Higgs Characterization with MEM
(in $X \to ZZ^* \to \ell^+\ell^-\ell^+\ell^-$ channel)

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This talk is based on
[“Geolocating the Higgs Boson Candidate at the LHC”]
(arXiv:1304.4936)

Motivations

- With a newly discovered particle, we want to measure its properties, especially the coupling structure to two Z bosons.
- We want to be general as much as possible.
- But we want to keep analyses manageable, thus we try to reduce parameters (eliminating unnecessary degree of freedom)
- In real analyses, we may not need to generate pseudo experiments with whole parameter space.

General spin 0 particle

- We consider a spin-0 particle X, which is a linear combination of CP eigenstates, CP even H (0^+) and CP odd scalar A (0^-),

$$X \equiv H \cos\alpha - A \sin\alpha.$$

- In general X is not a CP eigenstate, but a mixture of both.

$\alpha = 0$: Pure CP even state

$\alpha = \frac{\pi}{2}$: Pure CP odd state

- We assume that the other mass eigenstate is heavy, and can be neglected in our analysis of M_x ~125GeV study.

General structure

- The most general Lagrangian can be written with three terms,

$$\mathcal{L} \ni \boxed{\frac{M_Z^2}{v} H Z^\mu \hat{f}^{(H)}_{\mu\nu} Z^\nu} + \boxed{\frac{1}{2} H F^{\mu\nu} \hat{f}^{(H)}_{\mu\nu\rho\sigma} F^{\rho\sigma}} + \boxed{\frac{1}{2} A F^{\mu\nu} \hat{f}^{(A)}_{\mu\nu\rho\sigma} F^{\rho\sigma}}$$

with form factor fs,

CP even terms which violates gauge invariance.

CP even terms which preserves gauge invariance.

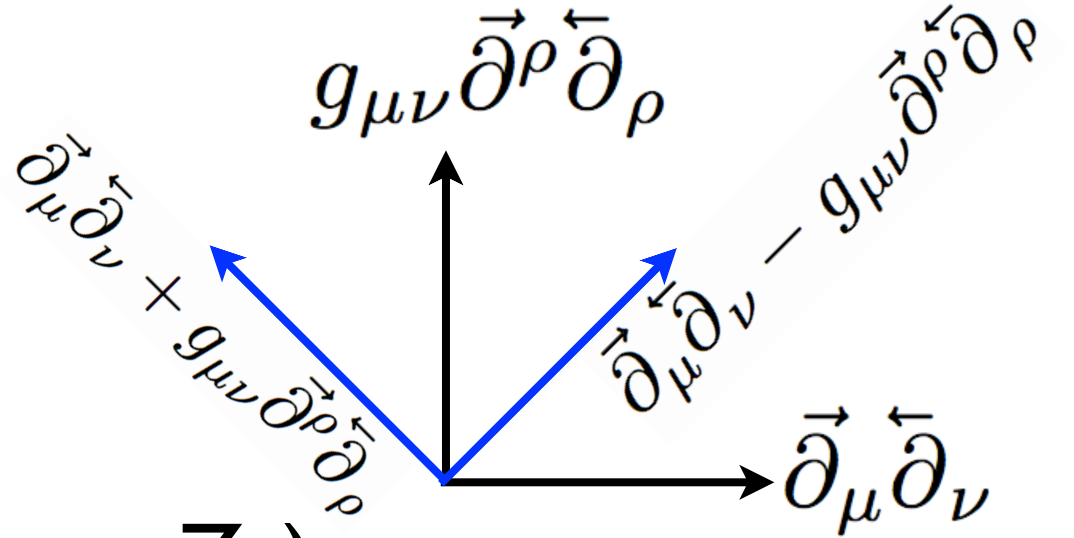
CP odd terms which preserves gauge invariance.

- In a mass eigenstate, we can rewrite the Lagrangian as

$$\mathcal{L} = X \left[\kappa_1 \frac{m_Z^2}{v} Z_\mu Z^\mu + \frac{\kappa_2}{2v} F_{\mu\nu} F^{\mu\nu} + \frac{\kappa_3}{2v} F_{\mu\nu} \tilde{F}^{\mu\nu} \right]$$

Form factors

- With $F^{\mu\nu}F_{\mu\nu} \ni Z^{\mu}\left(\vec{\partial}_{\mu}\overleftarrow{\partial}_{\nu} - g^{\mu\nu}\vec{\partial}^{\rho}\overleftarrow{\partial}_{\rho}\right)Z_{\mu}$



we do the change of basis to single out gauge invariant term (also symmetrization for two Zs)

$$\hat{f}^{(H)}_{\mu\nu} \equiv g_1 g_{\mu\nu} + \frac{g_5}{\Lambda^2}\left(\vec{\partial}_{\mu}\overleftarrow{\partial}_{\nu} + g_{\mu\nu}\vec{\partial}^{\rho}\overleftarrow{\partial}_{\rho}\right) + \frac{g_6}{\Lambda^2}g_{\mu\nu}\left(\overleftarrow{\Box} + \vec{\Box}\right) + \mathcal{O}\left(\frac{1}{\Lambda^4}\right),$$

SU(2)

$$\hat{f}^{(H)}_{\mu\nu\rho\sigma} \equiv \frac{g_2}{\Lambda}g_{\mu\rho}g_{\nu\sigma} + \frac{g_3}{\Lambda^3}g_{\mu\rho}\partial_{\nu}\partial_{\sigma} + \mathcal{O}\left(\frac{1}{\Lambda^5}\right)$$

SU(2) gauge inv.

CP even

$$\hat{f}^{(A)}_{\mu\nu\rho\sigma} = \frac{g_4}{\Lambda}\varepsilon_{\mu\nu\rho\sigma} + \mathcal{O}\left(\frac{1}{\Lambda^5}\right)$$

SU(2) gauge inv.

CP odd

Coupling structure

- If we consider only the first terms

$$\hat{f}^{(H)}_{\mu\nu} \equiv g_1 g_{\mu\nu} + \cancel{\frac{g_5}{\Lambda^2}}\left(\vec{\partial}_\mu \overleftarrow{\partial}_\nu + g_{\mu\nu}\vec{\partial}^\rho \overleftarrow{\partial}_\rho\right) + \cancel{\frac{g_6}{\Lambda^2}} g_{\mu\nu}\left(\overleftarrow{\Box} + \vec{\Box}\right) + \mathcal{O}\left(\cancel{\frac{1}{\Lambda^4}}\right) \longrightarrow \kappa_1 \equiv g_1 \cos\alpha$$

$$\hat{f}^{(H)}_{\mu\nu\rho\sigma} \equiv \frac{g_2}{\Lambda} g_{\mu\rho} g_{\nu\sigma} + \cancel{\frac{g_3}{\Lambda^3}} g_{\mu\rho}\partial_\nu \partial_\sigma + \mathcal{O}\left(\cancel{\frac{1}{\Lambda^5}}\right) \longrightarrow \kappa_2 \equiv g_2 \frac{v}{\Lambda} \cos\alpha$$

$$\hat{f}^{(A)}_{\mu\nu\rho\sigma} = \frac{g_4}{\Lambda} \varepsilon_{\mu\nu\rho\sigma} + \mathcal{O}\left(\cancel{\frac{1}{\Lambda^5}}\right) \longrightarrow \kappa_3 \equiv g_4 \frac{v}{\Lambda} \sin\alpha$$

$$\mathcal{L} = X\left[\kappa_1 \frac{m_Z^2}{v} Z_\mu Z^\mu + \frac{\kappa_2}{2v} F_{\mu\nu} F^{\mu\nu} + \frac{\kappa_3}{2v} F_{\mu\nu} \tilde{F}^{\mu\nu}\right]$$

We have three degree of freedom, and
we will reduce a d.o.f by factoring out overall normalization
from measured total rate.

Coupling structure

$$\mathcal{L} = X\left[\kappa_1 \frac{m_Z^2}{v} Z_\mu Z^\mu + \frac{\kappa_2}{2v} F_{\mu\nu} F^{\mu\nu} + \frac{\kappa_3}{2v} F_{\mu\nu} \tilde{F}^{\mu\nu}\right]$$

- These operators cover all possible Lorentz structure in the amplitude. [e.g., Gao, Gritsan, Guo, Melnikov, Schulze, Tran (2010), De Rújula, Lykken, Pierini, Rogan, Spiropulu (2010), Bolognesi, Gao, Gritsan, Melnikov, Schulze, Tran, Whitbeck (2012)]

$$A(X \to V_1 V_2) = v^{-1} \epsilon_1^{*\mu} \epsilon_2^{*\nu} \left(a_1 g_{\mu\nu} m_H^2 + a_2\, q_\mu q_\nu + a_3 \epsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta \right)$$

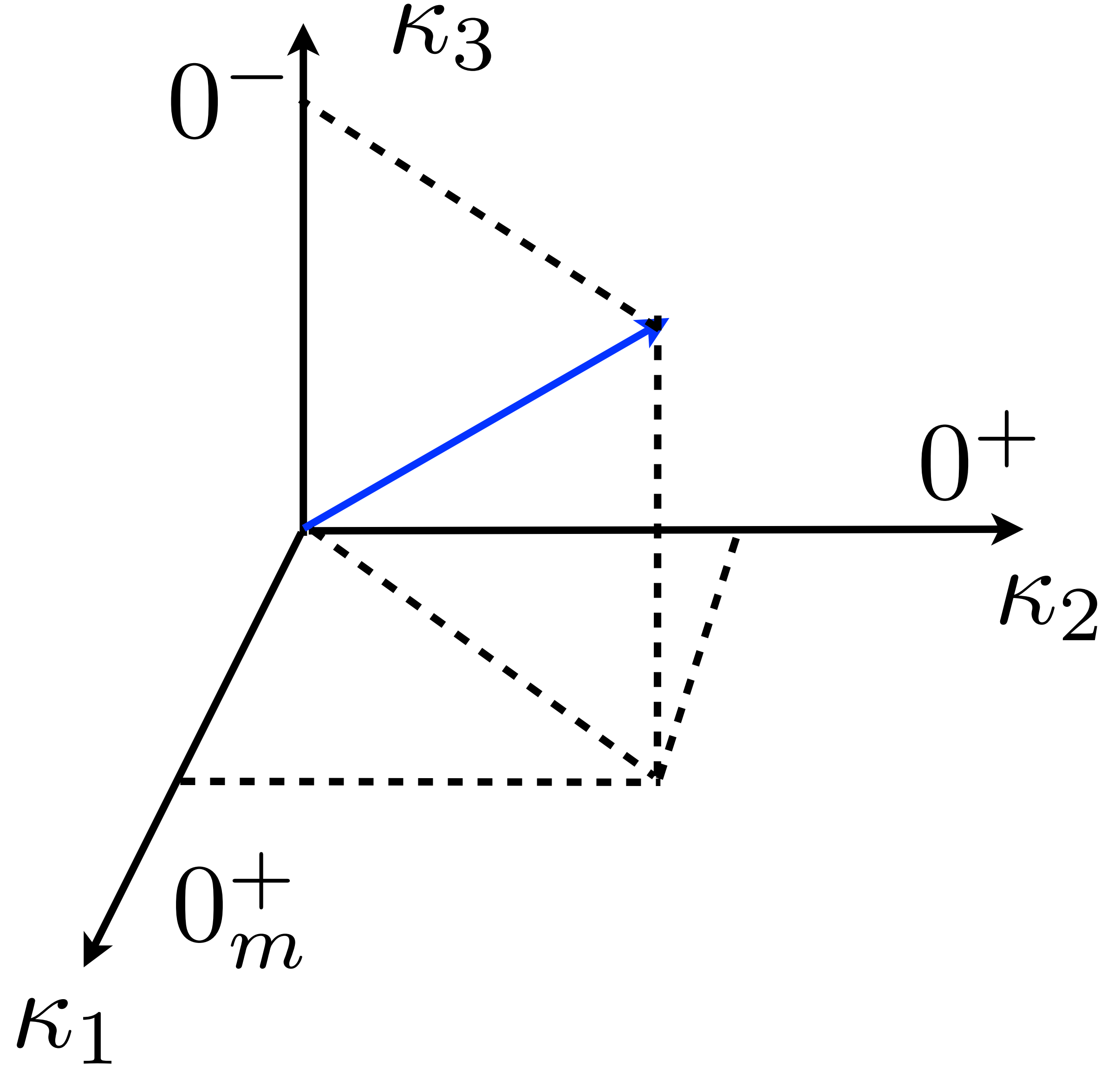
from Bolognesi.et.al, 2012

Parameter space

- Lagrangian must be real, so kappa are real.
- Amplitude receives corrections from loops

