Consistent minimal symmetry breaking model for Lorentz & CPT invariance violations

J. Alfaro, A.A. Andrianov, M. Cambiaso,
Paola Giacconi, & R.S.

July 8, 2007

Abstract :

The Chern-Simons modification of spinor QED involving a constant axial vector background is carefully analyzed. This field theoretic model appears to be fully consistent and represents the minimal modification of the standard QED that fulfill locality, renormalizability, unitarity and gauge invariance though leading to Lorentz and CPT invariance violations. Experimental limits as well as theoretical constraints arising from the structure of the radiatively induced modifications involving the photon and matter dynamics will be thoroughly focussed.

1. A. A. Andrianov & R. Soldati
Lorentz symmetry breaking in Abelian vector–field models with Wess–Zumino interaction
Phys. Rev. D**51** (1995) 5961

2. A. A. Andrianov & R. Soldati
Patterns of Lorentz symmetry breaking in QED by CPT-odd interaction
Phys. Lett. B**435** (1998) 449

3. A. A. Andrianov, L. Sorbo & R. Soldati
Dynamical Lorentz symmetry breaking from 3+1 axion-Wess-Zumino model
Phys. Rev. D**59** (1999) 025002

4. A. A. Andrianov, P. Giacconi & R. Soldati
Lorentz and CPT violations from Chern-Simons modifications of QED
JHEP **02** (2002) 030

5. J. Alfaro, A.A. Andrianov, M. Cambiaso, Paola Giacconi, R. Soldati
On the consistency of Lorentz invariance violation in QED induced by fermions in constant axial-vector background
Phys. Lett. B **639**, 586-590 (2006)

1. INTRODUCTION

2. MAXWELL-CHERN-SIMONS (**MCS**) FREE RADIATION FIELD

3. CPT-ODD FREE SPINOR FIELD

4. THE PHYSICAL ULTRAVIOLET CUT OFF AND THE RADIATIVELY INDUCED EFFECTIVE ACTION

5. CONCLUSIONS

Louvain La Neuve - August 2007

0. INTRODUCTION

- **Lorentz & CPT- symmetry breaking at the quantum gravity M_{Planck} scale**

  ◦ $\mathcal{STRING} \quad \& \quad \mathcal{BRANE} \quad \mathcal{THEORIES}$

  V.A. Kostelecky & S. Samuel (1989)

  ◦ $\mathcal{NON-COMMUTATIVE} \quad \mathcal{QFT}$

  V.A. Kostelecky & R. Potting (1995)

  ◦ $\mathcal{QUANTUM\ GRAVITY\ EFFECTS}$

  G. Amelino-Camelia et al. (1998)

- **Low-energy very small deviation from the Lorentz covariant Standard Model**

  ◦ $\mathcal{MAXWELL-CHERN-SIMONS}$ modification of classical electrodynamics
  `S.M.Carroll, G.B.Field, R.Jackiw (1990)`

  ◦ CPT-odd Lorentz symmetry breaking from the chiral U(1) Anomaly
  `A.A. Andrianov & R. Soldati (1995)`

  ◦ CPT & Lorentz spontaneous breaking
  `D. Colladay & V.A. Kostelecky (1998)`

  ◦ CPT-even Lorentz symmetry breaking
  `S.R. Coleman & S.L. Glashow (1999)`

- **Lorentz & CPT Invariance Violations**
  - Power counting renormalizable
  - $SU(3) \otimes SU(2) \otimes U(1)$ gauge invariant

- **Preferred Inertial Reference Frames**
  - Cosmic Microwave Background isotropic
  - **translation invariance**
  - **tiny violations in lab experiments**
  - Lorentz & CPT-breaking by **very small**
  - * **constant four-vectors** η_α & b_μ
  - **Experiments** versus benchmark value

$$|\eta_\alpha| \ll |b_\mu| < \frac{m_e^2}{M_{\text{Planck}}} = 2 \times 10^{-17} \text{ eV}$$

