Deformation of Quantum Dynamics in Phase Space

Olivier Mattelaer

CP3

Plan

- Introduction

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- Introduction
- Path Integral
  1. Definition
  2. Regularisation
  3. Deformation

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  1. Bosonic case
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- Conclusions and Perspectives

Introduction

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- non commutative geometry

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- Darboux theorem

$$\begin{array}{lcl} \{x_i, x_j\} = \theta^1_{ij} & & \{\tilde{x}_i, \tilde{x}_j\} = 0 \\ \{x_i, p_j\} = \theta^2_{ij} & \longmapsto & \{\tilde{x}_i, \tilde{p}_j\} = \delta_{ij} \\ \{p_i, p_j\} = \theta^3_{ij} & & \{\tilde{p}_i, \tilde{p}_j\} = 0 \end{array}$$

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Path Integral

Path Integral : Definition

- Quantum Mechanics : We have to Consider All trajectory

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- taking explicit limit on time
  $U(T) = \int \prod_{n=1}^{N-1} dx_n dp_n e^{\frac{\mathrm{i}}{\hbar}\int dt[\frac{1}{2}(\dot{x}p - x\dot{p}) - H(x,p)]}$

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  1. the limit of the measure is not defined
  2. the correspondance principle (linked to this) doesn't work in general
- We need a correct definition for the measure

Path Integral : Regularisation

- overlap function
  $\langle x_{n-1}, p_{n-1} | x_n, p_n \rangle \sim e^{\frac{i}{\hbar}\epsilon[\frac{1}{2}(\dot{x}p - x\dot{p})]}$

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  - recover usual quantum mechanics for $\lim_{\tau \longrightarrow 0}$
  - metric on phase space
  - more general quantification

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- cure UV divergencies

Some Deformation

Harmonic Oscillator

- Harmonic Oscillator

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\begin{aligned}
L(x,\dot{x}) &= \frac{m}{2}\dot{x}^2 - \frac{1}{2}m\omega^2 x^2 \\
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$$L = \frac{1}{2}(p\dot{x} - \dot{p}x) - \frac{p^2}{2m} - \frac{1}{2}m\omega^2 x^2 + i\frac{m\omega\tau_0}{2}[\dot{x}^2 + \frac{\dot{p}^2}{m^2\omega^2}]$$

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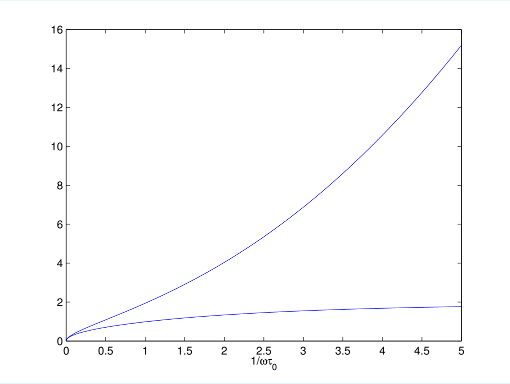
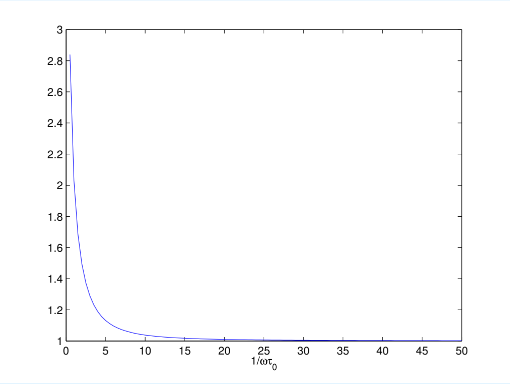
- 2 time scale : $\tau_- = \frac{2\tau_0}{R}$ $\quad\tau_+ = \frac{2\tau_0}{R-1}$





