Monte Carlo for top quark decays

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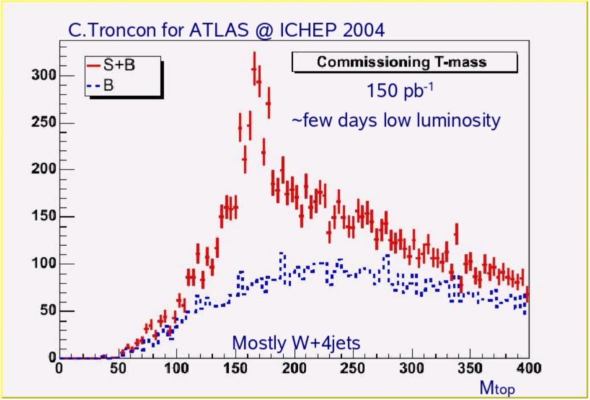
Outline

1. Top quarks in year 1 of LHC.
2. What to simulate: $t \to bW + gluons$.
3. How to simulate it: parton showering.
4. Soft radiation & QCD coherence.
5. Herwig++ fragmentation formalism.
6. Matrix element corrections.
7. Jet structure.
8. Production and decay: a multiscale parton shower.

Month 1: a possible first physics plot.

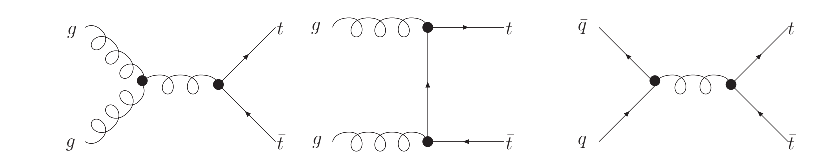
- Scenario : no b-tagging, no jet calibration, good lepton ID.
- Select semi-leptonic $t\bar{t}$ channel (30% of $t\bar{t}$ events) & reconstruct top mass:
  - Semi-leptonic $t\bar{t}$ (30%) cuts: isolated lepton (p_T>20GeV), 4 jets (p_T>40 GeV).
  - Reconstruct M_{top} with 3 maximal p_T jets & c.kinematic fit ($M_{W1} = M_{W2}$, $M_{T1} = M_{T2}$).



☞ Top quarks should be useful 'standard candles' at the LHC.

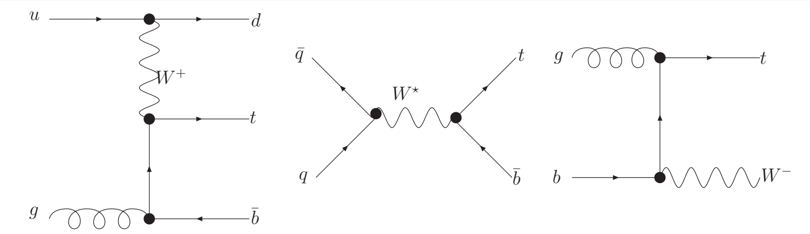
Year 1: $t\bar{t}$ pair and single top production in 10 fb^{-1}.

- **9M** $\bar{t}t$ pairs per 10fb^{-1}year $\sim$ 3/4 pairs per second



(NLO+NLL $\sigma\left(t\bar{t}\right) \approx 900 \pm 100$ *pb* - **90% gluon initiated**).

- **3M** single tops per 10fb^{-1}year



(Mainly t-channel "W-gluon fusion": $\sigma = 250 \pm 30$ *pb* (NLO)).

What to simulate: top decays fast ↯

- The top decays very quickly ‘semi-weak decay’ to Wb.
- About 75% of these W’s are longitudinal W’s

[Longitudinal decay mode is enhanced by: $\frac{m_t^2}{m_W^2} = 4.7$ (*i.e.* $\frac{m_t^2}{m_W^2} \propto \frac{g_t^2}{g_W^2}$)]

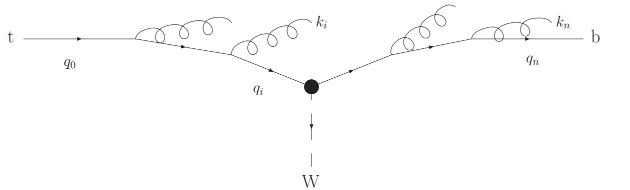
- Back of the envelope:
  - Decay time: $\boldsymbol{t_{decay}} \sim \mathbf{10^{-25}}$seconds.
  - Time for hadrons to form: $\boldsymbol{t_{hadronize}} \sim \frac{\mathbf{1}}{\boldsymbol{\Lambda_{QCD}}} > \mathbf{10^{-24}}$ seconds.

☞Top will decay before hadrons form.

☞Gluons can be radiated before the decay.

What to simulate: $t \to Wb + (n)\,g$.

- We wish to simulate the decay $t \to Wb + (n)\,g$:



- So we basically want to calculate $\Gamma\left(t \to Wb + (n)\,g\right)$:

$$\sum_n \int d\Gamma_n = \sum_n \frac{1}{2m_t} \int \mathrm{d}\Phi_{bW} \mathrm{d}\Phi_K \; (2\pi)^4 \, \delta^4 \left(p_t - q_b - q_W - \sum_{i=1}^{n} k_i \right) |\mathcal{M}_n|^2$$

- We work in the **narrow width approximation** *i.e.* production and decay processes don't talk to each other.

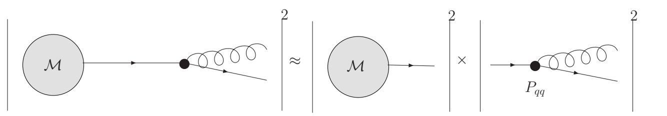
How to simulate it: chop it up into 1 $\longrightarrow$ 2 'decays'.

- Phase space can be rewritten as a **cascade** of 2-body '**decays**' $q \rightarrow qg$:

$$\int d\Phi_{bW} d\Phi_K = \int [dq_b]\,[dq_W]\,\left[\prod_{i=1}^{n} \int d\Phi_i\right]$$

$$\int d\Phi_i = \int dq_i^2 \int_{\mathcal{U}} [dk_i]\,[dq_i]\,\delta^4\left(q_{i-1} - q_i - k_i\right).$$

- Use **collinear factorization** and **strong virtuality ordering** $\left(q_{i-1}^2 > q_i^2\right)$ to factorize all parton emissions in $|\mathcal{M}_n|^2$:



$$\lim_{q_i \| k_i} |\mathcal{M}_{n+1}|^2 \propto \frac{1}{q_{i-1}^2 - m^2} P_{qq}(z_i)\,|\mathcal{M}_n|^2$$

$$P_{qq}(z_i) = C_F\left(\frac{1+z_i^2}{1-z_i}\right)$$

- In this **leading-collinear-log (LL)** approximation we can work out the **probability** that either quark **evolves** from one virtuality to another $Q^2 \rightarrow Q_0^2$ by emitting **unresolved** gluons ☞ Monte Carlo simulation.

How to simulate it: NLL Sudakov formfactor.

