\nu_e \rightarrow \nu_\mu}(L)$$



$$L_{\text{osc}}(km) = \frac{E(GeV)}{1.27\Delta m^2(eV^2)}$$

SNO

Sudbury Neutrino Observatory



1000 tonnes D₂O

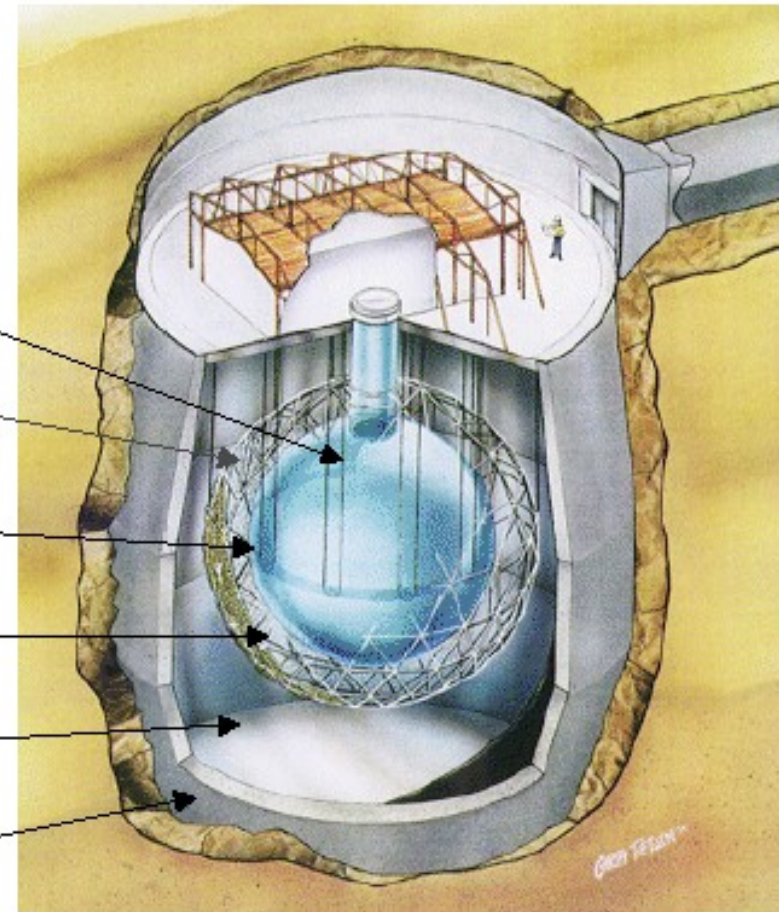
Support Structure
for 9500 PMTs,
60% coverage

