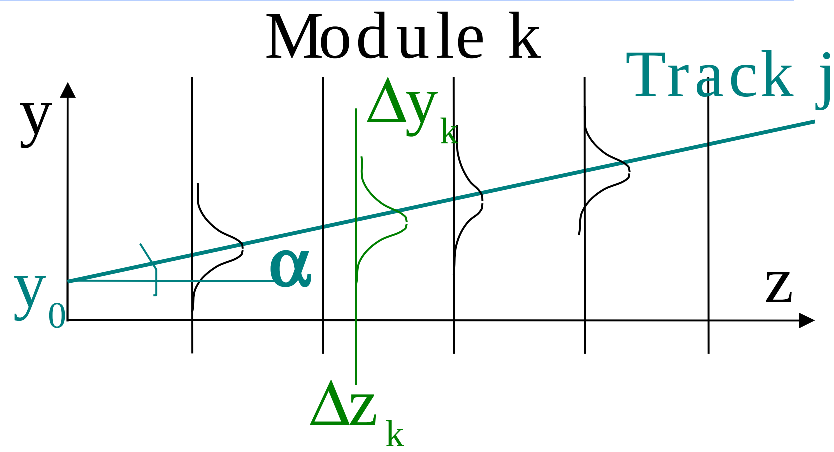
Alignment

The alignment problem

- Estimate 3 shifts + 3 rotation angles per detector module
  - Vector $\mathbf{a}_k$ of alignment parameters for N modules, $N \sim 10^4$
- Precision to acheive:
  - << detector resolution
  - <<< assembly precision

    → **Track- based alignment**
- Principle of track- based alignment
  - Fit simultaneously parameters of M tracks, and 6xN alignment parameters (identical for all tracks)



- M=100,000; 10 points per track = 10^6 measurements
- $5M+6N \sim 56.10^4$ unknowns
- Measurement equation for track j in module k:

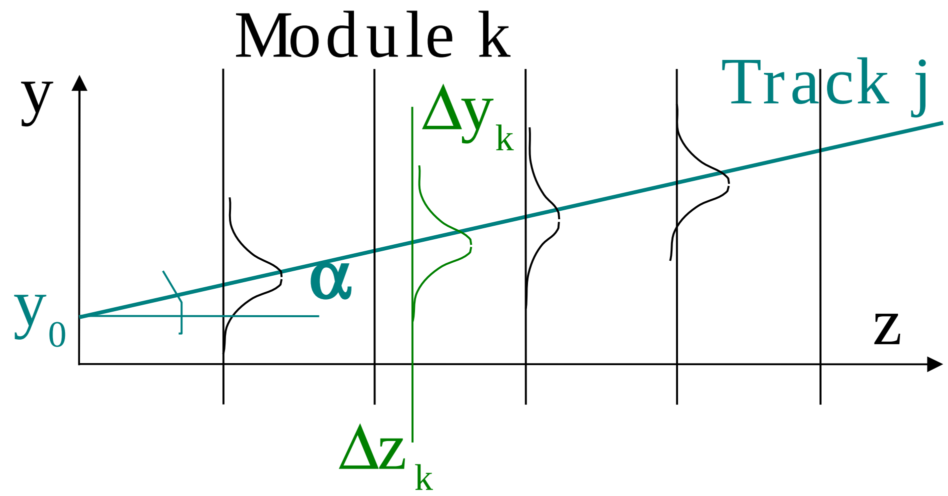
$$\boldsymbol{m}_{j,k} = h_k(\boldsymbol{p}_j, \boldsymbol{a}_k)$$

$$\boldsymbol{m}_{j,k} = p_j^1 + p_j^2 (z_k + \Delta z_k) + \Delta y_k$$

- Linearize, minimize $\sum_{j,k} \boldsymbol{r}^T \boldsymbol{R}^{-1} \boldsymbol{r}$

