The Matrix Element Method at NLO

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1st MEM workshop

Motivation for MEM@NLO

- Certain observables change definition at NLO.
- Greater theoretical confidence in result : Win-win since either
- 1) Large differences when using NLO => You need to use NLO
- 2) Small differences at NLO => Perturbative stability, its a good method (but you still need NLO to check)
- As a phenomenologist the phrase “only available at LO” is unacceptable.

Overview

- In my view there are two halves to the MEM@NLO
- 1) Defining the MEM : i.e. providing an algorithm to associate experimental events / MC input with LO matrix elements.
- 2) Extending any given 1) to be higher order in perturbation theory : i.e. providing NLO weights for LO phase space points.
- Of these halves 2) is much more rigorously defined and will be the focus of my talk. One can then apply NLO corrections to any algorithm of the form 1).

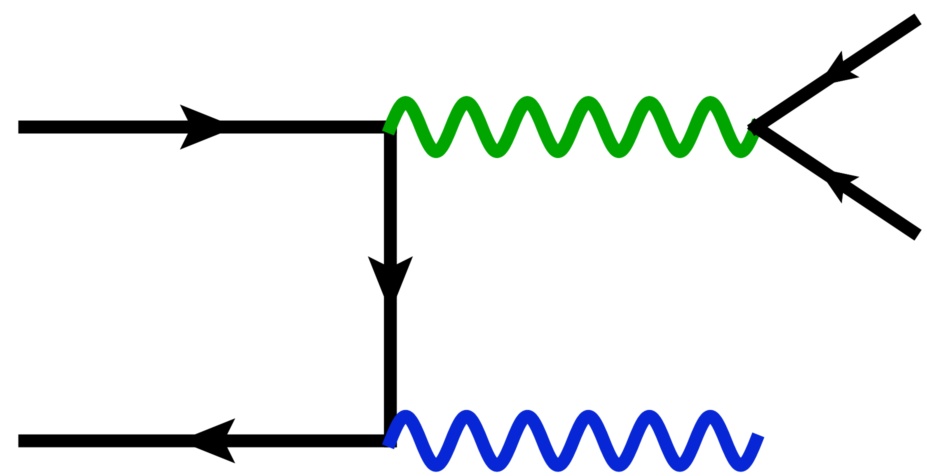
In the beginning

- Imagine a universe in which every event recorded at colliders is an exact Born phase space point.

- In this universe it would be pretty straightforward to provide event by events weights for searches and measurements.

- A final state phase space point (with no jets) is defined by the following quantities,

$$\Phi_B = (x_1, x_2, \{Q_n\}).$$



- Given this phase space point one can define a weight in a straightforward fashion,

$$\mathcal{P}_{LO}(\Phi_B) = \frac{f(x_1)f(x_2)}{2x_1x_2s}|\mathcal{M}^{(0)}(\Phi_B)|^2$$

- The total cross section is then obtained by integrating over all possible weights, (i.e. over all Born phase space points)

$$\sigma_{LO} = \int dx_1\, dx_2 \prod_{i=1}^{n} d^4p_i\; \delta^{(+)}(p_i^2 - m_i^2)\; \delta^{(4)}(\sum_i p_i - p_1 - p_2)\; \mathcal{P}_{LO}(\Phi_B)$$

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Helping Max out, by including NLO.

- Can split higher order corrections into two halves, defined by whether or not observed parton is above our jet definition or not.

- Call two regions resolved and un-resolved

- No problem for the MEM if we are in the resolved region, since its nothing other than a LO MEM with an additional jet. i.e. we can re-calculate the weights using a new phase space point

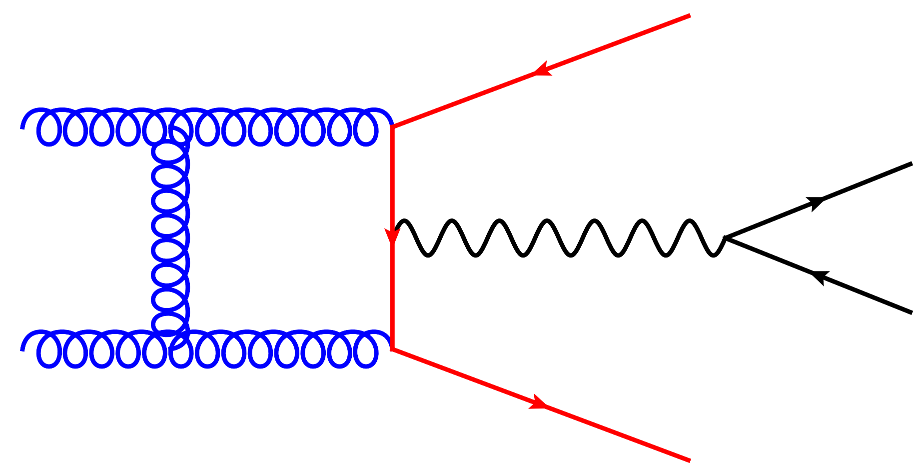
$$\Phi_B = (x_1, x_2, \{Q_i\}, \{J_j\})$$

- What about the un-resolved region? Much more tricky.... Our aim is to set the calculation up in the following way,

$$\mathcal{P}_{NLO}(\Phi_B) = K(\Phi_B)\mathcal{P}_{LO}(\Phi_B)$$

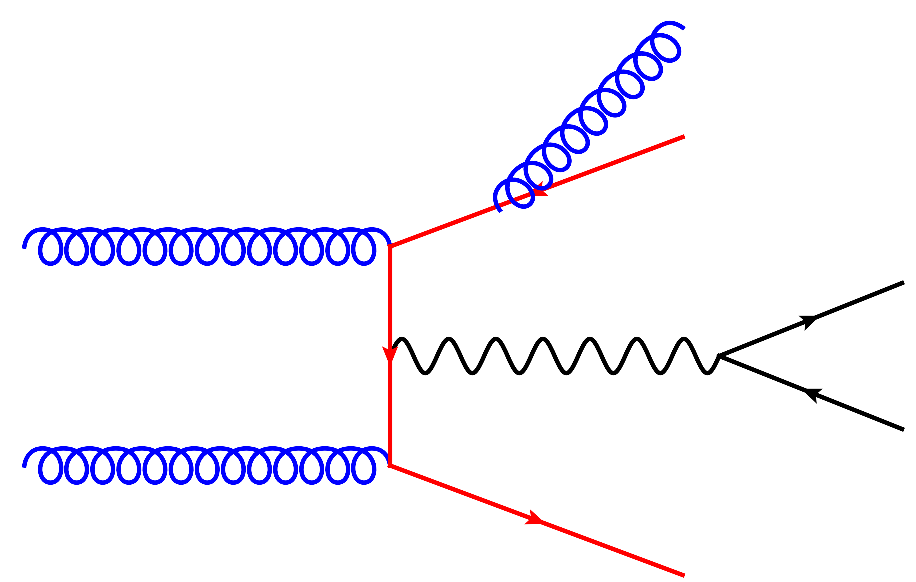
Virtual corrections to phase space points

The unresolved NLO calculation naturally contains two types of contributions. Virtual (loop) diagrams and real radiation.



$$\tilde{\mathcal{P}}_V(\Phi_B) = \frac{f(x_1)f(x_2)}{2x_1x_2s}\left(|\mathcal{M}^{(0)}(\Phi_B)|^2 + 2\mathrm{Re}\left\{\mathcal{M}^{(0)}\mathcal{M}^{(1)^\dagger}(\Phi_\mathrm{B})\right\}\right)$$

The virtual corrections can be readily incorporated into our weight since they share the same phase space as the Born (note the above formula is currently divergent).



Dealing with the real corrections: Phase space

- A generic real phase space point has the following parameterization

$$\hat{\Phi}_R = (\hat{x}_1, \hat{x}_2, \{\hat{Q}_n\}, \hat{p}_r)$$

- We want to group all of the real phase space points which contain our born event together (neglecting those which dont contain our Born event) i.e.

$$\Phi_R(\Phi_B) = (x_a, x_b, \{Q_n\}, p_r).$$

- Note that the x's have changed, this can't be avoided! But at least the final state EW particles (and hence all Lorentz invariant quantities associated with them) are kept invariant.

$$\text{cf} \quad \Phi_B = (x_1, x_2, \{Q_n\}).$$

The Forward Branching phase space (Giele, Glover; Giele, Stavenga, Winter).

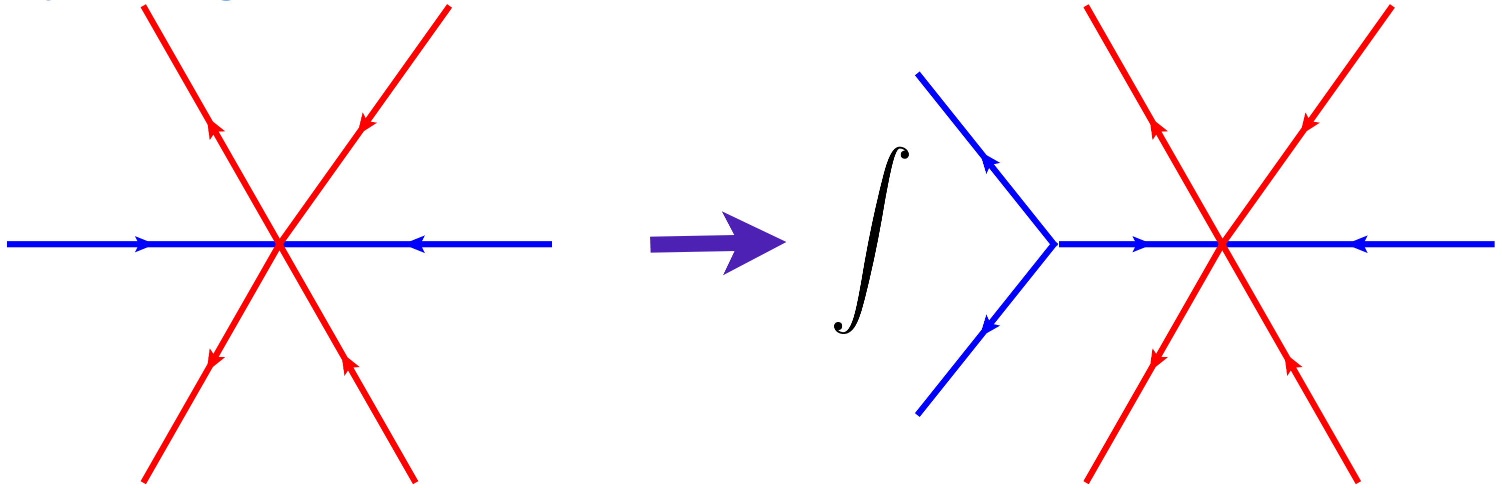
- Mathematically we need to factorize the real phase space into the following,

$$d\Phi(p_a + p_b \to Q + p_r) = d\,\Phi(\widehat{p}_a + \widehat{p}_b \to Q) \times d\,\Phi_{\text{FBPS}}(p_a, p_b, p_r) \times \theta_{\text{veto}}$$

- Then Q is identified with the observed final state, from this we derive the form of the FBPS integration

$$d\,\Phi_{\text{FBPS}}(p_a, p_b, p_r) = \frac{1}{(2\pi)^3}\left(\frac{\widehat{s}_{ab}}{s_{ab}}\right) d\,t_{ar} d\,t_{rb} d\,\phi\ ,$$

- We then explicitly integrate out these quantities for each event.



Putting it all together.

We can now write down our real weight, defined for an input Born phase space point.