  1. Contributions from heavy particles loops are real.

  2. Contributions from light particles loops are complex
- These complex contributions can be mimicked by complex kappas.



Ellipsoid of a higgs

With NWA, we will single out the overall scale.
The constraint will be expressed as a partial width

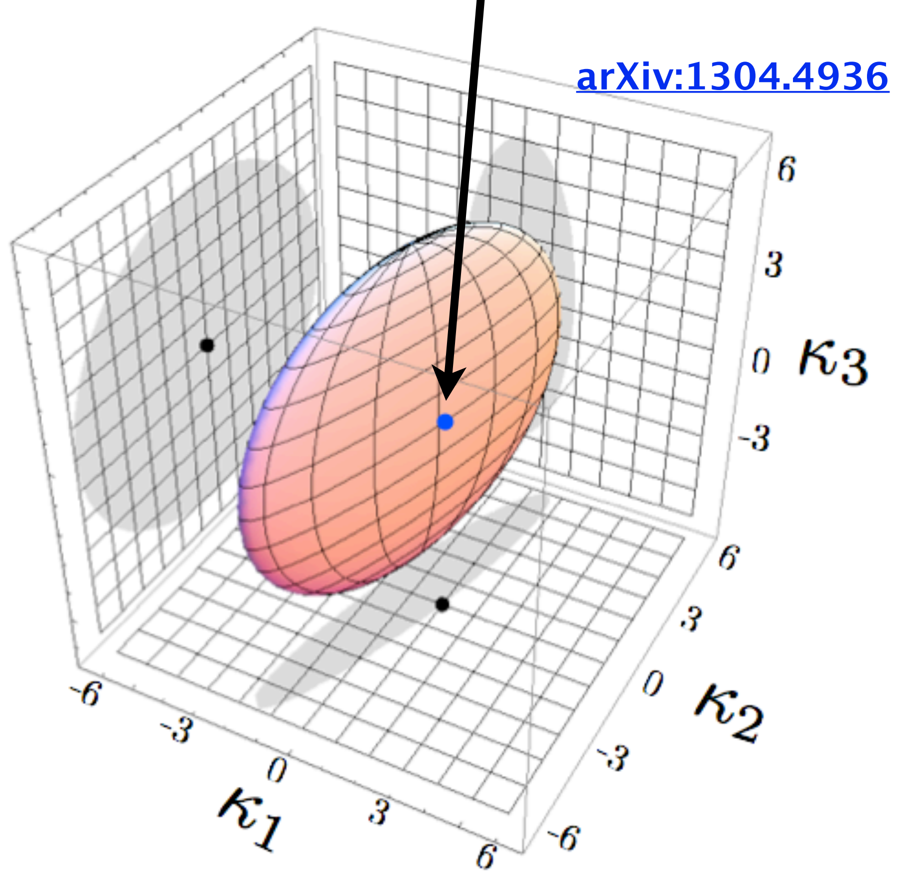
$$\Gamma(X \to ZZ) = \Gamma_{SM} \sum_{i,j} \gamma_{ij} \kappa_i \kappa_j$$

Process	γ_{11}	γ_{22}	γ_{33}	γ_{12}
$X \to ZZ$ (DF)	1	0.090	0.038	−0.250
$X \to ZZ$ (SF)	1	0.081	0.032	−0.243
$X \to \gamma\gamma$	0	1	1	0
$X \to WW$	1	0.202	0.084	−0.379

r13 and r23 will be 0 through the phase space integration.

$(k_1, k_2, k_3)=(1,0,0)$:
Standard model tree level process

arXiv:1304.4936



Larger(smaller) total rate will make the ellipsoid inflated (deflated), but the shape of ellipsoid remains same.

- Mathematically, from the phase space integrations for a width of a boson X, we get the analytical expression for r_{ij}.
(also in Sara Bolognesi et.al., arXiv:1208.4018v1)

- r_{13}, r_{23} will be 0 since terms in $|M|^2$ proportional to k_1k_3 or k_2k_3 are parity odd.

- But with cuts (limitation on the phase space integrations)
1. r_{ij} will be changed.
2. There may be non-zero r_{13} ,r_{23} terms through incomplete phase-space integration.

- If the cuts are even under parity (pt cut, eta cut, invariant-mass cut), then even after cuts, r_{13} ,r_{23} will be still 0.

Geolocating the Higgs boson!

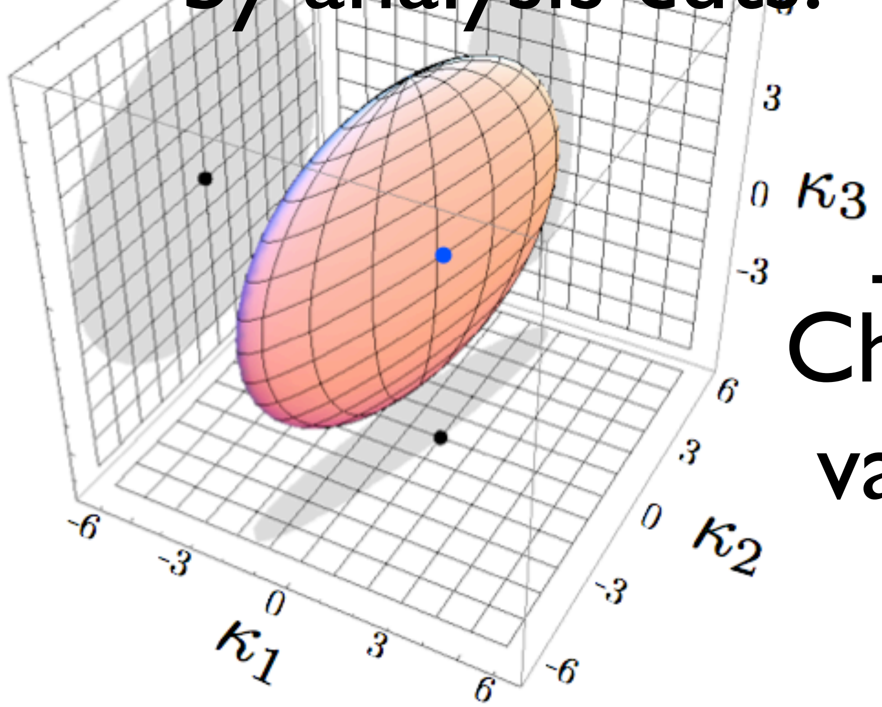
$$\mathcal{L} = X\left[\kappa_1 \frac{m_Z^2}{v} Z_\mu Z^\mu + \frac{\kappa_2}{2v} F_{\mu\nu}F^{\mu\nu} + \frac{\kappa_3}{2v} F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

$$\Gamma(X \to ZZ) = \Gamma_{SM} \sum_{i,j} \gamma_{ij}\kappa_i\kappa_j$$ (antipodal sym)

Process	γ_{11}	γ_{22}	γ_{33}	γ_{12}
$X \to ZZ$ (DF)	1	0.090	0.038	-0.250
$X \to ZZ$ (SF)	1	0.081	0.032	-0.243
$X \to \gamma\gamma$	0	1	1	0
$X \to WW$	1	0.202	0.084	-0.379
after cuts				
$X \to ZZ$ (DF)	1	0.101	0.037	-0.277

Will be distorted by analysis cuts.

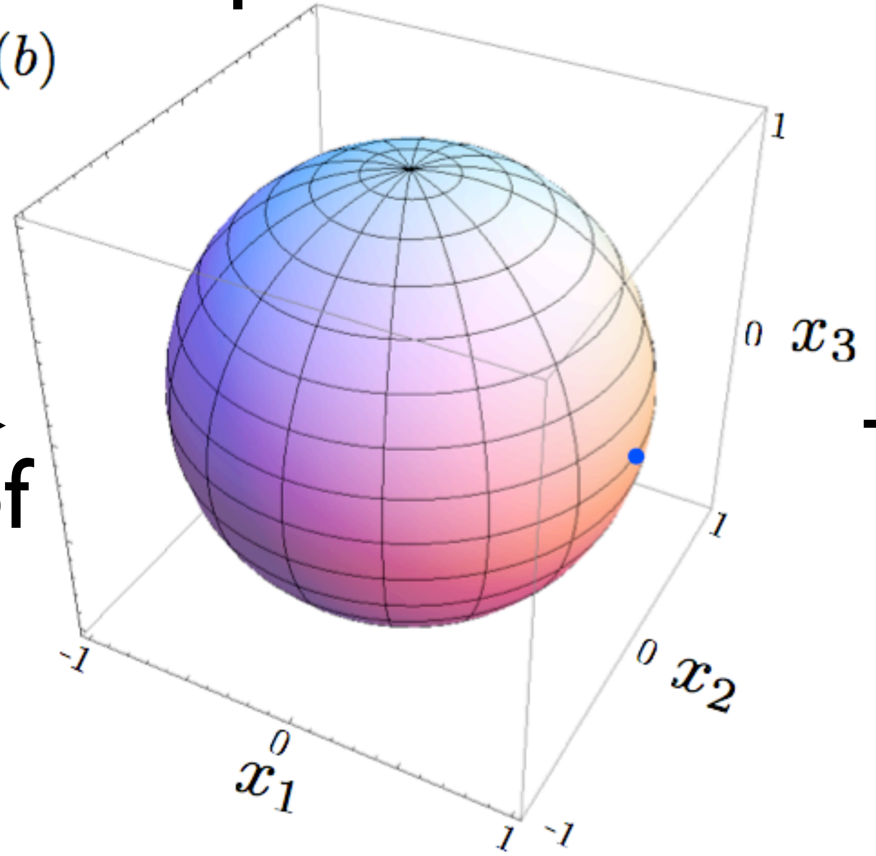
(a)



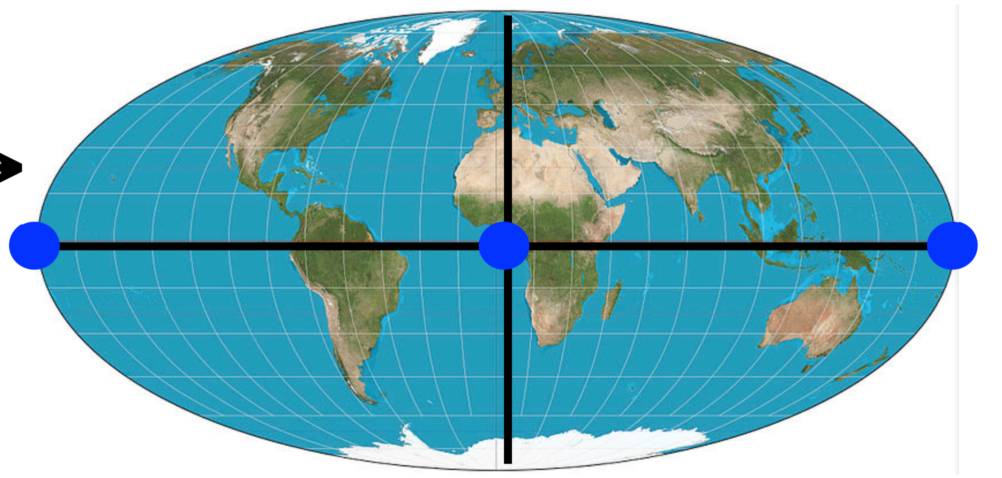
Change of variables

3D presentation

(b)



2D presentation, using Mollweide projection.