12 m Diameter
Acrylic Vessel

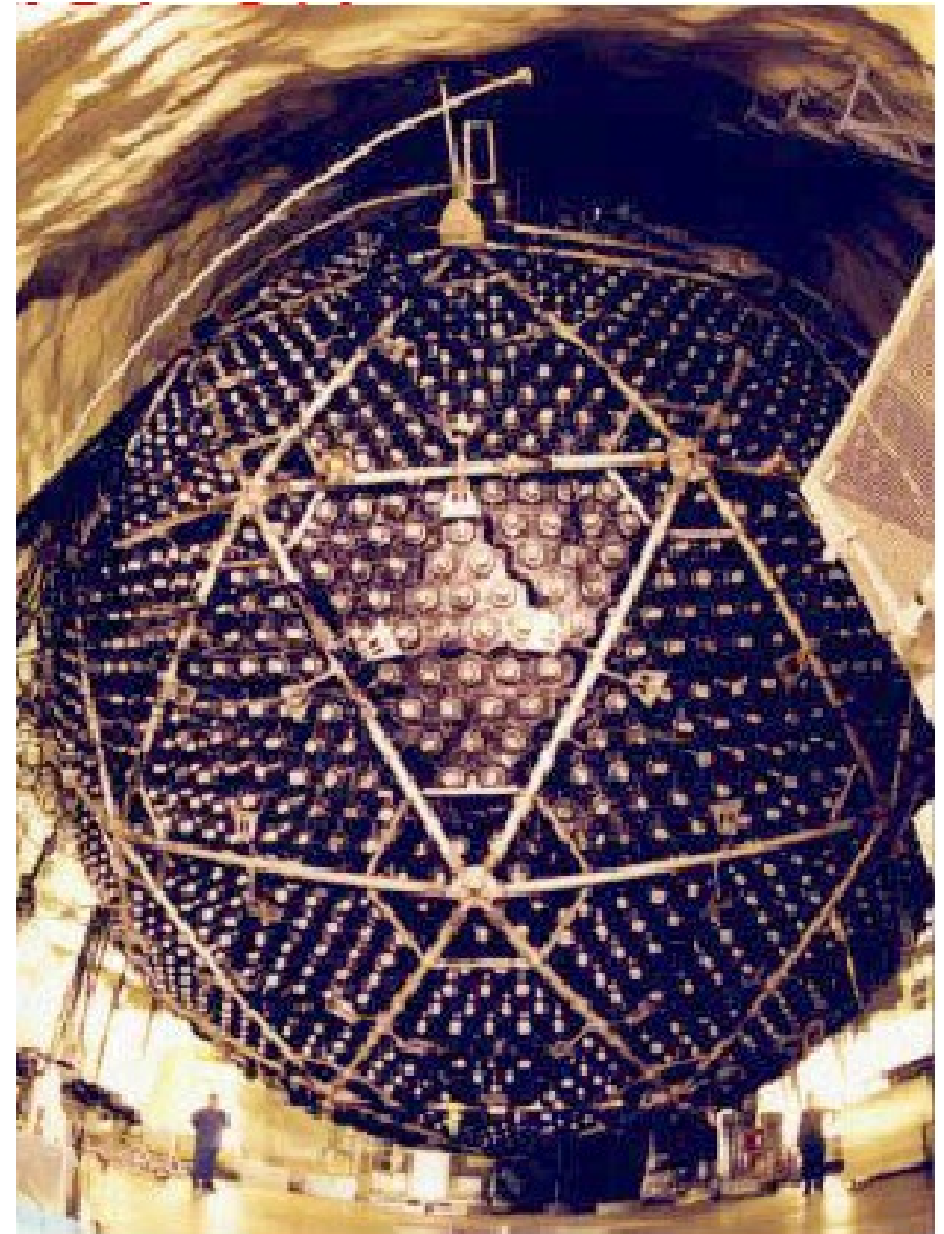
1700 tonnes Inner
Shielding H₂O

5300 tonnes Outer
Shield H₂O

Urylon Liner and
Radon Seal

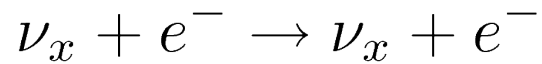


SNO detector



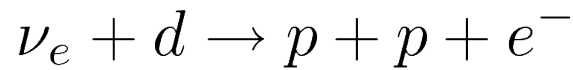
Signals in SNO

ES



Strong directional sensitivity

CC

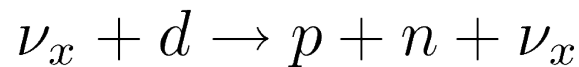