  → **Linear system of 56.10^4 eq.**

Systematic effects



- Unconstrained parameters
  - Global scale factors, rotations, shearings
    - In-situ survey of a few reference modules (Laser alignment systems etc.)
- Correlated displacements of modules due to mechanical structure
  - Added to sum of squared residuals with Lagrange multipliers
- Correlations bw. Alignment parameters of different modules, mediated by the tracks that cross both
  - Essentially 2 recent developments
    - Millepede algorithm [1,2]
    - KF-based method [3]

- Millepede algorithm [2]
  - Takes advantage of special structure of system of eq.
  - First fit each track separately
  - Then modify system of 6N equations accounting for track fit results (Schur complement matrix of $\sum_k C_k^{global}$)
    - NB: here k is track index
  - A system of 6N equations remains to be solved
    - Advanced iterative methods needed

6N | M blocks of 5x6N

6N

$$\left(\begin{array}{c||c|c|c} \sum_k C_k^{\text{global}} & \cdots & H_k^{\text{global-local}} & \cdots \\ \hline\hline \vdots & \ddots & 0 & 0 \\ \hline \left(H_k^{\text{global-local}}\right)^{T} & 0 & C_k^{\text{local}} & 0 \\ \hline \vdots & 0 & 0 & \ddots \end{array}\right) \times \left(\begin{array}{c} \Delta p^{\text{global}} \\ \hline\hline \vdots \\ \hline \Delta q_k^{\text{local}} \\ \hline \vdots \end{array}\right) = \left(\begin{array}{c} \sum_k b_k^{\text{global}} \\ \hline\hline \vdots \\ \hline b_k^{\text{local}} \\ \hline \vdots \end{array}\right).$$

References

- [1] V.Blobel, “Software alignment for tracking detectors”, Nucl. Instr. and Meth. A566 (2006) 5
- [2] V.Blobel, “Alignment algorithms”, 1st LHC Detector Alignment Workshop, CERN, Geneva, Switzerland, 4 - 6 Sep 2006, pp.5-12 (CERN Yellow Report)
- [3] R.Fruhwirth, E.Widl, “Track-based alignment using a Kalman filter technique”, 1st LHC Detector Alignment Workshop, CERN, Geneva, Switzerland, 4 - 6 Sep 2006, pp.13-19 (CERN Yellow Report)
- [4] P.Avery, lectures on tracking (on line)

Vertex fitting

Vertex fitting

Set of N tracks (+ beam constraint) → *Vertex fitting* → **3D point and N momentum vectors, most compatible with input tracks (and beam)**

Minimization problem:

- **Compatibility defined by a distance F between the vertex parameters x and the vertex "measurements", i.e. the track parameters p_i (more precisely, a function f of them).**
- **"Most compatible" implies minimization of this distance.**

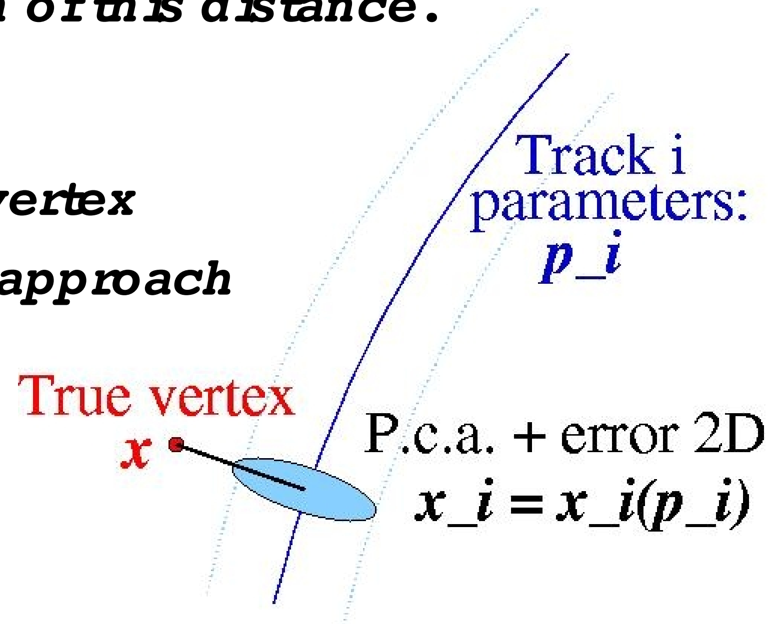
Example:

- $f(p_i) = x_i$ **point of closest approach to vertex**

  C_i **covariance matrix of point of closest approach**

- $F = \Sigma_{i=1}^{N} (x - x_i)^T C_i^{-1} (x - x_i)$

sum of squared reduced residuals



Least- Squares vertex fitting

F = sum of squared reduced residuals
Algorithm chosen: Kalman vertex fitting

- vertex parameters $x \equiv$ state vector, in Kalman parlance
- $f \equiv$ reciprocal of measurement function h, non-linear in presence of B-field

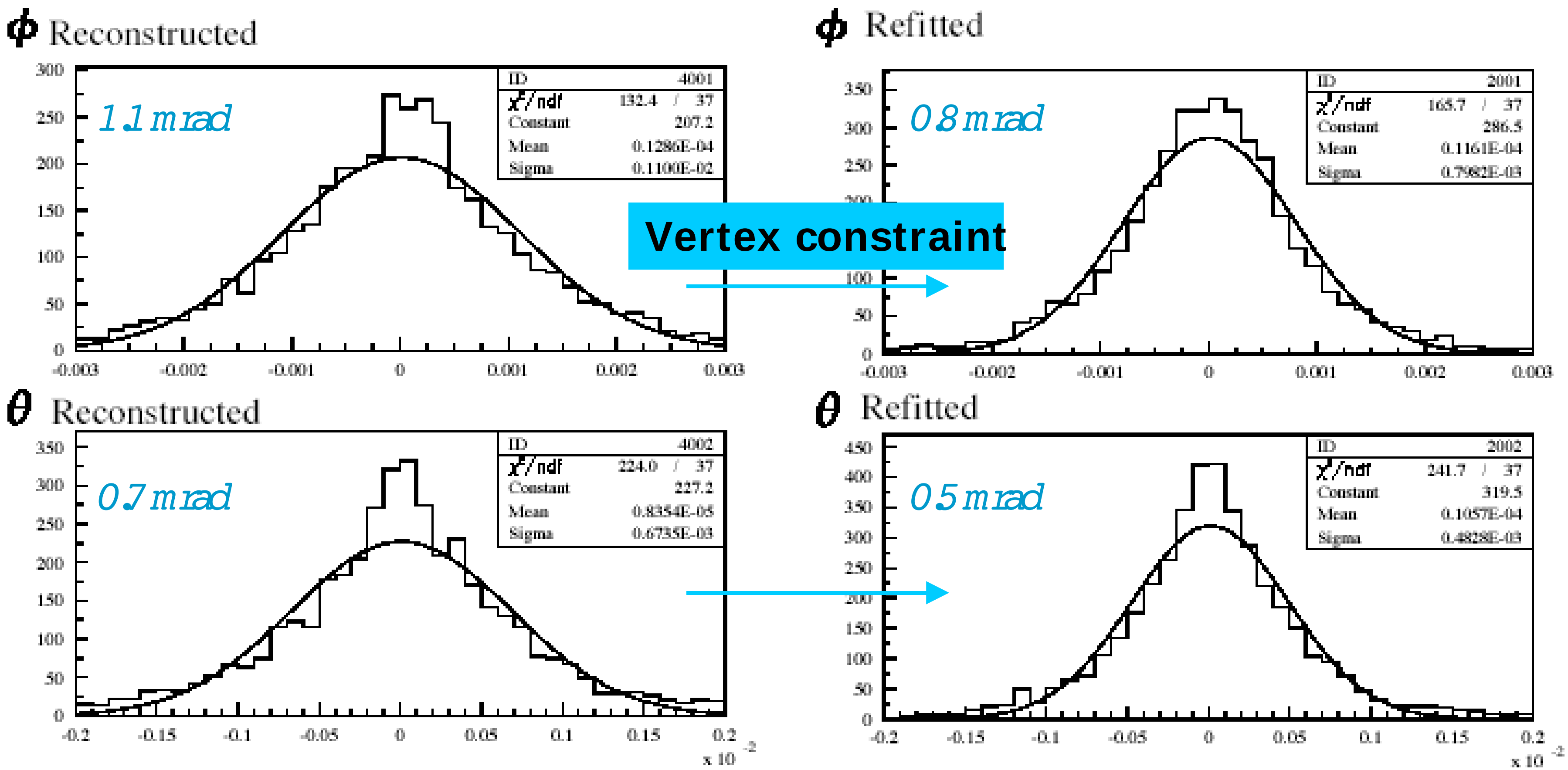
- problem is linearized:
  - h replaced by 1st order Taylor expansion
  - minimum of F is solution of a linear system of equations
    - explicit expression, no need for numerical minimization