$(\kappa_1, \kappa_2, \kappa_3) \leftrightarrow (-\kappa_1, -\kappa_2, -\kappa_3)$

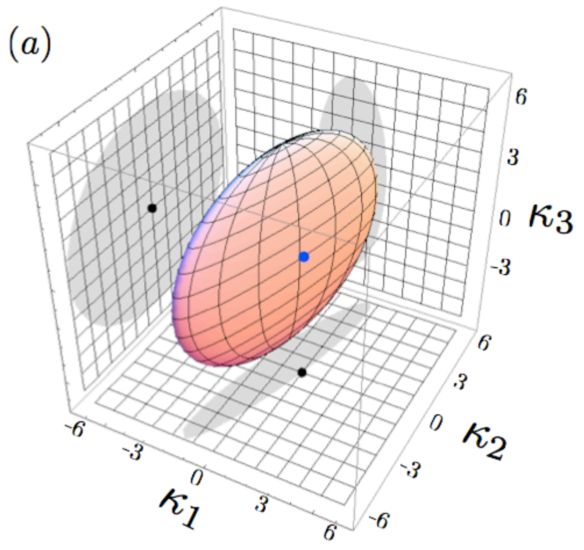
Geolocating the Higgs boson!

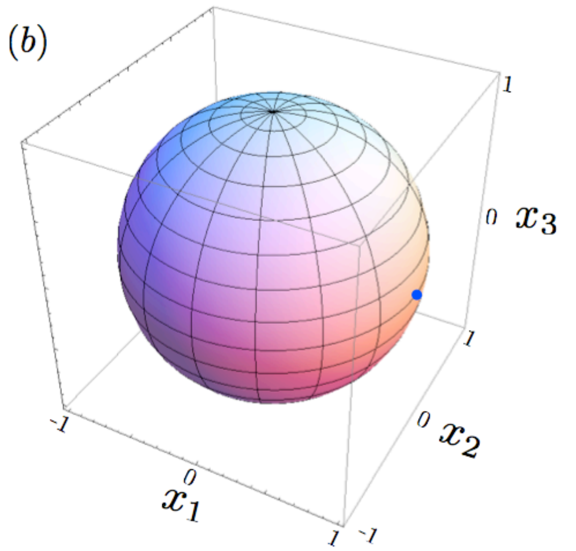
- There will be many ways to do a change of variable.
  In our case, we do the simple triangular transform: $x_i = \sum O_{ij}\kappa_j$

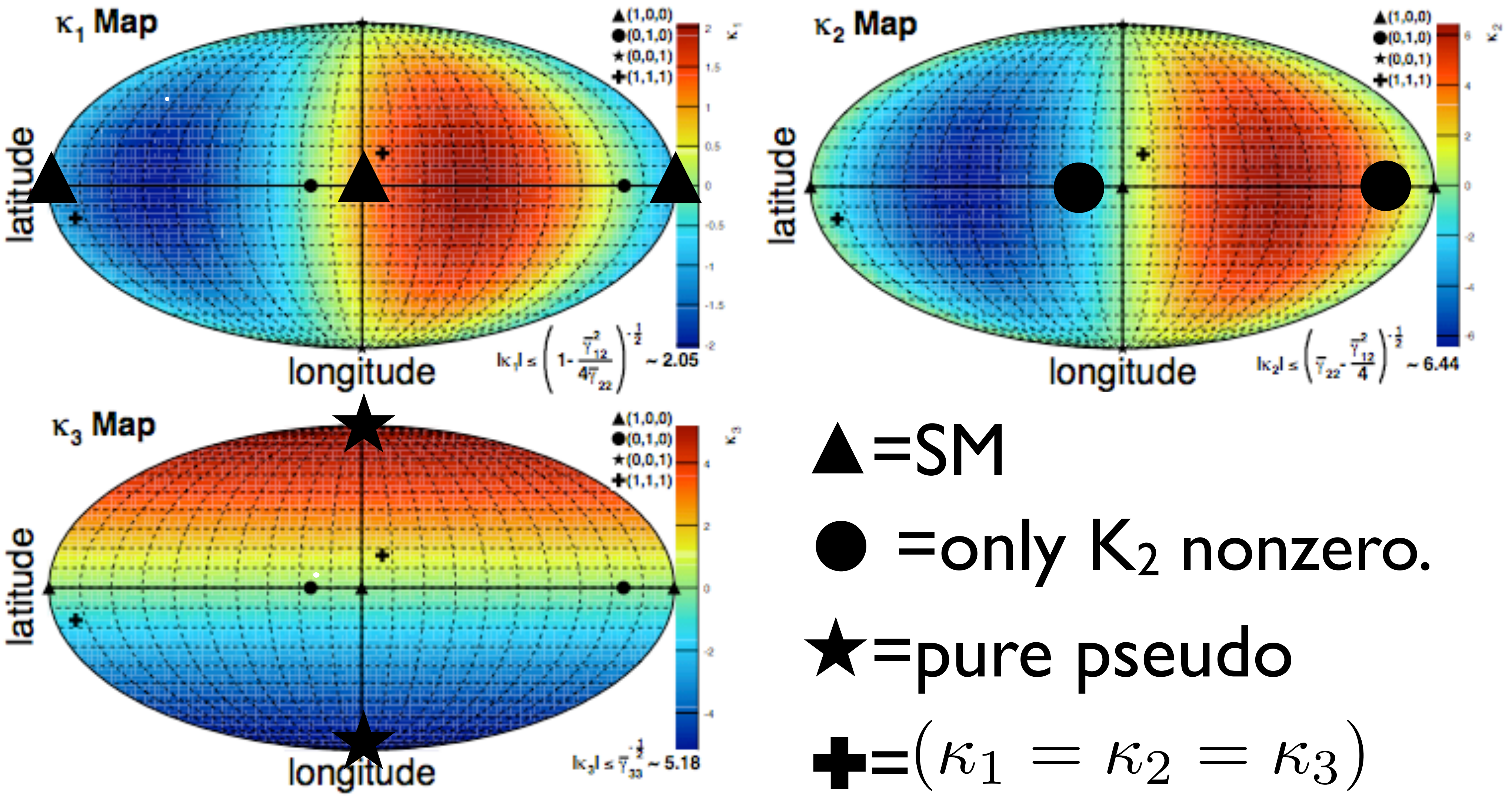
$$
\begin{aligned}
O_{1i} &= \gamma_{1i}/\sqrt{\gamma_{11}}, \quad (i=1,2,3),\\
O_{2i} &= \frac{\gamma_{11}\gamma_{2i}-\gamma_{12}\gamma_{1i}}{\sqrt{(\gamma_{11}\gamma_{22}-\gamma_{12}^2)\gamma_{11}}}, \quad (i=2,3),\\
O_{33} &= \sqrt{\det||\gamma_{ij}||/(\gamma_{11}\gamma_{22}-\gamma_{12}^2)}.
\end{aligned}
$$

$$x_1 = \kappa_1 - 0.25\kappa_2$$
$$x_2 = 0.17\kappa_2,\ x_3 = 0.19\kappa_3$$

arXiv:1304.4936







▲=SM

● =only K2 nonzero.

★=pure pseudo

✚=$(\kappa_1 = \kappa_2 = \kappa_3)$

Cut effects

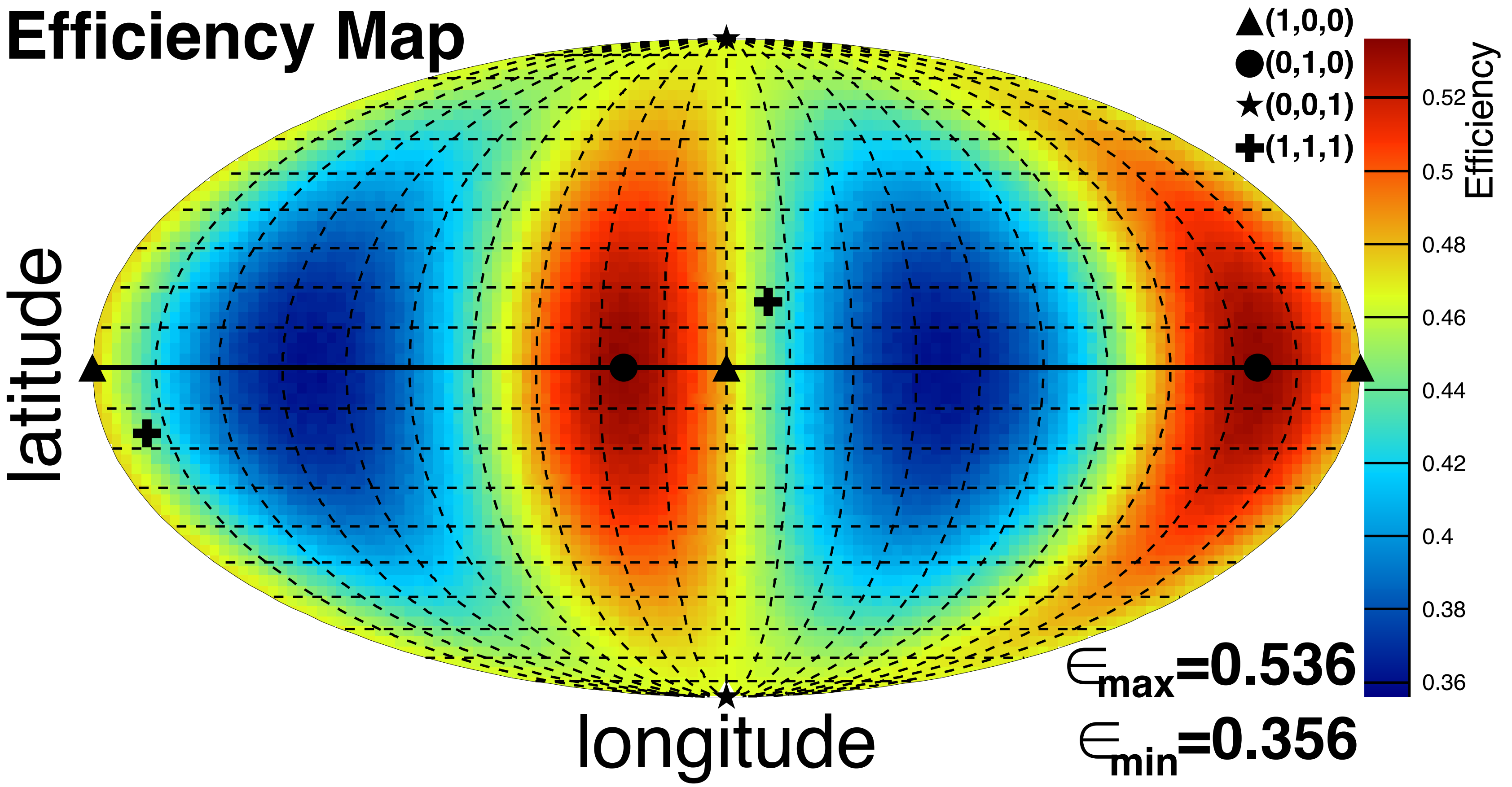
- What we observed is the one after analysis cuts.
- Efficiencies of cuts are different point by point on higgs' ellipsoid from 36% to 53%. Thus we need to consider cut effects when we make higgs' ellipsoid.

