

MadGraph 5

Olivier Mattelaer

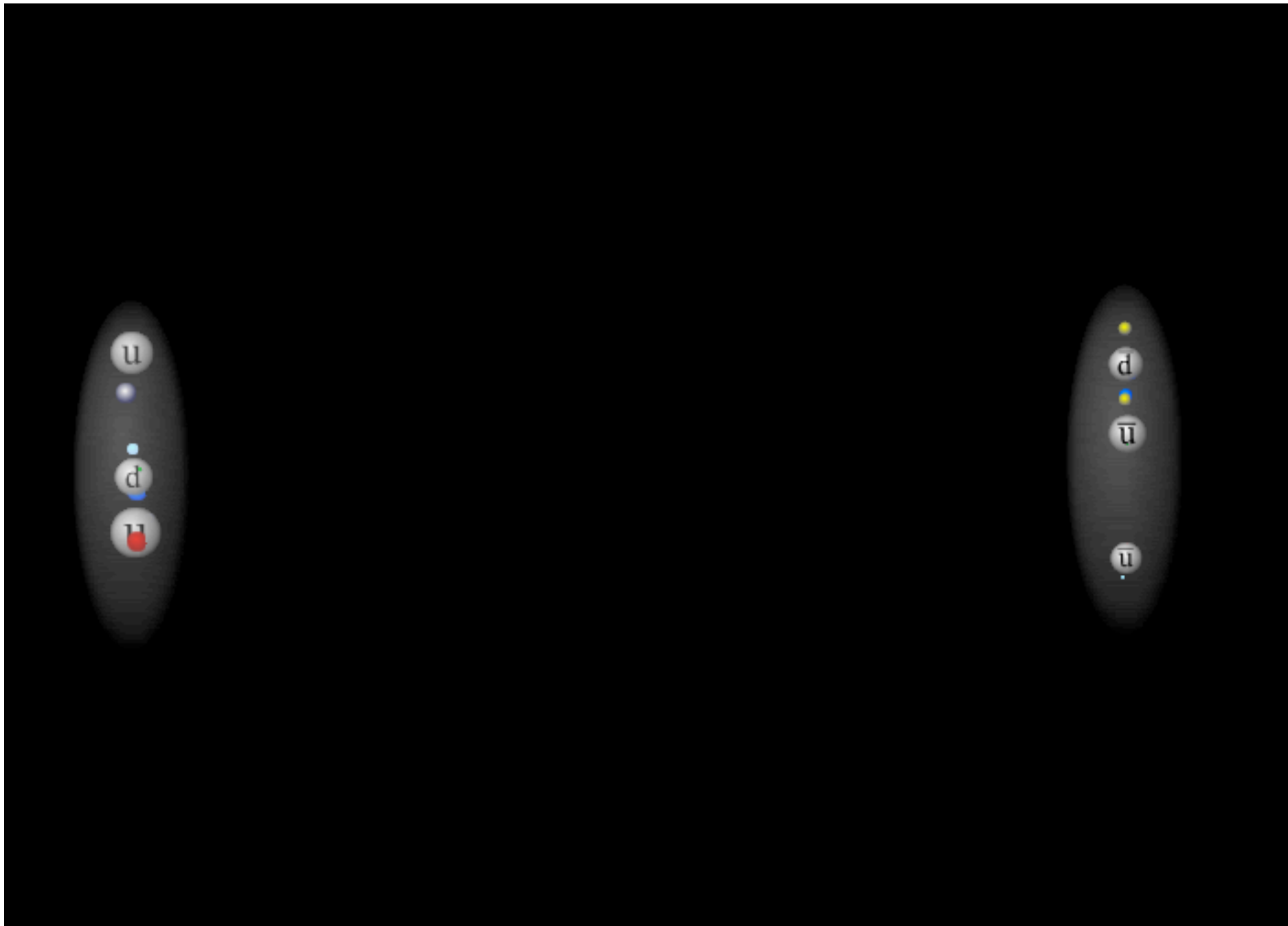
University of Illinois at Urbana Champaign

Special thanks to Fabio Maltoni / Johan Alwall from whom I have shamelessly borrowed many of the following slides