- $\alpha_S\left(\mathbf{p}_{\perp i}^2\right)$ instead of $\alpha_S(q_i^2)$, gives the soft divergent terms in the $\mathcal{O}\left(\alpha_S\right)$ part of the *NLO* splitting functions $\propto \frac{1}{1-z}\ln\left(1-z\right)$.
- Rescaling $\Lambda_{\overline{MS}}$ gives remaining soft divergent terms $\propto \frac{1}{1-z}$.
- Expand $\alpha_S\left(\left(\left(1-z\right)q^2\right)\right)$ and rescale $\Lambda_{\overline{MS}}$:
  - $\alpha_S\left(\left(1-z\right)q^2\right) = \alpha_S\left(q^2\right) - \beta_0 \ln\left(1-z\right)\alpha_S^2\left(q^2\right) + \mathcal{O}\left(\alpha_S^3\right)$.
  - $\Lambda_{MC} = \Lambda_{\overline{MS}} \exp\left(\frac{C_A\left(67-3\pi^2\right)-20T_F n_f}{3(11C_A-4T_F n_f)}\right)$.
- Substitute into LO expression to get:

$$\begin{aligned} lim_{z\to 1}\frac{\alpha_S}{2\pi}P_{qq}^{LO}\left(z\right) &= \frac{\alpha_S}{2\pi}\frac{2C_F}{1-z}\left\{1+\frac{\alpha_S}{2\pi}\left[-\frac{11C_A-4T_F n_f}{6}\ln\left(1-z\right)\right.\right. \\ &+ \left.\left. C_A\left(\frac{67}{18}-\frac{\pi^2}{6}\right)-T_F n_f\frac{10}{9}\right]\right\} \end{aligned}$$

- This α_S prescription is also used for orthodox NLL resummation [*e.g.* Cacciari *et al* for top decay].

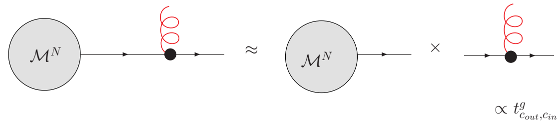
Soft radiation: matrix element factorisation.

- We have: leading collinear logs, leading soft-collinear logs, next-to-leading soft-collinear logs.

- Soft gluons give large logarithms irrespective of angle:

$$propagator \propto \frac{1}{2q_i.k_i} \propto \left(2E_i E_{gluon}\right)^{-1} .$$

- Amplitudes with *soft* gluon emissions from on-shell lines factorise:



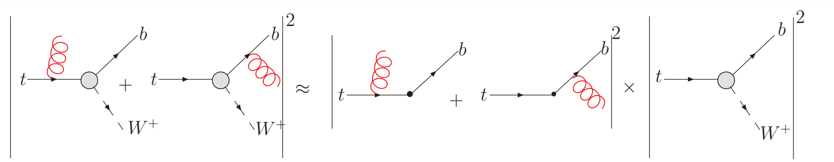
Long wavelength ☞insensitive to short distance phenomena.

- For each external coloured particle we get an **eikonal factor** *e.g.* from the *b*-quark:

$$lim_{k\to 0}\mathcal{M}^{(bWk)} \quad = \quad g_S \left(\frac{\epsilon^*.p_b}{p_b.k}\right) \hat{T}^a_{c_i b_i}\mathcal{M}^{(bW)}.$$

Soft radiation: dipole factorisation.

- Soft emissions are long wavelength so communicate over large distances ☞interference is v.important.
- On squaring the amplitude eikonal factors arrange themselves into functions representing **colour dipole radiation**:



$$lim_{k\to 0}\left|\mathcal{M}^{(bW+k)}\right|^2$$

$$\propto \quad -g_S^2 C_F\left(\frac{p_t}{p_t.k}-\frac{p_b}{p_b.k}\right)^2\left|\mathcal{M}^{(bW)}\right|^2.$$

- We call these "**dipole functions**"

$$W_{tb}=-\frac{E_k^2}{2}\left(\frac{p_t}{p_t.k}-\frac{p_b}{p_b.k}\right)^2.$$

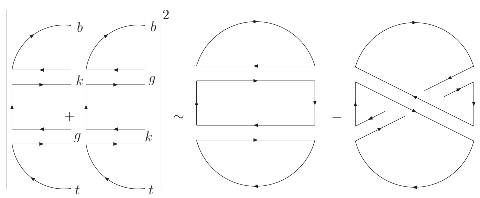
- Each pair of external coloured particles contributes a dipole function

Soft radiation: dipole factorisation.

- Imagine a hard gluon g is emitted *i.e.* $t \to Wbg$, if we now emit a soft gluon k what happens?
- Now have **3** pairings of external coloured particles ***tg, bg, tb*** ☞ **3** dipoles.
- ***Colour-connected pairs*** give the leading dipoles ($\mathcal{O}(1)$) in $\frac{1}{N_C^2}$ expansion.
- Radiation pattern for k in $t \to bWg + k$ is

$$\lim_{k\to 0} \left|\mathcal{M}^{(bWgk)}\right|^2 \propto \left(W_{bg} + W_{tg} - W_{tb}/N_C^2\right) \left|\mathcal{M}^{(bWg)}\right|^2$$

- Heuristically,



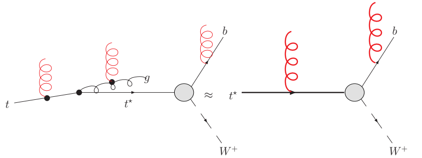
QCD coherence: dipole decomposition.

- Although interference terms, dipole functions contain collinear singularities:

$$W_{xy} = W_{xy}^{x} + W_{xy}^{y}$$
$$lim_{m\to 0} W_{xy}^{x} = \frac{1}{2\left(1 - cos\theta_{xk}\right)}\left(1 + \frac{cos\theta_{xk} - cos\theta_{xy}}{1 - cos\theta_{yk}}\right).$$

- Although interference terms we say W_{xy}^{x} $\sim$the probability of emission from x.
- Consider emission of gluon g very close to the top quark:

$$\left|\mathcal{M}^{(bWgk)}\right|^2 \approx \frac{2g_s^2}{E_k^2}\left(C_F W_{tg}^{t} + C_A W_{tg}^{g} + C_F W_{t^*b}^{b} + C_F W_{t^*b}^{t^*}\right)\left|\mathcal{M}^{(bWg)}\right|^2$$

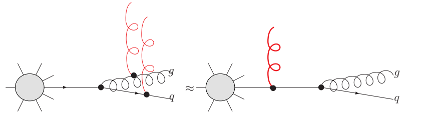


For wide angles 3 dipoles *tg*, *bg*, *tb* looks like 1 dipole t*b!

At small angles 3 dipoles looks like 3 independent emitters t,b,g!

QCD coherence: dipole decomposition.

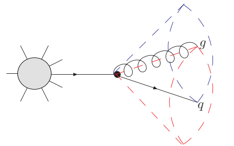
- Likewise for b close to g we find coherent radiation at wide angles:



- Recall $W_{xy} = W_{xy}^{x} + W_{xy}^{y}$, if we average W_{xy}^{x} over the azimuthal angle (with respect to x):

$$\begin{aligned} \langle W_{xy}^{x} \rangle &= \frac{1}{2\pi} \int d\phi_x \, W_{xy}^{x} \\ &= \begin{cases} \frac{1}{2(1-\cos\theta_{xk})} & \theta_{xk} < \theta_{xy} \\ 0 & \theta_{xk} > \theta_{xy} \end{cases} . \end{aligned}$$

- Radiation from x confined to a cone!