analysis cuts:

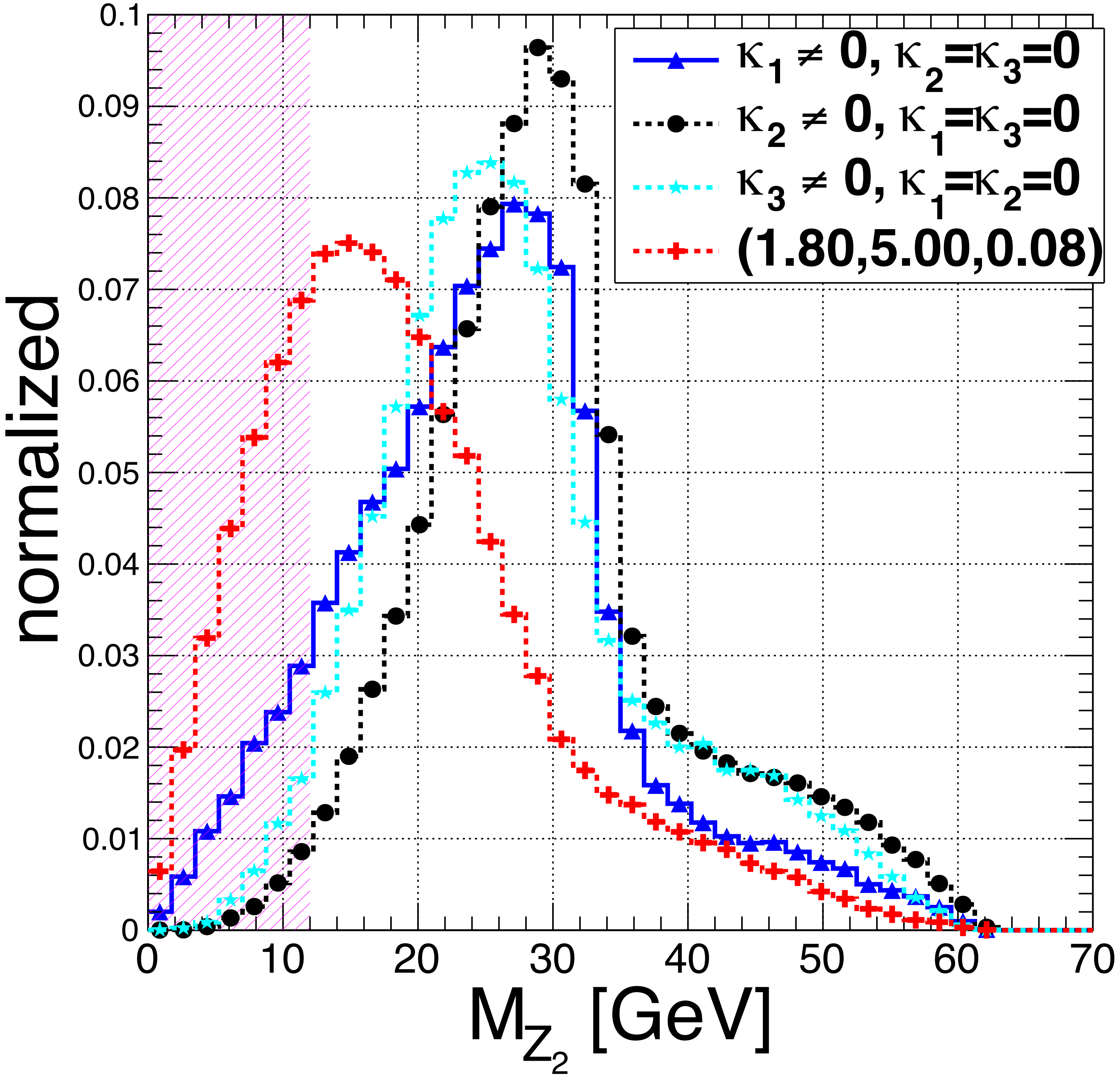
electrons: Pt> 7GeV and |eta|<2.5

mouns: Pt> 5GeV and |eta|<2.4

M_{Z1}> 40 GeV, M_{Z2} > 10GeV



Cut effects



- The main source of the change of efficiency on the ellipsoid seems to be the M_{z2} (the smaller invariant mass of two leptons).
- Actually the M_{z2} is one of the strong discriminator for higgs property studies.

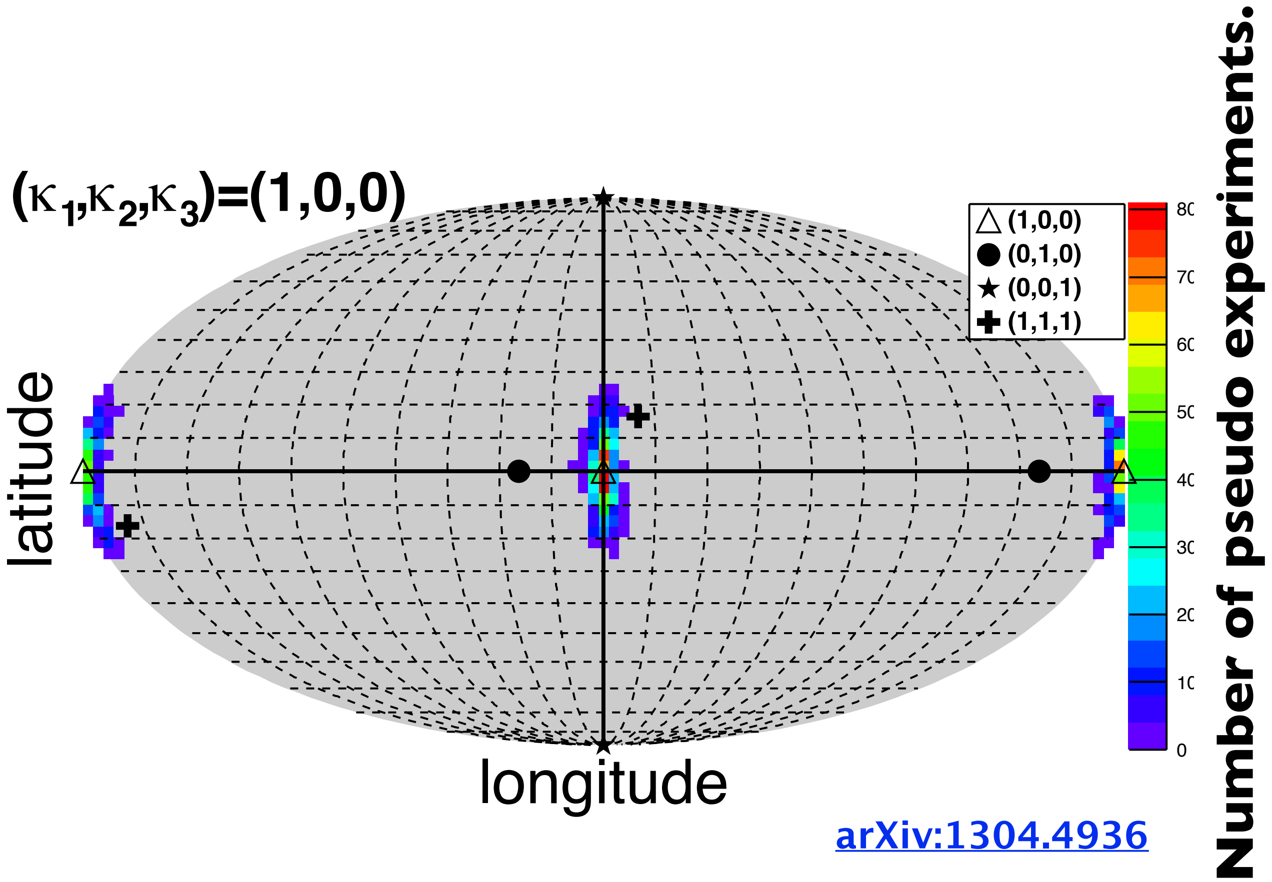
(For example, Choi, Miller, Muhlleitner, and Zerwas, 2003, Godbole, Miller, and Muhlleitner, 2007 , Boughezal, LeCompte, and F. Petriello, 2012, ...

Geolocating the Higgs boson!

- We simulated 1000 pseudo experiments.
  (for Opposite flavor 300 events after analysis cuts.)

- When data comes from
  a pure standard model higgs case,

$$\mathcal{L} = X\left[\kappa_1 \frac{m_Z^2}{v} Z_\mu Z^\mu \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \right]$$