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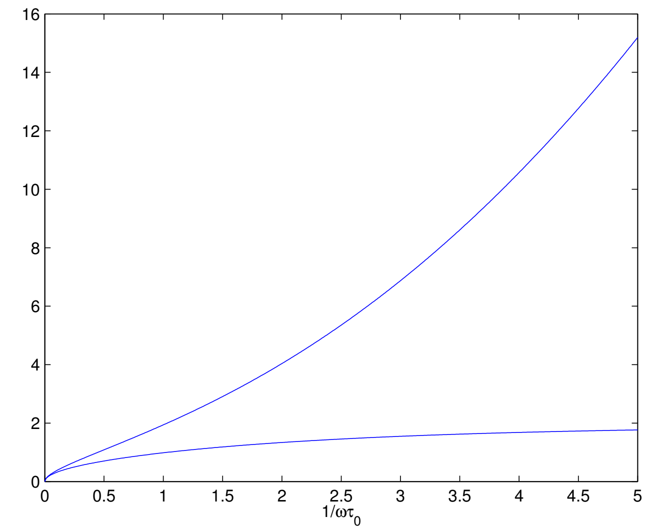

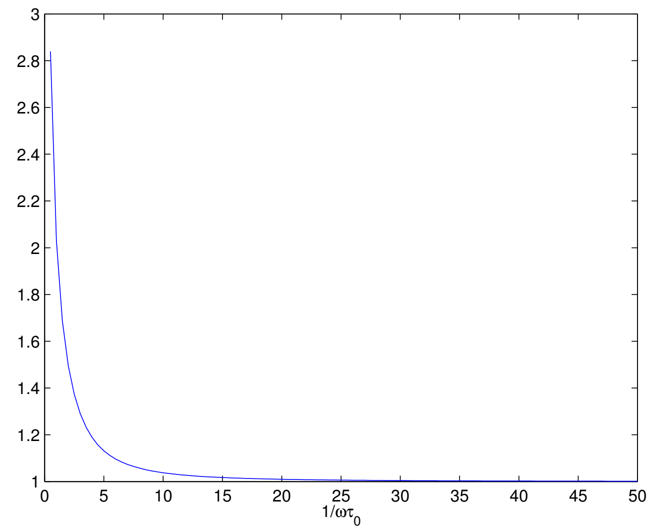

O.H. : Hilbert Space

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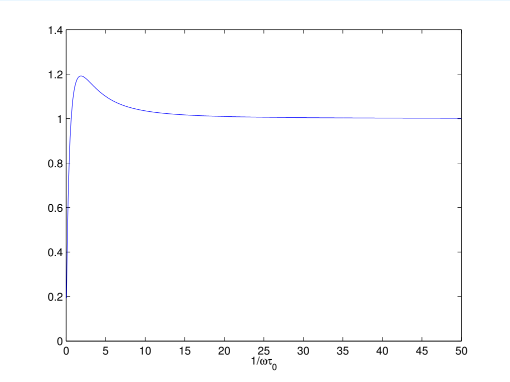
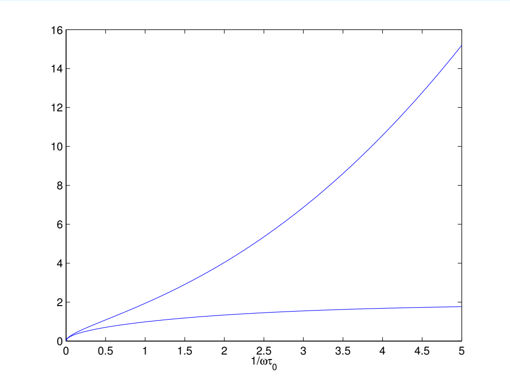
- Time Evolution

$$\begin{aligned} K_{nm} &= \langle n|\hat{K}(T_{21})|m\rangle \\ &= F_0^{n+1}(T)e^{-(n+\frac{1}{2})\frac{i\omega T}{R}}e^{-n\frac{T}{\tau_+}}\delta_{mn} \end{aligned}$$





