

SUSY: From the Basics to Collider Phenomenology

Sven Heinemeyer, IFCA (Santander)

Louvain, 05/2007

1. Introduction to SUSY
2. SUSY Lagrangians and the MSSM
3. “Simplified versions” and special sectors
4. SUSY Phenomenology

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SUSY lectures (I): Introduction to SUSY

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1. Motivation for SUSY
2. Symmetry and Algebra
3. Superfields and Superspace
4. Supersymmetric Lagrangians
5. Soft SUSY-breaking

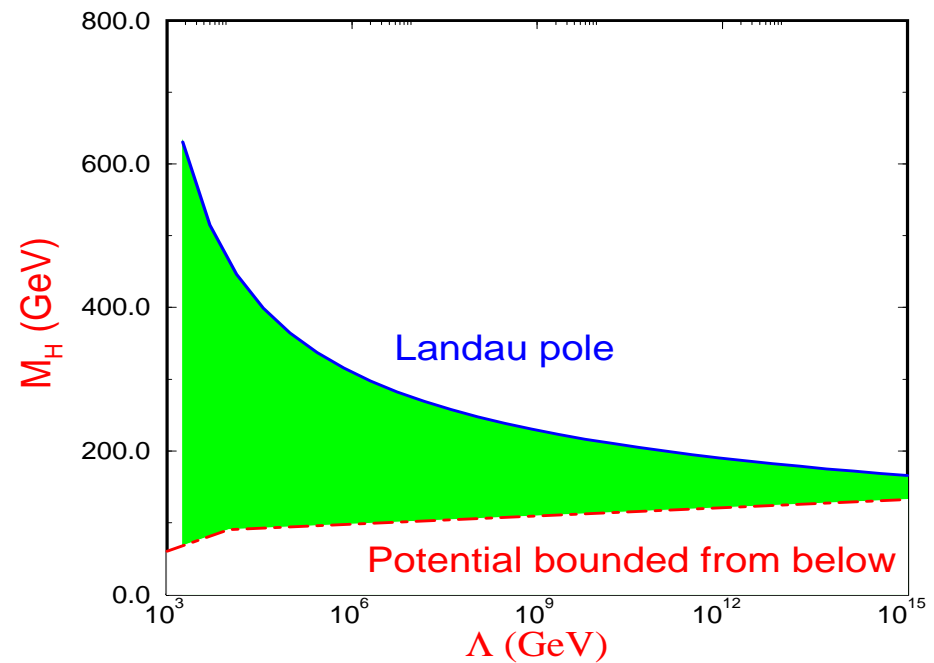
1. Motivation for SUSY

The Standard Model (SM) cannot be the ultimate theory

- The SM does not contain gravity
- Further problems: **Hierarchy problem**

Up to which energy scale Λ can the SM be valid?

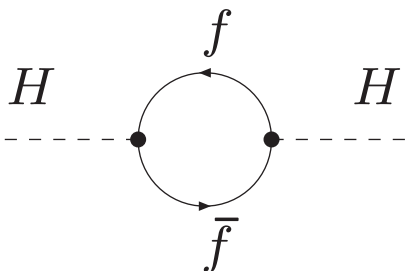
- $\Lambda < M_{\text{Pl}}$: inclusion of gravity effects necessary
- stability of Higgs potential: \Rightarrow
- **Hierarchy problem** :
Higgs mass unstable
w.r.t. quantum corrections
 $\delta M_H^2 \sim \Lambda^2$
(but what does this mean?)



Mass is what determines the properties of the **free propagation** of a particle

Free propagation: $\text{---} \overset{H}{\text{---}} \text{---}$ inverse propagator: $i(p^2 - M_H^2)$

Loop corrections: $\text{---} \overset{H}{\text{---}} \text{---}$ inverse propagator: $i(p^2 - M_H^2 + \Sigma_H^f)$



QM: integration over all possible loop momenta k

dimensional analysis:

$$\Sigma_H^f \sim N_f \lambda_f^2 \int d^4 k \left(\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2} \right)$$

$$\text{for } \Lambda \rightarrow \infty : \quad \Sigma_H^f \sim N_f \lambda_f^2 \left(\underbrace{\int \frac{d^4 k}{k^2}}_{\sim \Lambda^2} + 2m_f^2 \underbrace{\int \frac{dk}{k}}_{\sim \ln \Lambda} \right)$$

\Rightarrow quadratically divergent!