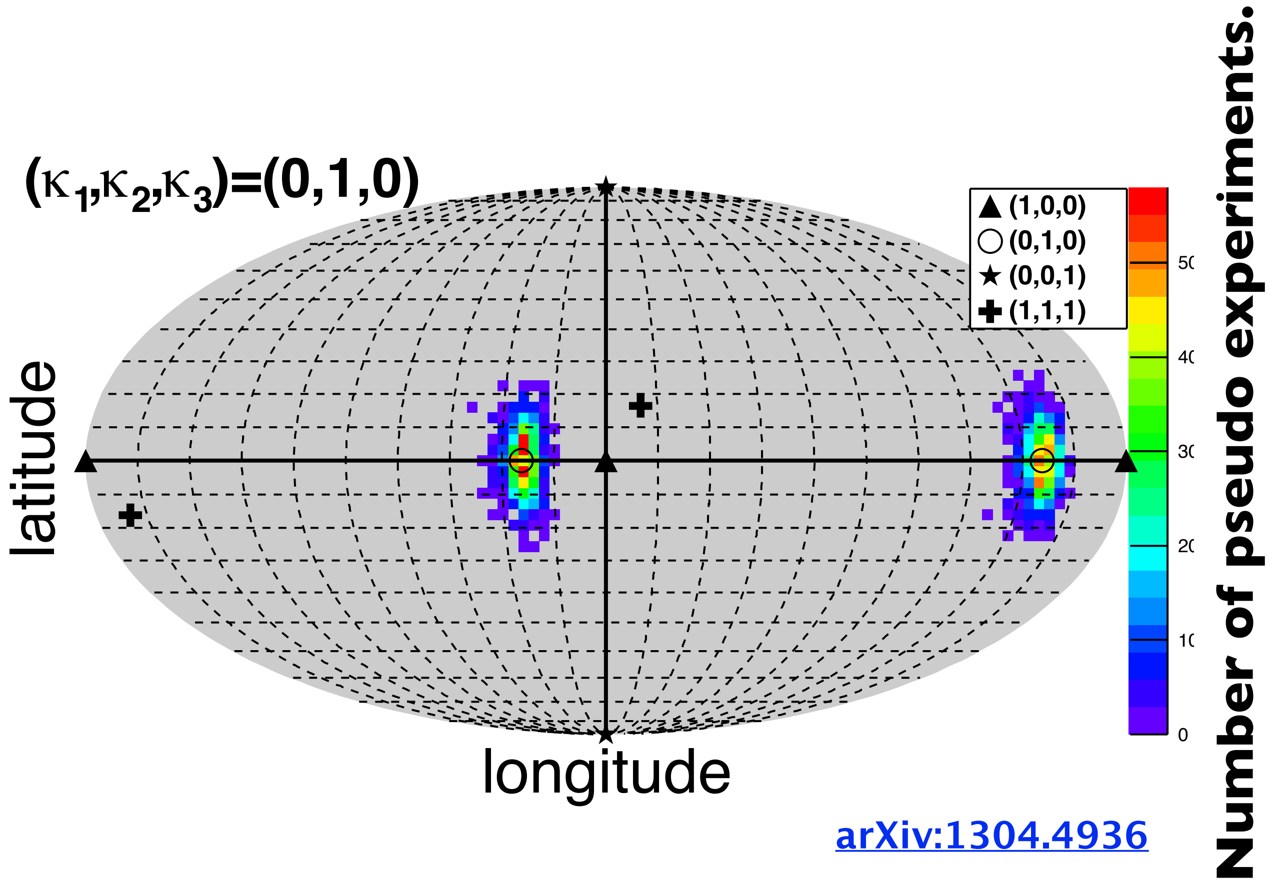


arXiv:1304.4936

Geolocating the Higgs boson!

- When data comes from XFF term,

$$\mathcal{L} = X \left[\qquad\qquad \frac{\kappa_2}{2v} F_{\mu\nu} F^{\mu\nu} \qquad\qquad \right]$$

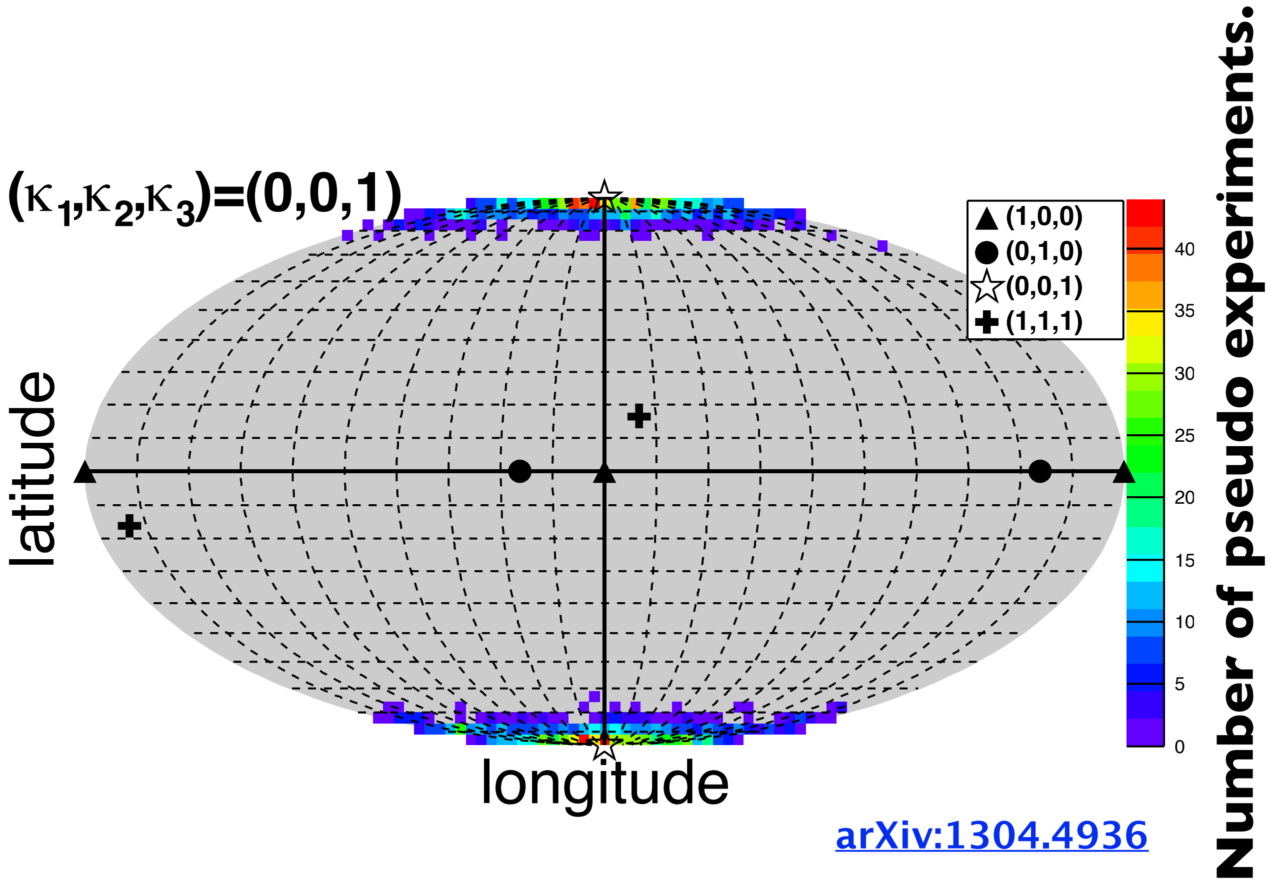


arXiv:1304.4936

Geolocating the Higgs boson!

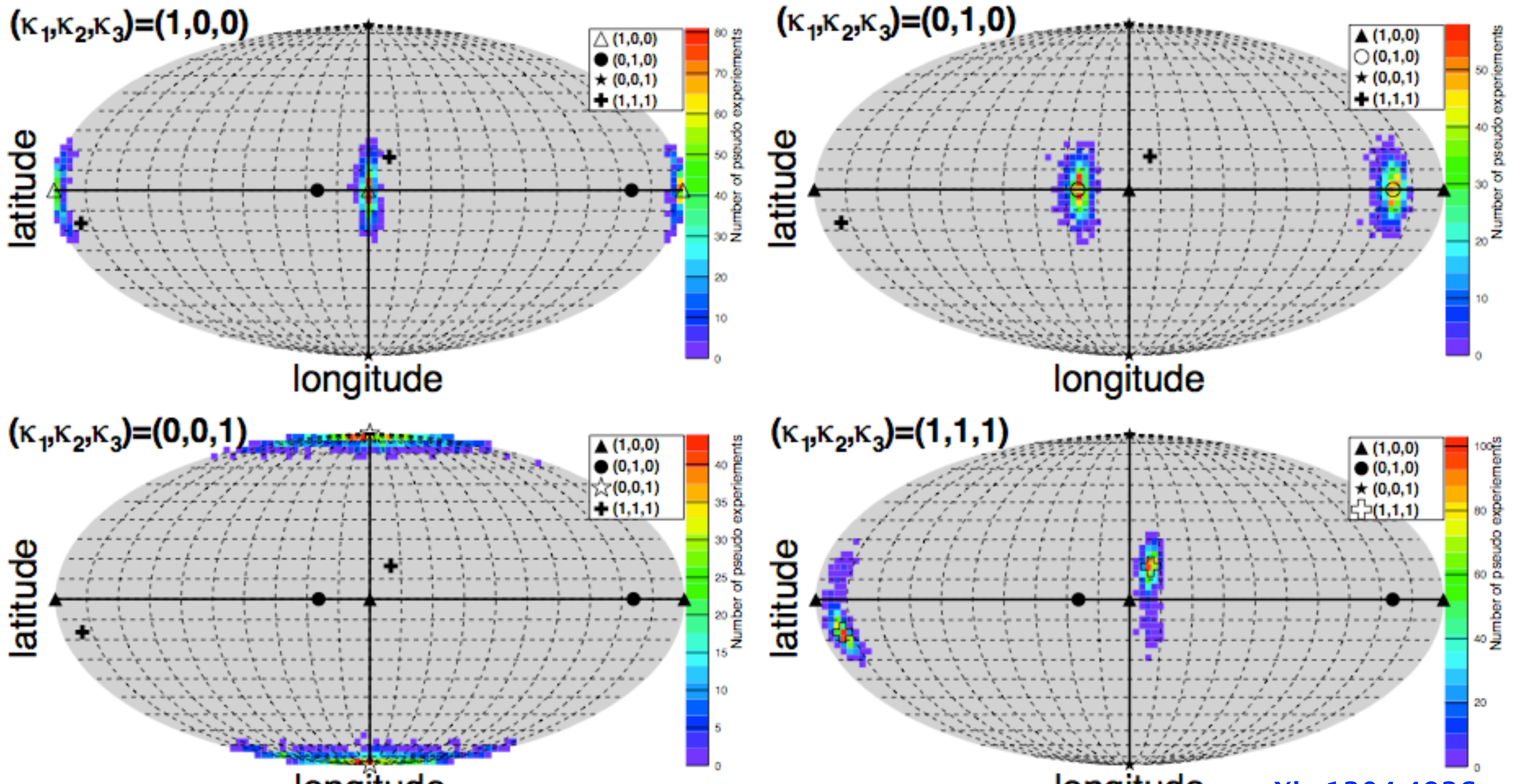
- When data comes from a pure pseudo scalar case,

$$\mathcal{L} = X \left[\qquad\qquad\qquad\qquad \frac{\kappa_3}{2v} F_{\mu\nu}\tilde{F}^{\mu\nu} \right]$$



arXiv:1304.4936

Geolocating the Higgs boson!