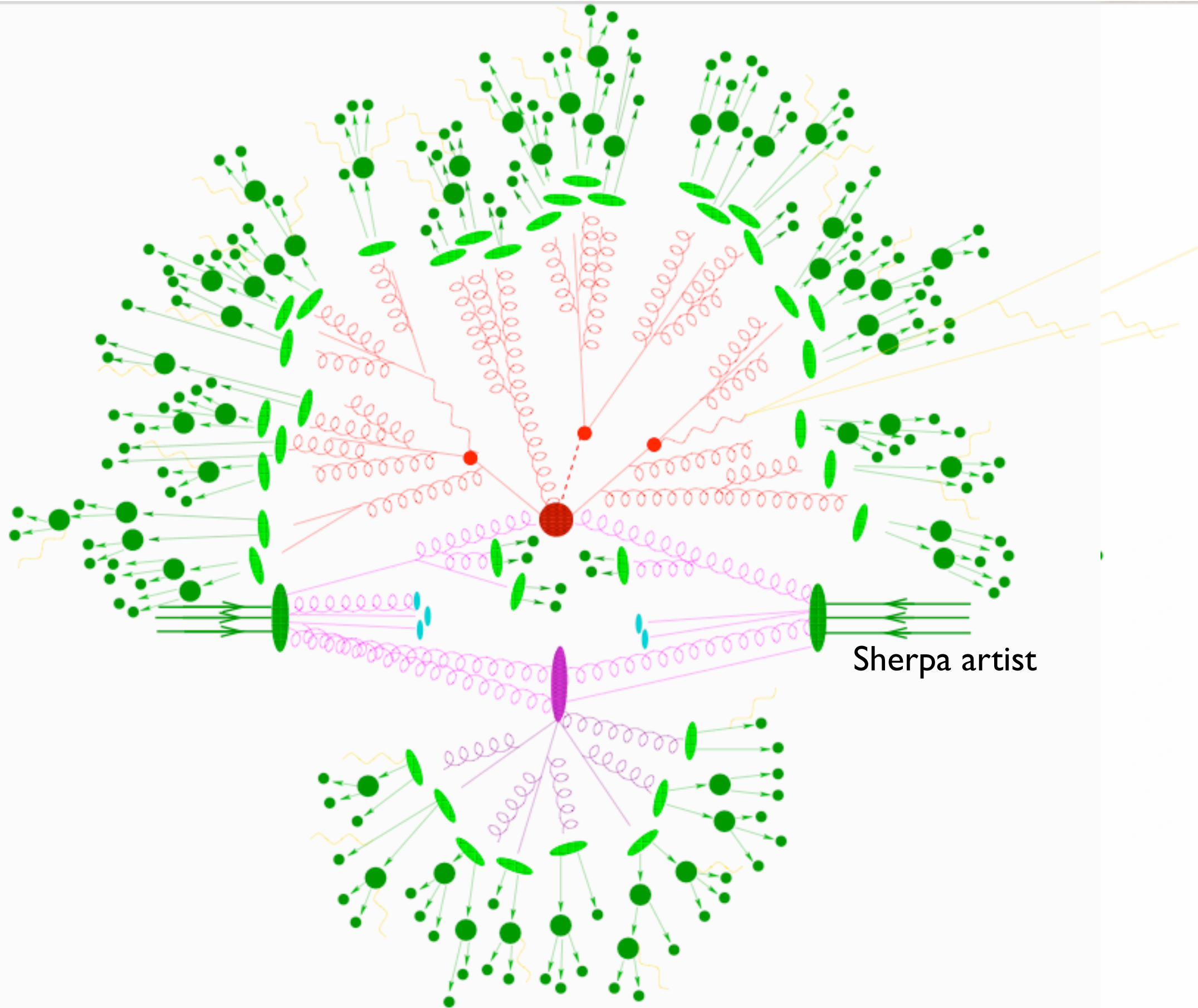
Aims for these lectures

- Get you **acquainted** with the concepts and techniques used in event generation
- Give you hands-on experience with matrix element generation, event generation and analysis
- **Answer as many of your questions as I can (so please ask questions!)**