Good measurement of ν_e spectrum

Weak directional sensitivity

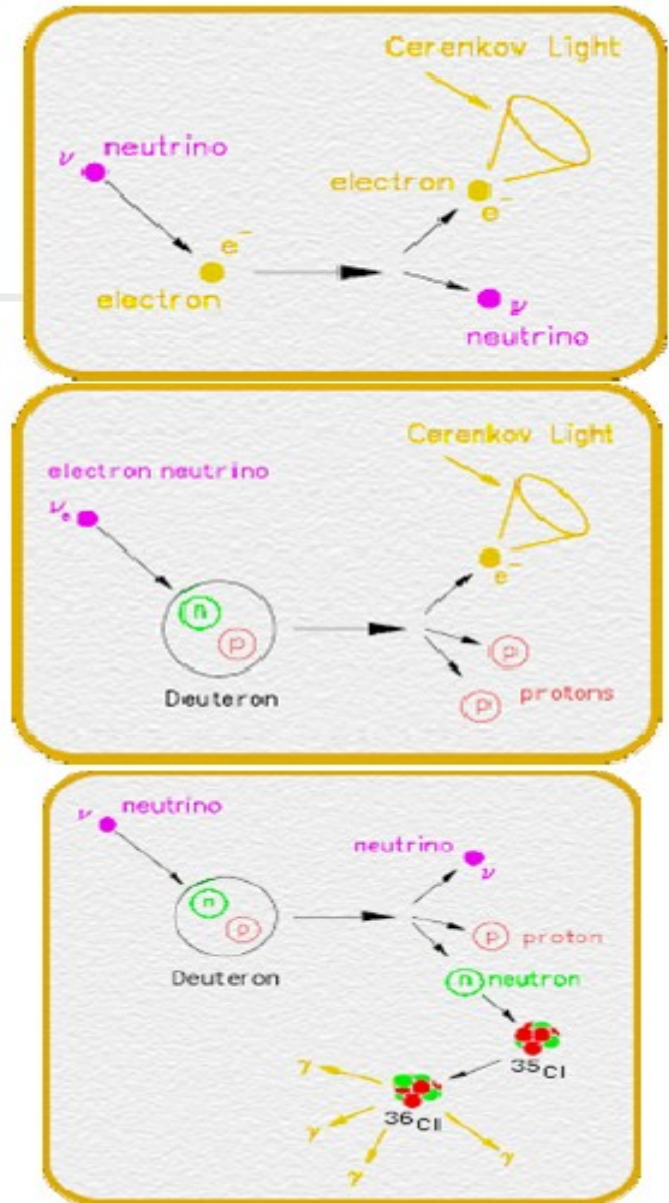
ν_e only

NC

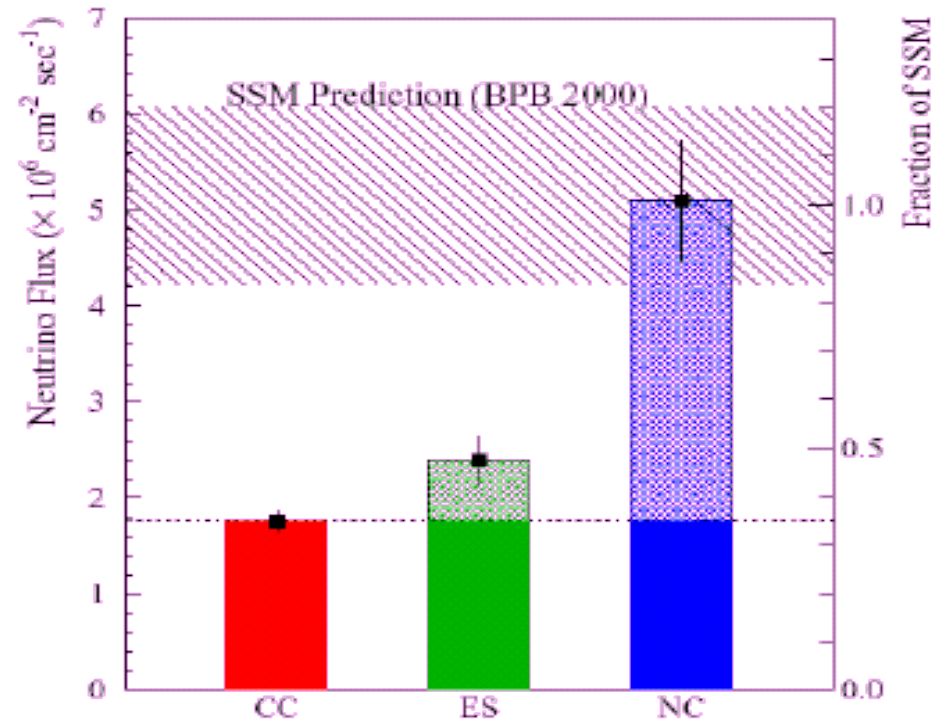
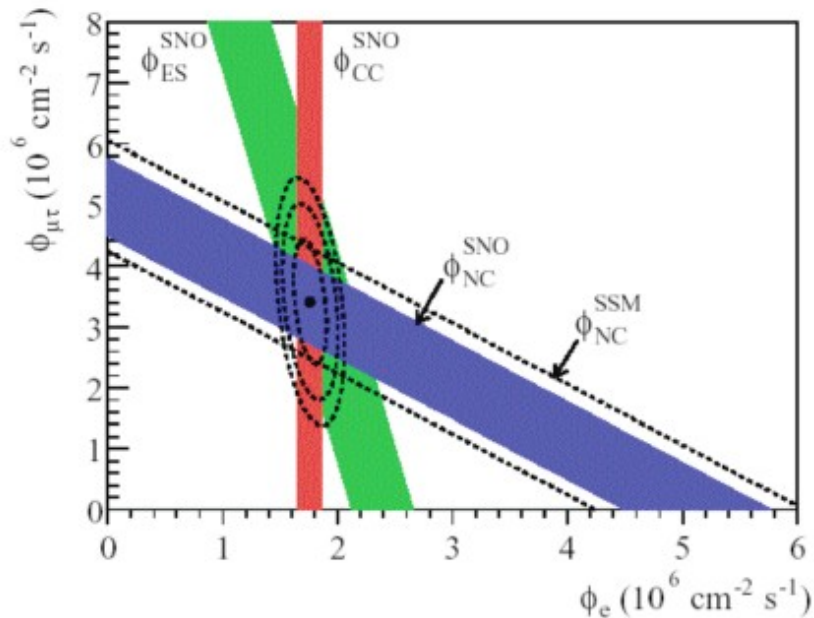
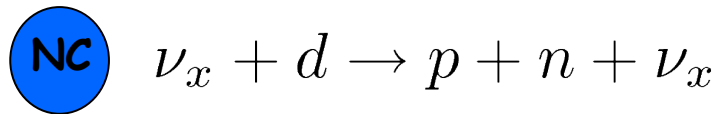
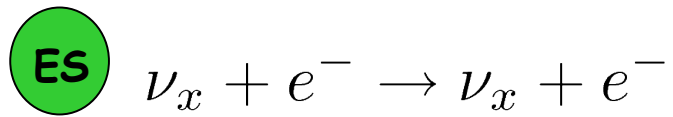
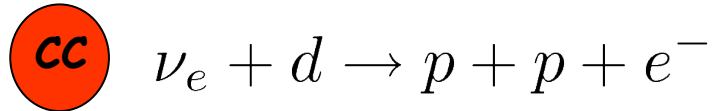