η_α & b_μ of **cosmological** origin

Cosmic $\mathcal{TORSION}$ axial-vector

$$b_\mu = \epsilon_{\mu\nu\rho\sigma} \langle T^{\nu\rho\sigma} \rangle_{\underline{0}}$$

Cosmic $\mathcal{QUINTESSENCE}$ vector

$$\eta_\alpha = \langle \partial_\alpha \Phi \rangle_{\underline{0}}$$

Quantum gravity *ad hoc* hypothesis
about photon dispersion relation
G.Amelino-Camelia et al.

$$\omega(k) \simeq \sqrt{(1-\epsilon)\, k^2 - A\, k^4\, M_{\mathsf{P}}{}^{-2}}$$

unbounded group velocity for large k

$$\lim_{k \to \infty} \frac{d\omega}{dk} = \infty$$

imaginary frequency for small k tachionic
and acausal: not from solid QFT

- **CPT-even tiny perturbations**

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \quad \mapsto \quad -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}(1-\epsilon)$$

$$v_{\text{light}} = c\sqrt{1-\epsilon} < c$$

  - **maximal velocity of a material body**

$$v_{\text{body}} \le c$$

  - **vacuum** *Čerenkov* **radiation**

$$v_{\text{light}} < v_{\text{body}} = c$$

UHECR $\quad k \approx 3 \times 10^{20}$ eV $\quad$ (HiRes I-II)*

$$\epsilon < 1.6 \times 10^{-23}$$

*15.10.1991 High Resolution Fly's Eye Air Shower Detector in Utah

- **Evading the *GZK* Cutoff ?**

UHECR : $E_{\mathrm{max}} \simeq m_p m_\pi / kT_\gamma \simeq 3 \times 10^{20}$ eV

CR arising outside our galaxy $\sim$ 100 Mpc

K. Greisen (1966)

G. Zatsepin & V. Kuz'min (1966)

- **CMB γ-proton scattering**

$$p + \gamma_{\mathrm{CMB}} \;\rightarrow\; \Delta(1232)$$

$$p + \gamma_{\mathrm{CMB}} \;\rightarrow\; p + \pi$$

- **Dispersion relation** of the a -particle

$$E_a = \sqrt{c_a^2 \vec{p}^{\,2} + m_a^2 c_a^4} \quad \Rightarrow \quad v_a \;\leq\; c_a$$

higher *GZK* cutoff $\quad \Leftrightarrow \quad c_\Delta > c_p > c_\pi$

- **Disagreement in UHECR detectors**

  ◦ 8 events above GZK in AGASA *

  M. Takeda et al. Phys.Rev.Lett. (1998)

  ◦ all events below GZK in HiRes Fly's Eye

  D.J. Bird et al. Phys.Rev.Lett. (1995)

  The difference remains unknown if only statistical errors are taken into account

  ◦ 3 events above GZK in P. AUGER Obs.

  H. Wahlberg, private comm. (2007)

  PAO DOES NOT CONFIRM AGASA

  * Akeno Giant Air Shower Array in Japan

1. Maxwell-Chern-Simons photon

- **MCS free Lagrange density**

$$\mathcal{L}_{\text{MCS}} = -\frac{1}{4} F^{\nu\lambda} F_{\nu\lambda} - B(x)\,\partial_\alpha A^\alpha - \frac{1}{2}\eta^\alpha A^\beta \widetilde{F}_{\alpha\beta}$$

$$\widetilde{F}_{\alpha\beta}(x) \equiv \frac{1}{2}\epsilon_{\alpha\beta\rho\sigma} F^{\rho\sigma}(x)$$

  - $B(x)$ is the gauge fixing auxiliary field
  - η_α **Chern-Simons constant** vector