- tracks added sequentially to the vertex (state vector update)

- problem is factorizable:
  - first fit vertex position, then constrain track momenta at vertex

Least- Squares vertex fitting results

Residuals of track parameters at vertex
$B_s \rightarrow J/\psi\ \varphi \rightarrow \mu^+\mu^-\ K^+K^-$
Full CMS tracker simulation and reconstruction



Linearization

Expansion of h only valid in vicinity of a given point:

- needs initial guess of vertex position
- iterations if fitted vertex too far from initial guess
- ...but convergence is quick (1 or 2 iterations for least-squares vertex fits)
  - track = helical constraint

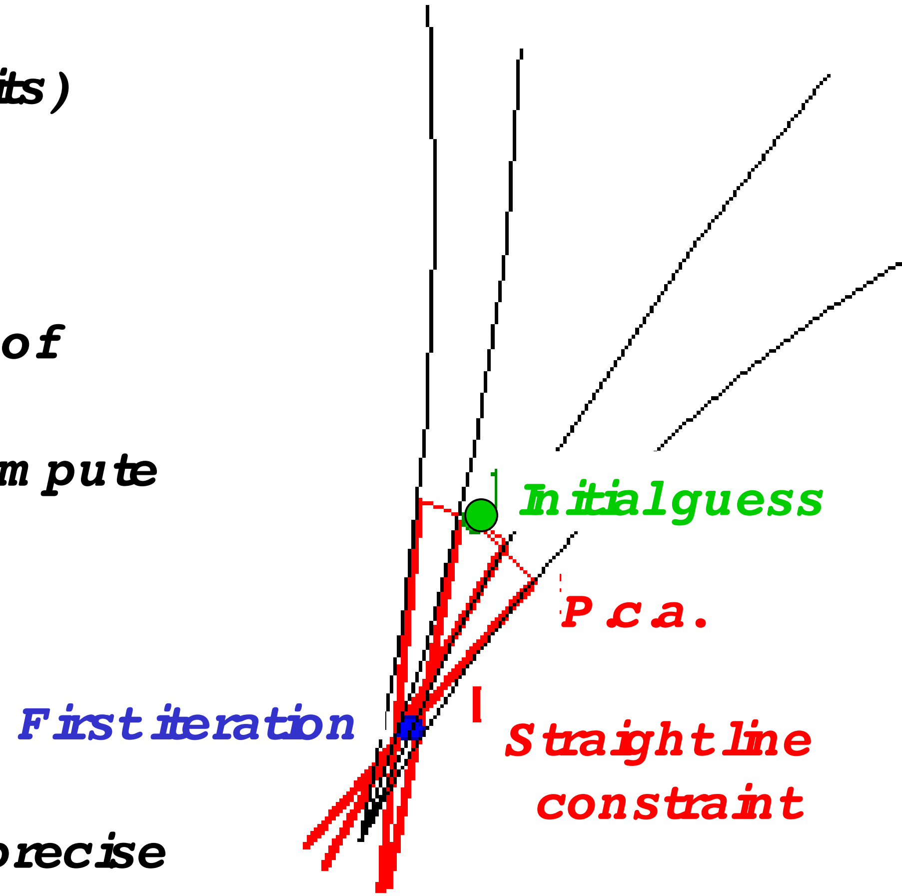
Initial guess:

- computed from average of points of closest approach of track pairs
  - fast numerical algorithm to compute p.c.a.'s of two helices
- made robust by a Half-Sample Mode finder

Linearization:

- Perigee parametrization fast and precise

Illustration of vertex fit convergence when tracks are approximated by straight lines



Robustification

Least squares estimators are efficient and unbiased if:

- *track parameter reduced residuals are N (0,1)*
  - *Gaussian, unbiased, covariance matrix perfectly known*
- *all tracks originate from fitted vertex*

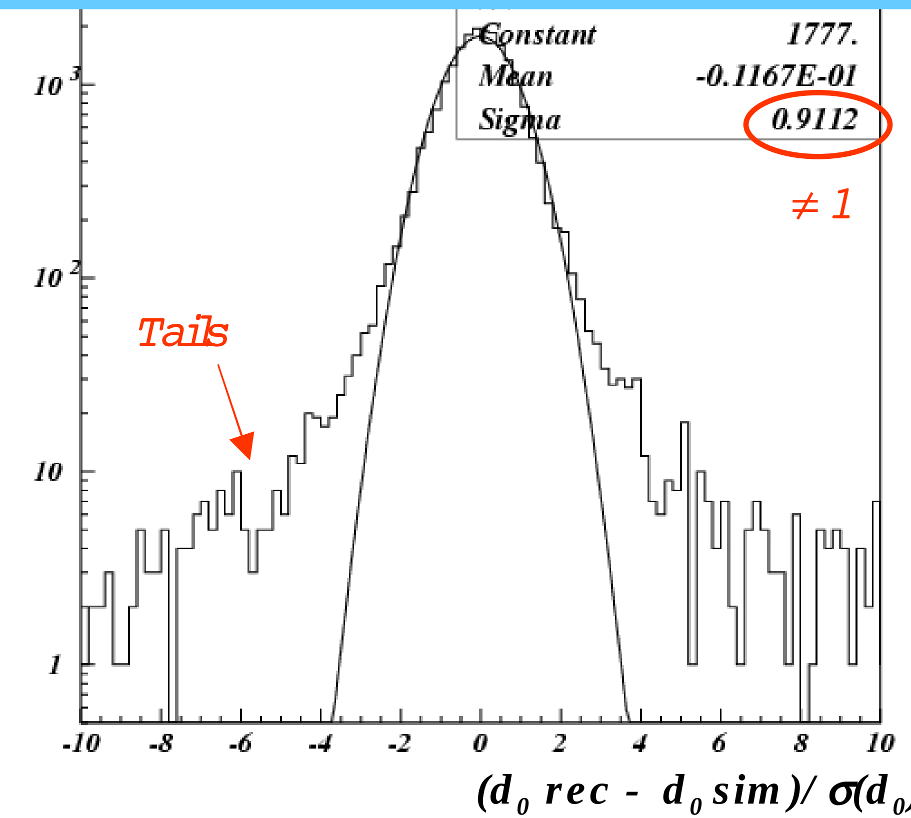
In real experiments and in detailed simulations:

- *detector resolutions not Gaussian*
- *distribution of dead material imperfectly known*
- *non-Gaussian multiple scattering*
- *track finding problems*
- *vertices not well separated ...*

...difficult to get it all right!