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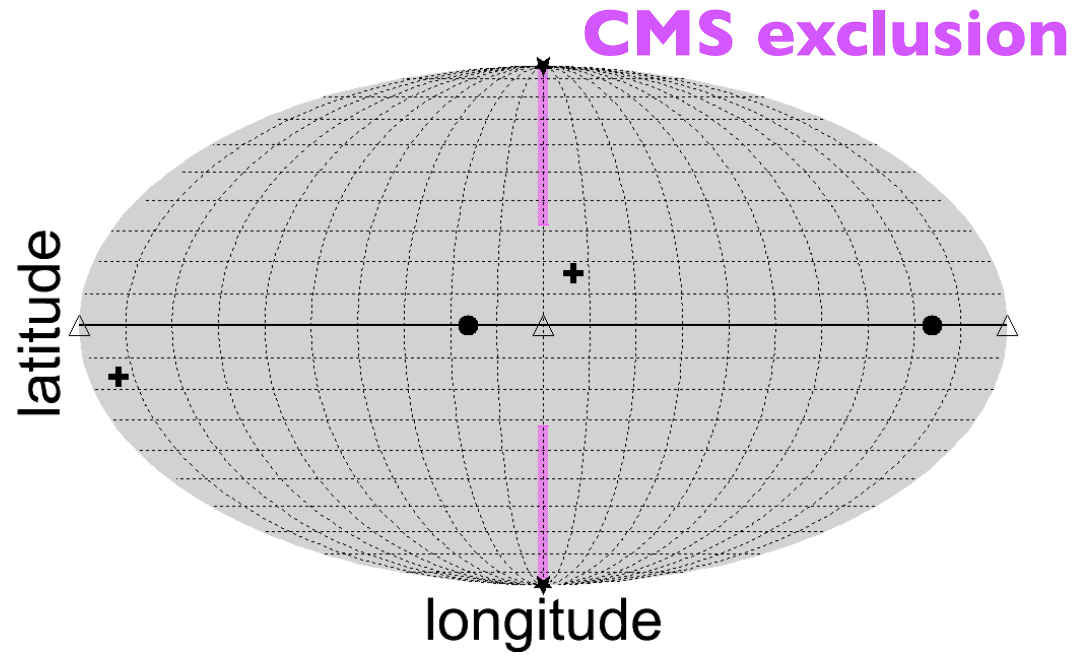
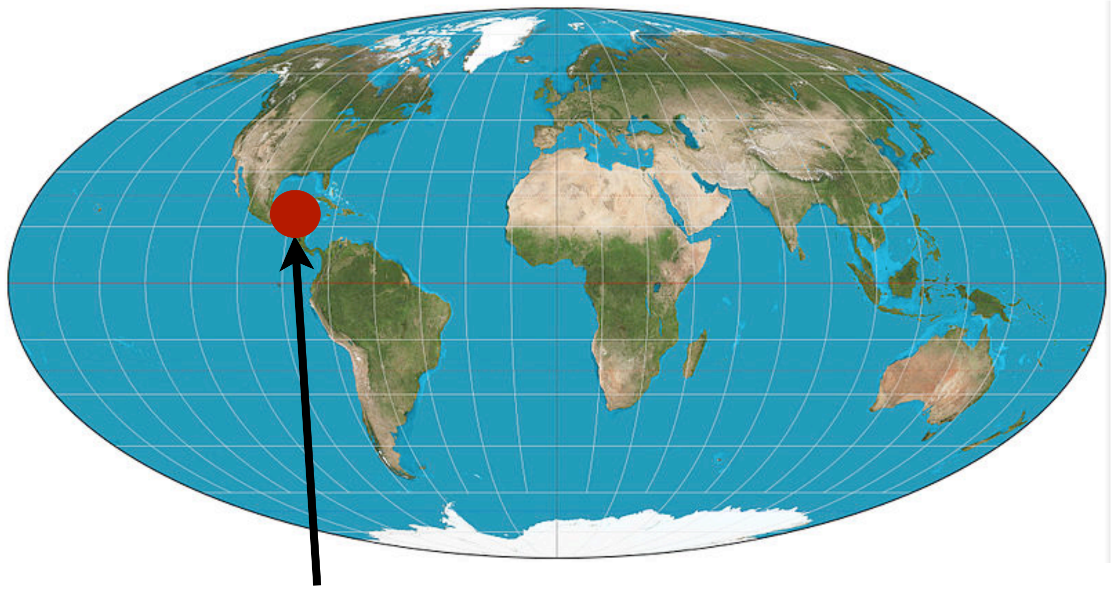


Exclusion region (exclusion-line) is plotted from the results of CMS-PAS-HIG-13-002

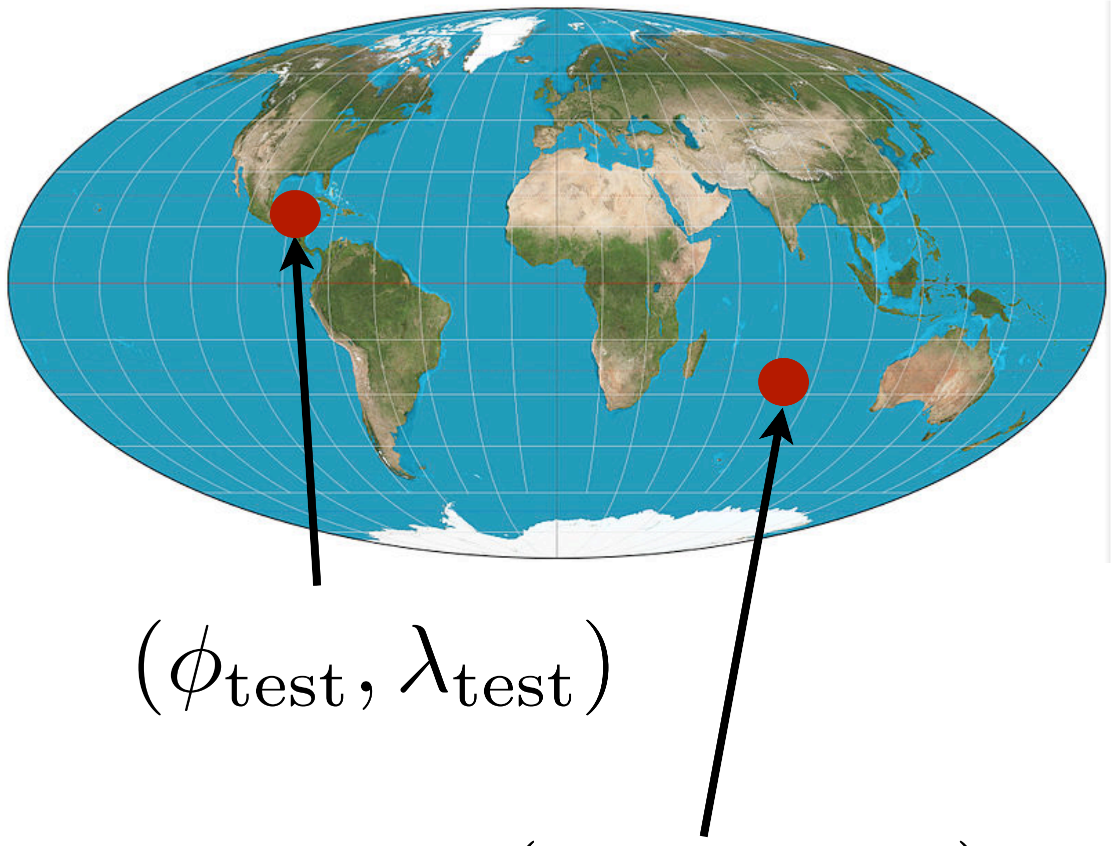
Matrix Element Re-weighting



$(\phi_{\text{test}}, \lambda_{\text{test}})$

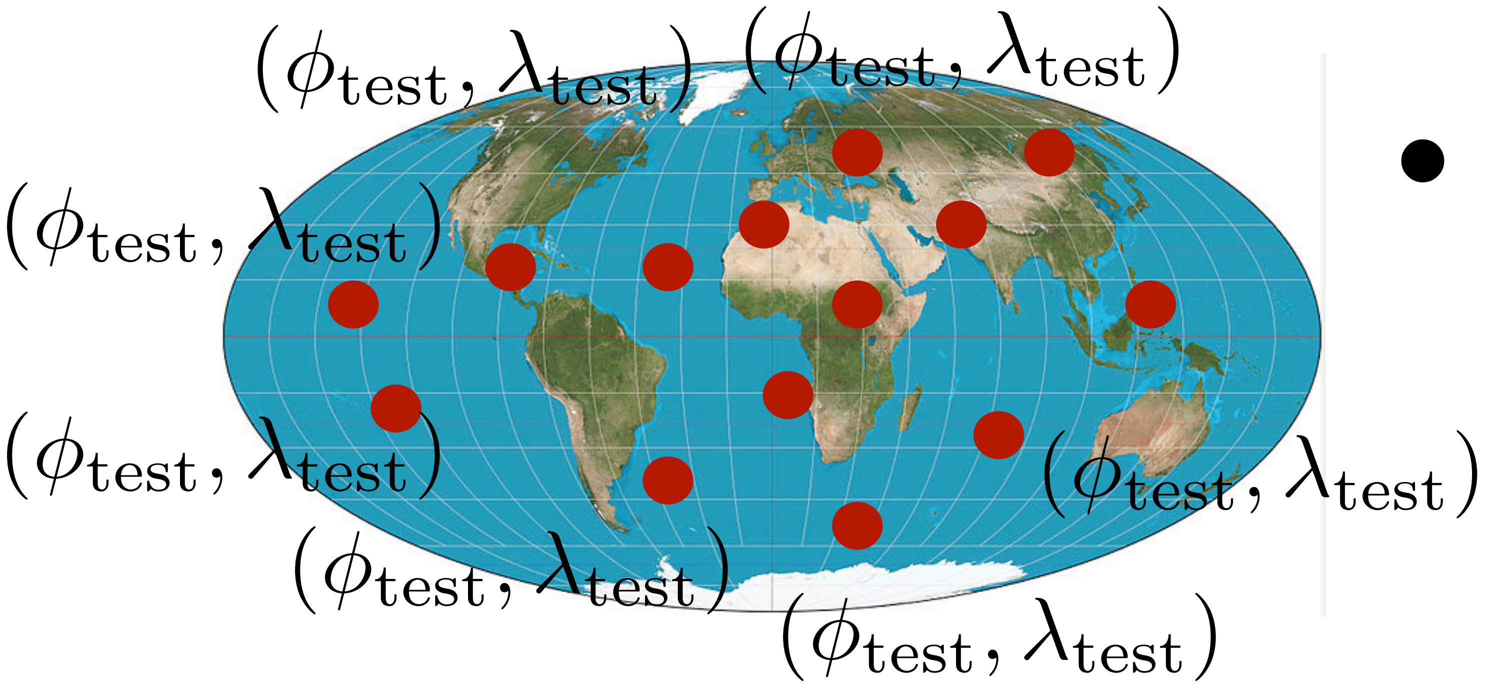
- Do we need to generate pseudo-experiments on each parameter point on this huge space???

Matrix Element Re-weighting



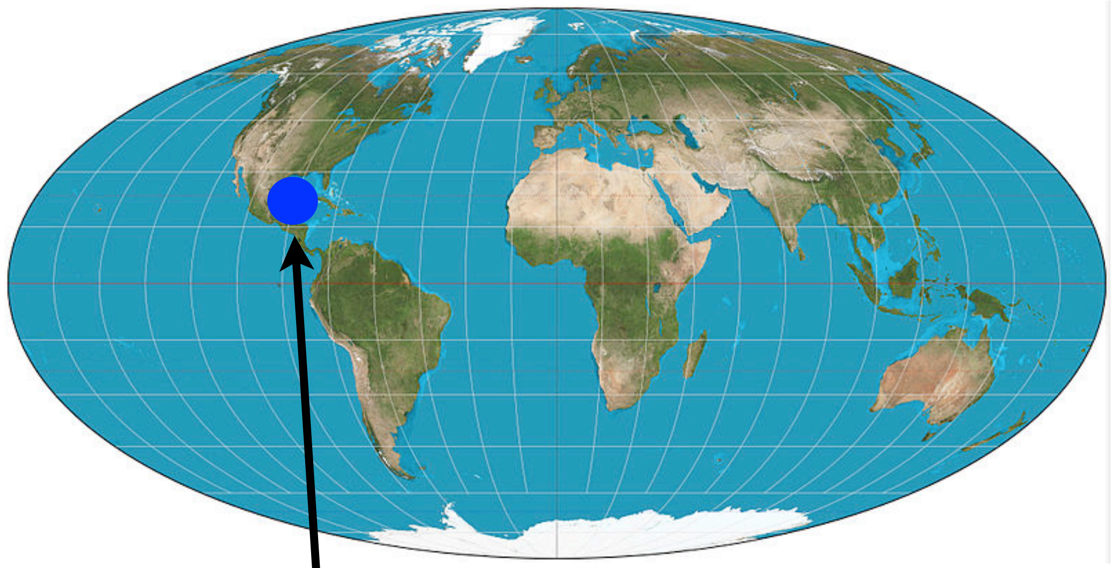
- Do we need to generate pseudo-experiments on each parameter point on this huge space???