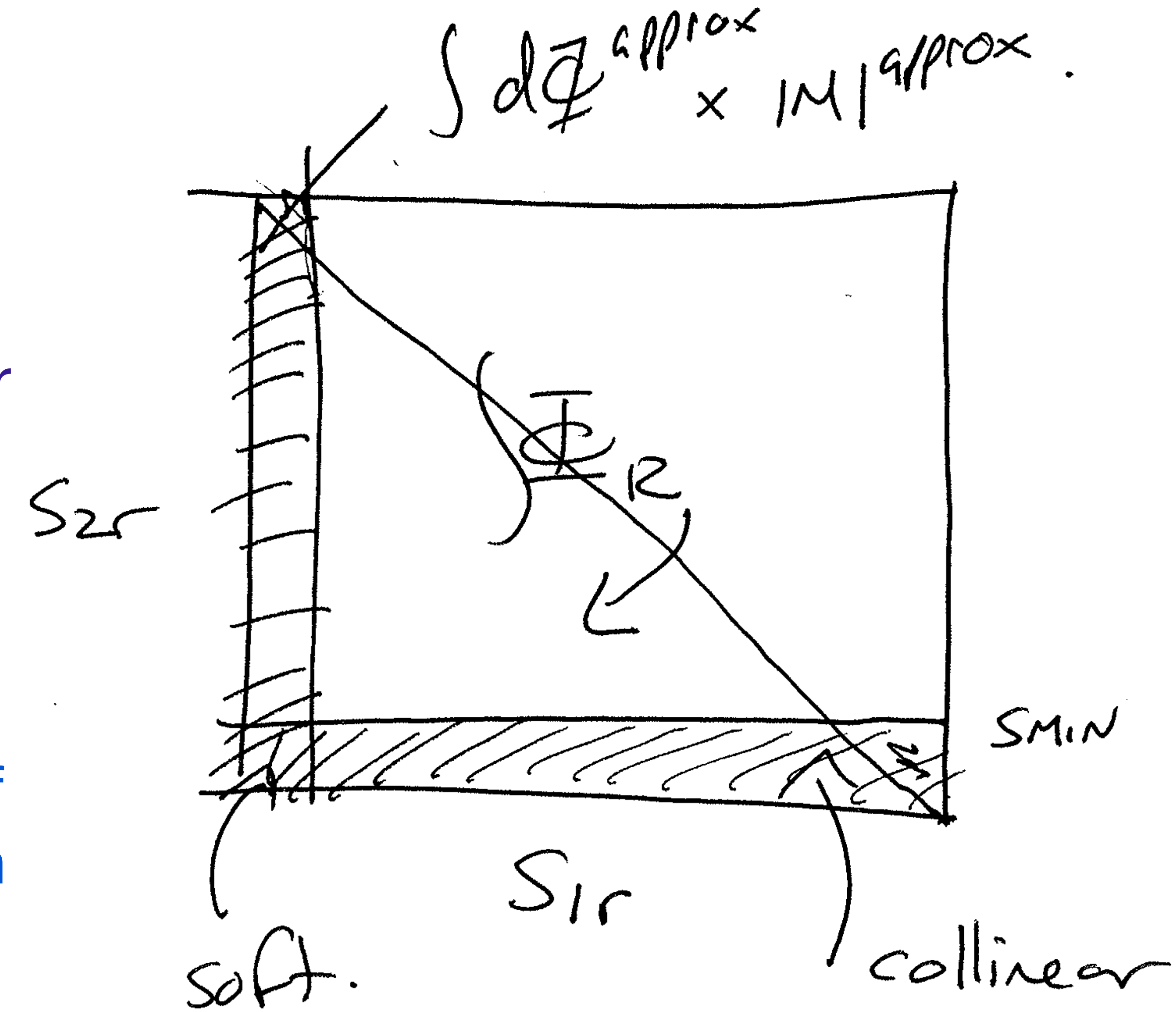
$$\tilde{\mathcal{P}}_R(\Phi_B) = \int d\,\Phi^{IS}_{\mathrm{FBPS}}(\Phi_B) J_x \frac{f(x_a)f(x_b)}{2x_a x_b s} |M_R^{(0)}(\Phi_R(\Phi_B))|^2$$

Jx represents the Jacobian factor which will take the integration in the Born x to the real x.

Note that we are still currently divergent! Not currently that useful for phenomenology! So we'd better regularize the weights!

Phase space slicing

- The issue of regularization is a thorny one.
- Need our regularzing functions to be exact functions of the Born phase space point (Not like Catani-Seymour dipoles)
- Simplest possible scheme is to use phase space slicing (Giele, Glover, Kosower), which naturally maps all of the singularities to the identified Born phase space point.



The full (finite) NLO weight

$$\mathcal{P}_{NLO} = \frac{f(x_1)f(x_2)}{2x_1x_2s}\Bigg((1+\mathcal{R}_v(s_{min}))|\mathcal{M}^{(0)}(\Phi_B)|^2 + 2\mathrm{Re}\Bigg\{\mathcal{M}^{(0)}\mathcal{M}^{(1)^\dagger}(\Phi_\mathrm{B})\Bigg\}\Bigg)$$
$$+\int_{s_{min}} d\,\Phi^{IS}_{\mathrm{FBPS}}(\Phi_B)J_x\frac{f(x_a)f(x_b)}{2x_ax_bs}|M_R^{(0)}(\Phi_R(\Phi_B))|^2 + \mathcal{O}(s_{min})$$

New addition here is the integrated approximate ME, Rv which renders the virtual pieces finite.

Can define this to be either Exclusive (integrate FBPS upto a veto scale). Or inclusive, in which we integrate overall of phase space.

What about jets?......

Dealing with Jets (minus transfer functions)

- We define our Born phase space in terms of jets (not partons)

$$\Phi_B = (x_1, x_2, \{Q_i\}, \{J_j\})$$

- Each jet is defined using the following variables

$$J_i = (p_{T,i}, \eta_i, \phi_i, m_i)$$

- A map to a fixed order result (in terms of partons) is thus defined by a jet-function.

- The transfer functions describe the generation of the jet parameters (to be used in the Born phase space point) and model the shower and the detector response. These will change at NLO (subtly) and this is beyond the scope of this talk. (Probably would proceed best through direct experimentalist/theorist collaboration)

The Jet function

The LO jet function is simple to define, since "every parton becomes a jet".

$$C^{LO}(\{p_m\}|\{J_m\}) = \sum_{i=1}^{m} \delta(p_{T,i} - J_{T,i})\delta(\phi_i - \phi_i^J)\delta(\eta_i - \eta_i^J)$$

At NLO things become a tad more complicated, since I have more ways of making the jet.

$$\begin{aligned} C(\{p_{m+1}\}|\{J_m\}) &= \sum_{i=1}^{m+1} \delta(p_{T,i} - J_{T,i})\delta(\phi_i - \phi_i^J)\delta(\eta_i - \eta_i^J) \\ &+ \sum_{i \neq j, i>j} \delta(p_{T,i+j} - J_{T,i})\delta(\phi_{i+j} - \phi_i^J)\delta(\eta_{i+j} - \eta_i^J) \\ &= \sum_{i=1}^{m+1} C_{IS}(i) + \sum_{i \neq j, i>j} C_{FS}(i,j) \end{aligned}$$

NLO event by event weights with jets.

- The first type of configuration is exactly the same as a EW final state, all but one final state partons are identified with final state jets and the free one is integrated over.
- The second configuration is new, and requires the integration over partons which cluster to form the observed jet.
- This second region is called the Final State Forward Brancher (FSFBPS).
- Note that in this region the jets acquire a mass (and this is integrated over).

Event by event NLO

We are now in a position to define our NLO event by event weight.

$$\mathcal{P}_{NLO} = \frac{f(x_1)f(x_2)}{2x_1x_2s}\left((1+\mathcal{R}_v(s_{min}))|\mathcal{M}^{(0)}(\Phi_B)|^2 + 2\mathrm{Re}\left\{\mathcal{M}^{(0)}\mathcal{M}^{(1)^\dagger}(\Phi_\mathrm{B})\right\}\right)$$

$$+\sum_{i=1}^{n_{jets+1}}\int_{s_{min}} d\,\Phi^{IS}_{\mathrm{FBPS}}(\Phi_B)J_x\frac{f(x_a)f(x_b)}{2x_ax_bs}|M_R^{(0)}(\Phi_R(\Phi_B))|^2C_{IS}(i)$$

$$+\sum_{i\neq j, i>j}\int_{s_{min}} d\,\Phi^{FS}_{\mathrm{FBPS}}(\Phi_B)J_x\frac{f(x_a)f(x_b)}{2x_ax_bs}|M_R^{(0)}(\Phi_R(\Phi_B))|^2C_{FS}(i,j) + \mathcal{O}(s_{min})$$

Which can be used to define an event by event K-factor.

$$\mathcal{P}_{NLO}(\Phi_B) = K(\Phi_B)\mathcal{P}_{LO}(\Phi_B)$$

Event by event NLO

We are now in a position to define our NLO event by event weight.

$\mathcal{P}_{NLO} = \frac{f(x}{}$ … $\Big\{\mathcal{M}^{(0)}\mathcal{M}^{(1)^\dagger}(\Phi_{\mathrm{B}})\Big\}\Big)$

… $(\Phi_B))|^2 C_{IS}(i)$

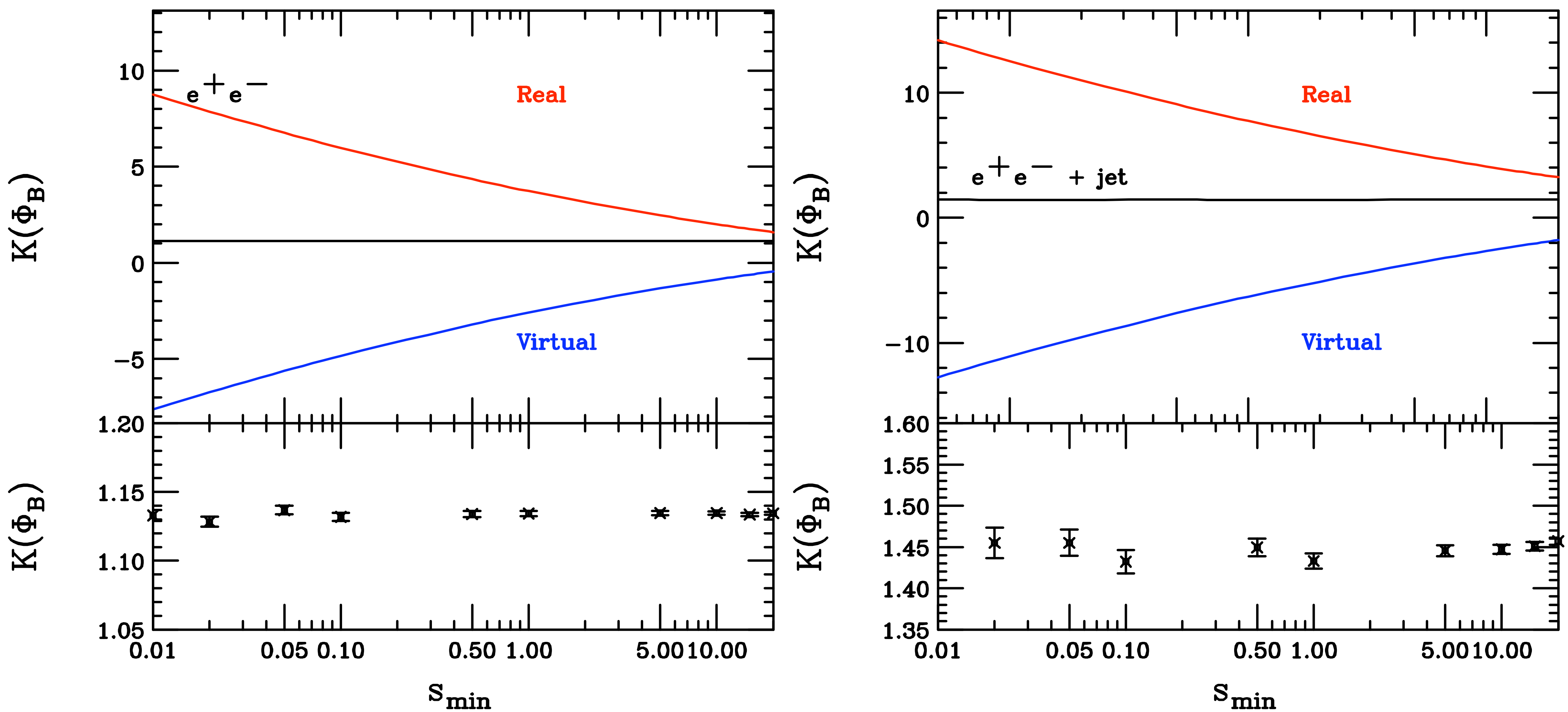
$+ \sum_{i \neq j, i > j}$ … $, j) + \mathcal{O}(s_{min})$

MEM@NLO!!!

Which can be … event K-factor.