For $\Lambda = M_{\text{Pl}}$:

$$\Sigma_H^f \approx \delta M_H^2 \sim M_{\text{Pl}}^2 \quad \Rightarrow \quad \delta M_H^2 \approx 10^{30} M_H^2$$

(for $M_H \lesssim 1 \text{ TeV}$)

- no additional symmetry for $M_H = 0$
- no protection against large corrections

\Rightarrow Hierarchy problem is instability of small Higgs mass to large corrections in a theory with a large mass scale in addition to the weak scale

E.g.: Grand Unified Theory (GUT): $\delta M_H^2 \approx M_{\text{GUT}}^2$

Note however: there is another fine-tuning problem in nature, for which we have no clue so far – **cosmological constant**

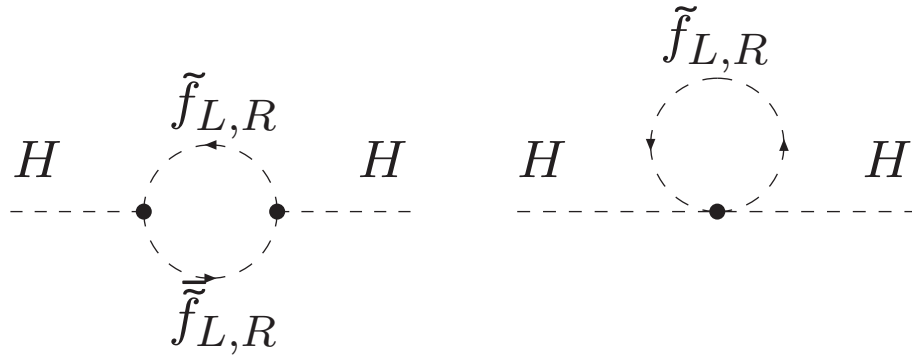
Supersymmetry:

Symmetry between fermions and bosons

$$\begin{aligned} Q|\text{boson}\rangle &= |\text{fermion}\rangle \\ Q|\text{fermion}\rangle &= |\text{boson}\rangle \end{aligned}$$

Effectively: SM particles have **SUSY partners** (e.g. $f_{L,R} \rightarrow \tilde{f}_{L,R}$)

SUSY: additional contributions from scalar fields:



$$\Sigma_H^{\tilde{f}} \sim N_{\tilde{f}} \lambda_{\tilde{f}}^2 \int d^4k \left(\frac{1}{k^2 - m_{\tilde{f}_L}^2} + \frac{1}{k^2 - m_{\tilde{f}_R}^2} \right) + \text{terms without quadratic div.}$$

for $\Lambda \rightarrow \infty$: $\Sigma_H^{\tilde{f}} \sim N_{\tilde{f}} \lambda_{\tilde{f}}^2 \Lambda^2$

⇒ quadratic divergences cancel for

$$N_{\tilde{f}_L} = N_{\tilde{f}_R} = N_f$$
$$\lambda_{\tilde{f}}^2 = \lambda_f^2$$

complete correction vanishes if furthermore

$$m_{\tilde{f}} = m_f$$

Soft SUSY breaking: $m_{\tilde{f}}^2 = m_f^2 + \Delta^2, \quad \lambda_{\tilde{f}}^2 = \lambda_f^2$

$$\Rightarrow \Sigma_H^{f+\tilde{f}} \sim N_f \lambda_f^2 \Delta^2 + \dots$$

⇒ correction stays acceptably small if mass splitting is of weak scale

⇒ realized if mass scale of SUSY partners

$$M_{\text{SUSY}} \lesssim 1 \text{ TeV}$$

⇒ SUSY at TeV scale provides attractive solution of hierarchy problem

2. Symmetry and Algebra

Symmetry: a **group** of transformations that leaves the Lagrangian invariant

Generators of the group fulfill certain **algebra**

Examples:

0. **Angular rotation**: $\psi \rightarrow \psi e^{i\theta^a L_a}$

theory is invariant under rotation

generators: L_a , **algebra**: $[L_a, L_b] = i\epsilon_{abc}L^c$

Quantum numbers: (max. spin)², spin $[l(l+1), m = +l \dots -l]$

1. **Internal symmetry**: $SU(3) \times SU(2) \times U(1)$

gauge symmetry for the description of the strong and electroweak force

generators: T_a , **algebra**: $[T_a, T_b] = if_{abc}T^c$

Quantum numbers: color, weak isospin, hyper charge

2. **Poincaré symmetry** (includes rotation)

space–time symmetries:

Lorentz transformations: $\Lambda^{\mu\nu}$, translations: P^ρ

Quantum numbers: mass, spin

Lorentz group: Representations of Lorentz group are labeled by two 'spins', $j_1, j_2 = 0$, where $j_1, j_2 = 0, \frac{1}{2}, 1, \dots$

Basic representations M_α^β act on:

$(\frac{1}{2}, 0)$: LEFT-handed 2-component Weyl spinor, ψ_α

$(0, \frac{1}{2})$: RIGHT-handed 2-component Weyl spinor, $\bar{\psi}^{\dot{\alpha}}$

The two component Weyl spinors ψ_α (left-handed) and $\bar{\psi}^{\dot{\alpha}}$ (right-handed) transform under Lorentz transformations as follows:

$$\begin{aligned}\psi'_\alpha &= M_\alpha^\beta \psi_\beta; & \bar{\psi}'_{\dot{\alpha}} &= (M^*)_{\dot{\alpha}}^{\dot{\beta}} \bar{\psi}_{\dot{\beta}} \\ \psi'^\alpha &= (M^{-1})_\beta^\alpha \psi^\beta; & \bar{\psi}'^{\dot{\alpha}} &= (M^{*-1})^{\dot{\alpha}}_{\dot{\beta}} \bar{\psi}^{\dot{\beta}}\end{aligned}$$

where $M = \exp(i\frac{\vec{\sigma}}{2}(\vec{\vartheta} - i\vec{\varphi}))$ and $\vec{\vartheta}$ and $\vec{\varphi}$ are the three rotation angles and boost parameters, respectively

⇒ spinors with undotted indices (first two components of Dirac spinor) transform according to $(\frac{1}{2}, 0)$ -representation of Lorentz group,
spinors with dotted indices (last two components of Dirac spinor) transform according to $(0, \frac{1}{2})$ -representation

Our world (the SM) is described by:

- internal symmetry: T_a
- Poincaré symmetry: $\Lambda^{\mu\nu}, P^\rho$

internal symmetry is a **trivial** extension of the Poincaré symmetry:

$$[\Lambda^{\mu\nu}, T^a] = 0, \quad [P^\rho, T^a] = 0$$

⇒ **direct product: (Poincaré group) \otimes (internal symmetry group)**

Particle states characterized by maximal set of commuting observables:

$$| \underbrace{m, s; \vec{p}, s_3}_{\text{space-time}}; \underbrace{Q, I, I_3, Y, \dots}_{\text{internal}} \rangle$$

quantum numbers

Theorem # 1: No-go theorem [Coleman, Mandula '67]

Any Lie-group containing Poincaré group P and internal symmetry group \tilde{G} must be **direct product** $P \otimes \tilde{G}$

$$| \underbrace{m, s; \vec{p}, s_3}_{\text{space-time}}; \underbrace{\tilde{g}, \dots}_{\text{internal}} \rangle$$

quantum numbers

New group \tilde{G} with generators Q^α and

$$[\Lambda^{\mu\nu}, Q^\alpha] \neq 0, \quad [P^\rho, Q^\alpha] \neq 0$$

impossible

Direct product \Rightarrow no irreducible multiplets can contain particles with different mass or different spin

\Rightarrow new symmetry **must** predict new particles with the same mass and spin as in the SM

\Rightarrow **experimentally excluded, no such symmetry possible :-)**

Theorem # 2: How-To-Avoid-the-No-go theorem

[Gol'fand, Likhtman '71] [Volkov, Akulov '72] [Wess, Zumino '73]

No go theorem can be evaded if instead of Lie-group (generators fulfill commutator relations):

$$[\dots, \dots] \rightarrow \{\dots, \dots\}$$

Anticommutator: $\{A, B\} = AB + BA$

⇒ Generator Q^α is fermionic (i.e. it has spin $\frac{1}{2}$)

⇒ Particles with different spin in one multiplet possible

$$Q|\text{boson}\rangle = |\text{fermion}\rangle, \quad Q|\text{fermion}\rangle = |\text{boson}\rangle$$

Q changes spin (behavior under spatial rotations) by $\frac{1}{2}$

E.g.:

spin 2	→	spin $\frac{3}{2}$	→	spin 1
graviton		gravitino		photon

Simplest case: only **one** fermionic generator Q_α (and conjugate \bar{Q}_β)

\Rightarrow $N = 1$ SUSY algebra:

$$[Q_\alpha, P_\mu] = [\bar{Q}_\beta, P_\mu] = 0$$

$$[Q_\alpha, M^{\mu\nu}] = i(\sigma^{\mu\nu})_\alpha^\beta Q_\beta$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_\alpha, \bar{Q}_\beta\} = 0$$

$$\{Q_\alpha, \bar{Q}_\beta\} = 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu$$

Energy = $H = P_0$, $\Rightarrow [Q_\alpha, H] = 0 \Rightarrow$ conserved charge

\Rightarrow SUSY: symmetry that relates bosons to fermions

unique extension of Poincaré group of symmetries of $D = 4$
relativistic QFT

\rightarrow A simple quantum mechanical SUSY example

Can SUSY be an exact symmetry?