- Soft emissions should be *angular* ordered - decrease away from subprocess!

QCD coherence: coherent branching algorithm.

- Azimuthal averaging "dipole splitting functions" W_{xy}^{x} gives rise to independent emissions up to boundary conditions:

$$\langle W_{xy}^{x} \rangle = \frac{1}{2(1-cos\theta_{xk})} \Theta\left(\theta_{xy} - \theta_{xk}\right).$$

- We can now radiate each parton **independently** into cones reaching out to their **colour partner**.

ALGORITHM.

1. Generate emissions in **nested cones** starting from the hard process working out to the external legs.
2. Emit into each cone according to $\frac{1}{2(1-cos\theta_{xk})}$.
3. Initial wide angle radiation is distributed correctly due to QCD coherence.

QCD coherence: coherent branching algorithm.

- Won't this mess up the collinear radiation? ☹
- Using $C_A \equiv 2C_F + \frac{1}{N_C}$ on W^b_{bg} and W^t_{tg} dipole splitting functions:

$$\begin{aligned}\left|\mathcal{M}^{(bWgk)}\right|^2 &\propto \frac{2g_s^2}{E_k^2}\Big[2C_F\left(W^b_{bg}+W^t_{tg}\right)+C_A\left(W^g_{bg}+W^g_{tg}\right)\\ &\quad+\frac{1}{N_C}\left(W^t_{tg}-W^t_{tb}+W^b_{bg}-W^b_{tb}\right)\Big]\left|\mathcal{M}^{(bWg)}\right|^2\\ &\approx \frac{2g_s^2}{E_k^2}\Big[2C_F\left(W^b_{bg}+W^t_{tg}\right)+C_A\left(W^g_{bg}+W^g_{tg}\right)\Big]\left|\mathcal{M}^{(bWg)}\right|^2\end{aligned}$$

- In the soft limit, for a massless b-quark, $E_k = (1-z)\,E_i$,

$$\begin{aligned}\int_{\mathcal{U}/\mathcal{R}} dP_i &\propto \int_{\mathcal{U}/\mathcal{R}} dq^2_{i-1}\,dz_i\,\frac{1}{q^2_{i-1}-m^2}\left(\frac{2C_F}{1-z_i}\right)\\ &= \int_{\mathcal{U}/\mathcal{R}} dcos\theta_{ik}\,dE_k\,\frac{1}{E_k}\left(\frac{2C_F}{1-cos\theta_{ik}}\right)\end{aligned}$$

- Identifying $W^b_{bg} \leftrightarrow P_{bb}$ *etc* gives correct collinear and soft behaviour.
- We only neglect **colour suppressed** $1/N_C^2$ terms... that are also **dynamically suppressed** (collinear cancellations: $W^x_{xy} - W^x_{xz}$).

QCD coherence: logs from the algorithm.

- Leading collinear logs: ☑.
- Leading soft-collinear logs: ☑.
- Next-to-leading soft-collinear logs: ☑ .
- Leading soft-non-collinear logs for massless partons: ☑

  (if in the leading N_C approximation)

  (and if not measuring jet azimuthal structure).
- Leading soft-non-collinear logs for ultra massive partons: approximate

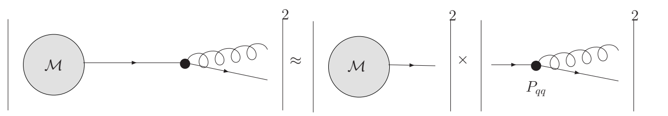
  (overestimated, dipole decomposition becomes unsound).

Herwig++ fragmentation formalism: shower kinematics.

- **Shower formalism rewritten covariantly & extended to massive partons**
  - New evolution variables correspond to those of Herwig77 in the massless limit.
  - Phase space mapping in terms of shower variables is exact and tractable.
  - Smooth coverage in the soft limit - no double counting.
  - Shower variables are Lorentz invariant (easy to go between different frames).
- **Example: *t*-quark evolution**
  - $q_i = \alpha_i p_t + \beta_i n + q_{\perp i}$; $n = \frac{1}{2}(1, \hat{\mathbf{p}}_b) \leftarrow$ "Sudakov decomposition".
  - $z_i \equiv \frac{\alpha_i}{\alpha_{i-1}} \leftarrow$Lorentz invariant$\leftarrow = E_i/E_{i-1}$ for n and p massless acolinear.
  - $\tilde{q}_i^2 \equiv \frac{m_t^2 - q_i^2}{1 - z_i} \leftarrow$Positive, Lorentz invariant.
- $\tilde{q}^2$ is monotonic in angle - preserves angular ordering ☺.
- n is simultaneously the gauge vector used for the massive splitting functions!

Herwig++ fragmentation formalism: massive quarks.

- Accounting for quark masses in the **dynamics** means generalizing from the collinear to the **quasi-collinear limit**.
- In the quasi-collinear limit $\mathbf{p}_\perp^2, \boldsymbol{m^2} \longrightarrow \mathbf{0}$ **at fixed** $\frac{\mathbf{p}_\perp^2}{\boldsymbol{m^2}}$:



$$\lim_{q_i \| k_i} |\mathcal{M}_{n+1}|^2 \propto \frac{1}{q_{i-1}^2 - m^2} P_{qq}\left(z_i, q_{i-1}^2\right) |\mathcal{M}_n|^2$$

$$P_{qq}\left(z_i, \tilde{q}_i^2\right) = C_F \left(\frac{1+z_i^2}{1-z_i} - \frac{2m^2}{q_{i-1}^2 - m^2} - \frac{(2q_i.k)\left(2zn^2\right)}{(2n.k)^2} \right)$$

$$z_i \approx n.q_i / n.q_{i-1}$$

- Quasi-collinear splitting functions and covariant kinematics means we **correctly model low p_T region** (no more spurious 'halo effect') ⚡.

Herwig++ fragmentation formalism: soft radiation.

- Choosing "colour partner gauge" means **all soft divergences** arise from considering emissions from just one leg:
  - ISR ($n \approx q_b$):

$$\lim_{soft} \frac{P_{qq}}{m_t^2 - q_i^2} \quad \rightarrow \quad \frac{C_F}{2} \left(\frac{q_b}{q_b.k} - \frac{q_t}{q_t.k} \right)^2 .$$

  - FSR ($n \approx q_t$):

$$\lim_{soft} \frac{P_{qq}}{q_{i-1}^2 - m_t^2} \quad \rightarrow \quad \frac{C_F}{2} \left(\frac{q_b}{q_b.k} - \frac{q_t}{q_t.k} \right)^2 .$$

- Quasi-collinear splitting functions give exact soft radiation pattern!
- Same eikonal x-section exponentiated in analytic NLL $t \rightarrow bW$ calculation

[JHEP 0212:015, Cacciari, Corcella, Mitov 2002].

- Generally the new formalism closely parallels that of orthodox resummation calculations, in particular recent work on massive quark fragmentation

[NPB617:253-290, Catani & Cacciari, 2001].

- Expect to model well semi-inclusive observables - *b*-quark energy spectrum.

Matrix element (ME) corrections.