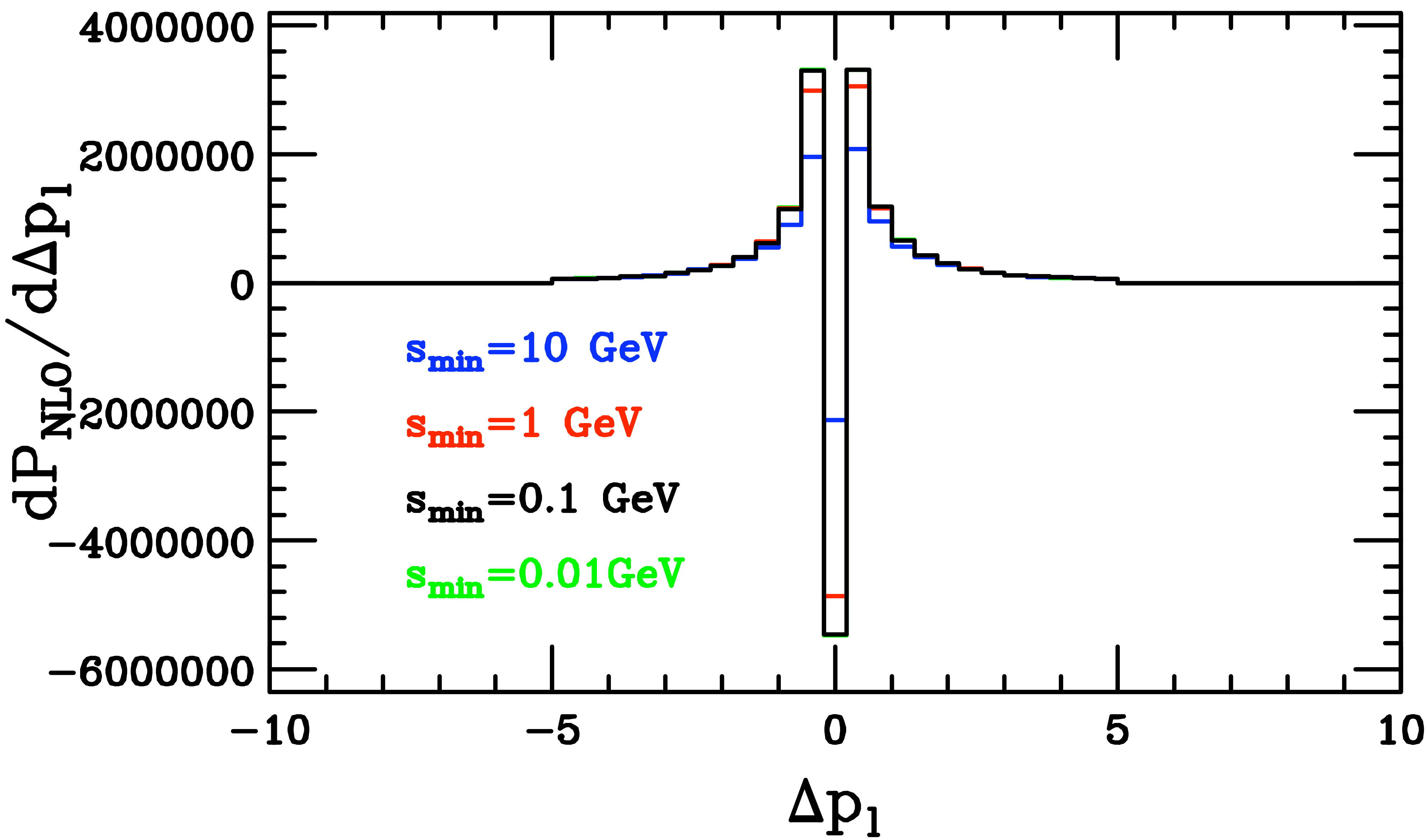
$$\mathcal{P}_{NLO}(\Phi_B) = K(\Phi_B)\mathcal{P}_{LO}(\Phi_B)$$

Examples : Z + 0,1 jets : event by event smin



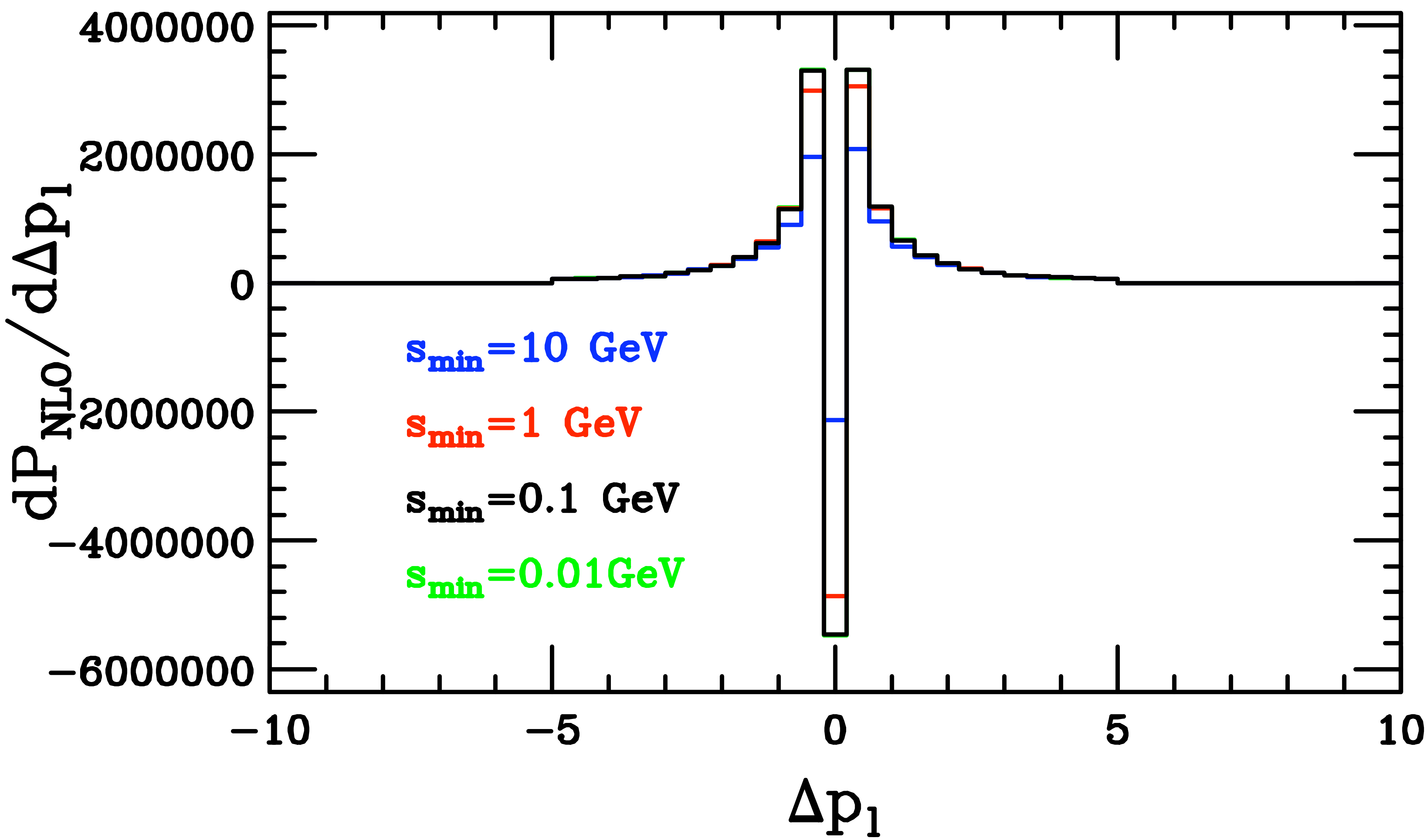
Proof is in the pudding, and the dessert of choice is smin independence at fixed Born phase space point.

Examples : Integration over the emissions.



This is a typical event, (for DY), we plot the pt difference from LO for one of the leptons.

Examples : Integration over the emissions.



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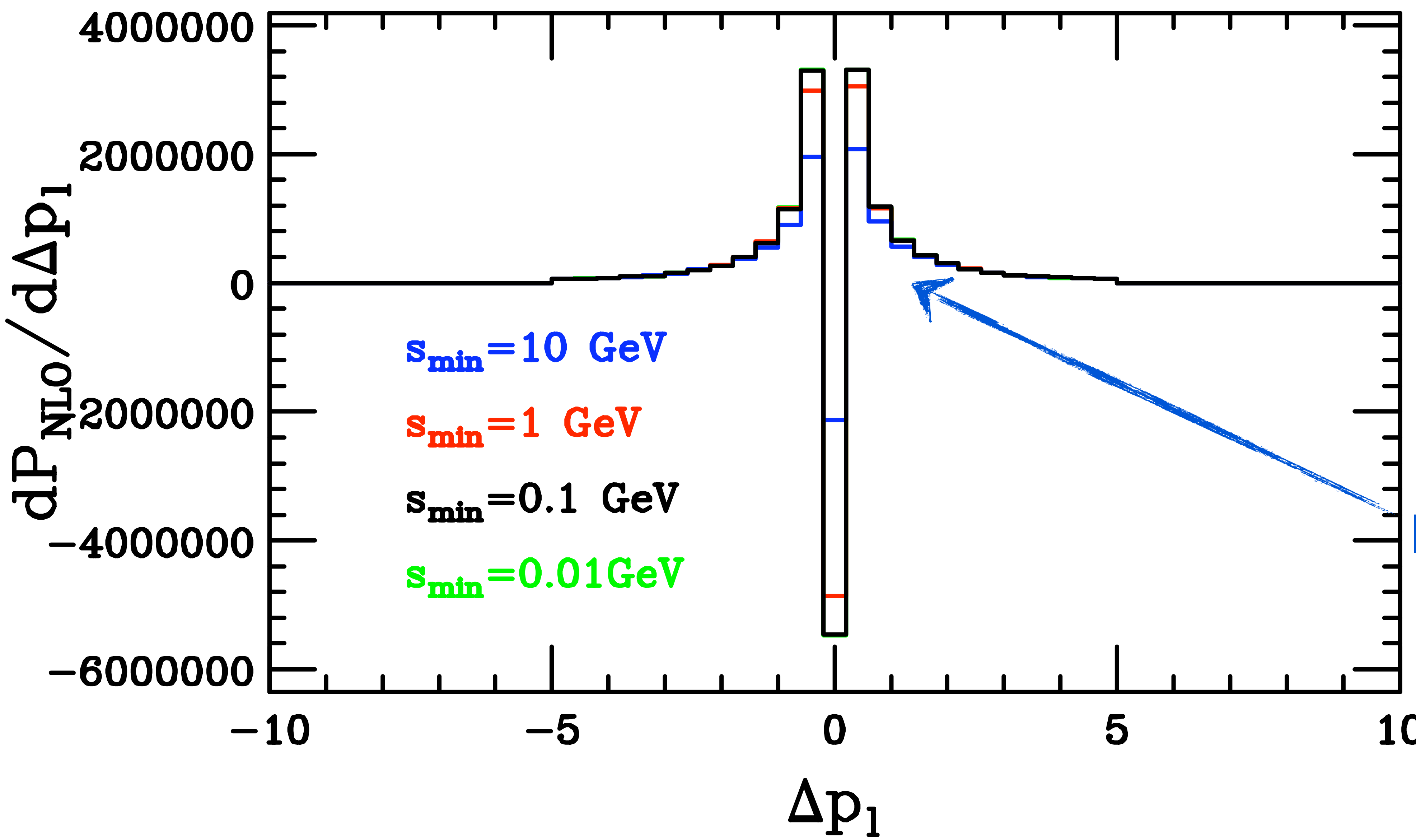
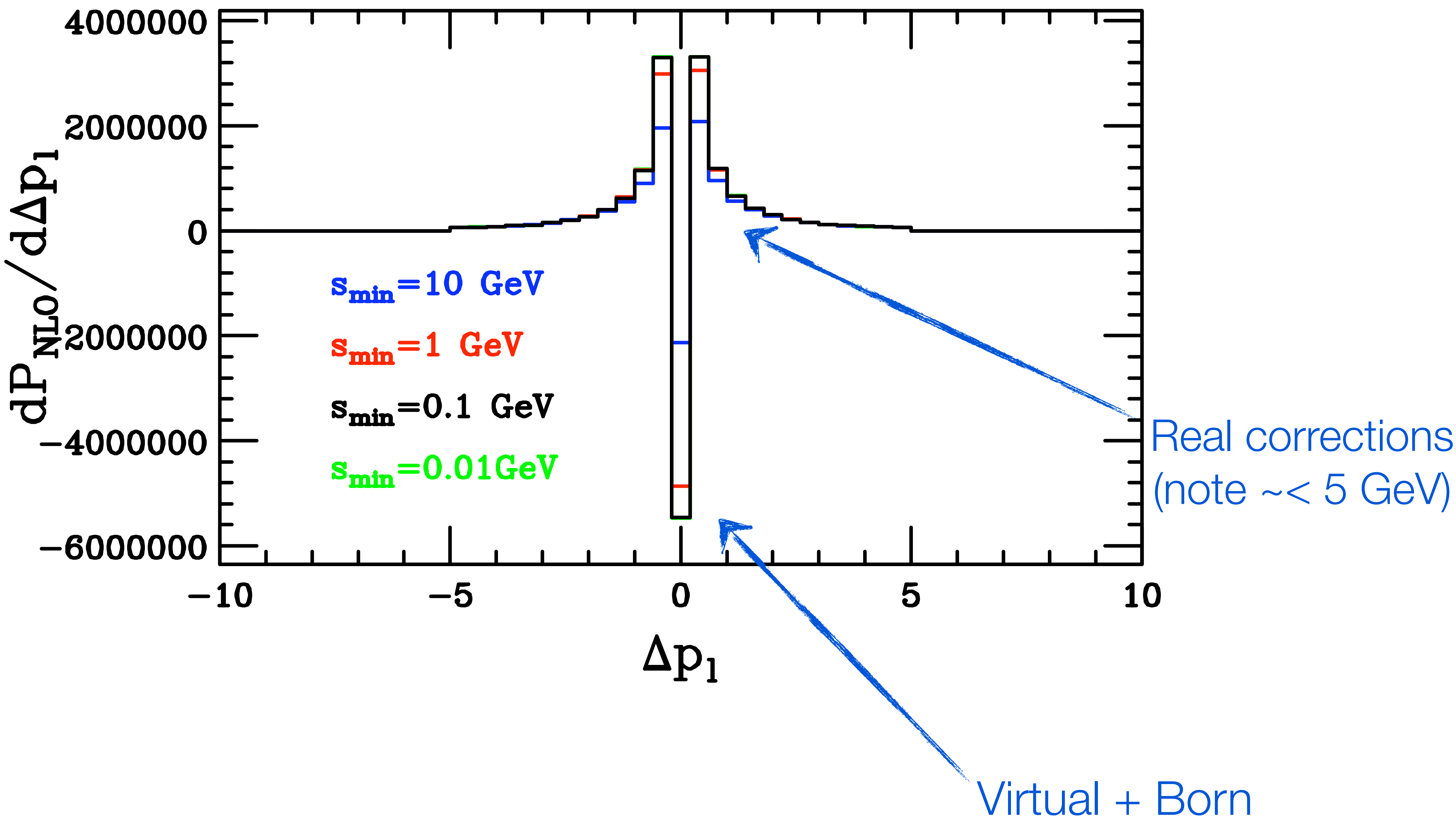
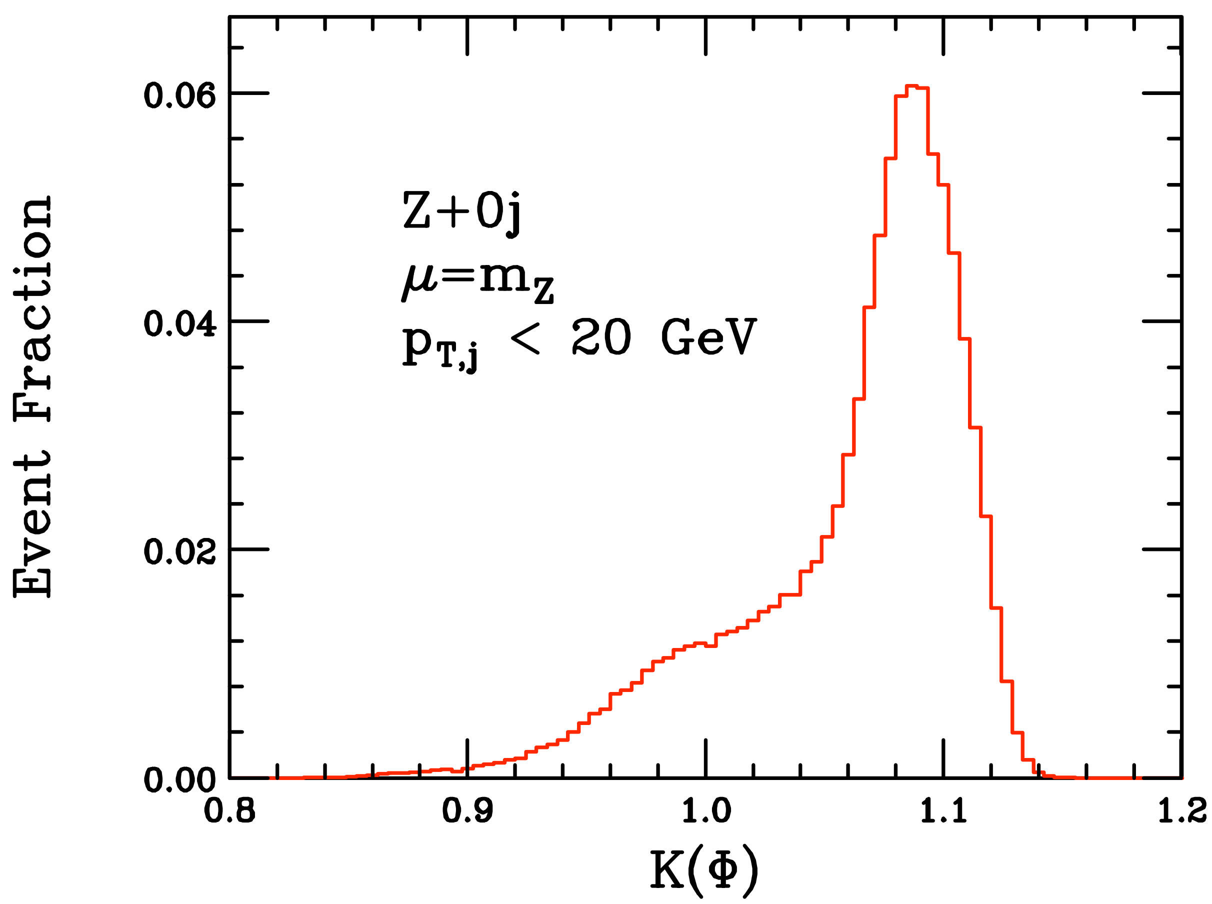