- PROTON COLLISIONS

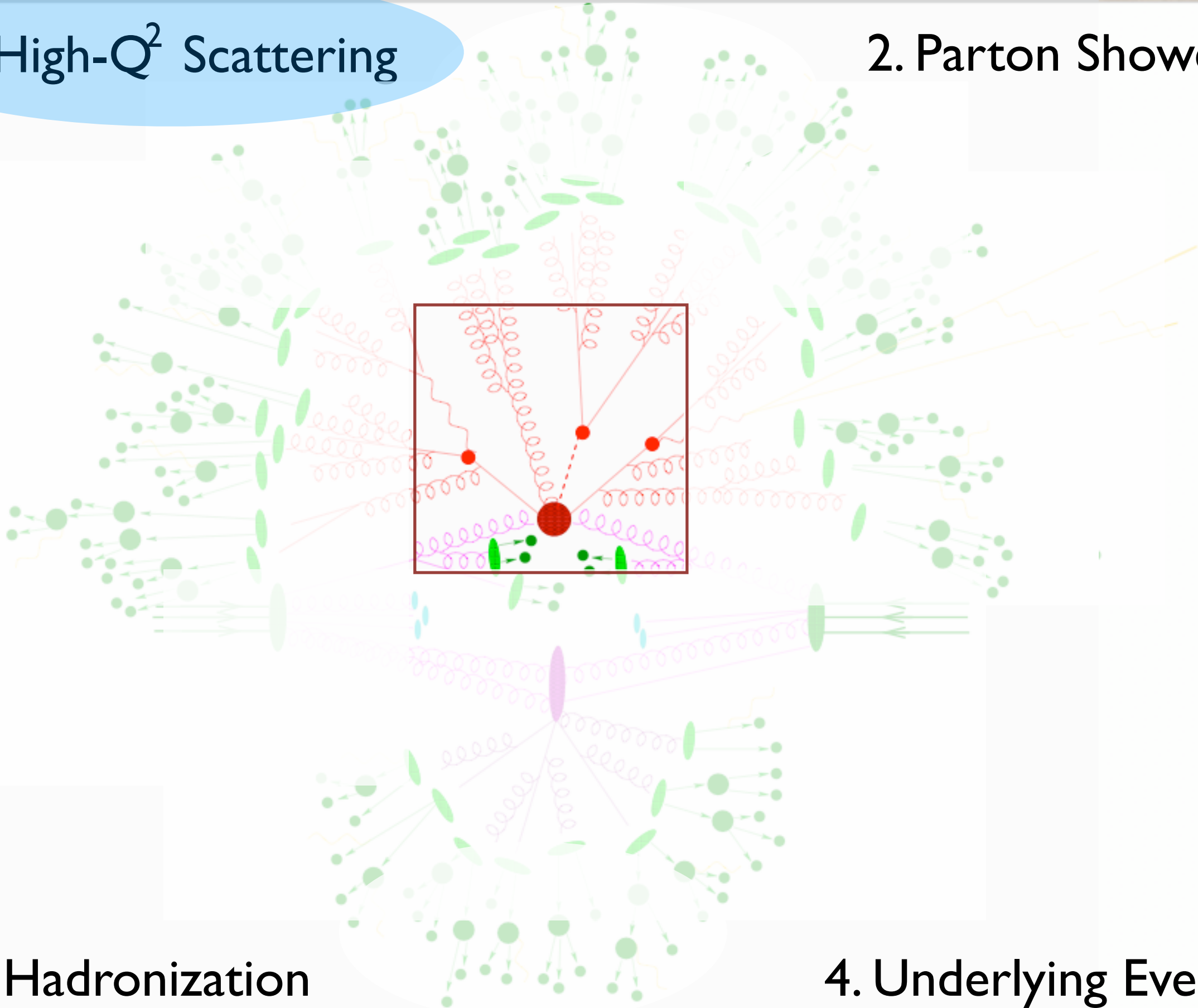


M. scott



I. High- Q^2 Scattering

2. Parton Shower



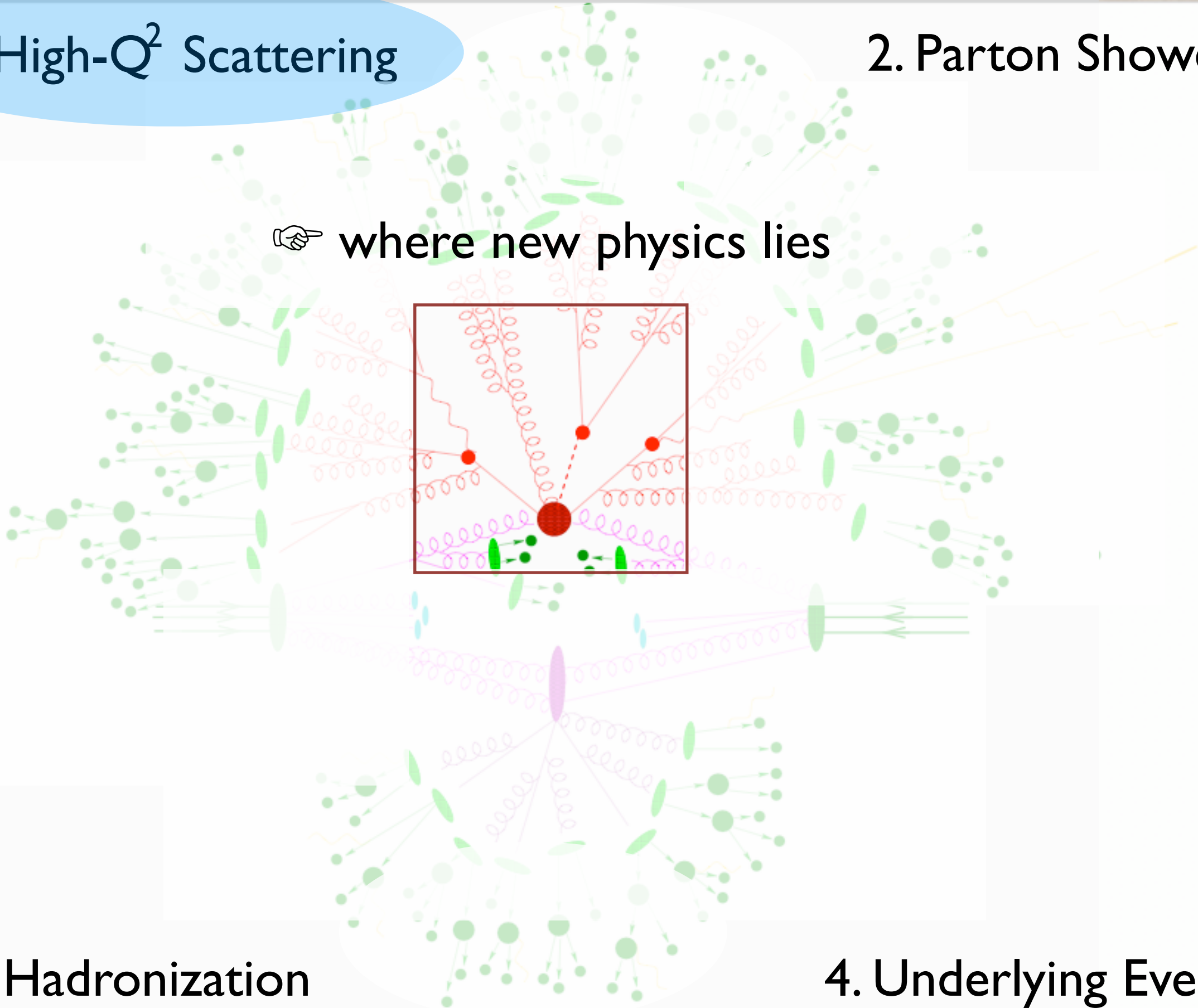
3. Hadronization

4. Underlying Event

I. High- Q^2 Scattering

2. Parton Shower