- Equations of motion in momentum space

$$\left(k^2 g^{\nu\sigma} - i\,\epsilon^{\mu\rho\nu\sigma}\eta_\mu k_\rho\right)\widetilde{A}_\sigma = 0$$

$$k^\sigma \widetilde{A}_\sigma(k) = 0$$

- MCS polarizations: 2D projector

$$
\begin{aligned}
\mathrm{e}^{\mu\nu} &\equiv g^{\mu\nu} - \frac{\eta\cdot k}{\mathtt{D}}\left(\eta^{\mu}k^{\nu} + \eta^{\nu}k^{\mu}\right) + \frac{\eta^2}{\mathtt{D}}k^{\mu}k^{\nu} \\
&+ \frac{k^2}{\mathtt{D}}\eta^{\mu}\eta^{\nu} \qquad \mathtt{D} \equiv (\eta\cdot k)^2 - \eta^2 k^2
\end{aligned}
$$

$$
\mathrm{e}\,\eta = \mathrm{e}\,k = 0 \qquad \mathrm{e}^2 = \mathrm{e} \qquad \mathtt{tr}\,\mathrm{e} = 2
$$

- Linear polarization real vectors

$$
\mathrm{e}_{\mu\nu} = -\sum_{a=1,2} \mathrm{e}_{\mu}^{(a)}\mathrm{e}_{\nu}^{(a)}
$$

- Chiral polarization complex vectors

$$
\varepsilon_{\mu}^{(L)} \equiv \frac{1}{2}\left(\mathrm{e}_{\mu}^{(1)} + i\mathrm{e}_{\mu}^{(2)}\right)
$$

$$
\varepsilon_{\mu}^{(R)} \equiv \frac{1}{2}\left(\mathrm{e}_{\mu}^{(1)} - i\mathrm{e}_{\mu}^{(2)}\right)
$$

$$
\mathrm{e}_{\mu\nu} = -\sum_{a=L,R}\left\{\varepsilon_{\mu}^{(a)}\varepsilon_{\nu}^{(a)*} + \varepsilon_{\nu}^{(a)}\varepsilon_{\mu}^{(a)*}\right\}
$$

ONLY CHIRAL POLARIZATIONS ARE THE **EIGENSTATES** OF THE **MCS** KINETIC OPERATOR

$$\left(k^2 g_\nu{}^\sigma - i\epsilon_\nu{}^{\mu\rho\sigma}\eta_\mu k_\rho\right)\varepsilon_\sigma^{(L)} = \left(k^2 - \sqrt{\mathtt{D}}\right)\varepsilon_\nu^{(L)}$$

$$\left(k^2 g_\nu{}^\sigma - i\epsilon_\nu{}^{\mu\rho\sigma}\eta_\mu k_\rho\right)\varepsilon_\sigma^{(R)} = \left(k^2 + \sqrt{\mathtt{D}}\right)\varepsilon_\nu^{(R)}$$

LET US SEARCH **FOUR REAL** SOLUTIONS OF THE QUARTIC ON SHELL CONDITION

$$(k^2)^2 - \mathtt{D} = (k^2)^2 - (\eta \cdot k)^2 + \eta^2 k^2 = 0$$

MCS CHIRAL POLARIZATIONS REDUCE TO MAXWELL'S CIRCULAR POLARIZATIONS ON THE LIGHT-CONE $k^2 = 0$

FREQUENCY SOLUTIONS

LEFT ($+$) RIGHT ($-$) polarizations

- **space-like** $\eta^{\mu} = (0, \vec{\eta}\,)$

$$\omega_L^2 = k^2 + \frac{1}{2}\eta^2 + \eta\sqrt{k^2\cos^2\varphi + \frac{1}{4}\eta^2}$$

$$\omega_R^2 = k^2 + \frac{1}{2}\eta^2 - \eta\sqrt{k^2\cos^2\varphi + \frac{1}{4}\eta^2}$$

$$\vec{\eta}\cdot\vec{k} \;=\; \eta\, k\cos\varphi$$

- **light-like** $\eta^{\mu} = (\eta, \vec{\eta}\,)$

$$\left(\omega_L - \frac{1}{2}\eta\right)^2 = \left(\vec{k} - \frac{1}{2}\vec{\eta}\right)^2$$

$$\left(\omega_R + \frac{1}{2}\eta\right)^2 = \left(\vec{k} + \frac{1}{2}\vec{\eta}\right)^2$$

monochromatic plane wave solutions are possible **only with** a **definite chiral polarization**