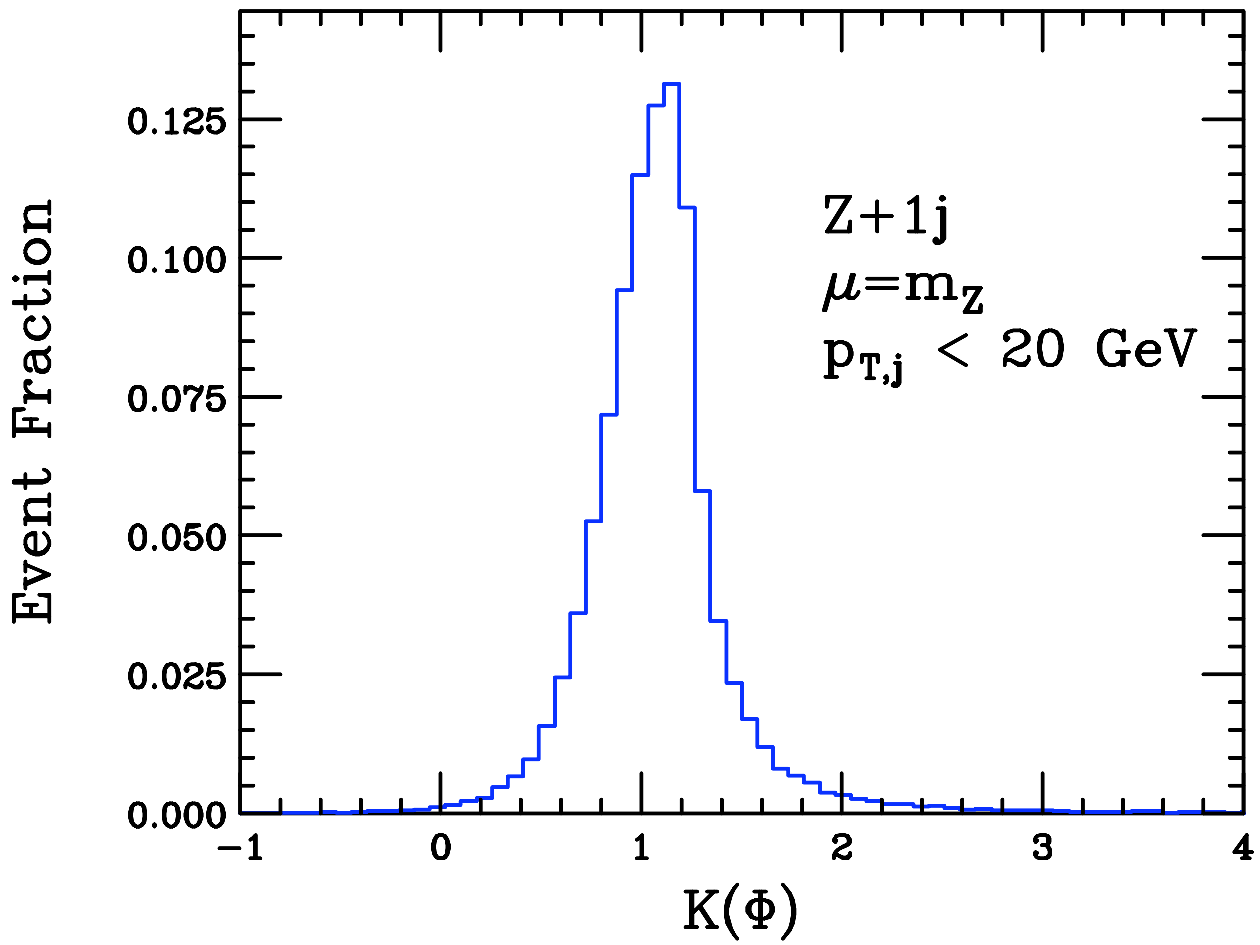
Real corrections
(note ~< 5 GeV)

Examples : Integration over the emissions.



Examples : Z+0,1 jets : differential K factors



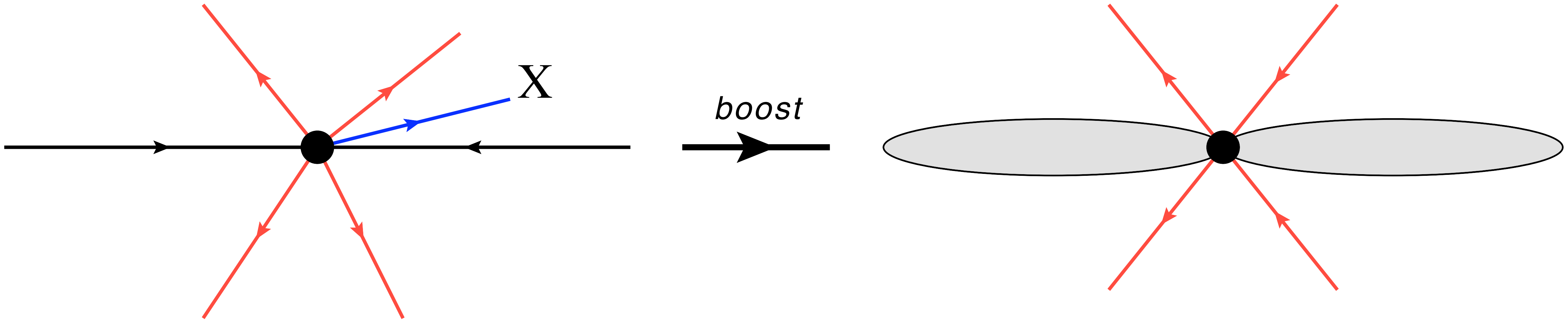


The pattern of the differential K-factor for the two types of processes (Z +0, or 1 jets) are markedly different. Both are defined exclusively, i.e. we veto radiation above > 20 GeV.

With a LO jet present the spread of K-factors is much larger (we also find some negative K-factors (fixed by changing scale).

Applications to the MEM

Our preferred MEM@NLO algorithm



- Take an input MC/data event, and generate parton level information through transfer functions (or lack thereof).

- Boost event such that transverse momentum of the final state balances, thus defining a Born phase space point, calculate LO and NLO weights.

- The boost was not unique (many longitudinally equivalent boosts) so integrate over all allowed boosts (all allowed x).

Boost integration

- We define our weights as follows,

$$\mathcal{P}^{MEM}_{LO}(\{Q_n\}) = \frac{1}{\sigma_{LO}} \int_{x_{min}}^{x_{max}} dx_1 \mathcal{P}_{LO}(\Phi_B)$$
$$\mathcal{P}^{MEM}_{NLO}(\{Q_n\}) = \frac{1}{\sigma_{NLO}} \int_{x_{min}}^{x_{max}} dx_1 \mathcal{P}_{NLO}(\Phi_B)$$

- The integration limits are found by solving for the maximum rapidity

$$\eta^{lab,i} = \frac{1}{2} \log\left(\frac{x_a^2 s}{s_{ab}} \frac{s_{ib}}{s_{ai}}\right)$$

- Here a and b are the beam particles.