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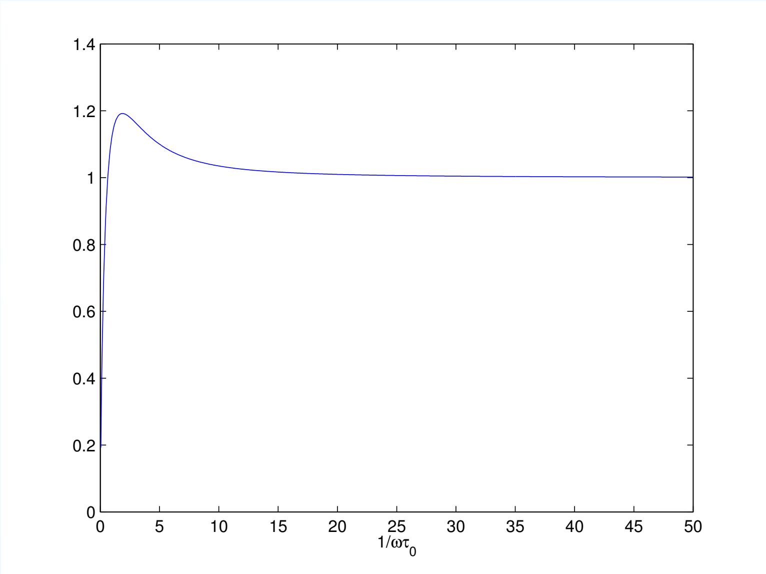

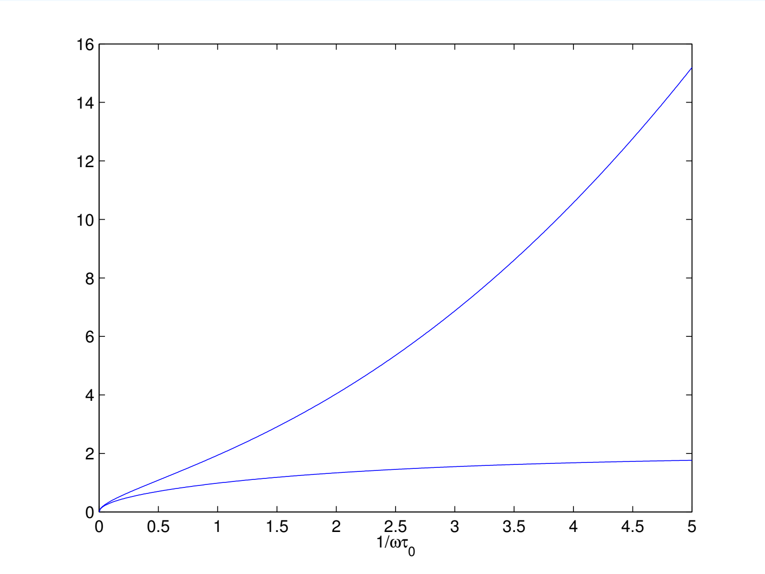

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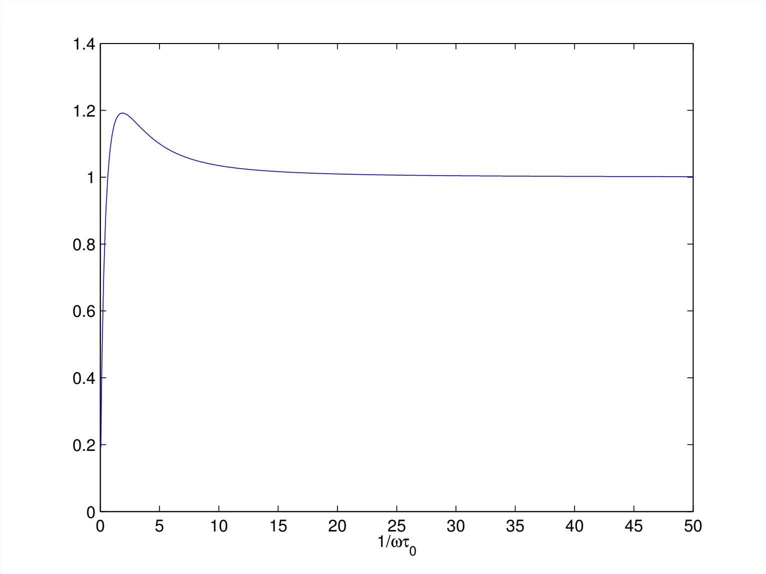