Consider fermionic state $|f\rangle$ with mass m

\Rightarrow there is a bosonic state $|b\rangle = Q_\alpha|f\rangle$

$$P^2|f\rangle = m^2|f\rangle$$

$$\Rightarrow P^2|b\rangle = P^2Q_\alpha|f\rangle = Q_\alpha P^2|f\rangle = Q_\alpha m^2|f\rangle = m^2|b\rangle$$

\Rightarrow for each fermionic state there is a bosonic state with the same mass

\Rightarrow states are paired bosonic \leftrightarrow fermionic

\Rightarrow still experimentally excluded

\Rightarrow SUSY must be broken

Further consequences of the SUSY algebra

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\sigma_{\alpha\beta}^\mu P_\mu$$

$$\Rightarrow \{Q_\alpha, \bar{Q}_\beta\} \bar{\sigma}_\nu^{\beta\alpha} = 2 \underbrace{\sigma_{\alpha\beta}^\mu \bar{\sigma}_\nu^{\beta\alpha}}_{2g^\mu{}_\nu} P_\mu = 4P_\nu$$

$$\nu = 0 \Rightarrow H = P_0 = \frac{1}{4} \{Q_\alpha, \bar{Q}_\beta\} \bar{\sigma}_0^{\beta\alpha} = \frac{1}{4} (\{Q_1, Q_1^\dagger\} + \{Q_2, Q_2^\dagger\})$$

where $\bar{Q}_{\dot{\alpha}} = (Q_\alpha)^\dagger$

$$\{Q_i, Q_i^\dagger\} = Q_i Q_i^\dagger + Q_i^\dagger Q_i: \text{ hermitian operator, eigenvalues } \geq 0$$

\Rightarrow for any state $|\alpha\rangle$: $\langle\alpha|H|\alpha\rangle \geq 0$

spectrum of H is bounded from below, ≥ 0

\Rightarrow no negative eigenvalues

State with lowest energy: vacuum state $|0\rangle$

if vacuum state is symmetric, i.e. $Q|0\rangle = 0$, $Q^\dagger|0\rangle = 0$ for all Q

\Rightarrow vacuum has zero energy, $\langle 0|H|0\rangle = E_{\text{vac}} = 0$

For spontaneous symmetry breaking: vacuum state is not invariant

\Rightarrow If (global) SUSY is spontaneously broken, i.e. $Q_\alpha|0\rangle \neq 0$,

then $\langle 0|H|0\rangle = E_{\text{vac}} > 0$

\Rightarrow non-vanishing vacuum energy

Further consequences for SUSY multiplets:

$$\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 \Rightarrow \bar{Q}_{\dot{\alpha}}^2 = 0 \text{ (and } Q_{\alpha}^2 = 0)$$

Consider multiplet, start with state of lowest helicity λ_0

application of $\bar{Q}_{\dot{\alpha}}$ \Rightarrow one additional state with helicity $\lambda_0 + \frac{1}{2}$

further application of $\bar{Q}_{\dot{\alpha}}$ $\Rightarrow 0$, no further state

\Rightarrow one fermionic + one bosonic state

(N SUSY generators $\Rightarrow 2^{N-1}$ bosonic and 2^{N-1} fermionic states)

SUSY multiplet contains equal number of bosonic and fermionic state

Most relevant multiplets (possess also CPT conjugate 'mirrors'):

- chiral supermultiplet: $-\frac{1}{2}, 0$

Weyl fermion (quark, lepton, ...) + complex scalar (squark, slepton)

- vector supermultiplet: $-1, -\frac{1}{2}$

Gauge boson (massless vector) + Weyl fermion (gaugino)

- graviton supermultiplet: $-2, -\frac{3}{2}$

graviton + gravitino

3. Superfields and Superspace

Translation transformation: P_μ , parameter: x^μ

SUSY transformation: $Q_\alpha, \bar{Q}_{\dot{\alpha}}$, parameter: $\theta, \bar{\theta} \rightarrow$ **anticommuting c-numbers**
(" Grassmann variables")