WAVE PACKETS ARE SPLITTED INTO LEFT ($+$) AND RIGHT ($-$) WAVE PACKETS MOVING WITH **DIFFERENT GROUP VELOCITIES**

$$\vec{v}_L = \nabla_{\vec{k}}\, \omega_L(\vec{k}\,) \qquad v_L \leq 1 \quad \forall \vec{k}$$

$$\vec{v}_R = \nabla_{\vec{k}}\, \omega_R(\vec{k}\,) \qquad v_R \leq 1 \quad \forall \vec{k}$$

* VACUUM BIREFRINGENCE *

Group Velocity Difference between Left and Right Wave Packets

$$\Delta v \equiv |\vec{v}_R - \vec{v}_L|$$

- **Vanishes at HIGH frequencies**

$$\lim_{k \to \infty} \Delta v = 0$$

- **Grows at LOW frequencies**

$$\lim_{k \to 0} \Delta v = 1$$

TIME DELAY of LEFT HANDED WAVES WITH RESPECT TO RIGHT HANDED ONES IS CALLED **VACUUM BIREFRINGENCE**

* UPPER BOUND *

EXPERIMENTAL ABSENCE OF
THIS EFFECT IN
RADIO-ASTRONOMY
OBSERVATIONS OF DISTANT
QUASARS AND RADIO GALAXIES

$$\Updownarrow$$

$$|\,\vec{\eta}\,| \;\leq\; 10^{-33}\ \mathrm{eV}$$

B. Nodland & J. P. Ralston (1997)

J.F.C.Wardle et al. (1997)

- **time-like** $\eta_\mu = (\eta, 0)$

$$\omega_L(k,\eta) = \sqrt{k\,(k + \eta)}$$

$$\omega_R(k,\eta) = \sqrt{k\,(k - \eta)}$$

WAVE PACKETS OF BOTH
POLARIZATIONS ARE
SUPERLUMINAL

LEFT ($+$) AND RIGHT ($-$)

WAVE PACKETS MOVE WITH

TACHION GROUP VELOCITIES

$$v_{L,R} = \frac{k}{\omega_{L,R}\,(k,\eta)}\left(1 \pm \frac{\eta}{2k}\right) > 1$$

$\omega_R(k,\eta_0)$ **imaginary** for $k < \eta$

UNPHYSICAL SOLUTIONS

2. CPT-ODD FREE SPINOR FIELD

- **CPT-ODD free spinor Lagrange density**

$$\mathcal{L}_{\text{spinor}} = \bar{\psi}(x)\left(i\gamma^{\mu}\partial_{\mu} - m - \gamma^{\mu} b_{\mu}\gamma^{5}\right)\psi(x)$$

- b_{μ} **constant** cosmic axial-vector
- **Lorentz & CPT**-violating term

$$\bar{\psi}(x)\gamma^{\mu} b_{\mu}\gamma^{5}\psi(x)$$

- **LIV & CPT-odd** spinor equation

$$\left(\gamma^{\mu} p_{\mu} - m - \gamma^{\mu} b_{\mu}\gamma^{5}\right)\widetilde{\psi}(p) \;=\; 0$$

◦ **Diagonalization** of the spinor operator

$$\widetilde{\psi}(p) \;\equiv\; \mathsf{A}\;\mathsf{B}\;\widetilde{\phi}(p)$$

$$\mathsf{A} \equiv p^2 + b^2 - m^2 + 2\,(b\cdot p + m b_\mu \gamma^\mu)\,\gamma^5$$

$$\mathsf{B} \equiv \gamma^\nu p_\nu + m + b_\nu \gamma^\nu \gamma^5$$

⋆ **quartic scalar free field equation** ⋆

$$\left[\left(p^2 + b^2 - m^2\right)^2 + 4b^2m^2 - 4(b\cdot p)^2\right]\widetilde{\phi}(p) = 0$$