👉 where new physics lies

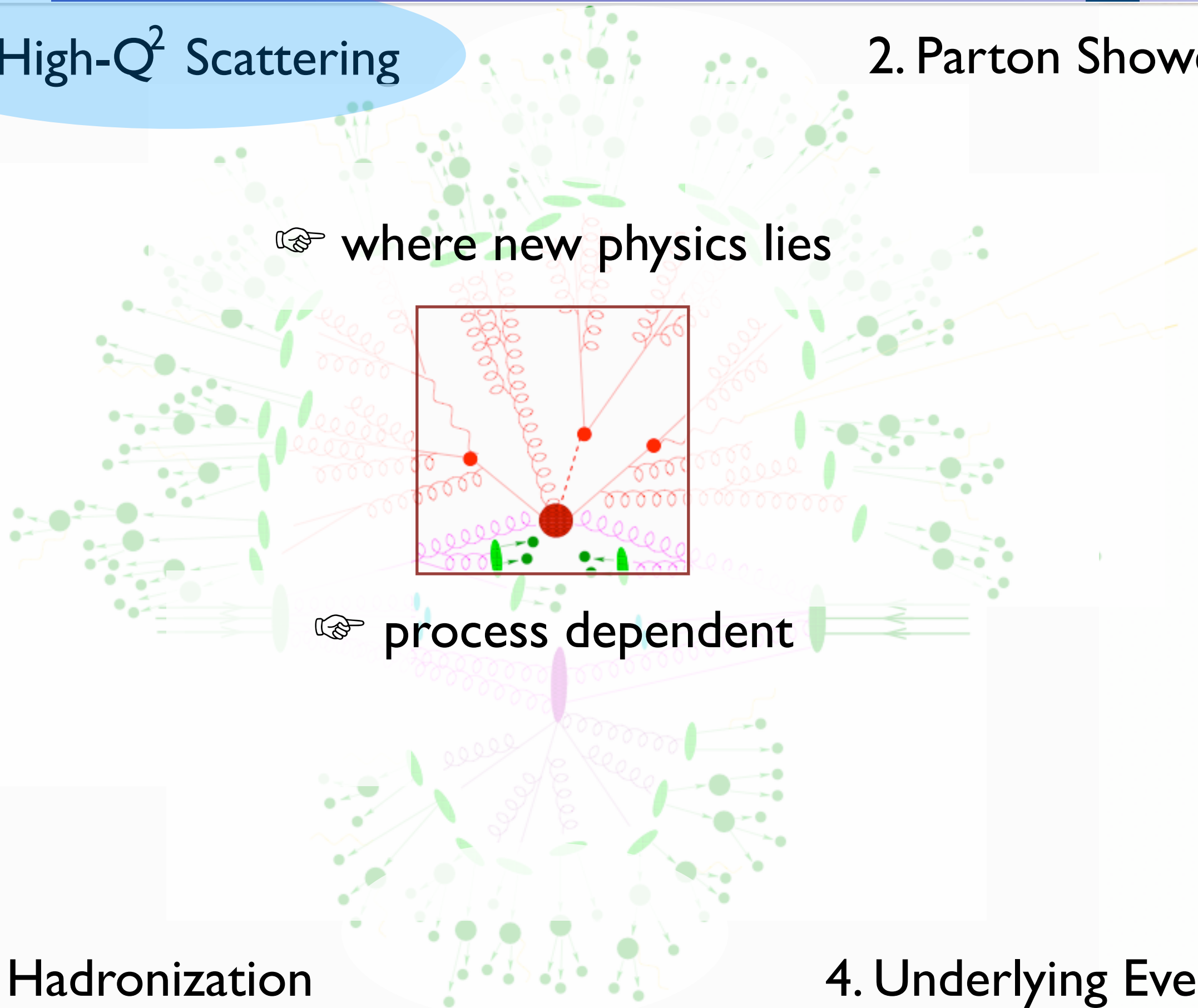


3. Hadronization

4. Underlying Event

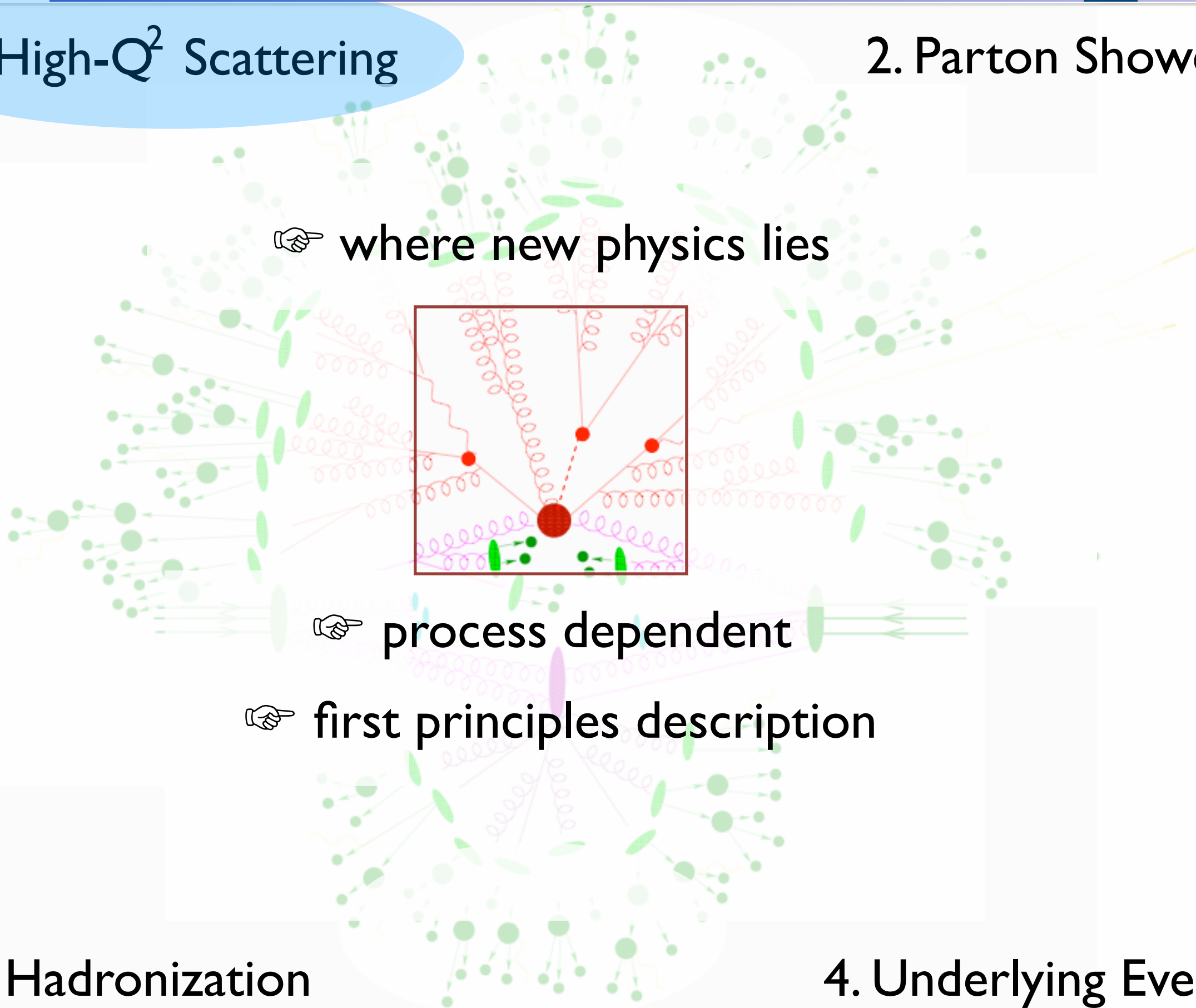
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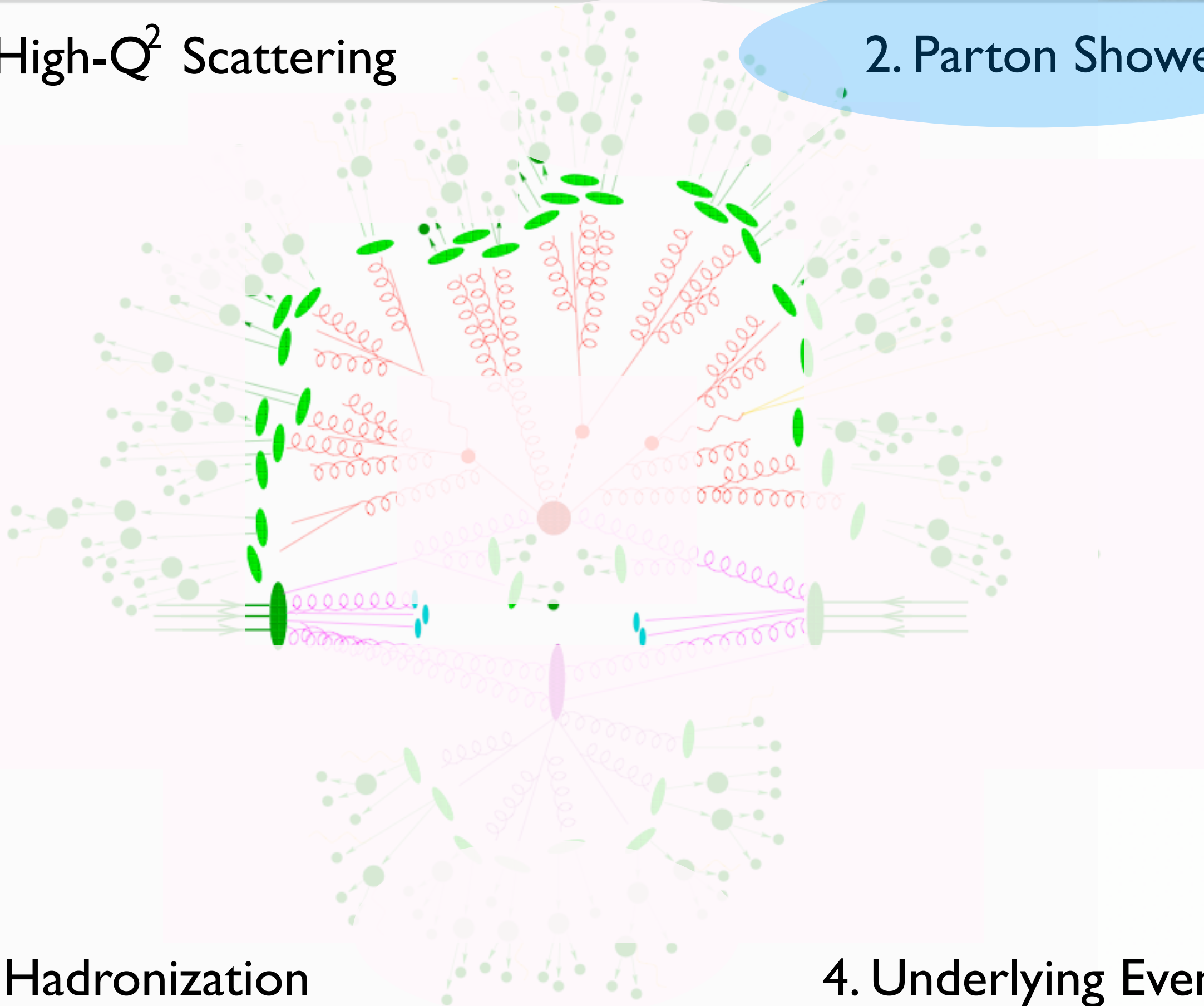