Measure total ${}^8\text{B}$ flux from the sun

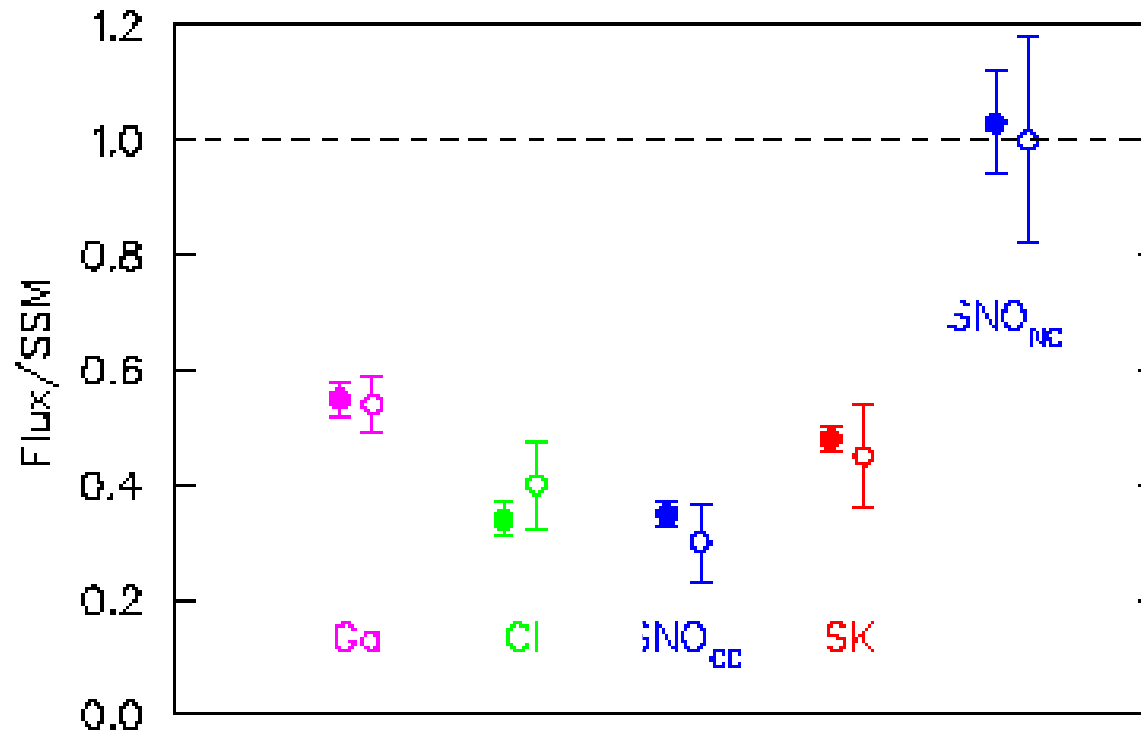
Equal cross section for all types



SNO observations

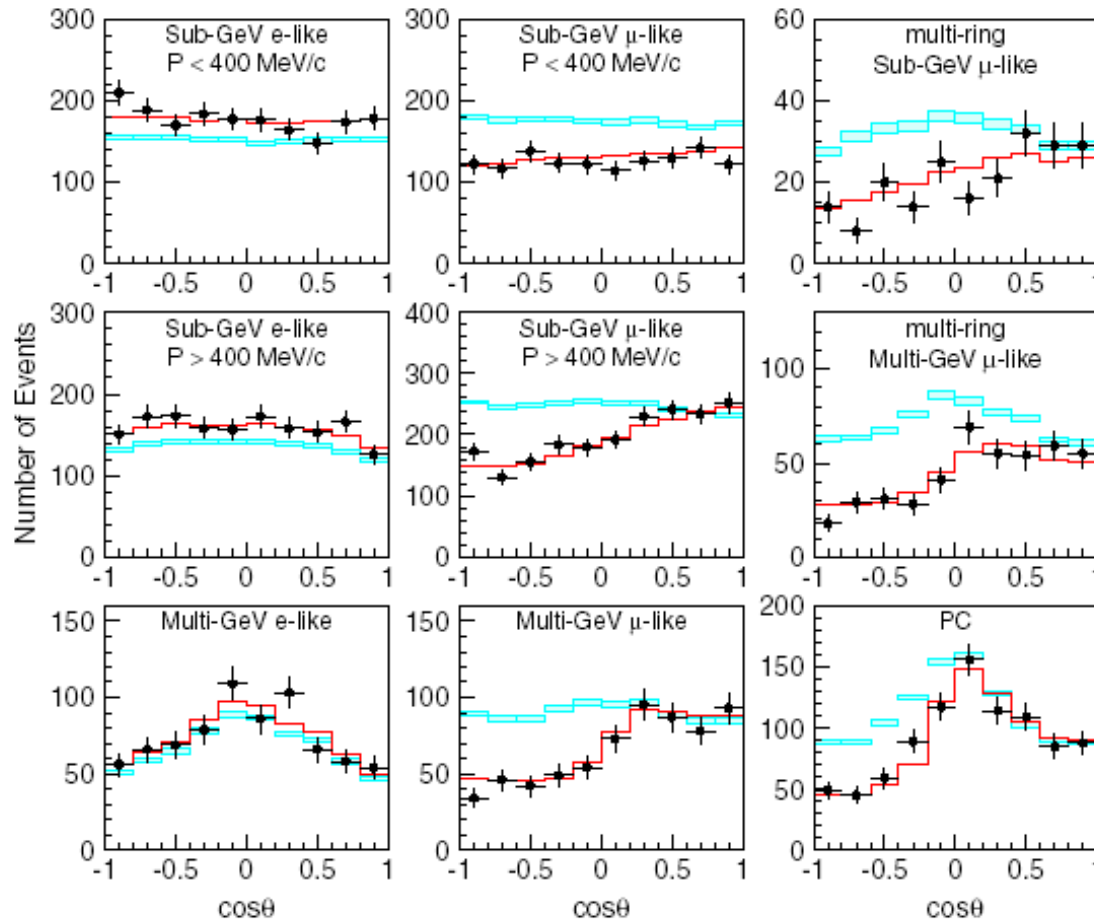


The solar neutrino problem



So the sun is shining the expected number of neutrinos but many of them are ν_{μ} and/or ν_{τ} ! Not only Davis, but also Bahcall was right!