\Rightarrow Extension of 4-dim. space–time by coordinates $\theta^\alpha, \bar{\theta}^{\dot{\alpha}}$: superspace

Point in <u>superspace</u> : $X = (x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$, <u>Superfield</u> : $\phi(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$
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Grassman variables:

SUSY transformation: $Q_\alpha, \bar{Q}_{\dot{\alpha}}$, parameter: $\theta, \bar{\theta} \rightarrow$ anticommuting c-numbers
("Grassmann variables")

Without spinor index:

$$\{\theta, \theta\} = 0 \Rightarrow \theta\theta = 0$$

With two-component spinor index (as it is our case):

$$\theta\theta \equiv \theta^\alpha\theta_\alpha = \epsilon_{\alpha\beta}\theta^\alpha\theta^\beta \Rightarrow \theta\theta \neq 0, (\theta^1\theta_2 \neq 0)$$

Taylor expansion in Grassmann variable: $\theta^\alpha\theta^\beta\theta^\gamma = 0$ ($\alpha, \beta, \gamma = 1, 2$)

\Rightarrow Taylor expansion terminates after second term, i.e. $\phi(\theta) = a + \theta\psi + \theta\theta f$

Integration: $\int d\theta = 0, \int d\theta \theta = 1$

$$d^2\theta = -\frac{1}{4}\epsilon_{\alpha\beta}d\theta^\alpha d\theta^\beta$$

$$\Rightarrow \int d^2\theta \phi(\theta) = \int d^2\theta (a + \theta\psi + \theta\theta f) = f$$

SUSY transformations:

(Lagrangian should be invariant!)

Group element of finite SUSY transformation:

$$S(y, \xi, \bar{\xi}) = \exp i (\xi Q + \bar{\xi} \bar{Q} - y^\mu P_\mu)$$

in analogy to group elements for Lie-groups

$\xi, \bar{\xi}$ are independent of y^μ : global SUSY transformation

Transformation of superfield: $S(y, \xi, \bar{\xi})\phi(x, \theta, \bar{\theta})$

$$\Rightarrow S(y, \xi, \bar{\xi})\phi(x, \theta, \bar{\theta}) = \phi(x^\mu + y^\mu - i\xi\sigma^\mu\bar{\theta} + i\theta\sigma^\mu\bar{\xi}, \xi + \theta, \bar{\xi} + \bar{\theta})$$

representations of generators obtained from infinitesimal transformation of superfield

$$\Rightarrow P_\mu = i\partial_\mu, \quad Q_\alpha = -i\partial_\alpha + (\sigma^\mu\bar{\theta})_\alpha\partial_\mu, \quad \bar{Q}_{\dot{\alpha}} = i\partial_{\dot{\alpha}} - (\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu$$

$$\text{with } \partial_\alpha = \frac{\partial}{\partial\theta^\alpha}, \quad \bar{\partial}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}$$

Definition of covariant derivatives:

$$D_\alpha = -i\partial_\alpha - (\sigma^\mu\bar{\theta})_\alpha\partial_\mu, \quad \bar{D}_{\dot{\alpha}} = i\bar{\partial}_{\dot{\alpha}} + (\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu$$

General superfield in component form

Most general form of field depending on $x, \theta, \bar{\theta}$:

$$\begin{aligned}\Phi(x, \theta, \bar{\theta}) = & \varphi(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta F(x) + \bar{\theta}\bar{\theta}H(x) + \theta\sigma^\mu\bar{\theta}A_\mu(x) \\ & + (\theta\theta)\bar{\theta}\bar{\lambda}(x) + (\bar{\theta}\bar{\theta})\theta\xi(x) + (\theta\theta)(\bar{\theta}\bar{\theta})D(x)\end{aligned}$$

Further terms vanish because of $\theta\theta\theta = \bar{\theta}\bar{\theta}\bar{\theta} = 0$

Components (can be complex):

φ, F, H, D : scalar fields

A_μ : vector field

$\psi, \bar{\chi}, \bar{\lambda}, \xi$: Weyl-spinor fields

⇒ Too many components in 4-dim. for irreducible representation of SUSY with spin ≤ 1 (chiral or vector multiplet)

⇒ representation is reducible

(not all component fields mix with each other under SUSY transf.)