3. Hadronization

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1. High- Q^2 Scattering

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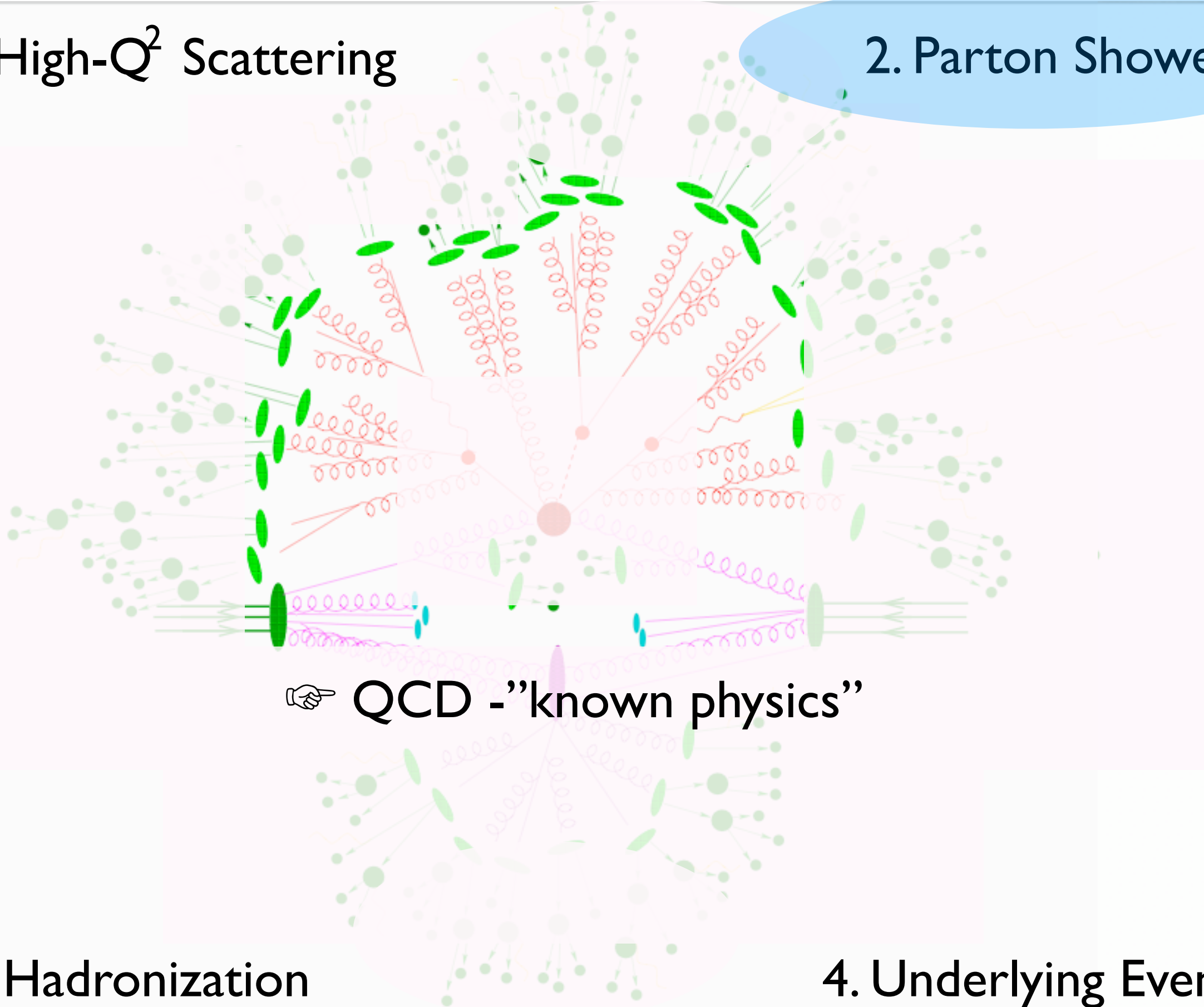


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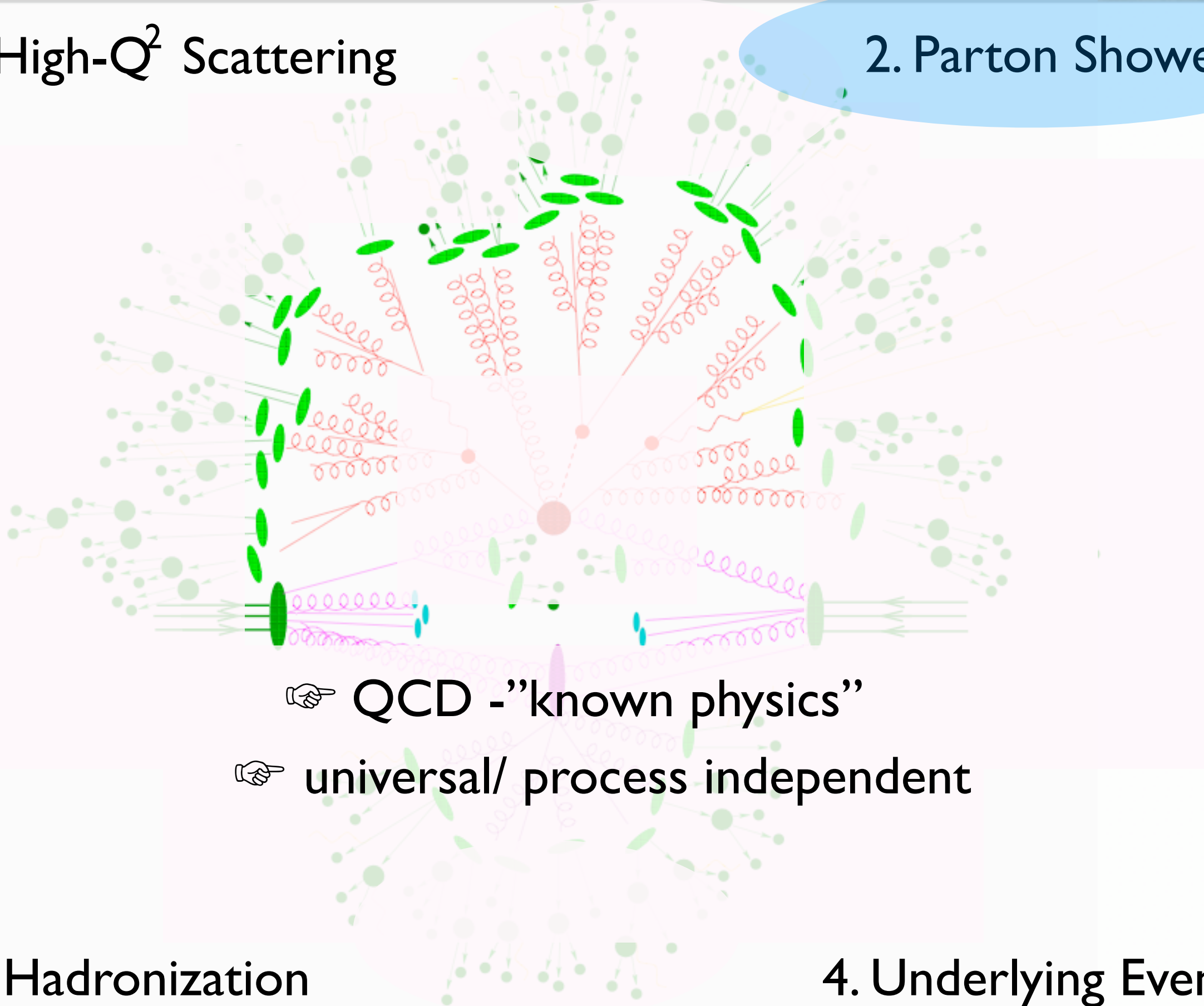