Matrix Element Re-weighting



- Do we need to generate pseudo-experiments on each parameter point on this huge space???

Matrix Element Re-weighting

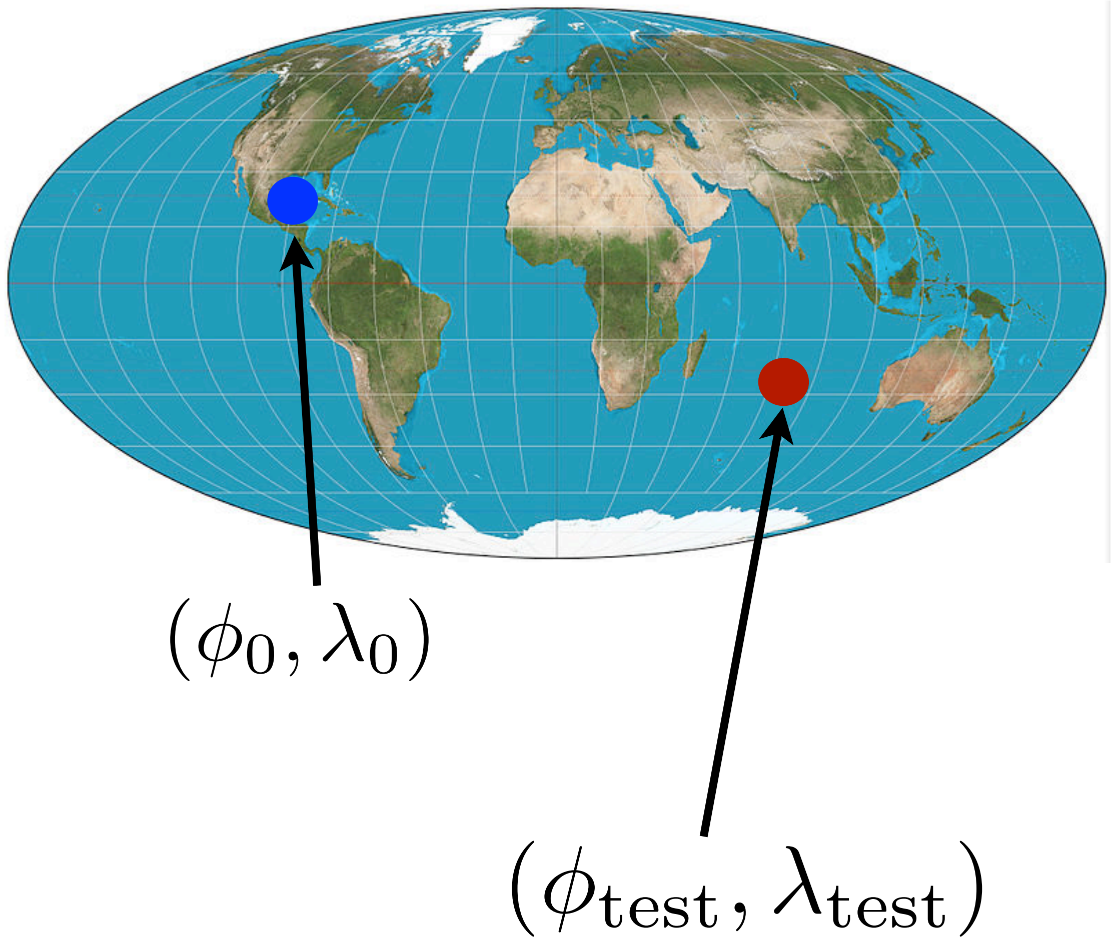


(ϕ_0, λ_0)

- Even though we need to cover the whole parameter space, we don't need generate pseudo experiments on each parameter point.

- We just need to generate pseudo-exps on a specific point. (here, labeled 0 for example.)

Matrix Element Re-weighting

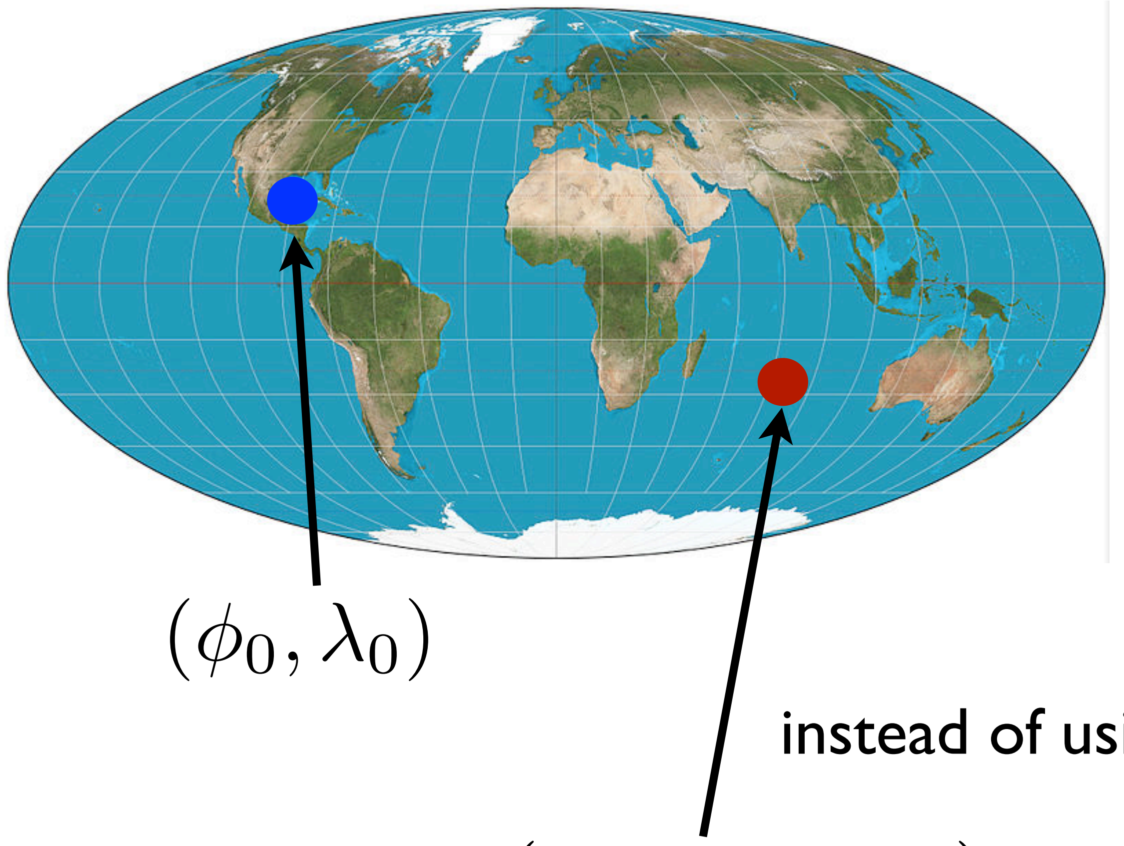


- We just need to generate pseudo-exps on a specific point. (here, labeled 0 for example.)
- Now for another parameter point (labeled as "test"), we simply need to re-weight events by the ratio:

$$\frac{P(\mathbf{p}_i, \phi_{test}, \lambda_{test})}{P(\mathbf{p}_i, \phi_0, \lambda_0)}$$

where p_i is the i-th event in sample generated on (ϕ_0, λ_0)

Matrix Element Re-weighting



(ϕ_0, λ_0)

$(\phi_{\text{test}}, \lambda_{\text{test}})$

- Now for another parameter point (labeled as “test”), we simply need to re-weight events by the ratio: $\dfrac{P(\mathbf{p}_i, \phi_{test}, \lambda_{test})}{P(\mathbf{p}_i, \phi_0, \lambda_0)}$
- Now to calculate a likelihood L;

$$\ln \mathcal{L}(\phi, \lambda) = \sum_i \ln P(\bar{\mathbf{p}}_i, \phi, \lambda)$$

instead of using $\bar{p}_i$, which is the i-th event in sample generated on $(\phi_{\text{test}}, \lambda_{\text{test}})$

$$\ln \mathcal{L}(\phi, \lambda) = \sum_i \boxed{\frac{P(\mathbf{p}_i, \phi_{test}, \lambda_{test})}{P(\mathbf{p}_i, \phi_0, \lambda_0)}} \ln P(\mathbf{p}_i, \phi, \lambda)$$

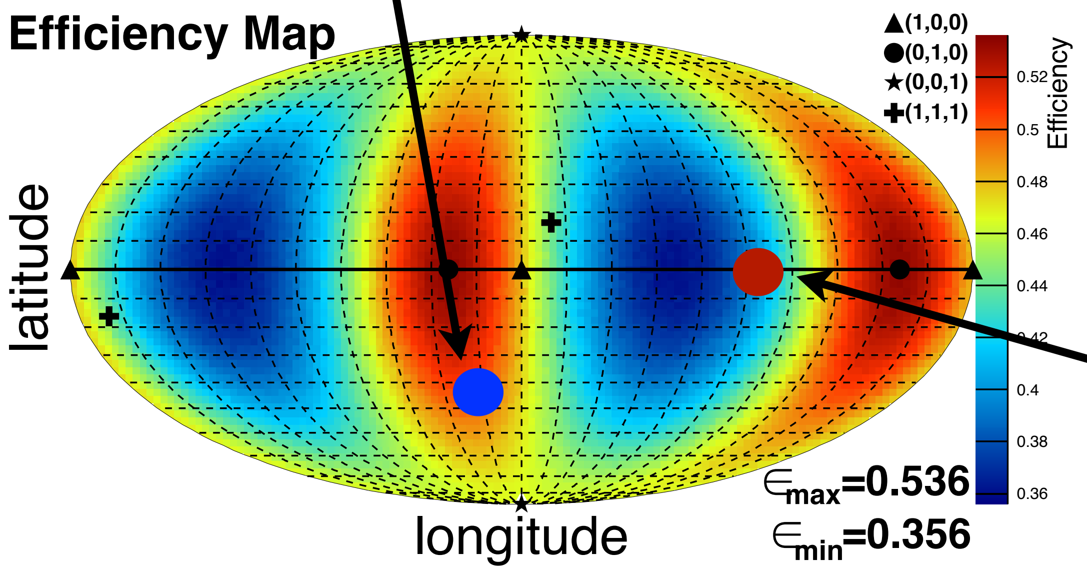
where p_i is the i-th event in sample generated on (ϕ_0, λ_0)

Matrix Element Re-weighting

- We can also apply to get the efficiencies of analysis cuts on the higgs' ellipsoid by re-weighting events.