⋆ **on shell condition for CPT odd spinors** ⋆

$$\left(p^2 + b^2 - m^2\right)^2 + 4b^2m^2 - 4(b\cdot p)^2 = 0$$

- **time-like** solution $\qquad b_\mu = (b, 0)$

$$(p^0_\pm)^2 = (p \pm b)^2 + m^2$$

- **light-like** solution $\qquad b_\mu = (b, \vec{b})$

$$p^0_\pm = \pm b + \sqrt{\left(\vec{p} \mp \vec{b}\right)^2 + m^2} \; > \; 0$$

$$\tilde{p}^0_\pm = \pm b - \sqrt{\left(\vec{p} \mp \vec{b}\right)^2 + m^2} \; < \; 0$$

Spinor free field quantization in the **space-like** case is **inconsistent** owing to the **lack of separation** between positive and negative frequency solutions

$(+)$ **Left** -handed particles

$$p^0_+ > 0 \qquad u_+(p) \qquad a^\dagger_+(\mathbf{p})|0\rangle$$

$(-)$ **Right** -handed particles

$$p^0_- > 0 \qquad u_-(p) \qquad a^\dagger_-(\mathbf{p})|0\rangle$$

$(+)$ **Right** -handed antiparticles

$$\tilde{p}^0_+ < 0 \qquad v_+(p) \qquad b^\dagger_+(\mathbf{p})|0\rangle$$

$(-)$ **Left** -handed antiparticles

$$\tilde{p}^0_- < 0 \qquad v_-(p) \qquad b^\dagger_-(\mathbf{p})|0\rangle$$

CPT-breaking removes energy-polarization degeneracy

$(+)$ 1P STATES

- **time-like** case $\qquad b_\mu = (b, 0)$

$$p_+^2 = b^2 + m^2 + 2b\,p \;>\; 0$$

- **light-like** case $\qquad b_\mu = (b, \vec{b}\,)$

$$p_+^2 = m^2 + 2b\,(p_+ - p\cos\varphi) \;>\; 0$$

$$\vec{b}\cdot\vec{p} = b\,p\,\cos\varphi$$

ALL $(+)$ 1P states are PHYSICAL

$$p_+^2 \;>\; 0 \qquad\qquad \forall\,\vec{p}\;\in\;\mathbf{R}^3$$

$(-)$ 1P STATES

- **time-like** case $\qquad b^{\mu} = (b, 0)$

$$p_{-}^{2} > 0 \quad \Leftrightarrow \quad 2bp < m^{2} + b^{2}$$

- **light-like** case $\qquad b^{\mu} = (b, \vec{b}\,)$

$$p_{-}^{2} > 0 \quad \Leftrightarrow \quad 2bp|\cos\varphi| \leq 2b^{2} + m^{2}$$

- **PHYSICAL UV cutoff** for fermions

$$p_{-}^{2} > 0 \quad \Leftrightarrow \quad p < \frac{m^{2}}{2b} = \Lambda_{f}$$

V.A. Kostelecký & R. Lehnert (2001)

A.A. Andrianov, P. Giacconi & R. S. (2002)

$\star$ GROUP VELOCITIES $\star$
OF POLARIZED WAVE PACKETS

$$\vec{v}_{+} = \nabla_{\vec{p}}\, \omega_{+}(\vec{p})$$

$$\vec{v}_{-} = \nabla_{\vec{p}}\, \omega_{-}(\vec{p})$$

$$\begin{cases} v_{+} \leq 1 \\ v_{-} \leq 1 \end{cases} \qquad \forall\ \vec{p}\ \in\ \mathbf{R}^3$$