→ **Robust estimators:** *insensitive to mis-measured or mis-associated tracks*

d_0 pull distribution
H (m_H = 300 GeV/c^2) → ZZ → $e^+e^-\mu^+\mu^-$
CMS full simulation and reconstruction - ORCA_7_6_0



Robust estimators

Discard (trimming) or down-weight (adaptive) distant tracks

Implemented as re-weighted least-squares fits:

$$F = \Sigma_{i=1}^{N} w_i \, r_i \qquad\qquad r_i = (x - x_i)^T C_i^{-1} (x - x_i)$$

Weight w_i ≡ probability of association of track i to vertex

Usually, time-consuming but fast approached algorithms exist (tried in CMS, ATLAS, H1, D0)

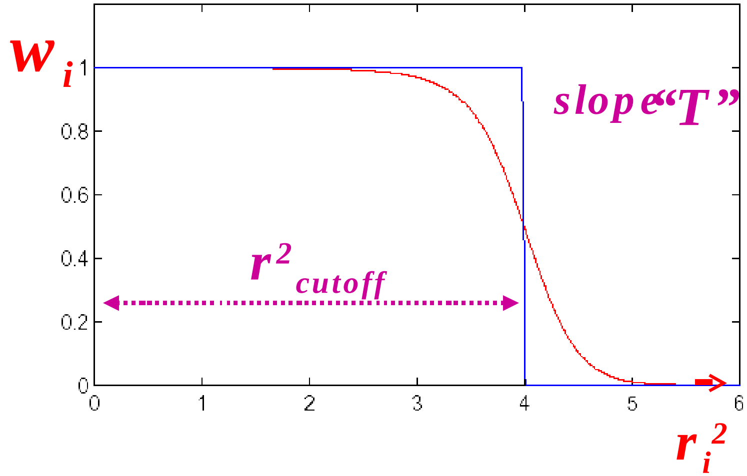
Trimming: $w_i = 0$ or 1

- Hard assignment
- Fixed trimming fraction: k out of N tracks are discarded
  - Fast Least-Trimmed-Squares
    - P.Rousseeuw et al., "Computing LTS regression for large data sets", Technical report, University of Antwerp, 1999.
- Fixed compatibility: tracks with probability of compatibility to the vertex below some cut discarded

Robust estimators (2)

Adaptive: $w_i = \{ 1 + \exp[(r_i^2 - r^2_{cutoff}) / 2T] \}^{-1}$

M.Ohlsson et al., Comput. Phys. Commun. 71 (1992) 77
R.Frühwirth and A.Strandlie, Comput. Phys. Commun. 120 (1999) 197



- **Soft assignment: association probabilities are fractional**
  - algorithm converges to optimal weight for each track

- **Annealing:**
  - avoids local minima
  - T starts high
  - decreases at each iteration
    - according to well-chosen schedule