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☞ QCD - "known physics"

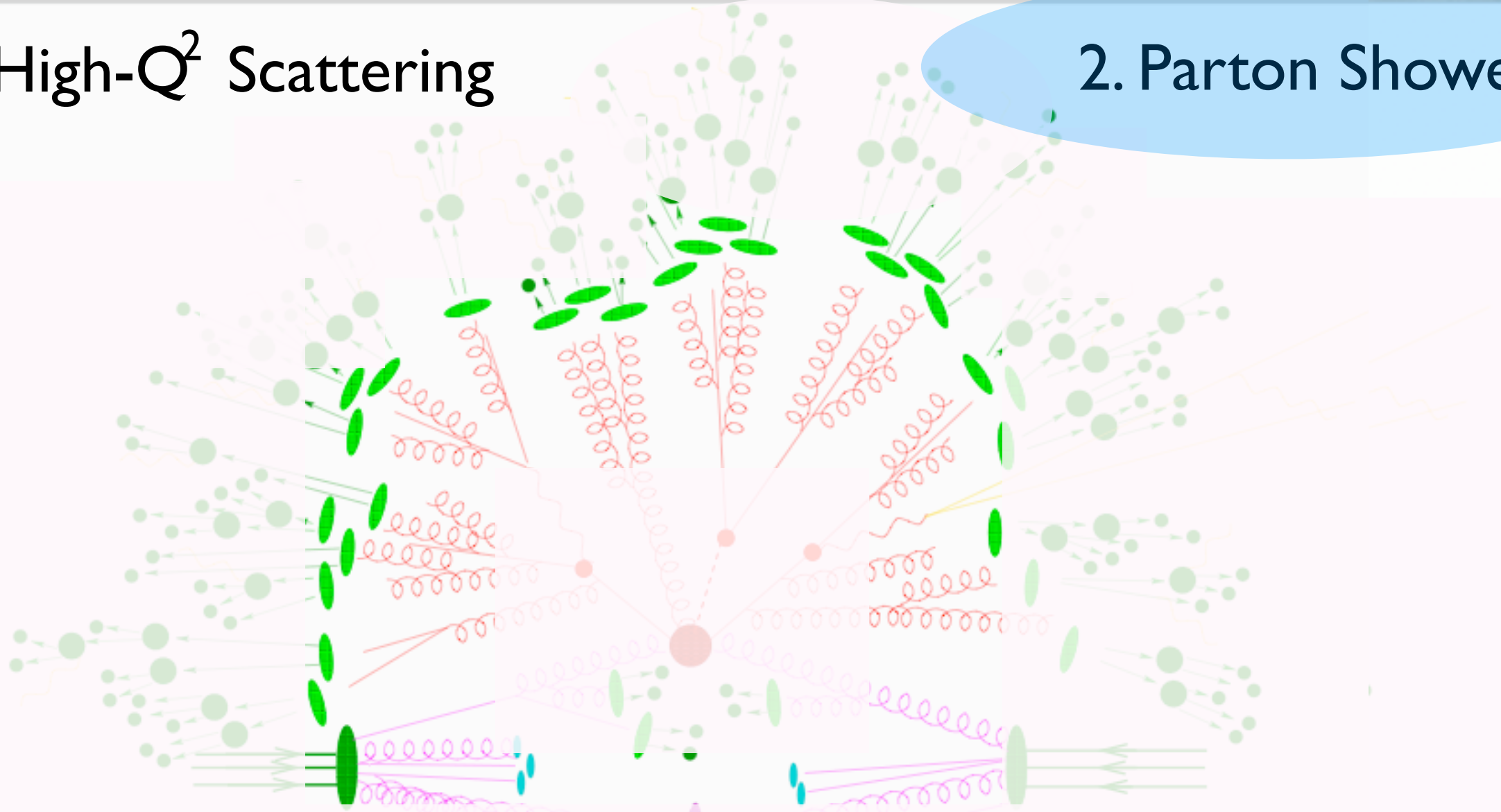
☞ universal/ process independent

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☞ universal/ process independent