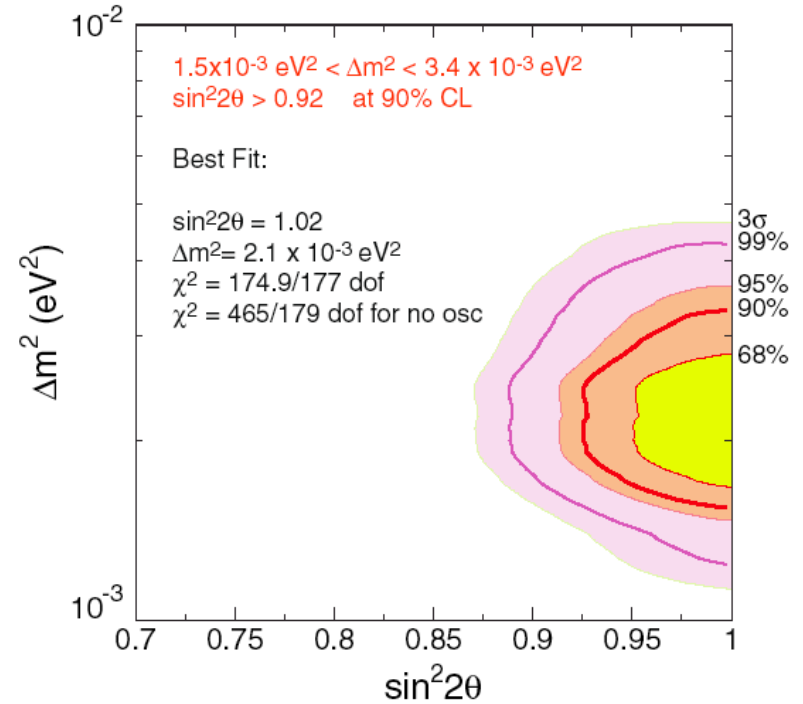
Atmospheric ν oscillations



ν_e

ν_μ

ν_μ

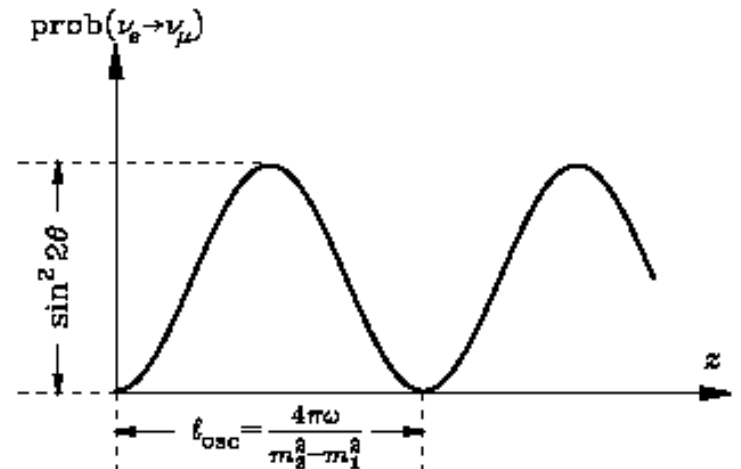
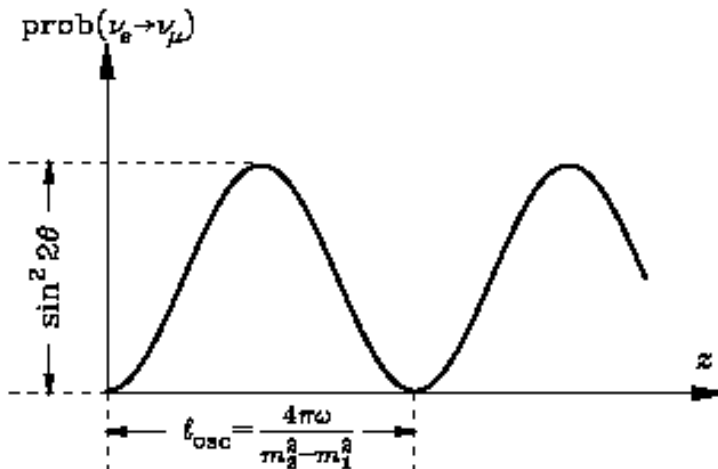
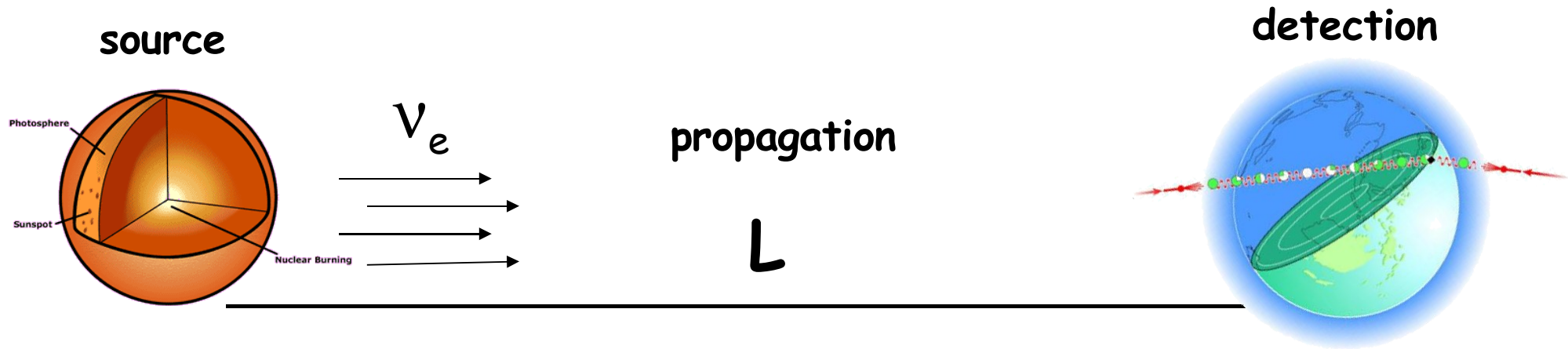


$\nu_\mu \rightarrow \nu_\tau$ oscillations

$$\Delta m^2 = 2.1 \cdot 10^{-3} \text{ eV}^2$$

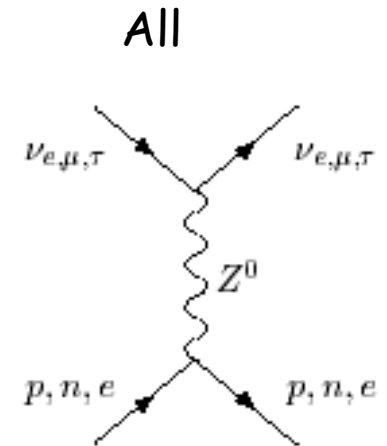
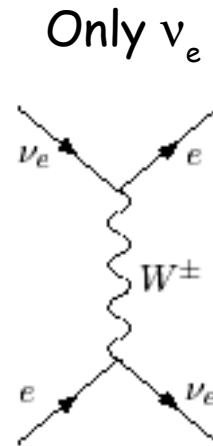
$$\sin^2 2\theta \approx 1$$

Fine-tuning oscillations ?



Neutrino oscillations in Matter

ν_e, ν_μ, ν_τ interact with e, p and n of matter via NC interactions (Z). Only ν_e interact via (CC) with the electrons of the medium



- Oscillation probability change in matter. There can be a resonant enhancement of the oscillation probability. The Mikheyev-Smirnov-Wolfenstein (MSW) effect.

- P_{osc}^{matter} can be large (≈ 1) even if mixing angle in vacuum is small.