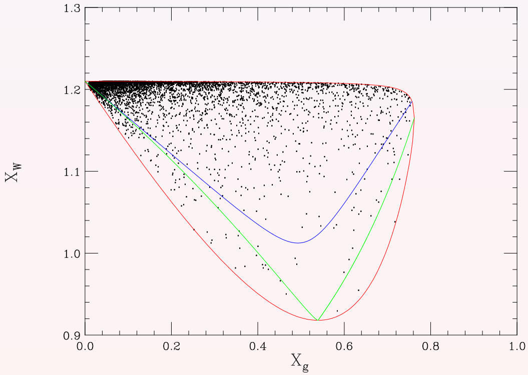
- Improve the approximation of the dynamics in the shower phase space.
- Large p_T emissions are not soft or collinear so they are not well modelled.
- Soft ME corrections correct the distribution of the shower's highest p_T emission to the **exact** single emission matrix element $\mathcal{M}\,(t \to bWg)$.
- Hard ME corrections populate very highest p_T regions of phase space that the shower can't reach.
- Equivalent to CKKW merging of matrix elements with parton showers.

Jet structure: Dalitz plot for $t \longrightarrow bWg$

- Three body decays have two degrees of freedom:

$$x_g = \frac{2p_t.q_g}{m_t^2} \qquad x_W = \frac{2p_t.q_W}{m_t^2}.$$

X axis: 2$\times$gluon energy fraction of m_t. Y axis: 2$\times$W energy fraction of m_t.



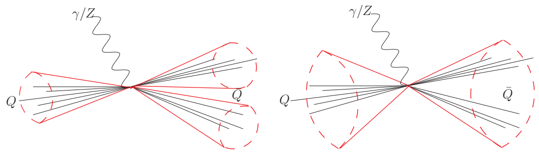
Jet Structure: $e^+e^- \to t\bar{t} \to b\bar{b}WW$ @ $\sqrt{s} = 360\ GeV$.

- **Analysis of $e^+e^- \to t\bar{t} \to b\bar{b}WW$ at $\sqrt{s} = 360\ GeV \approx 2m_t$.**

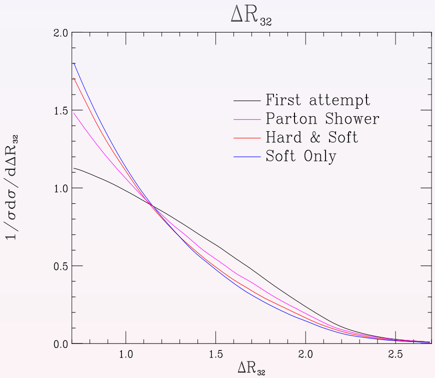
- All events are clustered into 3 jets using k_T jet algorithm.
  - NO hadronic W decays *i.e.* we get at least 2-jets ($b\bar{b}$ quarks).
  - QED ISR is switched off.
  - Radiation from $t\bar{t}$ production phase is negligible, inhibited as $\sqrt{s} \approx 2m_t$.
  - Cut out if $\Delta R \leq 0.7$ $\left(\Delta R^2 = \Delta\eta^2 + \Delta\phi^2\right)$ or if any jet has $E_T < 10\ GeV$.

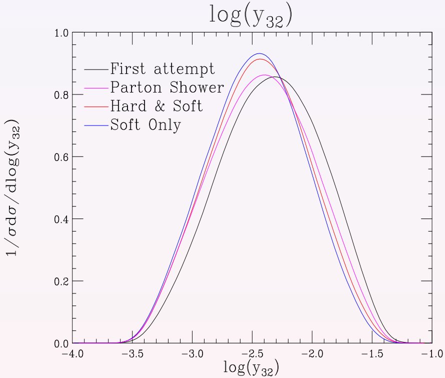
- **Relevant observables are sensitive to internal jet structure.**

- **Jet measures:**
  - $\frac{1}{\sigma}\frac{d\sigma}{d\Delta R_{32}}$ where ΔR_{32} is the "distance" between closest pair of jets in $\eta - \phi$ plane.
  - $\frac{1}{\sigma}\frac{d\sigma}{dlog(y_{32})}$ where $y_{ij} = \frac{2}{s}\, min_{ij}\left(min\left(E_i^2, E_j^2\right)(1 - cos\theta_{ij})\right)$ the Durham measure.



ΔR_{32}, $log\,(y_{32})$ for 3 jet events @ $\sqrt{s} = 360\ GeV$.



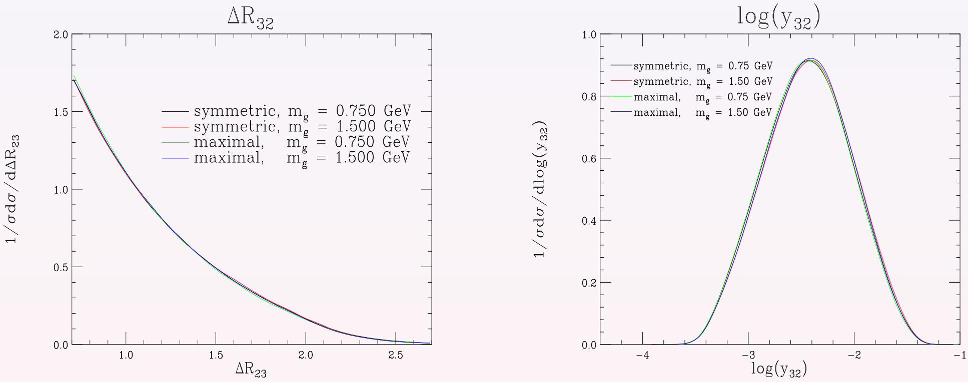


$$\frac{1}{\sigma}\frac{d\sigma}{d\Delta R_{32}}: \Delta R_{32}^2 = \Delta\eta^2 + \Delta\phi^2$$

$$\frac{1}{\sigma}\frac{d\sigma}{dlog(y_{32})}: y_{ij} = \frac{2}{s}\, min_{ij}\left(min\left(E_i^2, E_j^2\right)(1 - cos\theta_{ij})\right)$$

t & *b*-quark phase space is the same size *"symmetric"* partitioning.

Jet structure: ΔR_{32}, $log\,(y_{32})$ - varying gluon mass.



$\frac{1}{\sigma}\frac{d\sigma}{\mathrm{d}d\Delta R_{32}}$ and for 3-jet events @ $\sqrt{s} = 360\ GeV$.

Including matrix element corrections.

Conclusions.

- Parton shower formalism not previously been applied to the kinematic extremes found in top decay.

- New Herwig++ covariant parton shower formalism manages nicely.

    - Quasi-collinear P_{qq} screen collinear singularities: no more *dead-cone halo*.

    - We can shower the top in its rest frame populating infrared regions with the parton shower.

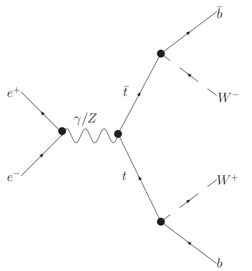
    - Our generalized quasi-collinear limit means the $q \to qg$ splitting function correctly reproduces the eikonal limit (soft gluon resummation).

- ME corrections distribute highest p_T emission according to $t \to bWg$ ME.
- Results for jet structure ΔR_{32} and y_{32} observables look good and show good agreement with the fortran program.

Things I didn't have time for...