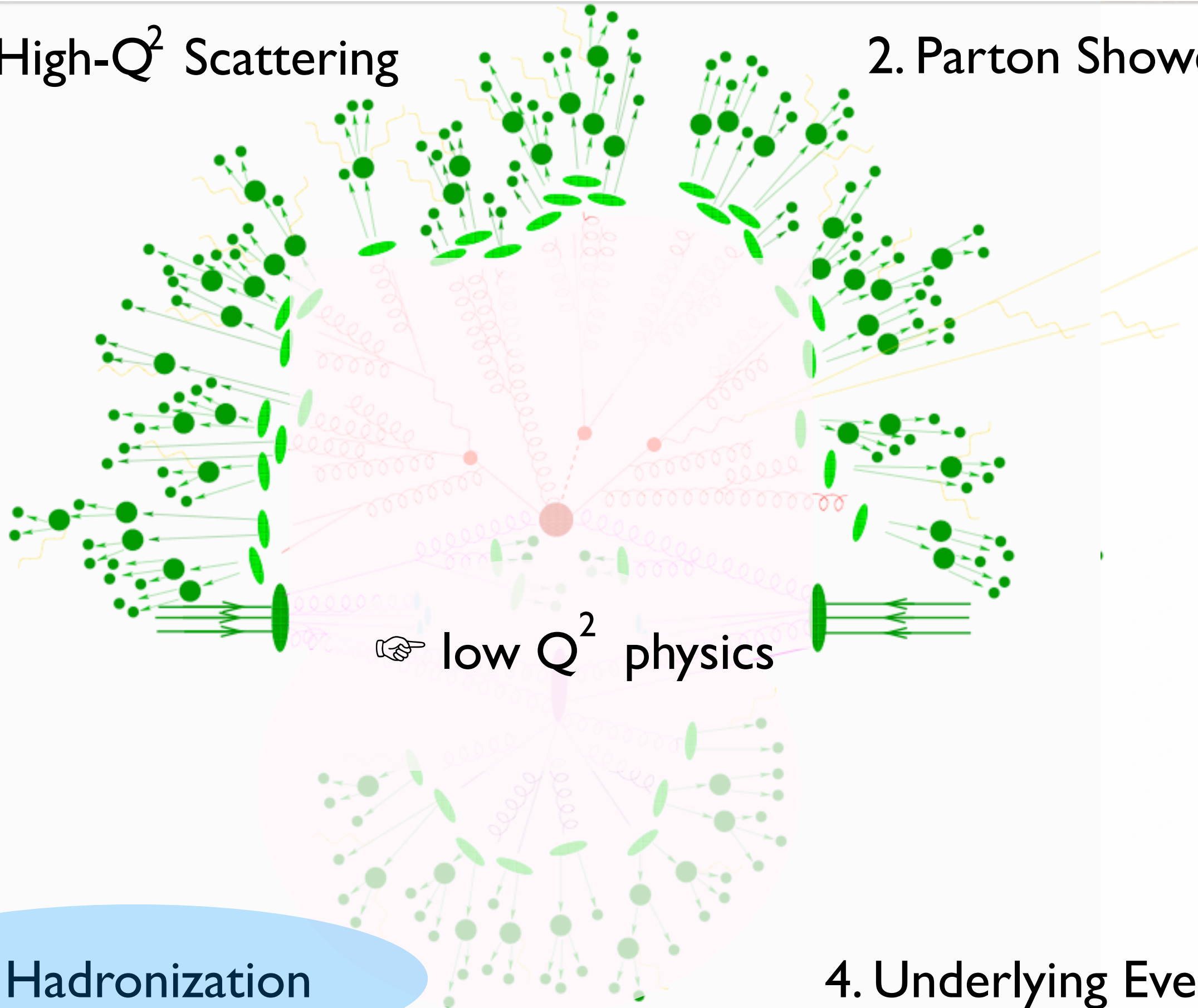
☞ first principles description

3. Hadronization

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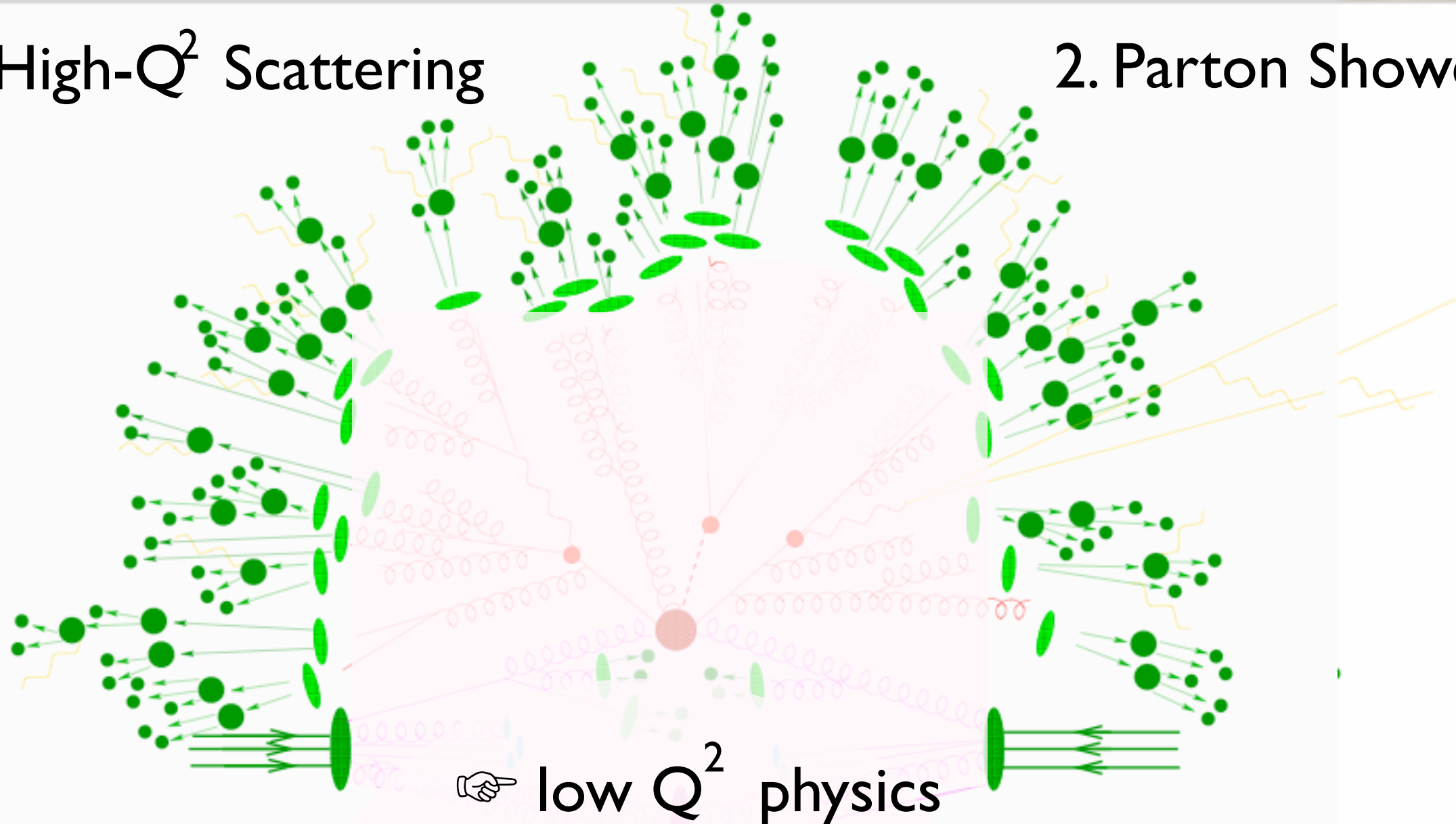