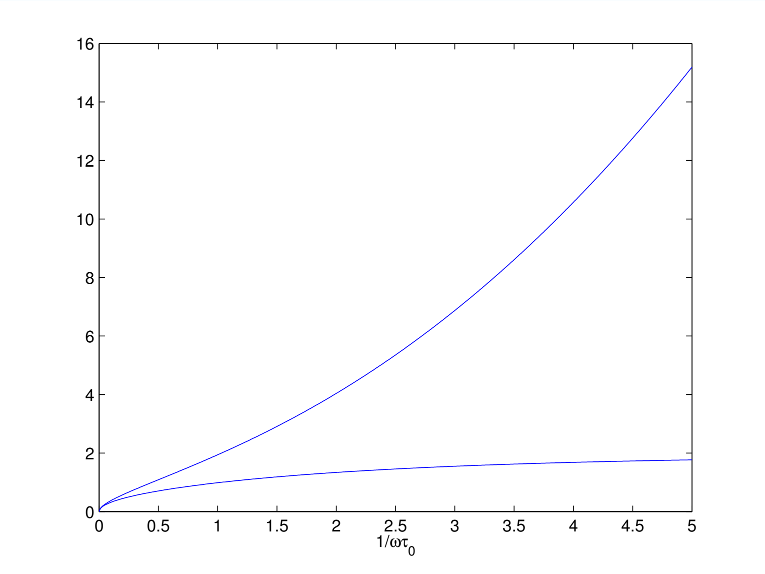

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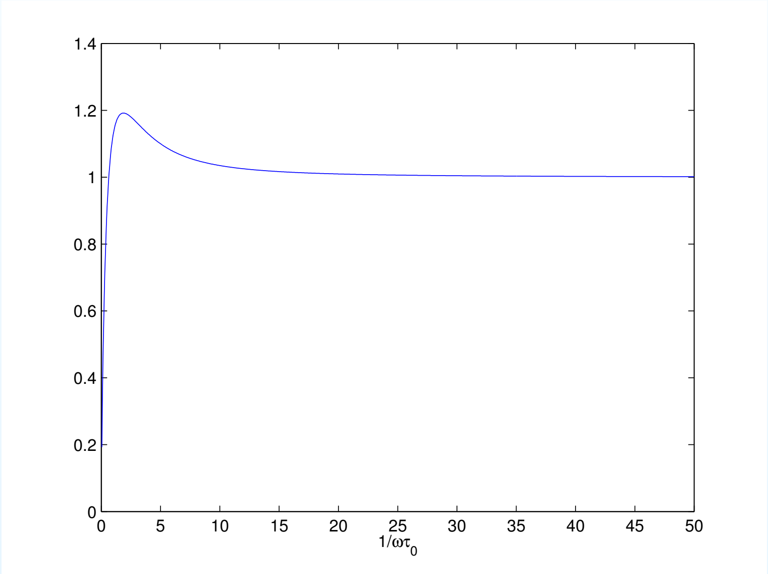

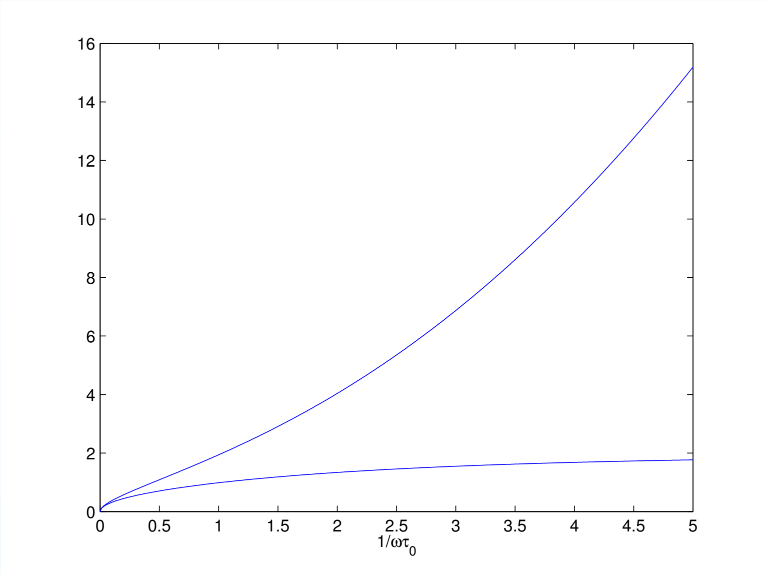

- suppresion of exited states
- problem for ground states

Commutativity

- Heisenberg representation

$$\hat{O}_H(t_2, t_1) = \hat{K}^\dagger(t_2, t_1)\hat{O}_S(t_1)\hat{K}(t_2, t_1)$$