CPT-odd free spinor quantization
CONSISTENT AND CAUSAL

$$\forall\ b^2\ \geq\ 0$$

$\star$ $(-)$ UNPHYSICAL STATES $\star$
very high momentum decay process

$$p_{-} \longrightarrow p_{-} + e^{-}_{+} + e^{+}_{-} \qquad |\vec{p}| \geq \Lambda_f$$

⋆ EXPERIMENTAL LIMITS ⋆

- **spatial components $\vec{b}$ constrained by**

◦ **POLARIZED ELECTRONS TESTS**

B.R. Heckel et al. (2006)

$$|\vec{b}| < 5.0 \times 10^{-21} \text{ eV} \ll m_e^2/M_{\text{Planck}}$$

- **temporal component b_0 constrained by**

◦ **CPT INVARIANCE TEST**

W.-M. Yao et al., J.Phys.G33 (2006)

$$(m_{e^+} - m_{e^-})/\bar{m}_e < 8 \times 10^{-9}$$

◦ **Absence of vacuum *Čerenkov* radiation**

$$\epsilon \simeq (\alpha\, b_0/m_e)^2 \;<\; 1.6 \times 10^{-23}$$

$b_0 < 10^{-2}$ eV (very soft bound!)

3. RADIATIVELY INDUCED CS ACTION

- **CPT-odd QED Lagrange density**

$$\mathcal{L} = \mathcal{L}_{\text{spinor}} + eq_f \bar{\psi}_f \not{A} \psi_f + \mathcal{L}_{\text{MCS}}$$

$$\mathcal{L}_{\text{spinor}} = \bar{\psi}_f \left(i\not{\partial} - m_f - \not{b}\gamma^5 \right) \psi_f$$

(f fermion species of charge eq_f)

$$\mathcal{L}_{\text{MCS}} = -\frac{1}{4} F^{\nu\lambda}(x) F_{\nu\lambda}(x) - B(x)\partial_\alpha A^\alpha(x) - \frac{1}{2}\eta^\alpha A^\beta(x)\widetilde{F}_{\alpha\beta}(x)$$

To be fully consistent with theoretical and experimental bounds and without loss of generality we can assume

$$\eta^2 \leq 0 \leq b^2$$

- **1Loop induced MCS polarization tensor**

$$\Pi^{\mu\nu}(k) = \frac{(-i)}{(2\pi)^4} \int \mathtt{tr}\left\{\gamma^\mu S(p)\gamma^\nu S(p-k)\right\} d^4p$$

- **Feynman's propagator of the spinor field**

$$S(p) = i \; \mathsf{A} \; \mathsf{B}\, \mathcal{D}^{-1}(\,p, b\,)$$

$$\begin{aligned} \mathcal{D}(\,p, b\,) \;\; &= \;\; \left(p^2 + b^2 - m^2 + i\varepsilon\right)^2 \\ &- \;\; 4\left[\,(b \cdot p)^2 - m^2 b^2\,\right] \end{aligned}$$

$$\mathsf{A} \equiv p^2 + b^2 - m^2 + 2\,(b \cdot p + m b_\mu \gamma^\mu)\,\gamma_5$$

$$\mathsf{B} \equiv \gamma^\nu p_\nu + m + b_\nu \gamma^\nu \gamma_5$$

- **UV divergencies $\longrightarrow$ REGULARIZATION**

$$\mathtt{reg}\,\Pi^{\mu\nu}(k) = \mathtt{reg}\,\Pi^{\mu\nu}_{\mathsf{even}}(k) + \mathtt{reg}\,\Pi^{\mu\nu}_{\mathsf{odd}}(k)$$

- **physical UV cutoff for fermions**

$$
\begin{aligned}
\mathtt{reg}\Pi^{\mu\nu}(k) \;&=\; \lim_{\Lambda_f\to\infty}\int\frac{d\vec{p}}{i(2\pi)^3}\,\vartheta\left(\vec{p}^{\,2}-\Lambda_f{}^2\right) \\
&\times\;\int_{-\infty}^{\infty}\frac{dp_0}{2\pi}\,\mathtt{tr}\left\{\gamma^\mu S(p)\gamma^\nu S(p-k)\right\}
\end{aligned}
$$