⇒ Irreducible superfields (smallest building blocks) from imposing conditions on general superfield conditions need to be invariant under SUSY transformations:

$\bar{D}_{\dot{\alpha}}\Phi = 0$: left-handed chiral superfield (LH χ SF)

$D_{\alpha}\Phi = 0$: right-handed chiral superfield (RH χ SF)

$\Phi = \Phi^{\dagger}$: vector superfield

⇒ chiral superfields describe left- or right-handed component of SM fermion
+ scalar partner

simplified LH χ SF in components:

$$\phi(x, \theta) = \varphi(x) + \sqrt{2}\theta\psi(x) - (\theta\theta)F(x)$$

φ, F : scalar fields , ψ : Weyl-spinor field

→ Explicit evaluation for the simplified case

⇒ Irreducible superfields (smallest building blocks) from imposing conditions on general superfield conditions need to be invariant under SUSY transformations:

$\bar{D}_{\dot{\alpha}}\Phi = 0$: left-handed chiral superfield (LH χ SF)

$D_{\alpha}\Phi = 0$: right-handed chiral superfield (RH χ SF)

$\Phi = \Phi^{\dagger}$: vector superfield

⇒ chiral superfields describe left- or right-handed component of SM fermion
+ scalar partner

LH χ SF in components:

$$\begin{aligned} \phi(x, \theta, \bar{\theta}) = & \varphi(x) + \sqrt{2}\theta\psi(x) - i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}\varphi(x) + \frac{i}{\sqrt{2}}(\theta\theta)(\partial_{\mu}\psi(x)\sigma^{\mu}\bar{\theta}) \\ & - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial^{\mu}\partial_{\mu}\varphi(x) - (\theta\theta)F(x) \end{aligned}$$

φ, F : scalar fields , ψ : Weyl-spinor field

LH χ SF: Transf. of component fields with infinitesimal SUSY param. $\xi, \bar{\xi}$:

$$\delta\phi(x, \theta, \bar{\theta}) = i(\xi Q + \bar{\xi}\bar{Q})\phi(x, \theta, \bar{\theta})$$

Comparison with

$$\begin{aligned} \delta\phi(x, \theta, \bar{\theta}) = & \delta\varphi(x) + \sqrt{2}\theta\delta\psi(x) - i\theta\sigma^\mu\bar{\theta}\partial_\mu\delta\varphi(x) + \frac{i}{\sqrt{2}}(\theta\theta)(\partial_\mu\delta\psi(x)\sigma^\mu\bar{\theta}) \\ & - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial^\mu\partial_\mu\delta\varphi(x) - (\theta\theta)\delta F(x) \end{aligned}$$

\Rightarrow determination of $\delta\varphi$, $\delta\psi$, δF :

$$\delta\varphi = \sqrt{2}\xi\psi$$

boson \rightarrow fermion

$$\delta\psi_\alpha = -\sqrt{2}F\xi_\alpha - i\sqrt{2}(\sigma^\mu\bar{\xi})_\alpha\partial_\mu\varphi$$

fermion \rightarrow boson

$$\delta F = \partial_\mu(-i\sqrt{2}\psi\sigma^\mu\bar{\xi})$$

$F \rightarrow$ total derivative

RH χ SF: analogously

Vector superfield in components:

$$\begin{aligned}
 V(x, \theta, \bar{\theta}) &= c(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + \theta\sigma^\mu\bar{\theta}v_\mu(x) \\
 &+ \frac{i}{2}(\theta\theta)(M(x) + iN(x)) - \frac{i}{2}(\bar{\theta}\bar{\theta})(M(x) - iN(x)) \\
 &+ i(\theta\theta)\bar{\theta}\left(\bar{\lambda}(x) + \frac{i}{2}\partial_\mu\chi(x)\sigma^\mu\right) - i(\bar{\theta}\bar{\theta})\theta\left(\lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)\right) \\
 &+ \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})\left(D(x) - \frac{1}{2}\partial^\mu\partial_\mu c(x)\right)
 \end{aligned}$$

Number of components can be reduced by SUSY gauge transformation:

Wess-Zumino gauge: $\chi(x) = c(x) = M(x) = N(x) \equiv 0$

Vector SF: $V(x, \theta, \bar{\theta}) = \dots + i(\theta\theta)\bar{\theta}\bar{\lambda}(x) - i(\bar{\theta}\bar{\theta})\theta\lambda(x) + \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})D(x) + \dots$

$$\delta D = -\xi\sigma^\mu\partial_\mu\bar{\lambda}(x) - \partial_\mu\lambda(x)\sigma^\mu\bar{\xi} \quad D \rightarrow \text{total derivative}$$