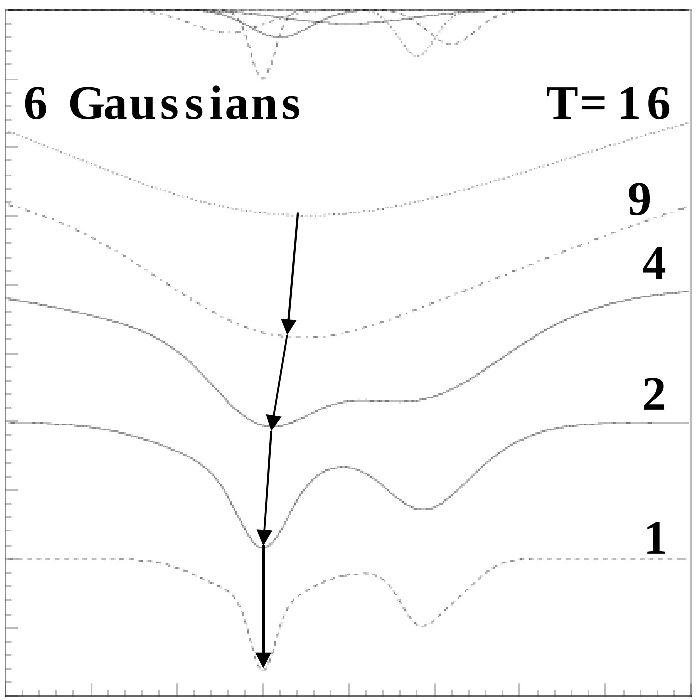
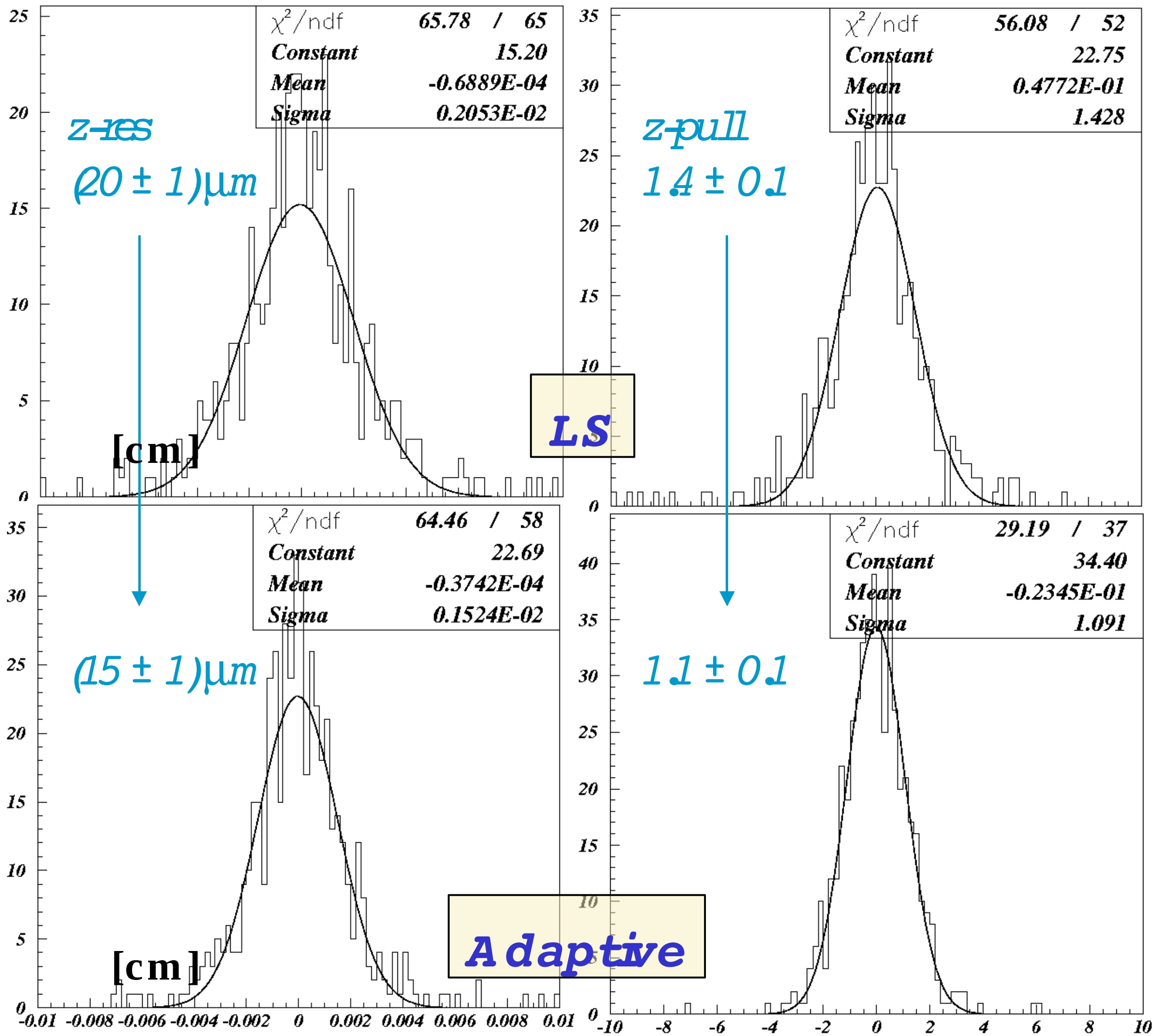