- **dimensional regularization**

$$
\mathtt{reg}\Pi^{\mu\nu}(k) \;=\; \lim_{\omega\uparrow 2}\,(-i)\,\mu^{\,4-2\omega}(2\pi)^{\,-2\omega}
$$

$$
\times\int\mathtt{tr}\left\{\gamma^\mu S(p)\gamma^\nu S(p-k)\right\}\,d^{2\omega}p
$$

BOTH regularizations guarantee the *MINIMAL* LORENTZ & CPT SYMMETRY BREAKING SOLELY DUE

to the presence of $b^2 \geq 0$

- **1Loop induced parity odd tensor**

$$\mathtt{reg}\,\Pi^{\mu\nu}_{\text{odd}}(k; b, m) \;=\; 2i\left(\frac{\alpha}{\pi}\right)\epsilon^{\mu\nu\rho\sigma}\, b_\rho\, k_\sigma \sum_f q_f^2$$

BOTH REGULARIZATIONS GIVE THE SAME RESULT **FOR THE** INDUCED PARITY-ODD EFFECTIVE ACTION

INDUCED Chern-Simons lagrangean

$$\Delta\mathcal{L}_{\text{odd}} \;=\; -\,\frac{1}{2}(\Delta\eta_\alpha)A^\mu\widetilde{F}_{\mu\nu}$$

$$\Delta\eta_\alpha = \frac{2\alpha}{\pi}\, b_\alpha \sum_f q_f^2 \equiv b_\alpha\,\zeta$$

$$\sum_f q_f^2 \equiv q^2 = 3\cdot 3\left(\frac{1}{9}+\frac{4}{9}\right)+3=8$$

- **1Loop induced parity even tensor**

$$
\begin{aligned}
\mathtt{reg}\,\Pi^{\nu\sigma}_{\text{even}} &= (k^2 g^{\nu\sigma} - k^\nu k^\sigma)\,\Pi_{\text{div}}(k,m) \\
&+ \frac{2\alpha}{3\pi}\sum_f q_f^2 \Big(b^2\, g^{\nu\sigma} - m_f^{-2}\, S_f^{\nu\sigma} \Big)
\end{aligned}
$$

$$
\begin{aligned}
S_f^{\nu\sigma} &\equiv g^{\nu\sigma}\Big[\,(b_f\cdot k)^2 - b_f^2\, k^2\,\Big] \\
&- (b_f\cdot k)\Big(b_f^\nu\, k^\sigma + b_f^\sigma\, k^\nu \Big) \\
&+ k^2\, b_f^\nu\, b_f^\sigma + b_f^2\, k^\nu k^\sigma \\
&+ \text{higher orders in } b^\mu \text{ and } k^\mu
\end{aligned}
$$

$$
\begin{aligned}
\Pi_{\text{div}} &= -\,8e^2\,(4\pi)^{-\,\omega}\ \mu^{4-2\omega}\,\Gamma(2-\omega) \\
&\times \int_0^1 dx\, x(1-x)[\,m^2 - x(1-x)k^2\,]^{\omega-2}
\end{aligned}
$$

- **Induced photon effective lagrangean**

$$\mathcal{L}_{\gamma} = \mathcal{L}_{\mathrm{MCS}} + \triangle\mathcal{L}_{\mathrm{even}} + \triangle\mathcal{L}_{\mathrm{odd}}$$

$$\begin{aligned}\mathcal{L}_{\gamma} &= -\frac{1}{4}(1+\varepsilon)F^{\nu\lambda}F_{\nu\lambda} + B(x)\,\partial_{\nu}A^{\nu} \\ &+ g_{\mu\nu}\,\varepsilon^{\mu\rho}\,F^{\nu\lambda}F_{\rho\lambda} - \frac{1}{2}m_{\gamma}^{2}\,A_{\nu}A^{\nu} \\ &- \frac{1}{2}(\eta_{\nu} + \zeta\,b_{\nu})A_{\lambda}\widetilde{F}^{\nu\lambda}\end{aligned}$$