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- time dependant non commutative geometry

Susy Oscillator

- Lagrangian

$$L_{susy} = L_{bos} + L_{ferm}$$

Susy Oscillator

- Lagrangian

$$\begin{aligned} L_{susy} &= L_{bos} + L_{ferm} \\ &= \frac{1}{2}\left(\dot{\bar{z}}z - \dot{z}\bar{z}\right) - \frac{\omega}{2}\bar{z}z + i\frac{\tau_0}{2}[\dot{\bar{z}}\dot{z}] \end{aligned}$$

- $z = x + \mathrm{i}p$

Susy Oscillator

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$$
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$$

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- θ Grassman Variable ($\theta^2 = 0$)

Susy Oscillator

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- different integral
  $\int dxdpe^{a(x^2+p^2)\sim a^{-1}}$
  $\int d\bar{\theta}d\theta e^{a\bar{\theta}\theta} \sim a$

Susy Oscillator

- Time-evolution

$$K^0_{mn} = F_0(T)^n e^{-(n)\frac{i\omega T}{R}} e^{-n\frac{T}{\tau_+}} \delta_{mn}$$

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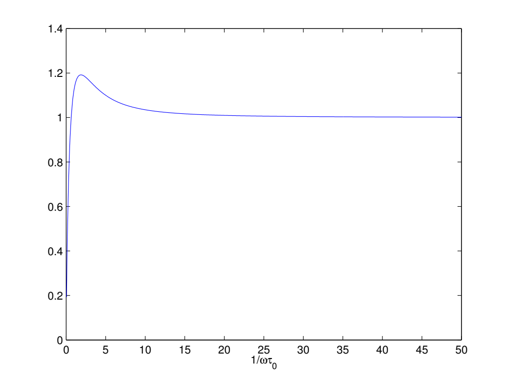

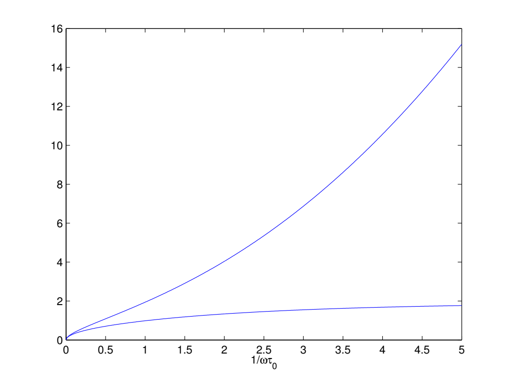

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Field Theory

- hamiltonian representation

$$H = \frac{c^2\chi^2}{2\hbar^2} + \frac{\hbar^2}{2}(\vec{\nabla}\phi)^2 + \frac{m^2c^2}{2}\phi^2$$

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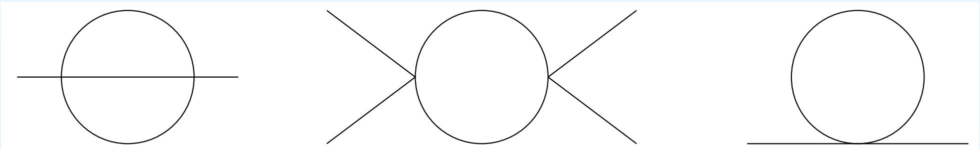

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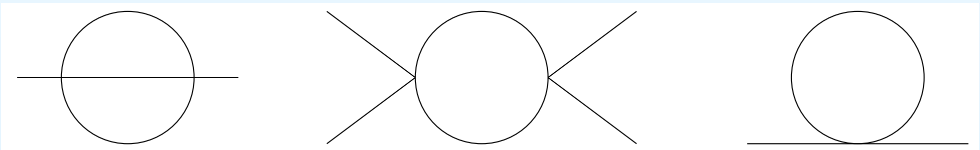

- But this not work for “tadpole” diagram

Conclusions and Perspectives

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  - intrinsic development of non commutative geometry
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- Perspectives
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  - cosmological applications
  - introduction of gravitations ?