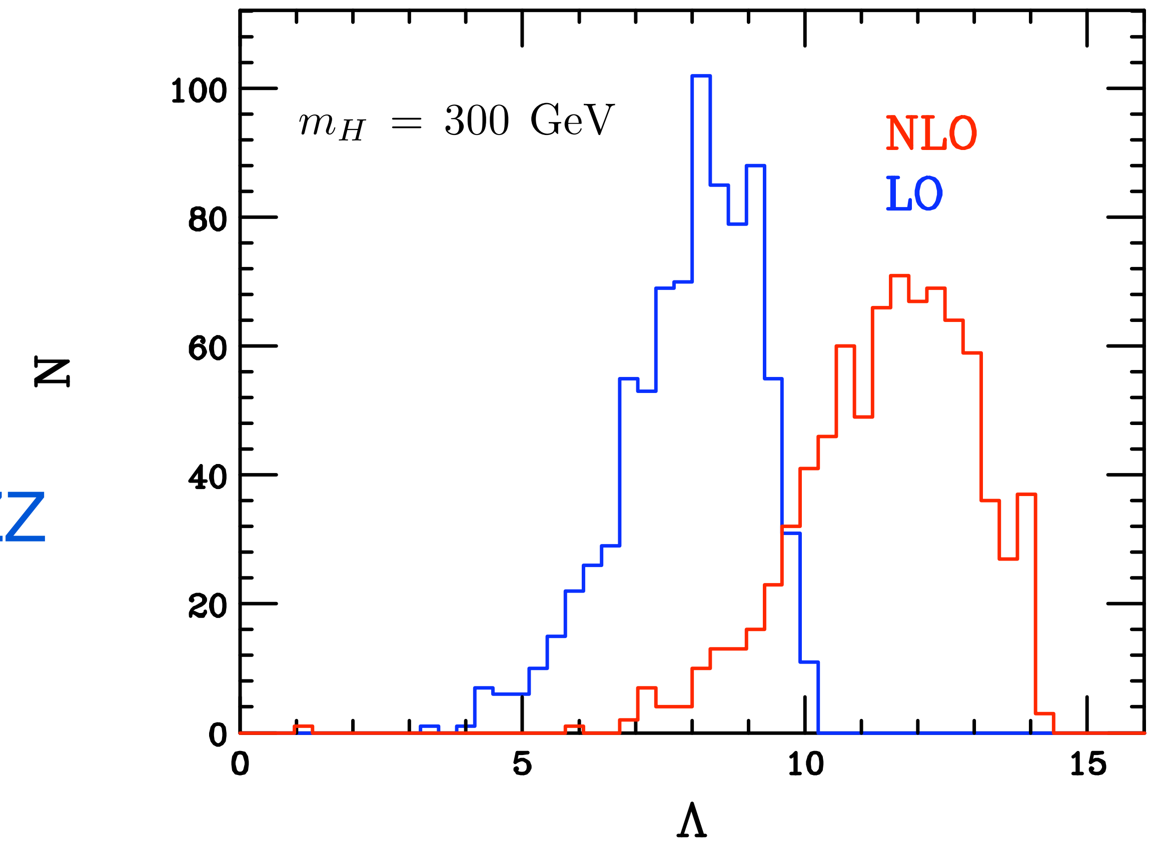
Applications so far

- Applications so far have been somewhat limited to theoretical study, and mostly focused on Higgs physics,

- Used (as a backup analysis) in CMS H=>ZZ

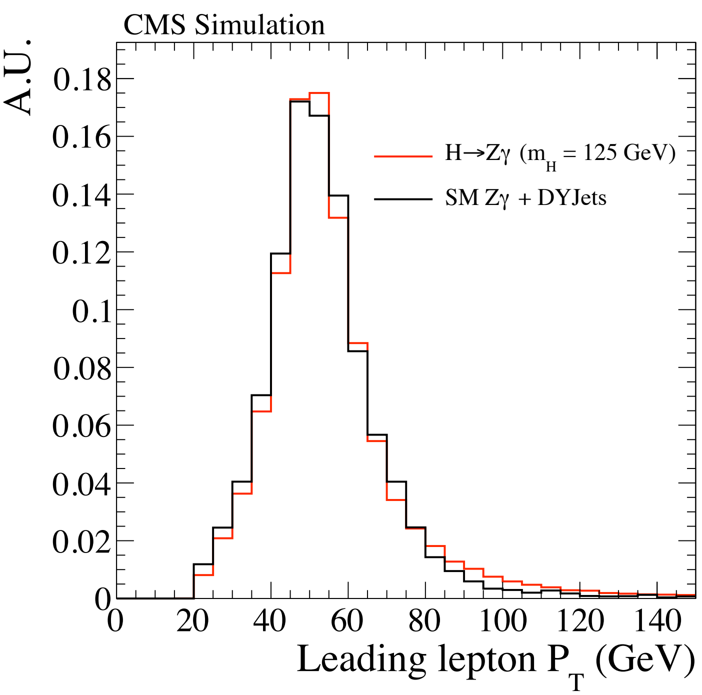
- Currently being implemented in CMS H=>Zγ analysis

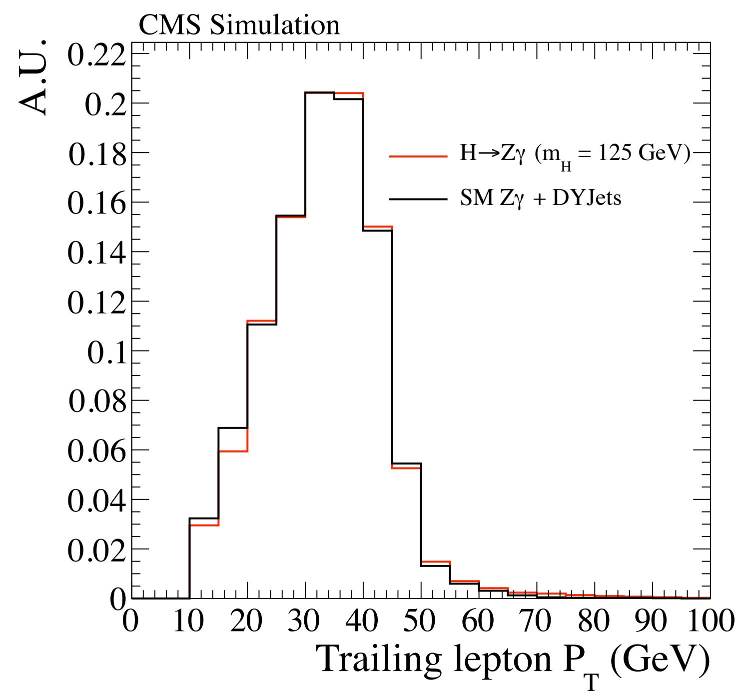
- Hope to release “MemCFM” in the latter half of this year, which will be a version of MCFM which can provide NLO weights event by event for every process in MCFM.



$$\Lambda = \log(\mathcal{L}_B/\mathcal{L}_{S+B})$$

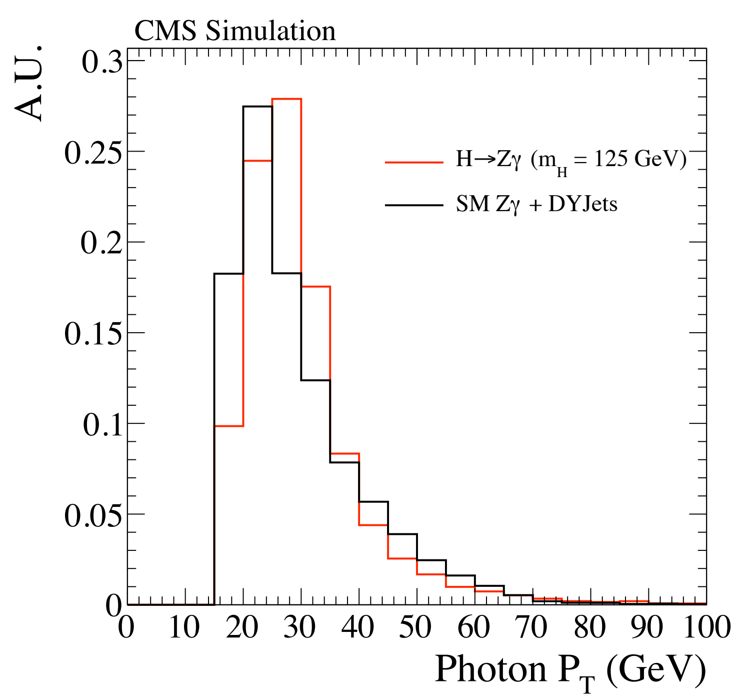
Searches for H=>Zγ





CMS-PAS-HIG-12-049

Phase space restrictions force the photon to be fairly soft.

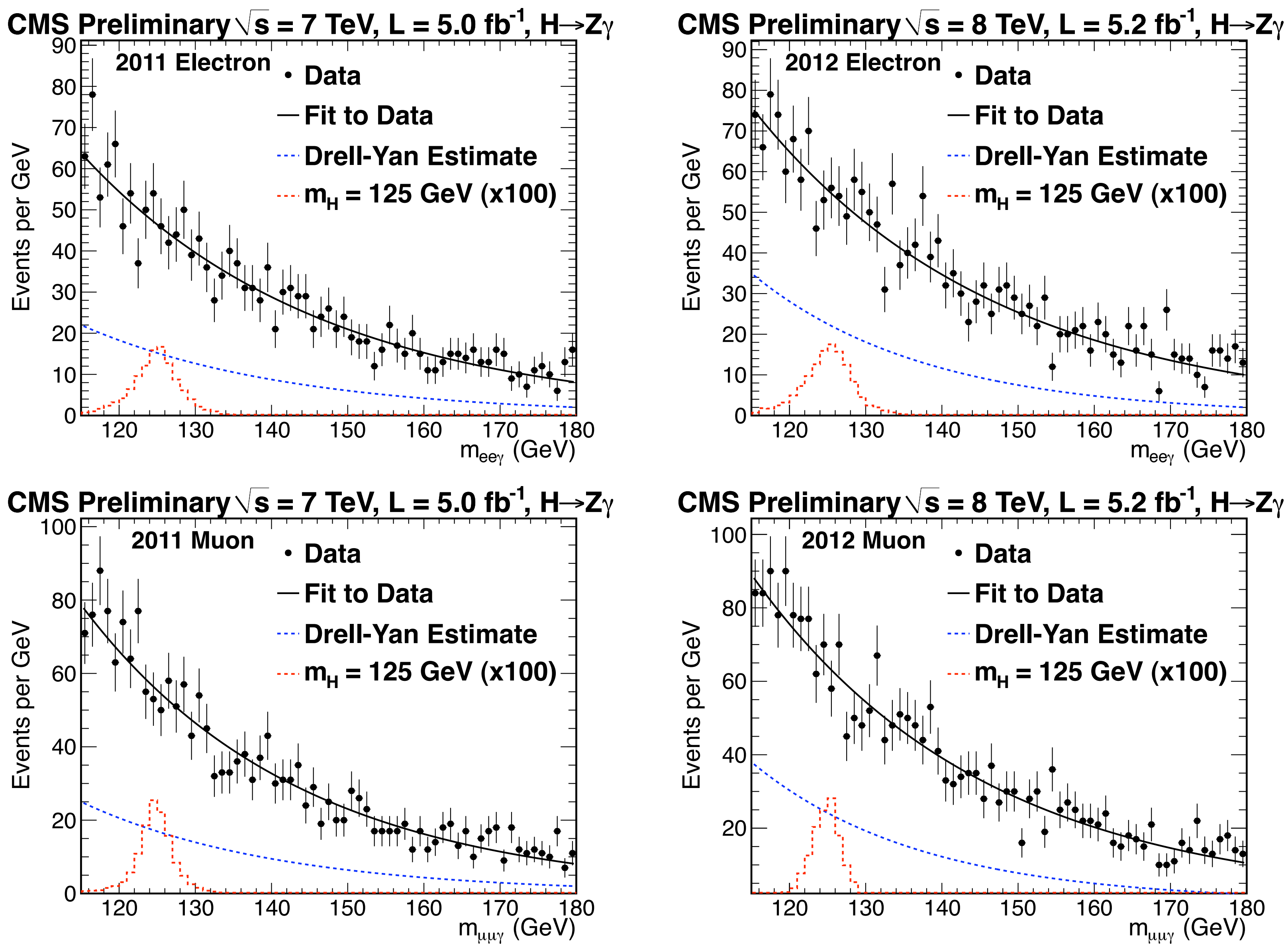


$$s \sim m_H^2 = m_Z^2 + 2(p_T^\gamma)^2$$

$$p_T^\gamma \lesssim 40$$

Very difficult Channel.