$$\varepsilon = \frac{2\alpha\,b^{2}}{3\pi m_e^2} \qquad \varepsilon^{\mu\nu} = \frac{\alpha\,b^{\mu}b^{\nu}}{3\,m_e^2}$$

$$m_{\gamma}^{2} = \frac{16\alpha}{3\pi}\;b^{2} \qquad \zeta = 16\,\frac{\alpha}{\pi}$$

anomalous photon mass in spite of dimensional regularization

- **time-like** case $\qquad b^{\mu} = (b, 0)$

$$m_{\gamma} < 6 \times 10^{-17} \text{ eV} = \frac{3\, m_e^2}{M_{\text{Planck}}}$$

$$b < 3 \times 10^{-15} \text{ eV}$$

∘ *longitudinal polarization dispersion law*

$$k_0^2 = k^2 + m_{\gamma}^2 \qquad \text{(covariant)}$$

∘ *transverse polarizations dispersion law*

$$k_0^2 = (1 + \varepsilon)\, k^2 + m_{\gamma}^2 \pm \zeta\, b\, k$$

∘ *group velocities* $\qquad (r = \pm 1)$

$$(d\omega_r / dk) \; < \; 1 \qquad \Leftrightarrow \qquad b\, k \leq \; m_e\, m_{\gamma}$$

absence of time dispersion in TeV γ ray burst of the active galactic nucleus Markarian 421

$$b < 10^{-24} \text{ eV}$$

- **light-like** case $\qquad b^{\mu} = (b, \vec{b})$

photons keep **massless with two transverse polarizations**

- *transverse polarizations dispersion law*

$$k_0^2 - k^2 = \varepsilon^{\mu\nu} k_\mu k_\nu - r\,\zeta\, b^{\mu} k_\mu$$

$$r = \begin{cases} +1 & \text{Left polarization} \\ -1 & \text{Right polarization} \end{cases}$$

- *group velocities* $\qquad (r = \pm 1)$

$$(d\omega_r/dk) \leq 1 \qquad \forall\ k \geq 0$$

- *memento : polarized electron tests*

$$|\vec{b}_e| < 5.0 \times 10^{-21}\ \text{eV} \ll m_e^2/M_{\text{Planck}}$$

for any other flavour f there is not yet such a stringent test so that $|\vec{b}_f|$ could be much bigger for $f \neq e$

- **induced Chern-Simons vector**

$$\eta^{\mu} + \zeta\, b^{\mu} = \eta^{\mu} + 2\left(\frac{\alpha}{\pi}\right)\sum_f q_f^2\, b_f^{\mu}$$

- **theoretical BOUNDS**

$$\eta^{0} + \zeta\, b^{0} \leq |\vec{\eta} + \zeta\,\vec{b}\,|$$

consistent quantization requires a space-like or light-like induced Chern-Simons vector

- **experimental BOUNDS**

$$|\vec{\eta} + \zeta\,\vec{b}\,| < 10^{-33}\ \text{eV} \ll m_e^2\, M_{\text{Planck}}^{-1}$$

absence of the vacuum birefringence requires fine tuned cancellation

4. CONCLUSIONS

- **free field consistent quantization**

$$\eta^{\mu}\eta_{\mu} \leq 0 \leq b^{\mu}b_{\mu}$$

experimental bounds from cosmological universality of constant vectors

$$|\vec{\eta}| < 10^{-33} \text{ eV} \qquad |\vec{b}| < 10^{-22} \text{ eV}$$

- **radiatively induced effects**

$$\eta^{\mu}\eta_{\mu} = 0 = b^{\mu}b_{\mu}$$

to keep the photons transverse and massless

- **cosmological diversity**

$$|\vec{b}_f| > m_e^2 M_{\text{Planck}}^{-1} \gg |\vec{b}_e|$$

not yet excluded empirically for massive flavours other than electrons