- In practice this implies that (if MSW is at work) ν_e can oscillate to ν_μ, ν_τ BEFORE exiting the sun

Oscillation Probability in matter

The probability of oscillation in matter has the same form as in vacuum.

$$p(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta^* \sin^2\left(2\pi \frac{L}{L^*}\right)$$
$$L^* = \frac{2\pi E(\text{GeV})}{1.27 \Delta m^{*2}(\text{eV}^2)}$$

MSW resonance condition

For constant matter density there is an energy such that mixing in matter is maximal independently from the vacuum value.

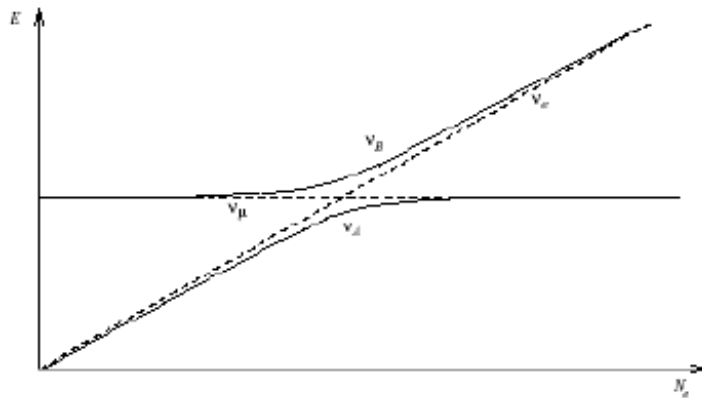
$$\text{if } \Delta m^2 \cos 2\theta = A = \sqrt{2} E G N_e$$

$$\sin^2 2\theta^* = \frac{(\Delta m^2)^2 \sin^2 2\theta}{(\Delta m^2 \cos^2 2\theta - A)^2 + (\Delta m^2)^2 \sin^2 2\theta} = 1$$

Thus the probability of neutrino transition in matter can be large even if the mixing angle is small

Adiabatic approximation

In the sun N_e is not constant. However if the variation is sufficiently slow the eigenstates of H change slowly with the density and one can assume that the neutrino remains an eigenstate along the trajectory:
adiabatic approximation



$$\tilde{v}_1 = v_e \cos \tilde{\theta} + v_\mu \sin \tilde{\theta}$$

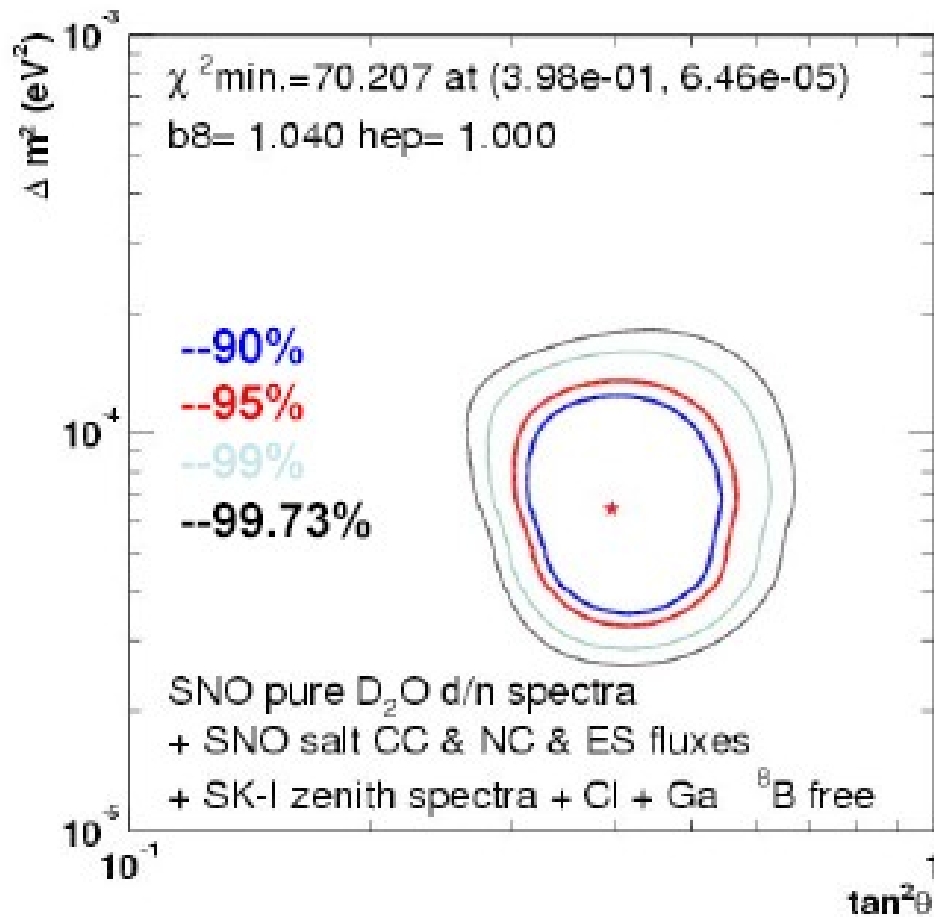
$$\tilde{v}_2 = -v_e \sin \tilde{\theta} + v_\mu \cos \tilde{\theta}$$

$$x = 0 \quad \text{if } A \gg \Delta m^2 \cos 2\theta \rightarrow \tilde{\theta} \approx \frac{\pi}{2} \Rightarrow v_e \approx \tilde{v}_1$$

$$x = R_{sun} \quad N_e = 0 \rightarrow \tilde{\theta} \approx \theta \Rightarrow v_\mu \approx \tilde{v}_2$$

A v_e produced at the sun core is the eigenstate v_2 but this eigenstate outside the sun is mostly v_μ . There is maximum $v_e \rightarrow v_\mu$ conversion