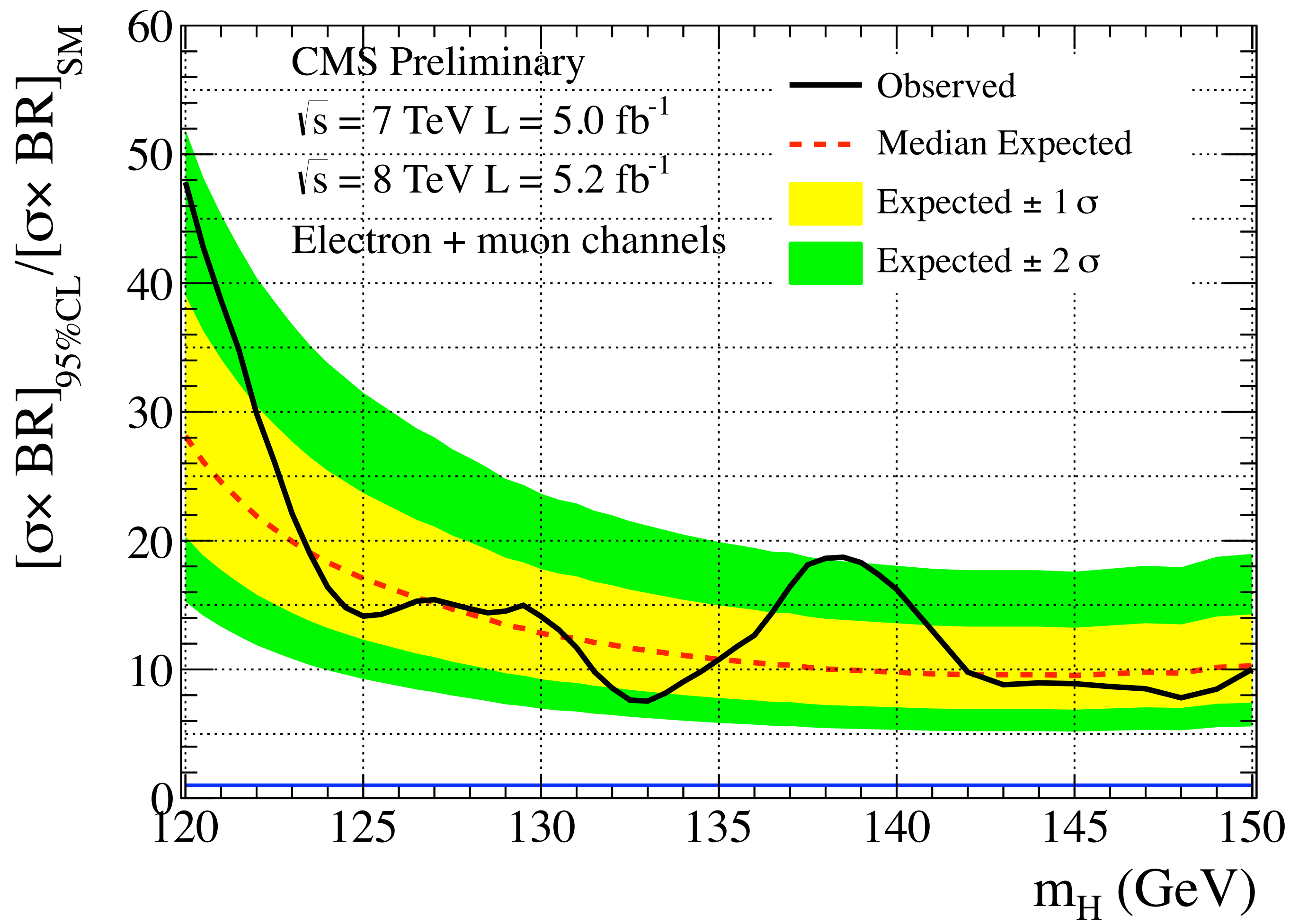
CMS search



CMS have recently released a first search in this channel.

Using the same sort of techniques as 2 photons.

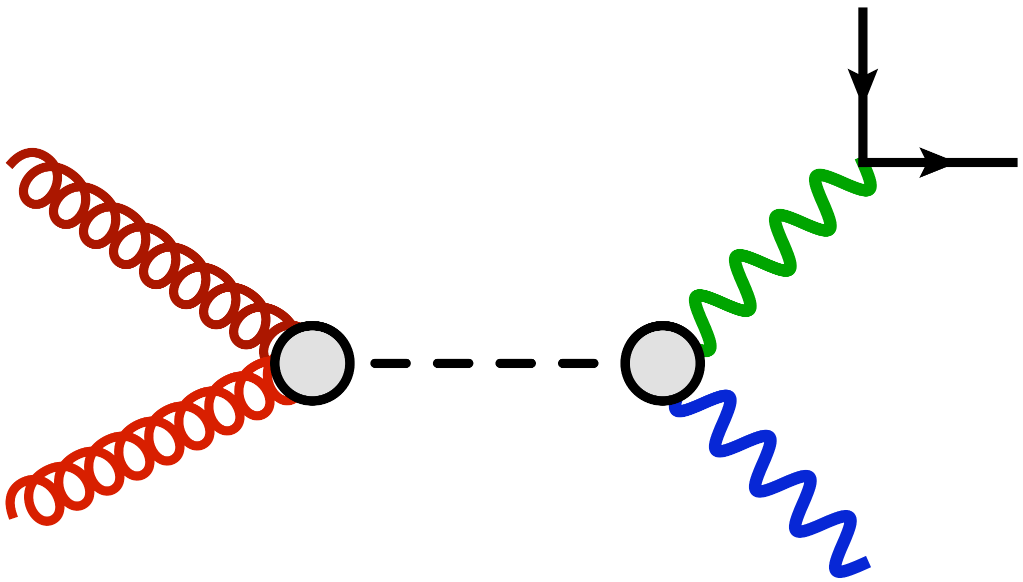
Limits on BR

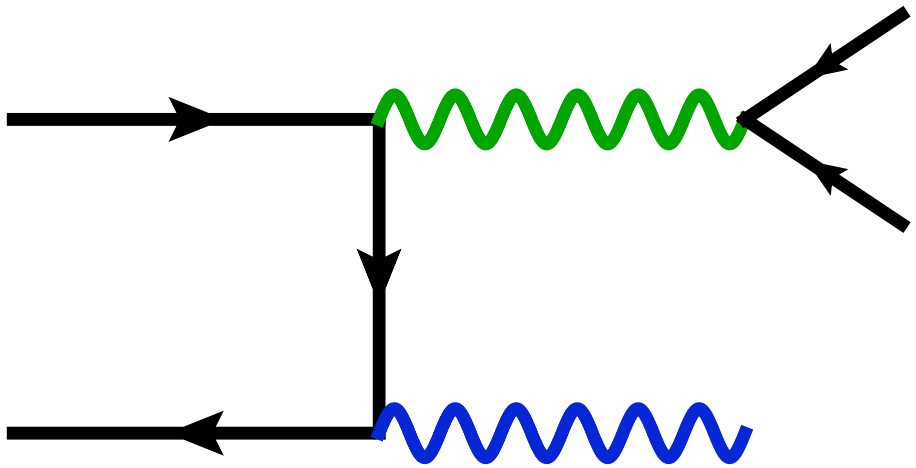


The CMS analysis was able to set limits at around 10-50 time the SM BR.

Its interesting to see how the MEM does here, since its really really hard!

Some hope,





- Production mechanisms are different
- Photon prefers to be radiated collinear to quark line in background.
- As a result rapidity/polar angle distributions different.
- Can we utilize this?

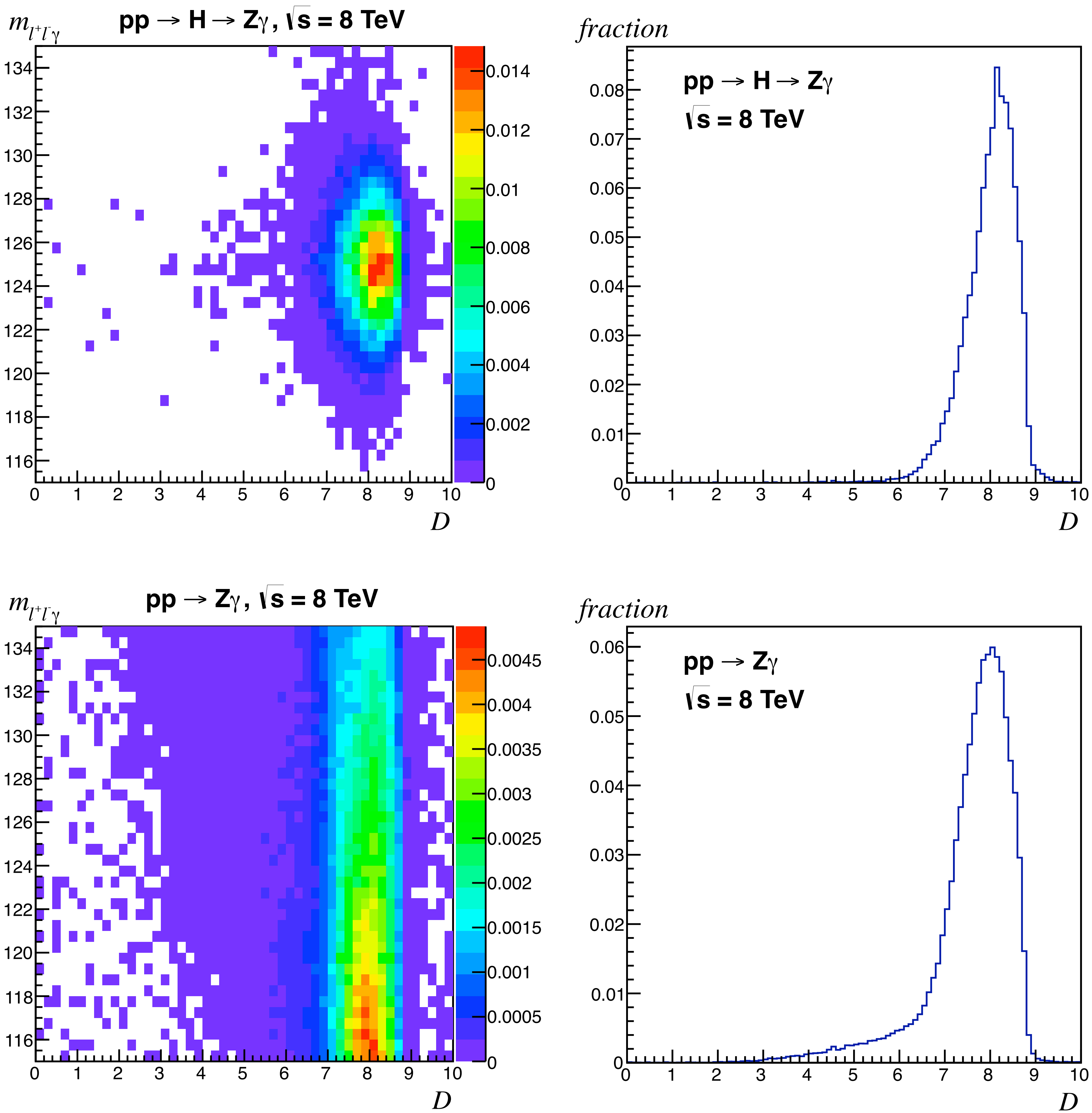
The kinematic discriminant

- There are numerous possibilities, we will choose the following,

$$\mathcal{D} = -\log\left(\frac{P_B}{P_S + P_B}\right).$$

- When the event is more like a background event D is closer to 0. Larger D correspond to events which look more Higgs like