$$\epsilon(\phi_{\text{test}}, \theta_{\text{test}}) = \frac{1}{N} \sum_i \frac{P(p_i, \phi_{\text{test}}, \theta_{\text{test}})}{P(p_i, \phi_0, \theta_0)} \Theta(p_i|\text{cuts})$$

(ϕ_0, λ_0)



$(\phi_{\text{test}}, \lambda_{\text{test}})$

with a normalization factor

$$N = \sum_i \frac{P(p_i, \phi_{\text{test}}, \theta_{\text{test}})}{P(p_i, \phi_0, \theta_0)}$$

Other spheres

- Now we allow to have complex kappas.

$$\mathcal{L} = X\left[\boxed{\kappa_1 \frac{m_Z^2}{v} Z_\mu Z^\mu} + \boxed{\frac{\kappa_2}{2v} F_{\mu\nu} F^{\mu\nu}} + \boxed{\frac{\kappa_3}{2v} F_{\mu\nu} \tilde{F}^{\mu\nu}}\right]$$

- Scenario 1: k_1=0, other kappas can be complex.
- Scenario 2: k_2=0. Mixing of SM scalar with pseudo scalar
- Scenario 3: k_3=0. Arbitrary CP even scalar

Example

$$\mathcal{L} = X\left[\boxed{\kappa_1 \frac{m_Z^2}{v} Z_\mu Z^\mu} + \boxed{\cancel{\frac{\kappa_2}{2v} F_{\mu\nu} F^{\mu\nu}}} + \boxed{\frac{\kappa_3}{2v} F_{\mu\nu} \tilde{F}^{\mu\nu}}\right]$$

- Degree of freedoms: 2 magnitude + 2 phase
- One overall phase is irrelevant.
- We can call a relative phase as ϕ_{13}.
- Rate restrict overall magnitude of couplings.
- One of remaining degree of freedoms is ratio of couplings:

$$x_{13} = \frac{|\kappa_3|^2}{|\kappa_1|^2 + |\kappa_3|^2} = \sin^2\theta_{13}$$

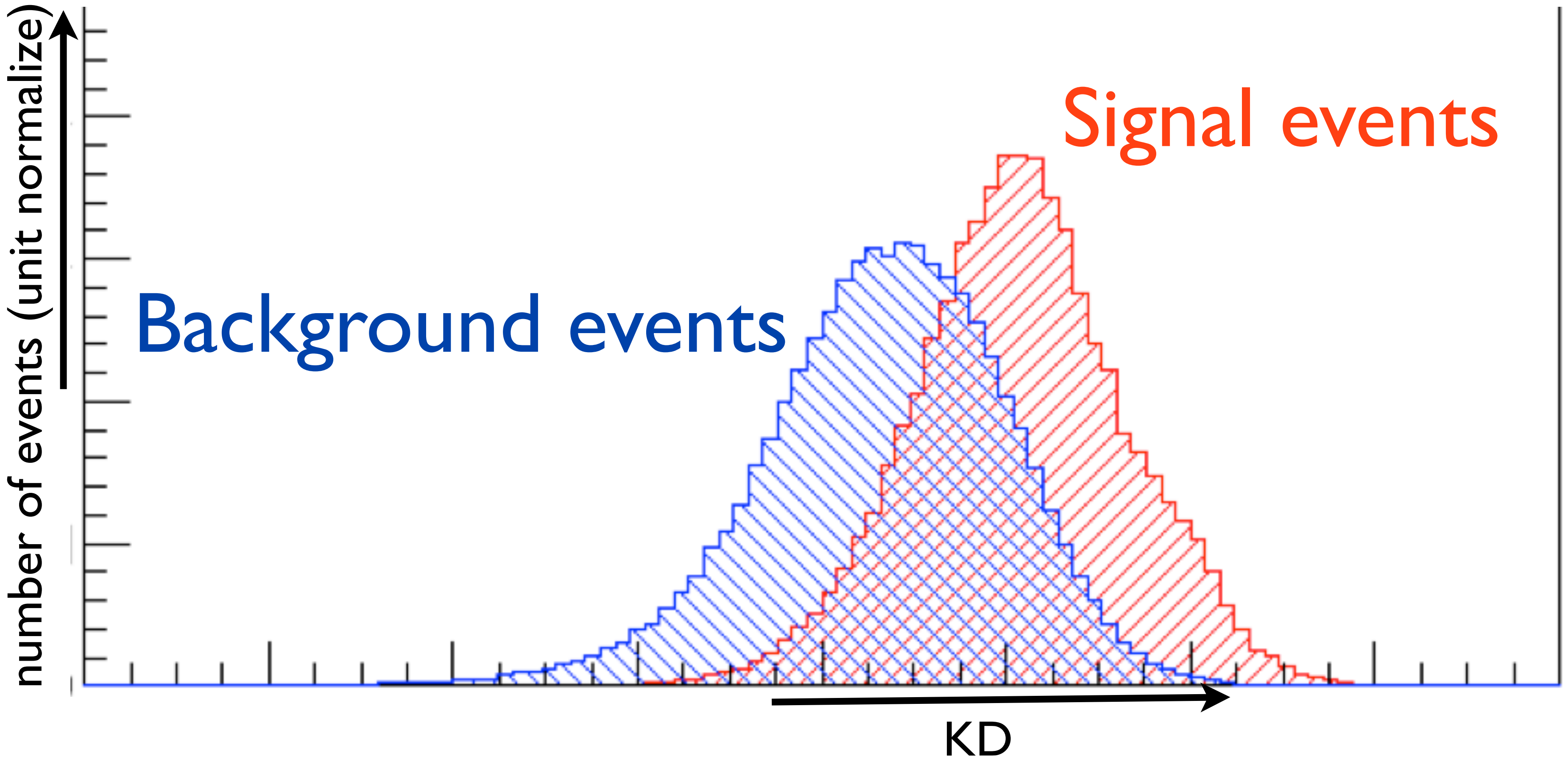
Conclusions

- Matrix Element Method can be very useful, especially when we can reconstruct events. We can go beyond four leptons
  -X to two photons
  -X to WW to two leptons+invisibles: Need to integrate over unknown neutrinos' momentum.(A. Freitas, J. S. Gainer, arxiv:1212.3589)
- While many operators may affect the coupling between spin 0 particle and bosons, it is reasonable to focus on three lowest dimensional operators from each class of couplings.
- With measured total rate, we can eliminate one degree of freedom and can give constraints among couplings $k_i \rightarrow$ Analysis on the closed hyper surface.
- We can re-weight events by matrix elements to cover various study points.

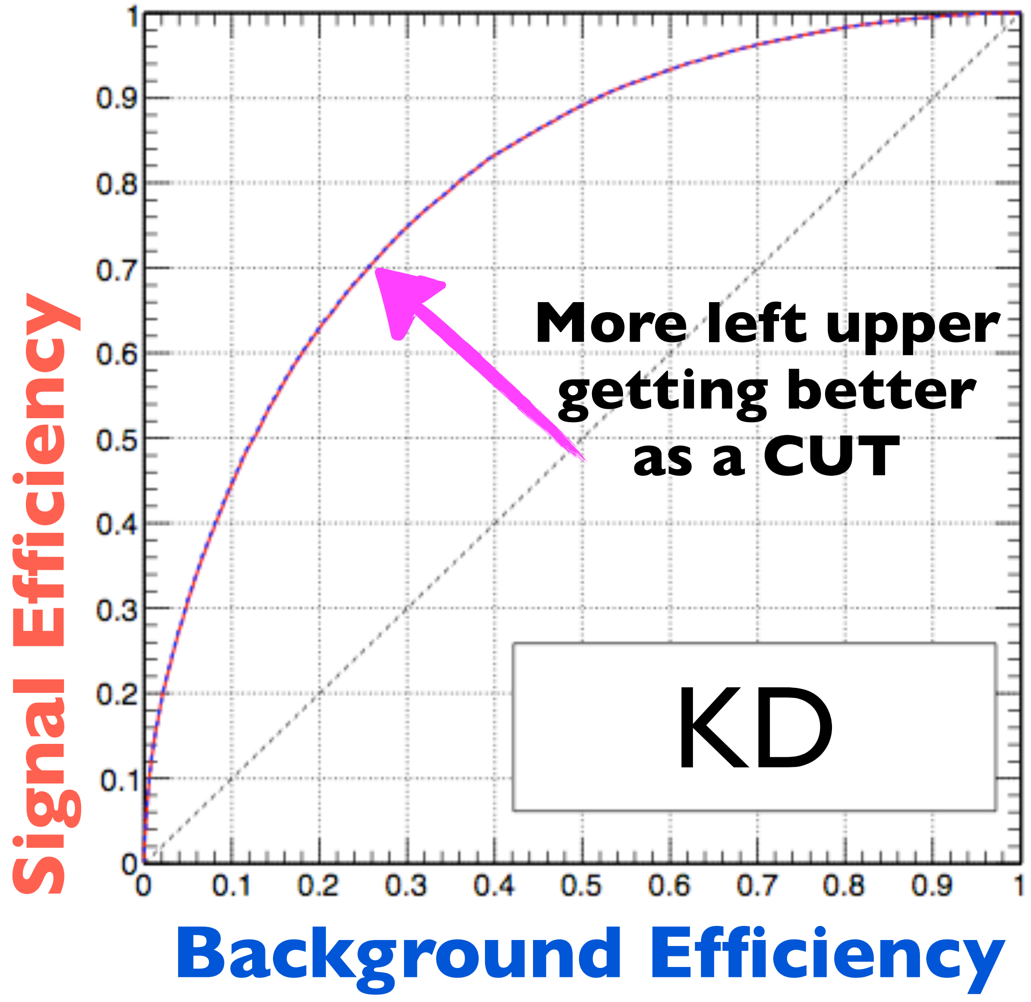
MEM as a cut

arXiv:1210.0896

- How much can we separate signals from background?

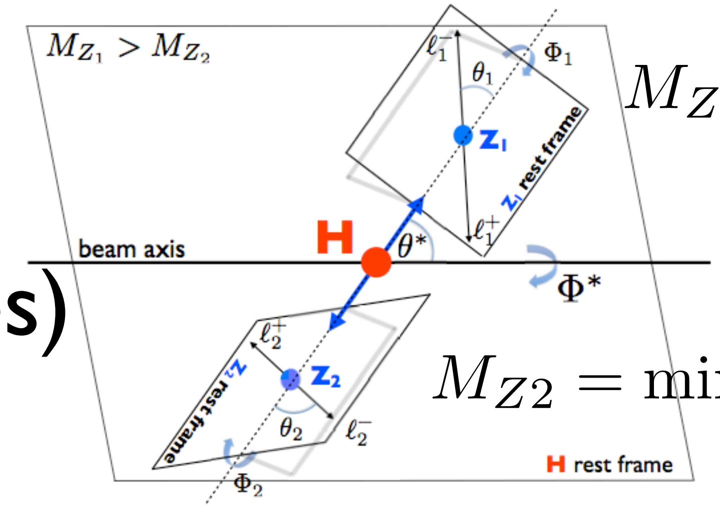


ROC curve



Comparison with projected variables

We plotted several projected variables
(angular variables, Invariant mass variables)



$$M_{Z1} = \max(m_{\ell^-\ell^+}, m_{\ell'^-\ell'^+})$$

$$M_{Z2} = \min(m_{\ell^-\ell^+}, m_{\ell'^-\ell'^+})$$

ROC curve