Solar oscillations



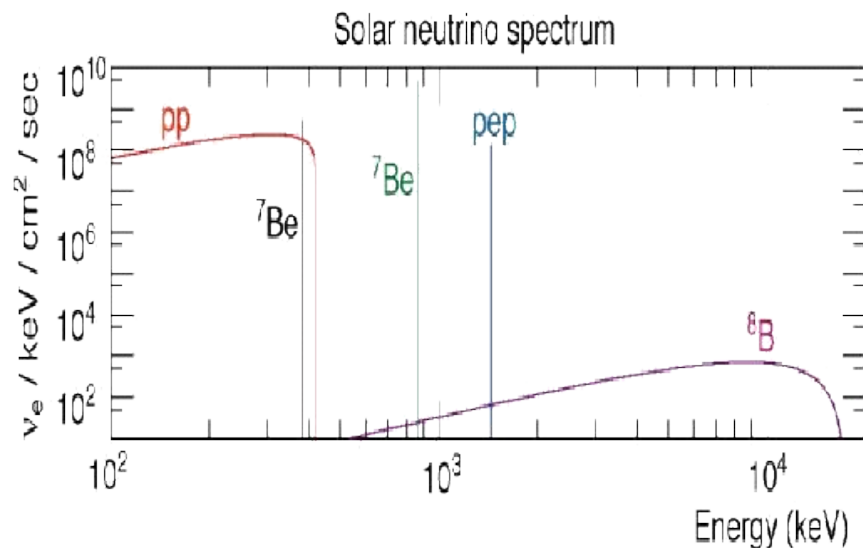
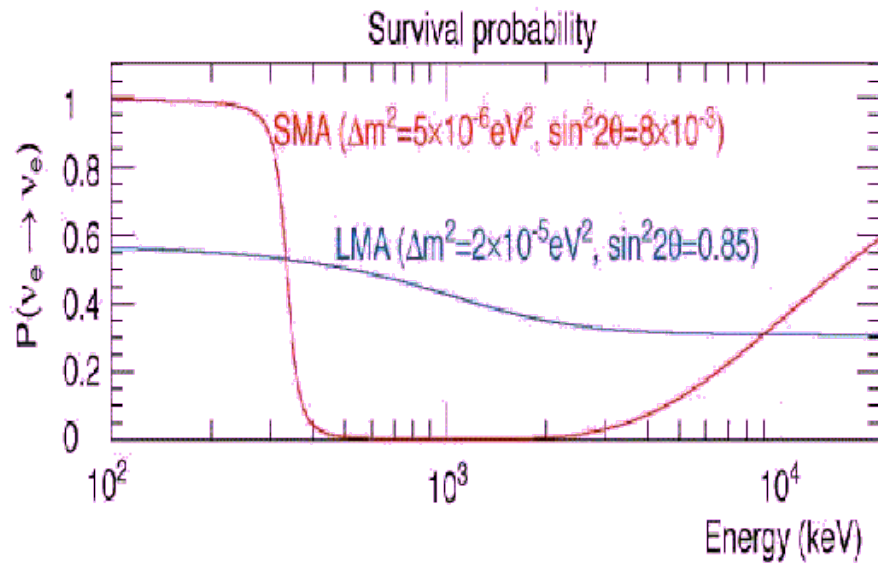
Neutrinos produced at the sun (ν_e) oscillate to other neutrinos via matter-enhanced MSW.