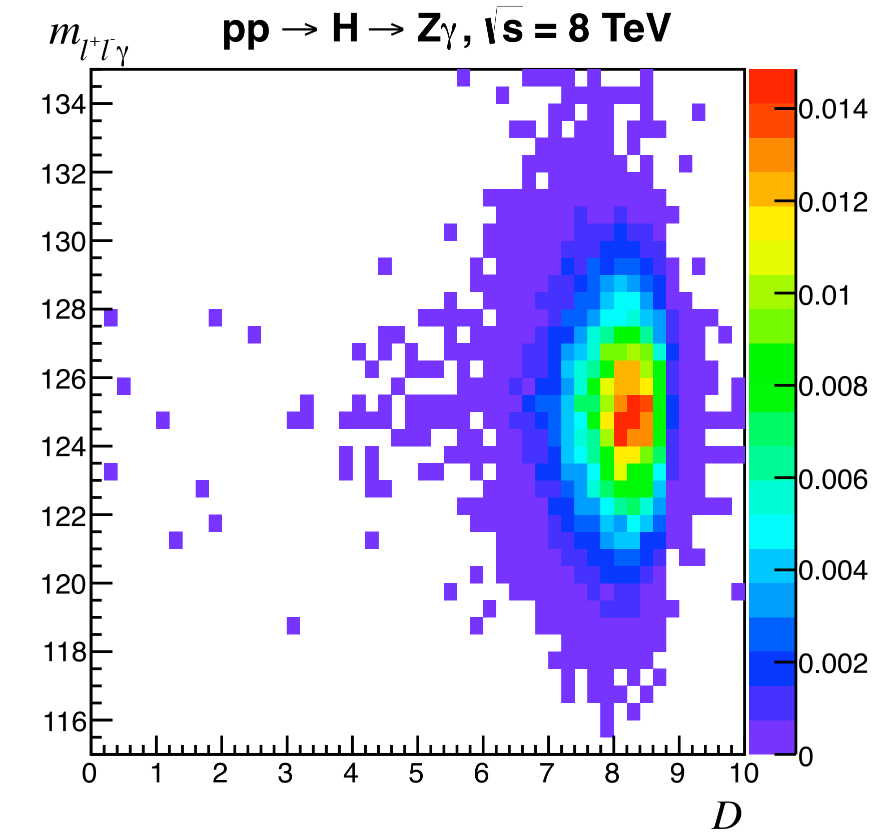
Results

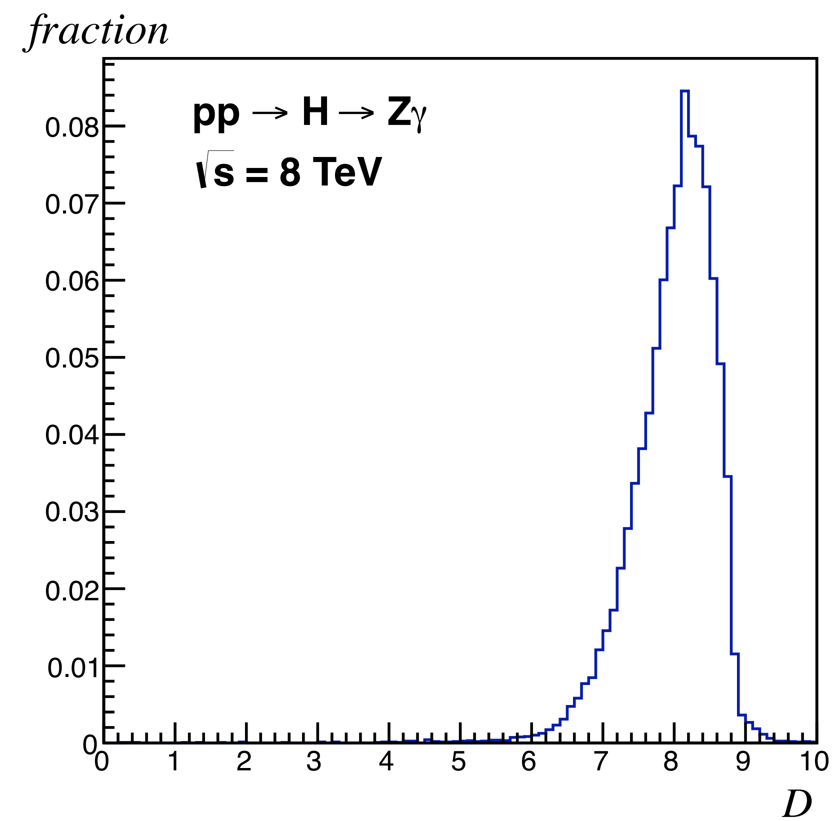


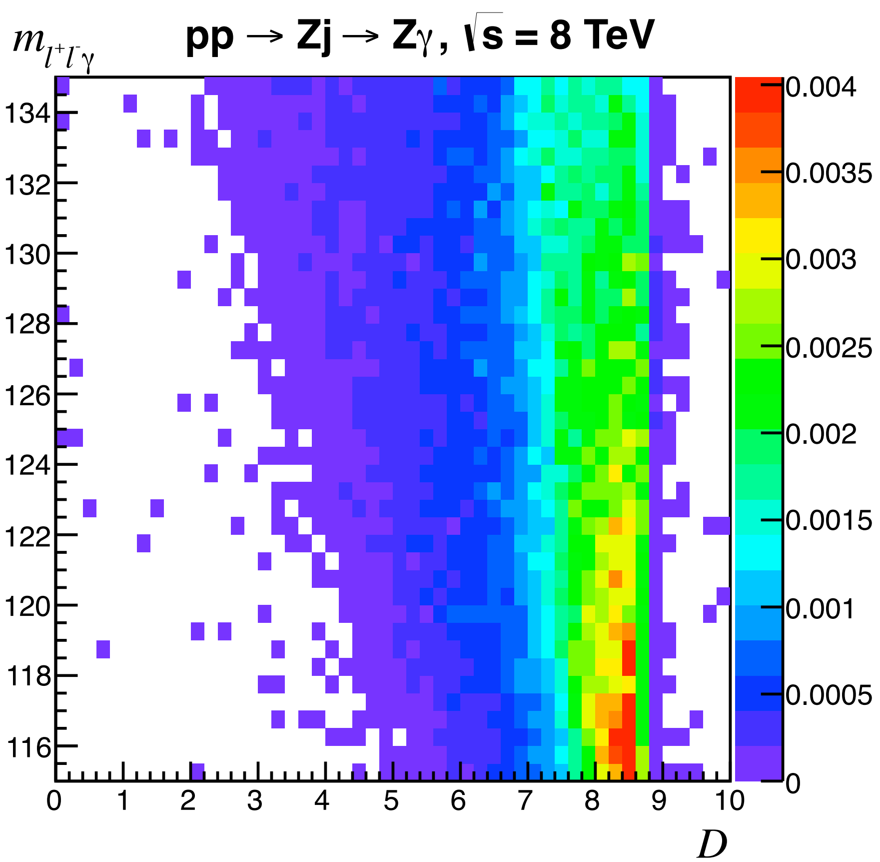
We can make a two dimensional histogram in the invariant mass, D plane.

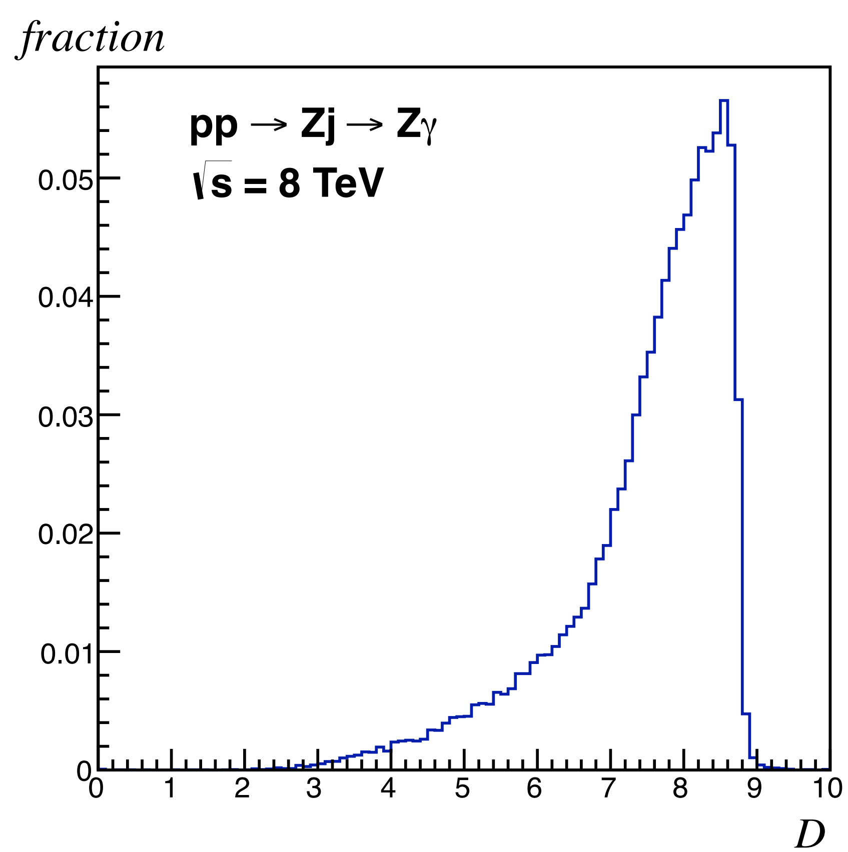
Background events have a longer tail and are move evenly distributed in invariant mass.

Results









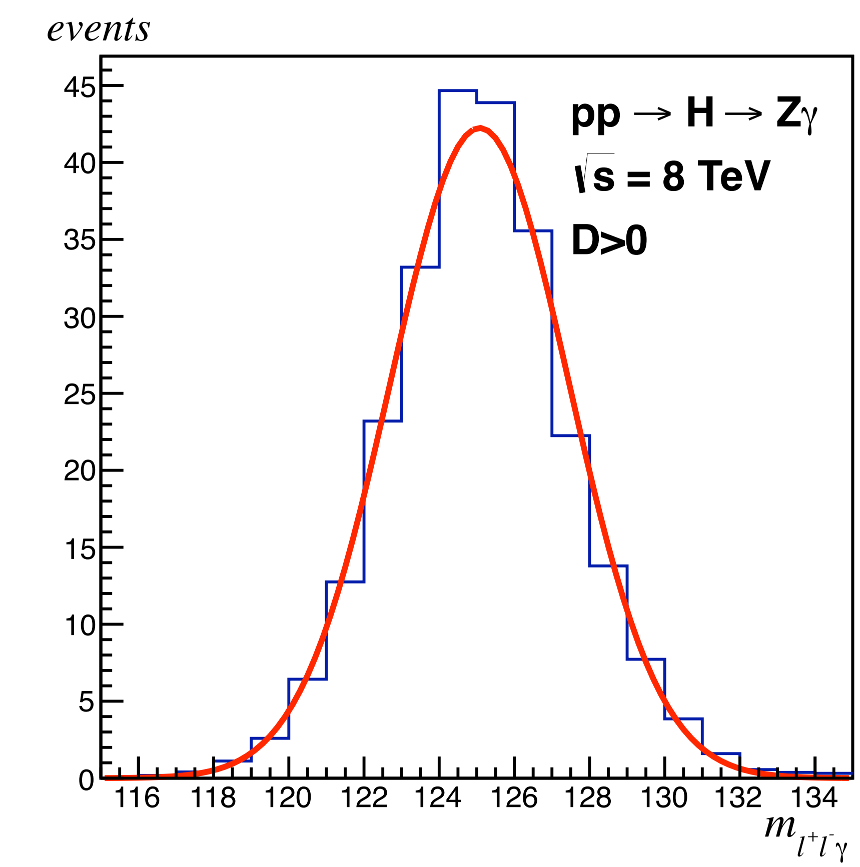
The fakes are similar to the irreducible background.
The effects of the larger smearing is are also apparent.

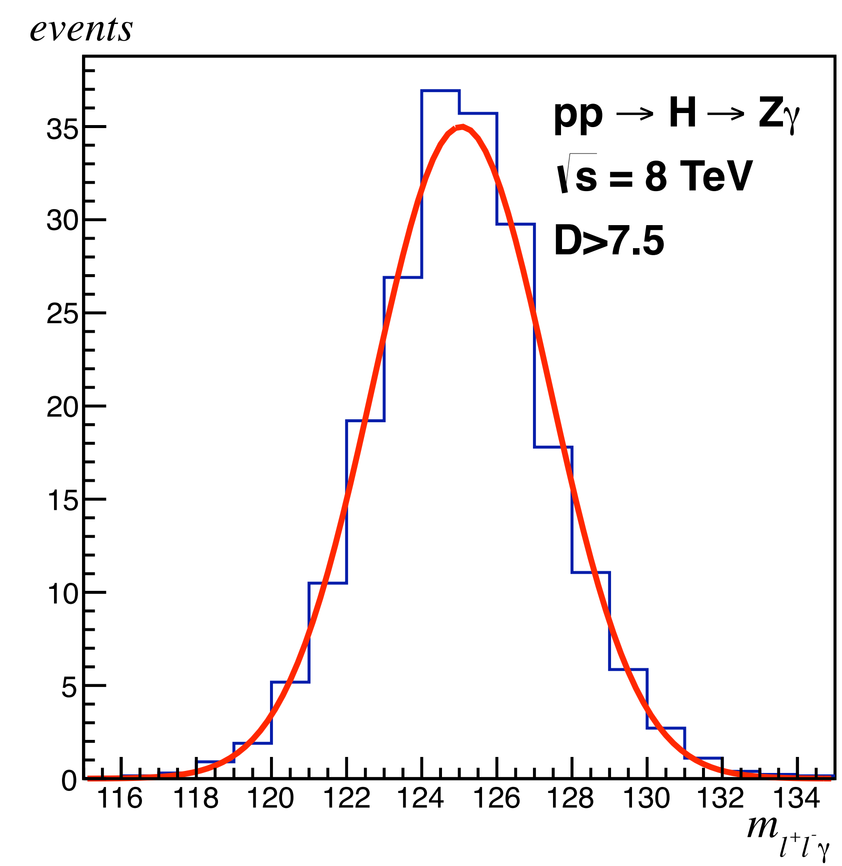
We will use these results to enhance the S/B by requiring the event to pass a minimum D requirement.