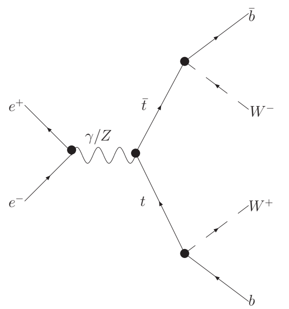
Coming soon: multiscale showering.

- $m_t = 172\ GeV$, $\Gamma_t \approx 1.4\ GeV$, $Q \approx 360\ GeV \to \mathcal{O}(TeV)$.



- $t\bar{t}$ production & decay processes are not well separated from the point of view of soft gluons.
- Soft gluons ‘feel’ the dipole structure of production & decay processes *simultaneously.*
- Showering $t\bar{t}$ production & $t \to Wb$ decays *independently* ignores cross talk.
- *Multiscale showering* aims to describe soft emissions from general $tb\bar{t}\bar{b}$ charge distribution including finite width (multiscale) effects.

Coming soon: multiscale showering.

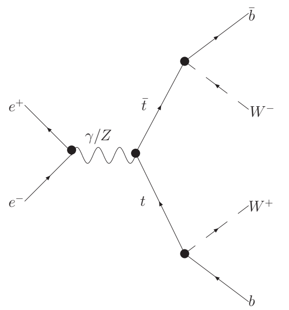


$$
\begin{aligned}
d\sigma^{(N+1)} \quad \propto \quad & \frac{2C_F}{E_k^2}\left(\left(1 + c_{t\bar{t}} - c_t - c_{\bar{t}}\right) W_{t\bar{t}} + \left(1 - c_t\right) W_{tb} + \left(1 - c_{\bar{t}}\right) W_{\bar{t}\bar{b}}\right. \\
& \left. + c_{t\bar{t}} W_{b\bar{b}} + \left(c_t - c_{t\bar{t}}\right) W_{\bar{t}b} + \left(c_{\bar{t}} - c_{t\bar{t}}\right) W_{t\bar{b}}\right) d\sigma^{(N)} \, [dk] \, .
\end{aligned}
$$

$$
c_{t/\bar{t}} = \frac{m^2\Gamma^2}{\left(p_{t/\bar{t}}.k\right)^2 + m^2\Gamma^2} \quad c_{t\bar{t}} = \frac{m^2\Gamma^2\left(\left(p_t.k\right)\left(p_{\bar{t}}.k\right) + m^2\Gamma^2\right)}{\left(\left(p_t.k\right)^2 + m^2\Gamma^2\right)\left(\left(p_{\bar{t}}.k\right)^2 + m^2\Gamma^2\right)}
$$

[Khoze, Orr, Stirling]

Coming soon: multiscale showering.



$$
\begin{aligned}
& d\sigma^{(N+1)} \\
\propto\ & \frac{2C_F}{E_k^2}\Bigg(\left(W^t_{t\bar{t}} + W^t_{tb}\right)(1-c_t) + \left(W^{\bar{t}}_{t\bar{t}} + W^{\bar{t}}_{\bar{t}\bar{b}}\right)(1-c_{\bar{t}}) \\
+\ & (1-c_{t\bar{t}})\left(W^b_{tb} + W^{\bar{b}}_{\bar{t}\bar{b}}\right) + c_{t\bar{t}}\left(W^b_{b\bar{b}} + W^{\bar{b}}_{b\bar{b}}\right) \\
+\ & (c_t - c_{t\bar{t}})\left(W^b_{\bar{t}b} - W^b_{tb} + W^{\bar{t}}_{\bar{t}b} - W^{\bar{t}}_{t\bar{t}}\right) + (c_{\bar{t}} - c_{t\bar{t}})\left(W^{\bar{b}}_{t\bar{b}} - W^{\bar{b}}_{\bar{t}\bar{b}} + W^t_{t\bar{b}} - W^t_{t\bar{t}}\right)\Bigg) \\
& d\sigma^{(N)}\,[dk]
\end{aligned}
$$

As $c_{t/\bar{t}} \leq 1$ all emissions are suppressed relative to naive expectation!

Emissions will need to be vetoed *e.g.* for top, with probability $1 - c_t$.

Coming soon: multiscale showering today.

- But what happens once one gluon is emitted and the colour line is snapped?
- Did 2nd order calculation $e^+e^- \to bW^+\bar{b}W^-gg$ with strongly ordered (gauge invariant) gluon emission:

$$
\begin{aligned}
& d\sigma^{(N+2)} \\
= \;& \frac{C_A C_F}{E_k^2 E_g^2}\left(\left(\mathcal{I}_{t\bar{t}}-\mathcal{C}_t\right)W^t_{t\bar{t}}+\left(\mathcal{I}_{tb}-\mathcal{C}_t\right)W^t_{tb}\right)+\left(\left(\mathcal{I}_{t\bar{t}}-\mathcal{C}_{\bar{t}}\right)W^{\bar{t}}_{t\bar{t}}+\left(\mathcal{I}_{\bar{t}\bar{b}}-\mathcal{C}_{\bar{t}}\right)W^{\bar{t}}_{\bar{t}\bar{b}}\right) \\
+ \;& \left(\mathcal{I}_{tb}-\mathcal{C}_{t\bar{t}}\right)W^b_{tb}+\left(\mathcal{I}_{\bar{t}\bar{b}}-\mathcal{C}_{t\bar{t}}\right)W^{\bar{b}}_{\bar{t}\bar{b}}+\mathcal{C}_{t\bar{t}}\left(W^b_{b\bar{b}}+W^{\bar{b}}_{b\bar{b}}\right) \\
+ \;& \left(\mathcal{C}_t-\mathcal{C}_{t\bar{t}}\right)\left(W^b_{\bar{t}b}-W^b_{tb}+W^{\bar{t}}_{\bar{t}b}-W^{\bar{t}}_{t\bar{t}}\right)+\left(\mathcal{C}_{\bar{t}}-\mathcal{C}_{t\bar{t}}\right)\left(W^{\bar{b}}_{t\bar{b}}-W^{\bar{b}}_{\bar{t}\bar{b}}+W^t_{t\bar{b}}-W^t_{t\bar{t}}\right) \\
& d\sigma^{(N+2)}\,[dk]\,[dg]+\mathcal{O}\left(1/N_C^2\right)
\end{aligned}
$$

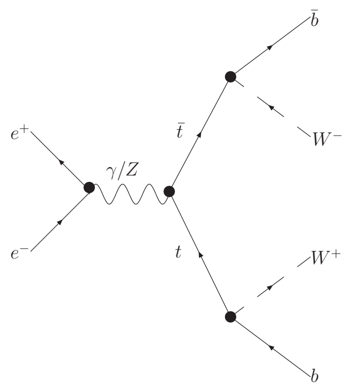
- Now each dipole splitting function has a *form factor* describing for emission of the second gluon (these look complicated).
- Reduces to the intuitive result recovered in the limit the second emission is very soft namely

$$
d\sigma^{(N+2)} = d\sigma^{(N+1)} \times \frac{2C_A}{\omega_k^2}\left(w_{bg}+w_{\bar{b}g}-\frac{1}{N_C^2}w_{b\bar{b}}\right).
$$

Things I really didn't have time for...

Production mechanisms: $t\bar{t}$ at the ILC.