$$\Delta m^2 = 8 \times 10^{-5} \text{ eV}^2$$

$$\theta \approx 30^\circ$$

Solar neutrino oscillations

Matter effect on ν_e from Sun to Earth

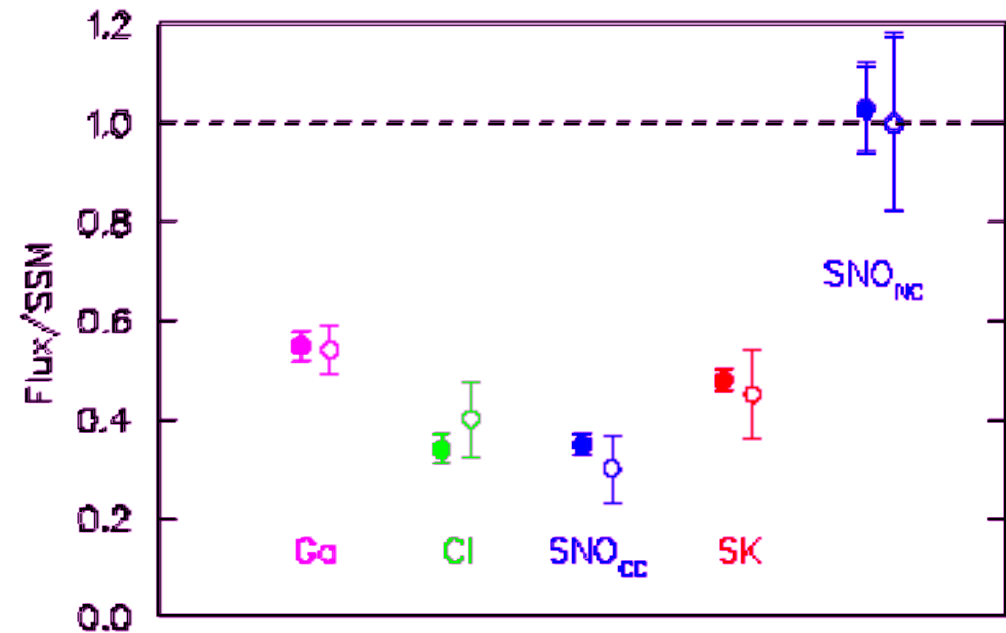


The LMA solar solution

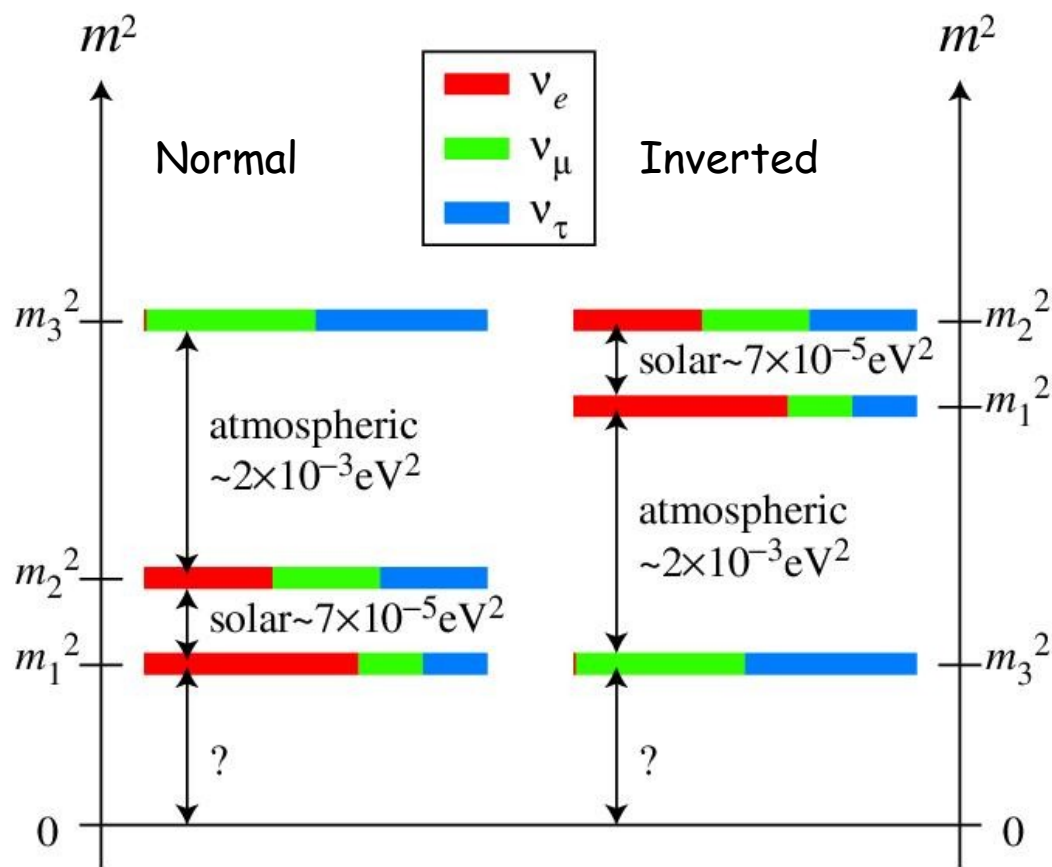
+

matter effects

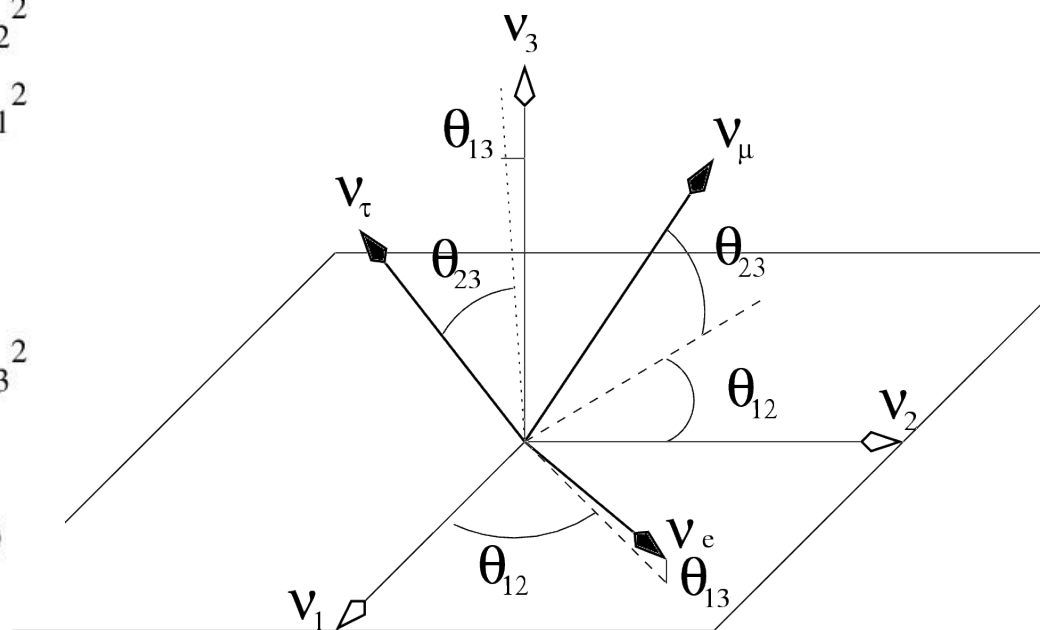
explain beautifully
all solar neutrino experiments



Oscillations revisited



| Parameter | Best-fit value | 3σ range |
|---------------|----------------|-------------------------------------|
| θ_{12} | 33.2° | $28.7^\circ \text{ .. } 38.1^\circ$ |
| θ_{23} | 45.0° | $35.7^\circ \text{ .. } 55.6^\circ$ |
| θ_{13} | 0.0° | $0^\circ \text{ .. } 12.5^\circ$ |



The neutrino mixing matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Unless the other two angles θ_{13} is small
(experimental upper limit $\theta_{13} < 10^\circ$)

If $\delta \neq 0, \pi, 2\pi \dots$ then weak interactions violate CP symmetry in the lepton sector (as in the quark sector)

atmospheric

CP violation phase

solar

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Links atmospheric & solar sectors