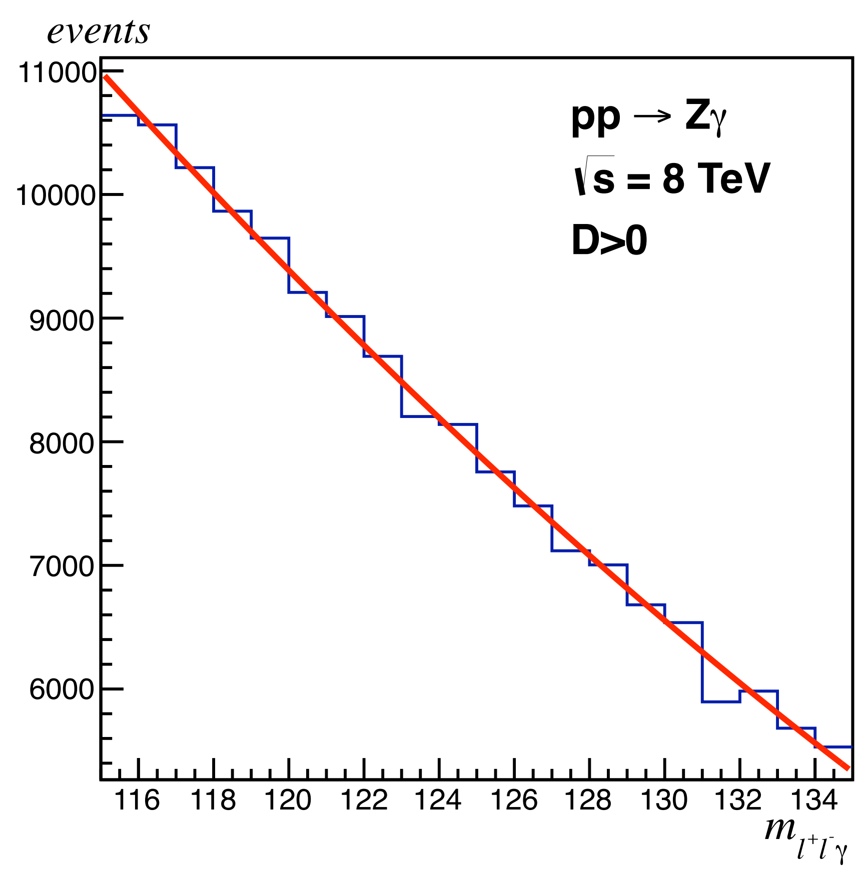
Invariant Mass

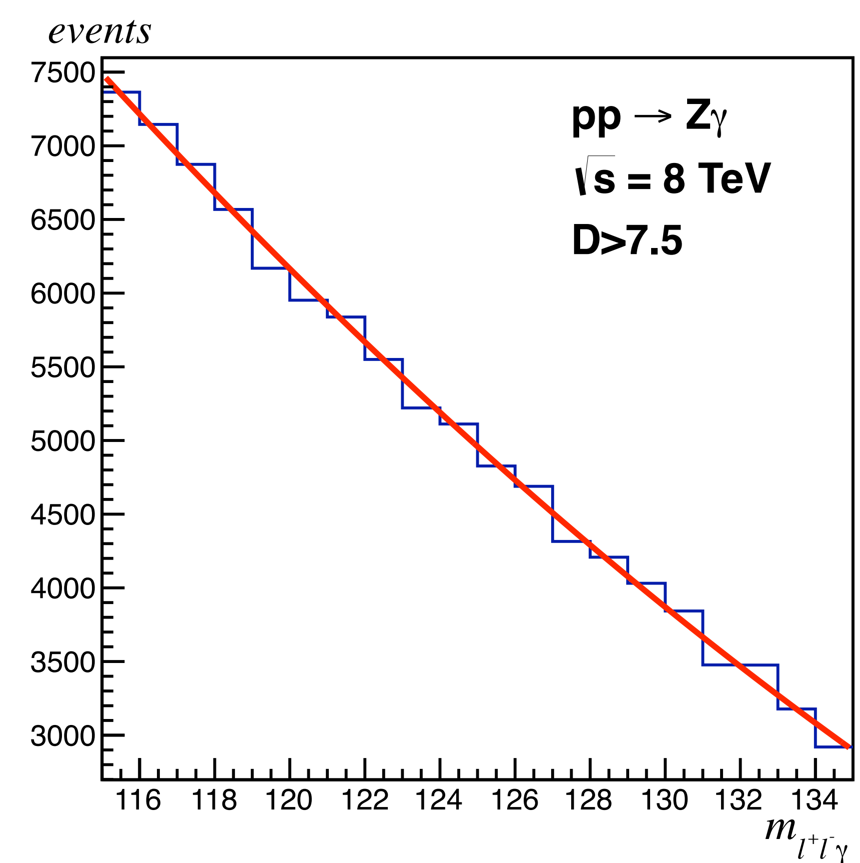
We make the invariant mass distribution before and after the cut, it is clear that S/B has increased.

With D > 7.5 we keep ~80% of the signal and lose ~40% of the background.





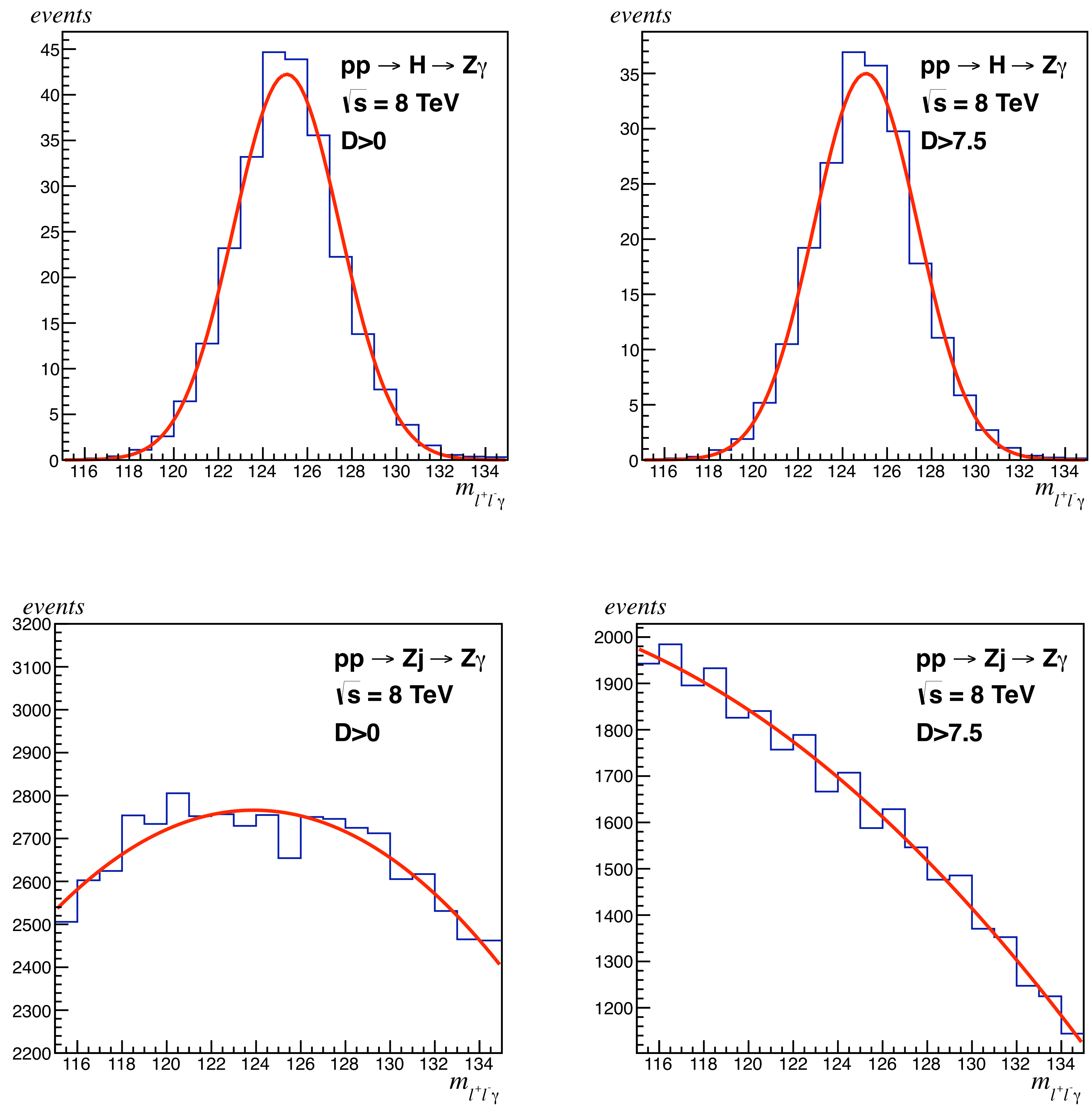




Invariant Mass

The fakes and background change shape after the cut, but the signal remains the same.

Could imagine doing a cut on D then redoing the initial CMS analysis. What would be the improvement?



S/√B scaling.

- Define the following parameter

$$\alpha = \frac{\sqrt{N_{Z\gamma} + N_{fakes}}}{N_H} \ ,$$

- Where N is the number of events in the window of invariant mass,

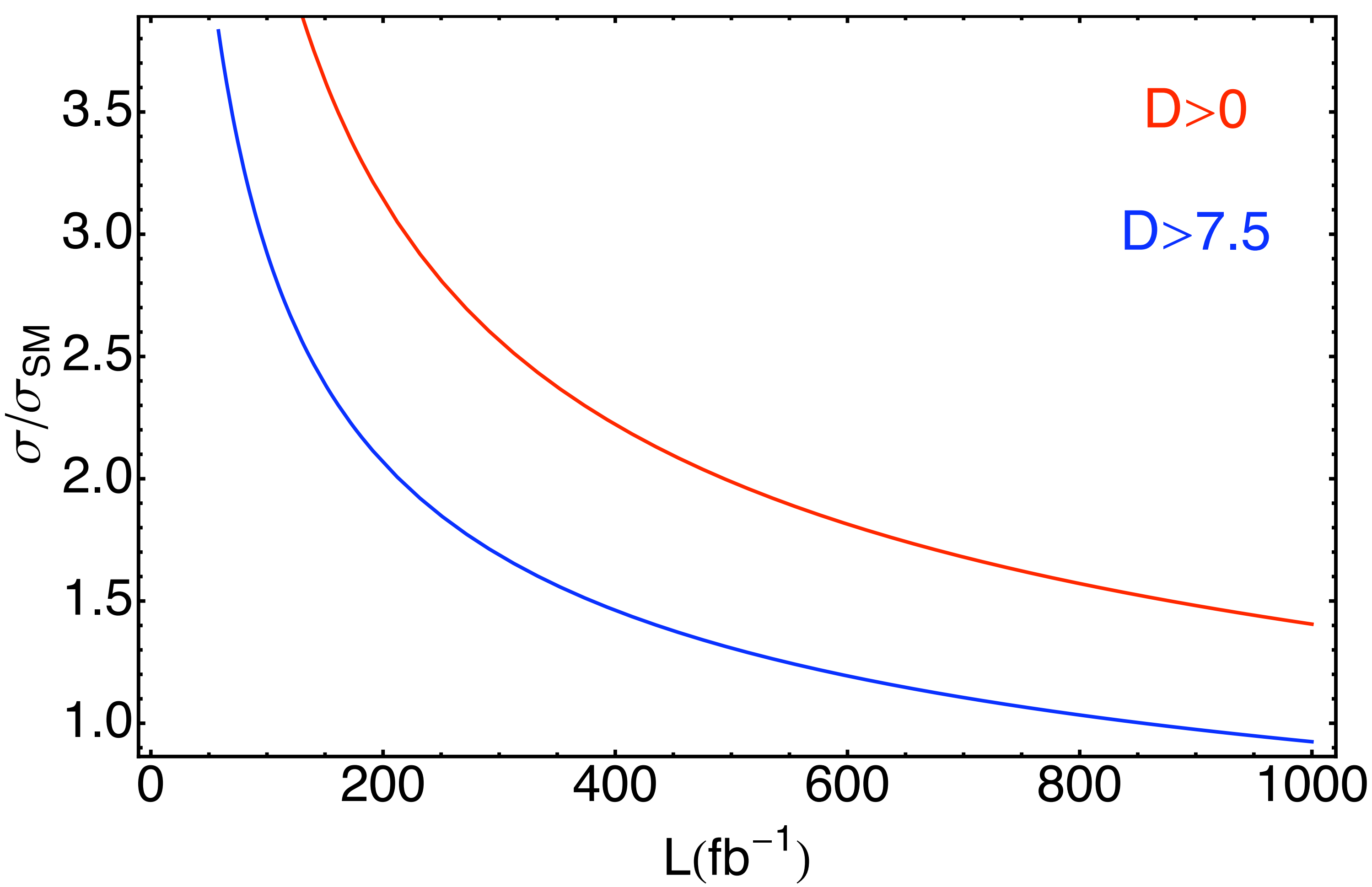
$$122 < m_{\ell\ell\gamma} < 128 \text{ GeV} \ ,$$

- We find

$$\frac{\alpha_{D>0}}{\alpha_{D>7.5}} = 1.52.$$

Luminosity Improvement

- Since alpha scales as root Luminosity this increase is (statistically) equivalent to taking 2.3 times more data.



My estimate of the limits on the cross section as a function of Luminosity.

Difference between LO and NLO is around 10 % (not shown here).

Conclusions

- I have shown how one can reweight LO events to NLO accuracy, on an event by event basis. Allowing one to define NLO corrections for the MEM.

- The method is general, and should work with *any* MEM algorithm which uses a LO Matrix Element.

- Transfer functions will (in principle) change at NLO although I had no time to discuss this. But these corrections should be relatively small. This probably needs good interplay between theorists and experimentalists to produce useful functions.

- Distributions of event by event K-factors seem to vary significantly process to process and are far from being a delta function (ongoing study).