- Dedicated $t\bar{t}$ threshold scans are carried out for precision mass and cross section measurements.
- Dominant production mode is via γ/Z exchange.



$$\sigma_{t\bar{t}}\left(\sqrt{s} = 500\ GeV\right) \approx 0.6\ pb$$

~**60,000** $t\bar{t}$ **pairs per year** ($100 fb^{-1}$).

Reasons for study: m_t.

- $m_t = 171.4 \pm 2.1\ GeV$: August'06 TEVEWWG "world average".
- SM has mass dependent couplings: top coupling to Higgs and longitudinal W's is enhanced.
- Non-decoupling: top loops give large corrections to gauge boson self energies $\propto m_t^2$:
$$\Pi_V\left(q^2\right) \sim q^2 A\left(q^2\right) + m_t^2 B\left(q^2\right).$$
- Gauge boson self energies are ubiquitous in precision electroweak predictions.
- Typically the experimental error on input parameter m_t gives the largest uncertainty to SM precision electroweak predictions *e.g.*:

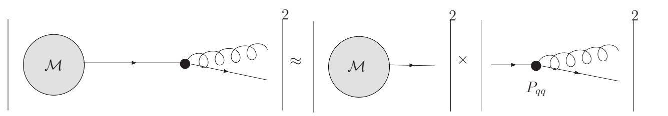
$$\delta m_t = 2.3\ GeV \text{ ☞ } \delta M_W^{(m_t)} = 14\ MeV.$$

Collinear radiation: matrix element factorisation.

- Quasi-collinear parton configurations are enhanced:

$$\textit{propagator} \propto \frac{1}{2q_i.k_i} \propto (1-\beta_i cos\theta)^{-1}\ . \qquad (1)$$

- In the quasi-collinear limit $\mathbf{p}_\perp^2, m^2 \to 0$ for fixed $\frac{\mathbf{p}_\perp^2}{m^2}$:



$$\begin{aligned} lim_{q_i \| k_i} \left|\mathcal{M}_{n+1}\right|^2 &= \frac{8\pi\alpha_S}{z_i(1-z_i)\tilde{q}_i^2} P_{qq}\left(z_i, \tilde{q}_i^2\right) \left|\mathcal{M}_n\right|^2 \\ P_{qq}\left(z_i, \tilde{q}_i^2\right) &= C_F\left(\frac{1+z_i^2}{1-z_i} - \frac{2m^2}{z_i(1-z_i)\tilde{q}_i^2}\right). \end{aligned}$$

$$z_i \approx E_i/E_{i-1} \qquad z_i\left(1-z_i\right)\tilde{q}_i^{\,2} = q_{i-1}^2 - m^2.$$

- For strongly ordered virtualities, $\left(FSR:\ q_{i-1}^2 \gg q_i^2\right)$, this factorization can be applied to all emissions.
- Herwig (massless partons) regulated collinear (and soft) divergences with angular cut (1). Herwig++ (massive partons), regulates soft divergence with a gluon mass: $z_i\left(1-z_i\right)\tilde{q}_i^{\,2} = 2q_i.k_i + \mathbf{m}_g^2$ (34).

Collinear radiation: Sudakov formfactor

- For illustrative purposes just consider gluons emitted by t.
- Leading collinear log width for $t \to bW + (n)\, g$ (unresolved phase space: $\mathcal{U}$):

$$\int d\Gamma_n = \prod_{i=1}^{n} \left(\int_{\mathcal{U}} dP_i\,(t \to tg)\right) \int d\Gamma_0$$

$$\int_{\mathcal{U}/\mathcal{R}} dP_i = \int_{\tilde{q}_{i-1}^2}^{\tilde{q}_{max}^2} d\tilde{q}_i^2 dz_i \frac{\alpha_S}{2\pi\tilde{q}_i^2} P_{qq}\left(z_i, \tilde{q}_i^2\right) \Theta\left(\mathcal{U}/\mathcal{R}\right).$$

- Integrating over everything all mass singular (big) logs have to vanish (KLN theorem/unitarity):

$$\int_{\mathcal{U}} dP_i\,(t \to tg) + \int_{\mathcal{R}} dP_i\,(t \to tg) = 0.$$

- This together with nesting of $\tilde{q}^2$ integrals means:

$$\int d\Gamma_n = \frac{1}{n!}\left(-\int_{\mathcal{R}} dP_1\,(t \to tg)\right)^n \int d\Gamma_0.$$

Collinear radiation: Sudakov formfactor.

- Total decay width for $t \to bW + (n)\,g$ is the sum over n:

$$\Gamma_{\mathcal{U}} = \Gamma_0 exp\left(-\int_{\mathcal{R}} dP_1\,(t \to tg)\right) \ .$$

- *Probability* no resolvable emissions in phase space as top ‘evolves’ from $\tilde{q}_0^2 \to \tilde{q}_{max}^2$ is

$$\frac{\Gamma_{\mathcal{U}}}{\Gamma_{\mathcal{U}}+\Gamma_{\mathcal{R}}} = \frac{\Gamma_{\mathcal{U}}}{\Gamma_0} \equiv S\left(\tilde{q}_0^2, \tilde{q}_{max}^2\right)$$

  since $\Gamma_{\mathcal{U}} + \Gamma_{\mathcal{R}} = \Gamma_0$ in this leading log approximation.

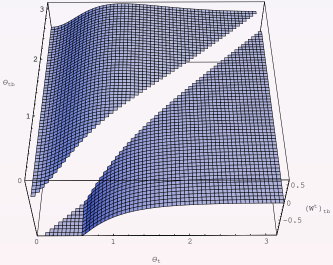
- The probability of that there is emission between the two scales is $1 - S$, in particular, for emission at scale $\tilde{q}^2$ it is:

$$-\frac{dS\left(\tilde{q}_0^2, \tilde{q}^2\right)}{d\tilde{q}^2} d\tilde{q}^2 = d\tilde{q}^2 dz\, \frac{\alpha_S}{2\pi\tilde{q}^2} P_{qq}\left(z, \tilde{q}\right) S\left(\tilde{q}_0^2, \tilde{q}^2\right) \Theta\left(\mathcal{R}\right).$$

- Sampling this distribution produces $t \to Wb + (n)\,g$ events including the effects of all orders leading collinear logs.

Soft radiation: massive particle problems.

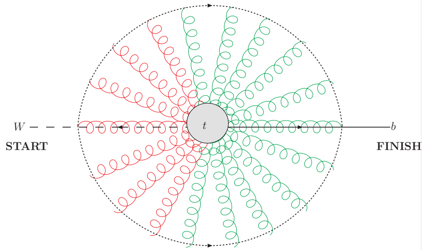
- Unfortunately angular ordering is only **exact** for massless partons.
- For radiation from the top quark with $v_t \sim 0.65c$ we have $W^t_{tb}(m_b = 0)$:



- Radiation is no longer $\propto \Theta\left(\theta_{tb} - \theta_{tk}\right)$ but radiation outside the $t \leftrightarrow b$ cone almost vanishes anyway.
- True for any mass of the emitter! Discontinuity only smoothed by the *spectator* mass.
- $\left\langle W^t_{tb} \right\rangle$ is independent θ_{tb} for $m_b \to 0$ ☞generate all emissions the same way.
- Angular ordering omits the relatively small (negative) bit outside the cone.