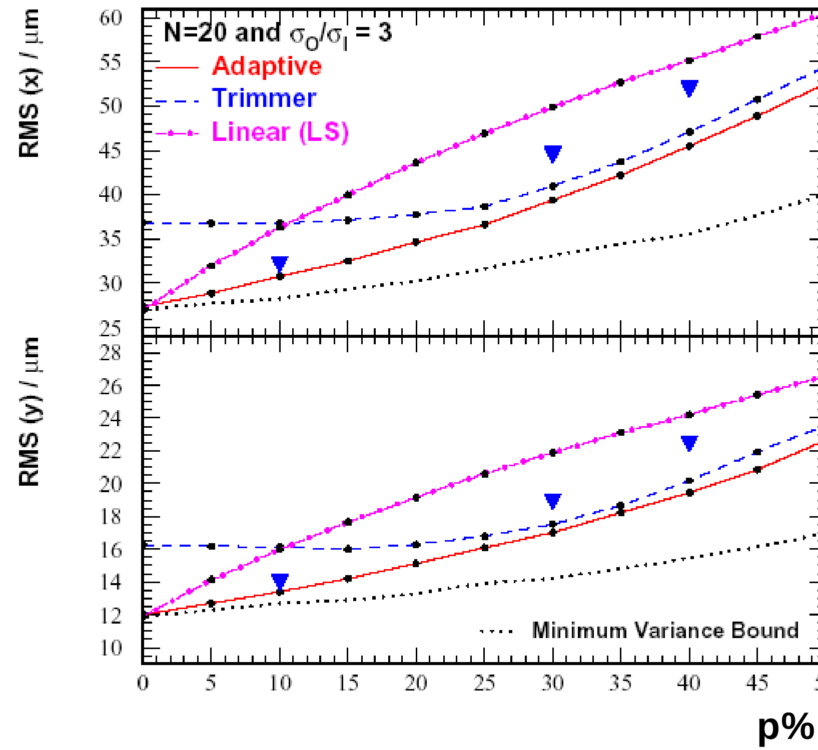
Illustration: minimization of sum of 6 Gaussians with annealing

Robustness against mis- measured tracks

Primary vertex z- residuals and pulls
H (m_H = 300 GeV/ c²) → ZZ → $e^+e^-\mu^+\mu^-$
Only primary tracks used - ORCA_7_6_0



High multiplicity vertex (20 tracks, p% with covariance underestimated by factor 3)
x- and y- residuals - VertexGun



References

- [1] R.Fruhwirth, Nucl. Instr. and Meth. A262 (1987) 444
- [2] P.Billoir, S.Qian, Nucl. Instr. and Meth. A311 (1992) 139
- [3] P.Avery, lectures on tracking (on line)
- [4] J. D'Hondt, R. Fruhwirth, P. Vanlaer and W. Waltenberger, “Sensitivity of Robust Vertex Fitting Algorithms”, IEEE Transactions on Nuclear Science Vol.51 (2004), 2037
- [5] R. Fruhwirth, W. Waltenberger and P. Vanlaer, “Adaptive Vertex Fitting”, CMS Note 2007/008

Conclusions

Conclusions

- Detector design studies needed in order to meet physics requirements
  - Cost-driven optimization, compromises...
  - Detailed detector simulations needed
- Track fitting; vertex fitting; alignment
  - Well-defined statistical problems
- Lot of recent developments and creativity in:
  - Handling of combinatorics
  - Handling of ambiguities, robustness
  - Application of known mathematical algorithms to high-energy physics
    - Sometimes in extreme cases like alignment of 10,000 modules