Herwig++: angular ordering in top decay.

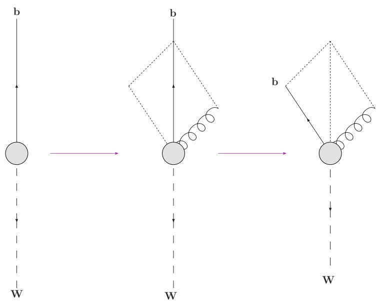
- In the top rest frame:
  - Symmetric choice: θ_{bg} cut-off is $\approx 100^{\circ}$ in the top rest frame, $\approx 55^{\circ}$ in the W rest frame.
  - Maximal choice: θ_{bg} cut-off is $\approx 110^{\circ}$ in the top rest frame, $\approx 65^{\circ}$ in the W rest frame.



Herwig++: ISR kinematics reconstruction.

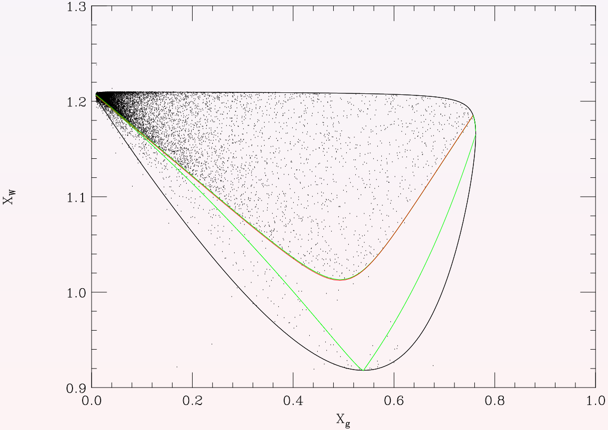
- b and W were initially generated according to a tree level decay $t \rightarrow bW$.
  - We now have extra jets as well as b and W.
    - We need to pay for their momentum so the momentum balances: $(m_t, \mathbf{0})$.
    - ISR recoil is absorbed by

1. Rescaling the 3-momenta of the b-jet and W so energy balances.
2. Absorb the transverse momentum in b.



Dalitz plots: first attempt.

- when we applied the formalism out of the box and it all went wrong...



Initial results: back to the drawing board.

- Is the quasi collinear limit sensible?
  - We said m^2 and $k_\perp^2$ had to be small (compared to $n.p$).
  - $n = \frac{1}{2} m_t \left(1, \hat{\mathbf{p}}_b\right)$ & $p = m_t \left(1, \mathbf{0}\right)$: $n.p = \frac{1}{2} m_t^2$ ↯.
- What logs should we get?
  - Soft logs ☑.
  - Collinear (mass singular) logs? But for the top there aren't any.
- What is the weird excess of points at wide angles due to?

Herwig++: lim_{soft}massive splitting function $\sim$ dipoles.

- $q_i = \alpha_i p + \beta_i n + q_{\perp i}$: $z \equiv \alpha_i / \alpha_{i-1}$

$$:z \equiv n.q_i / n.q_{i-1}.$$

- Now take n seriously as a physical vector not just a gauge vector.
- ISR ($n \approx q_b$):

$$\begin{aligned} \frac{P_{qq}}{m_t^2 - q_i^2} &= \frac{C_F}{2q_{i-1}.k_i}\left(\frac{n.k_i}{n.q_{i-1}} + \frac{2n.q_i}{n.k_i} - \frac{n.q_i m_t^2}{(n.q_{i-1})(q_{i-1}.k_i)}\right) \\ &\to \frac{C_F}{2}\left(\frac{2n.q_t}{(q_t.k_i)(n.k_i)} - \frac{m_t^2}{(q_t.k_i)^2}\right). \end{aligned}$$

- FSR ($n \approx q_W$):

$$\begin{aligned} \frac{P_{qq}}{q_{i-1}^2 - m_t^2} &= \frac{C_F}{2q_i.k_i}\left(\frac{n.k_i}{n.q_{i-1}} + \frac{2n.q_i}{n.k_i} - \frac{m_b^2}{q_i.k_i}\right) \\ &\to \frac{C_F}{2}\left(\frac{2n.q_b}{(q_b.k_i)(n.k_i)} - \frac{m_b^2}{(q_b.k_i)^2}\right). \end{aligned}$$

- Massive splitting functions reminiscent of soft dipole functions $\sim -\frac{C_F}{2}\left(\frac{q_t}{q_t.k_i} - \frac{q_b}{q_b.k_i}\right)^2$!

Herwig++: solutions.

- Amazingly, given $n \approx q_b$,

$$\frac{P_{qq}}{m_t^2 - q_i^2} \rightarrow \frac{C_F}{2}\left(\frac{2n.q_t}{(q_t.k_i)\,(n.k_i)} - \frac{m_t^2}{(q_t.k_i)^2}\right),$$

  ☞we exactly exponentiate the *tb* soft dipole function in the top quark Sudakov form factor.
- This is what Cacciari, Corcella, Mitov exponentiate in their NLL calculation ☺.

Preliminary: our results should agree for **all** leading logs (and NLL soft-collinear?).

[M.Caccari - private communication]

- Expect to model well sufficiently inclusive observables - *b*-quark energy spectrum.

STOP we haven't checked plots yet STOP

- Can we do the same for the *b*-quark?
- As above the *b*-quark is just wrong: the singularity structure is badly wrong (bad gauge invariance violation) - we need *n* to be massive.

Herwig++: generalising the massive splitting function.

- From polarization sum:

$$\sum_{phys} \epsilon^{\mu}(g)\,\epsilon^{\nu *}(g) = -\eta^{\mu\nu} + \frac{g^{\mu} n^{\nu} + n^{\mu} g^{\nu}}{n.g} - \frac{n^2 g^{\mu} g^{\nu}}{(n.g)^2}.$$

- Now the soft limit is precisely a dipole function:

$$lim_{q\|k} \overline{\left|\mathcal{M}^{(n+1)}\right|^2} = \frac{2g_s^2}{(2q_i.k)} C_F \left(\frac{1+z^2}{1-z} - \frac{2m_q^2}{(2q_i.k)} - \frac{(2q_i.k)\,(2zn^2)}{(2n.k)^2} \right) \overline{\left|\mathcal{M}^{(n)}\right|^2}$$

- Using $n_{ISR} = q_b$ and $n_{FSR} = q_t$ ☞ *exactly* soft radiation dipole!
- Confirmed by explicit calculation in soft limit.
- Image charge method?

Soft matrix element (ME) correction.

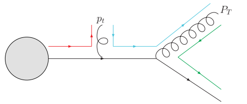
- Improve the approximation of the dynamics in the shower phase space.
  - Large $p_T$emissions are not well modelled.
  - Want largest p_T distributed according to $\mathcal{M}\left(t \rightarrow bWg\right)$ *i.e.*

$$\frac{1}{\Gamma_0}\frac{d^2\Gamma}{dzd\tilde{q}^2}\, exp\left(-\int_{\tilde{q}^2_{i-1}}^{\tilde{q}^2_{max}} d\tilde{q}^2 \mathrm{d}z\, \frac{1}{\Gamma_0}\frac{d^2\Gamma}{dzd\tilde{q}^2}\Theta\left(\mathcal{R}\right)\right).$$

  - Ansatz: assume all other emissions are (infinitely) soft.
  - Simple to generate: if it is the hardest so far veto if

$$\mathcal{R} \geq \frac{1}{\Gamma_0}\frac{d^2\Gamma}{dzd\tilde{q}^2}\Big/\frac{\alpha_S}{2\pi\tilde{q}^2_i}P_{qq}\left(z_i, \tilde{q}^2_i\right).$$

  - *Coherence* means this is OK even if the hardest isn't first



Hard ME correction.

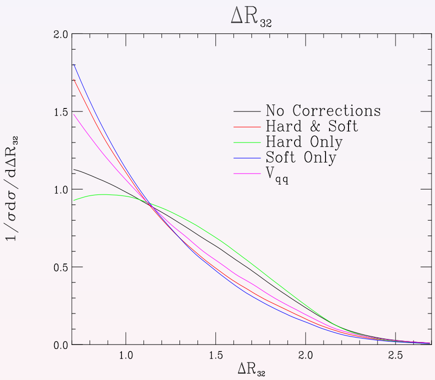
What about the dead region?

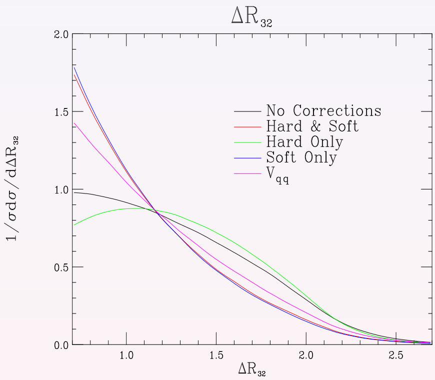
- This is simple in principle we just generate a point in the dead region according to

$$\frac{1}{\Gamma_0}\frac{d^2\Gamma}{dzd\tilde{q}^2}$$

  - The only thing to worry about is what precisely where the dead region is to avoid over counting the soft region.

Jet structure: ΔR_{32}.



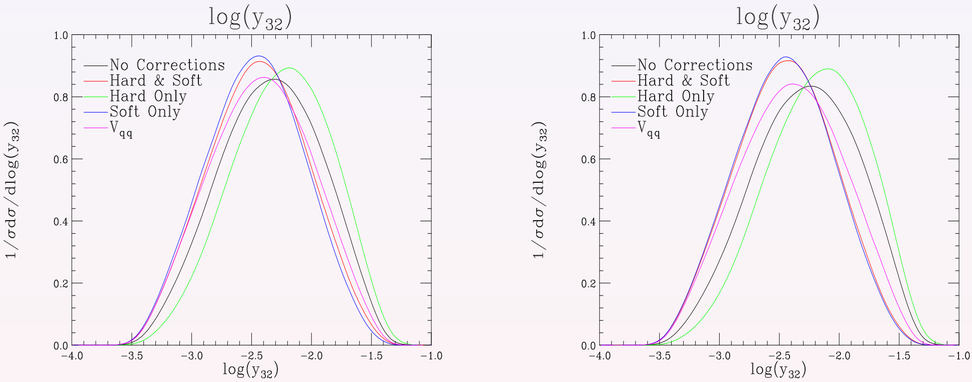


$$\frac{1}{\sigma}\frac{d\sigma}{d\Delta R_{32}}: \Delta R_{32}^2 = \Delta\eta^2 + \Delta\phi^2 \text{ for three jet events @ } \sqrt{s} = 360\ GeV.$$

Left: *t & b*-quark phase space is the same size *"symmetric"* partitioning.

Right: *b*-quark populates most of the phase space *"maximal"* partition.

Jet structure: $log\,(y_{32})$hard and soft ME corrections.

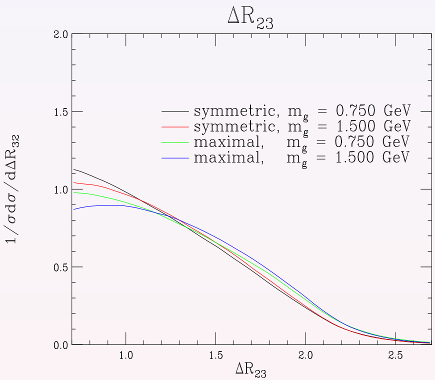


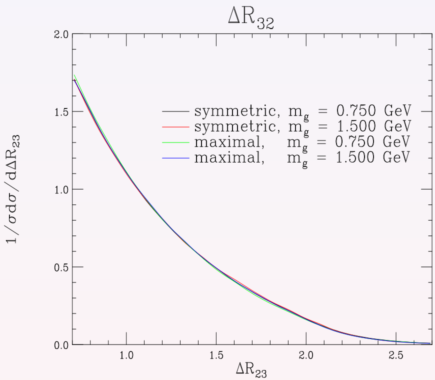
$$\frac{1}{\sigma}\frac{d\sigma}{dlog(y_{32})}: \; y_{ij} = \frac{2}{s}\, min_{ij}\left(min\left(E_i^2, E_j^2\right)\left(1 - cos\theta_{ij}\right)\right) \text{ for 3-jet events @ } \sqrt{s} = 360\ GeV$$

Left: *t & b*-quark phase space is the same size *"symmetric"* partitioning.

Right: *b*-quark populates most of the phase space *"maximal"* partition.

Jet structure: ΔR_{32} - varying gluon mass.



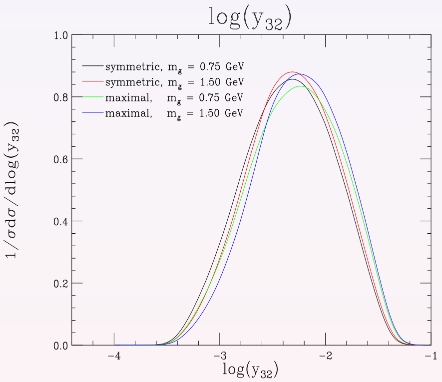


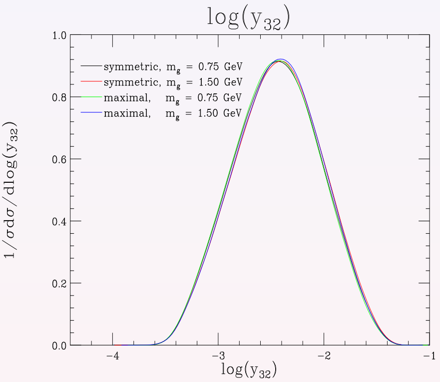
$\frac{1}{\sigma}\frac{d\sigma}{\mathrm{d}d\Delta R_{32}}$: $\Delta R^2 = \Delta\eta^2 + \Delta\phi^2$ for 3-jet events @ $\sqrt{s} = 360\ GeV$.

Left: parton shower approximation.

Right: parton shower with matrix element corrections.

Jet Structure: $log\,(y_{32})$ - varying gluon mass.





$\frac{1}{\sigma}\frac{d\sigma}{dlog(y_{32})}$: $y_{32} = \frac{2}{s}\, min_{ij}\left(min\left(E_i^2, E_j^2\right)(1 - cos\theta_{ij})\right)$ for 3-jet events @ $\sqrt{s} = 360\ GeV$

Left: parton shower approximation.

Right: parton shower with matrix element corrections.