3. Hadronization

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low Q^2 physics

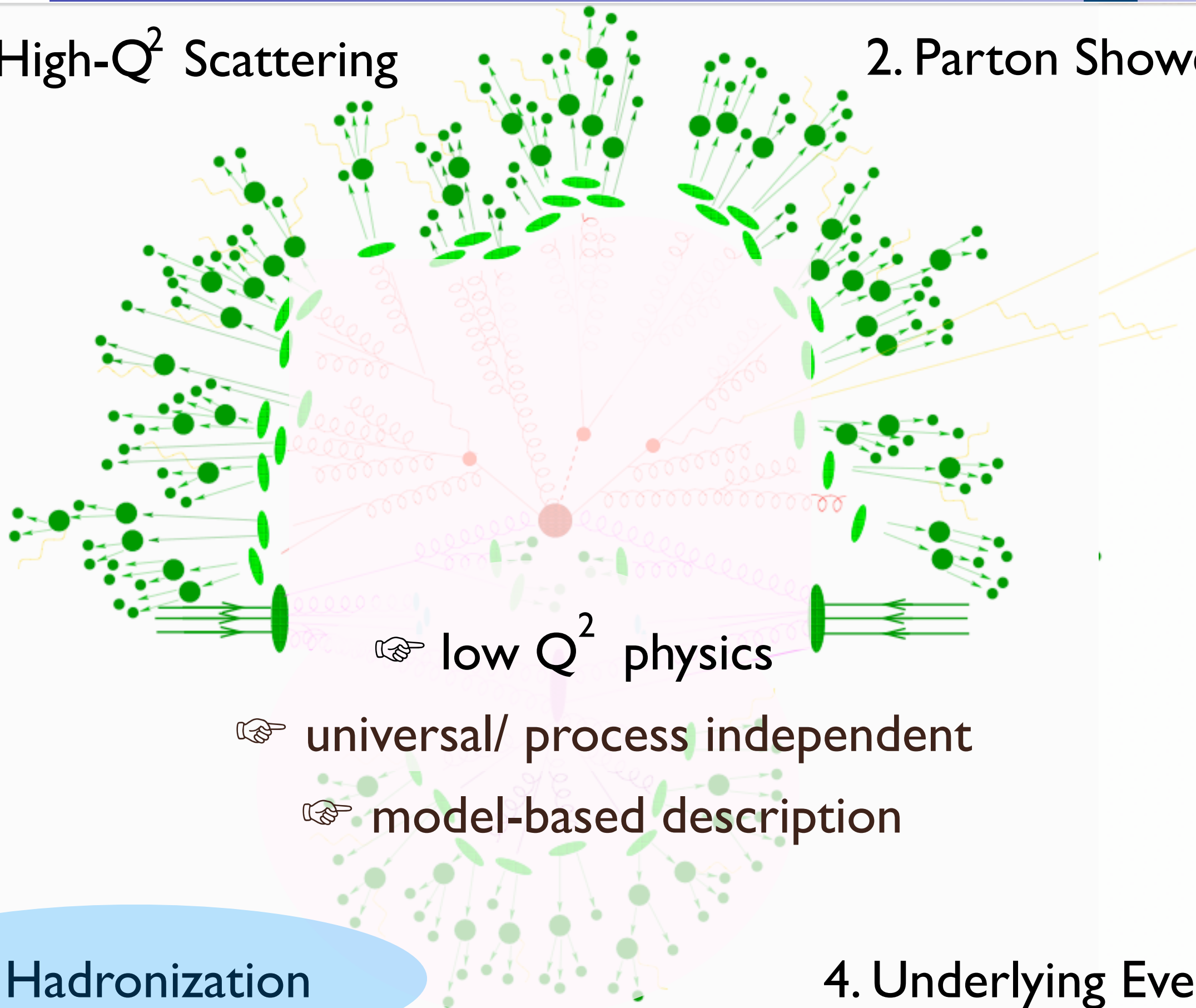
universal/ process independent

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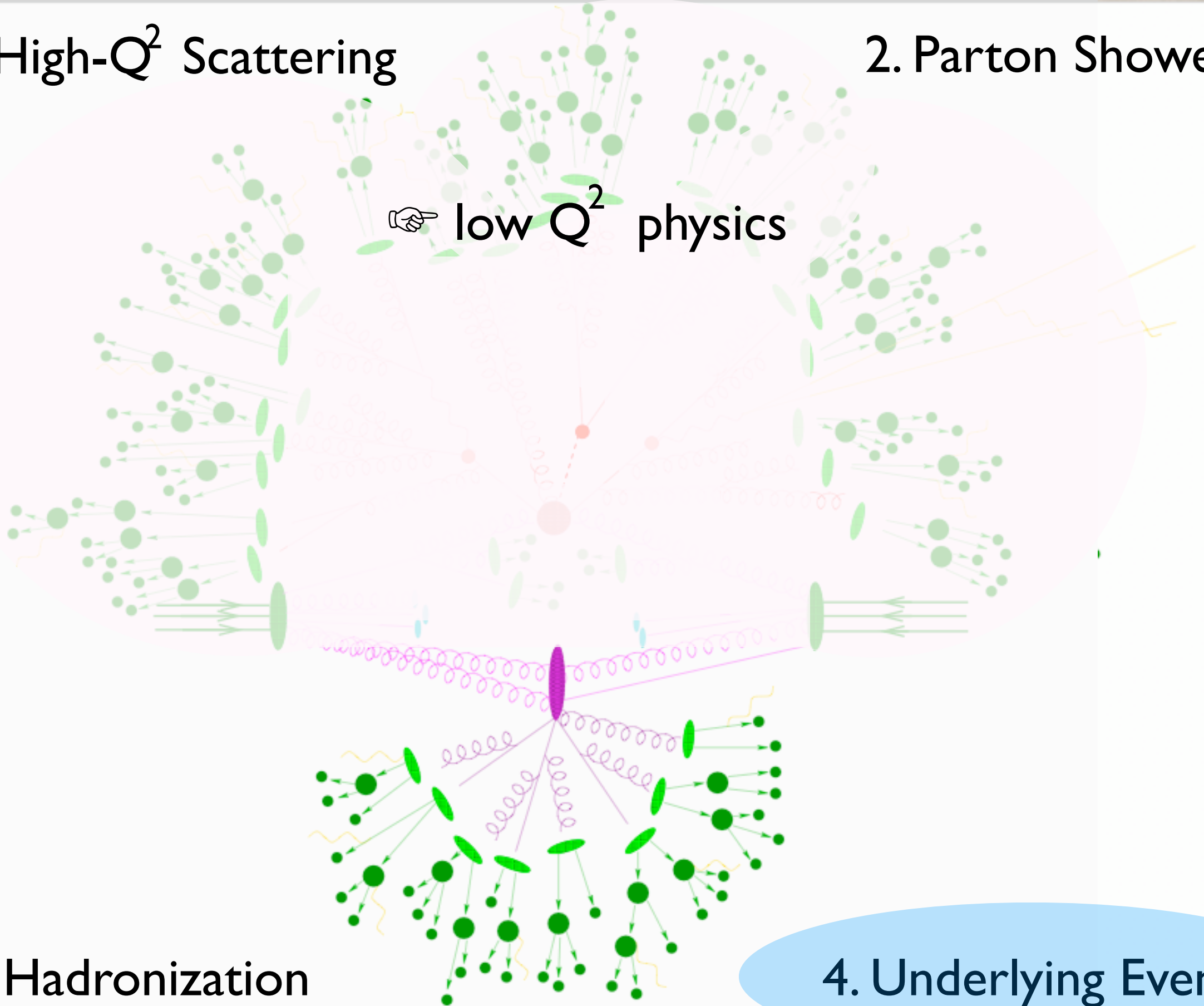
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low Q^2 physics

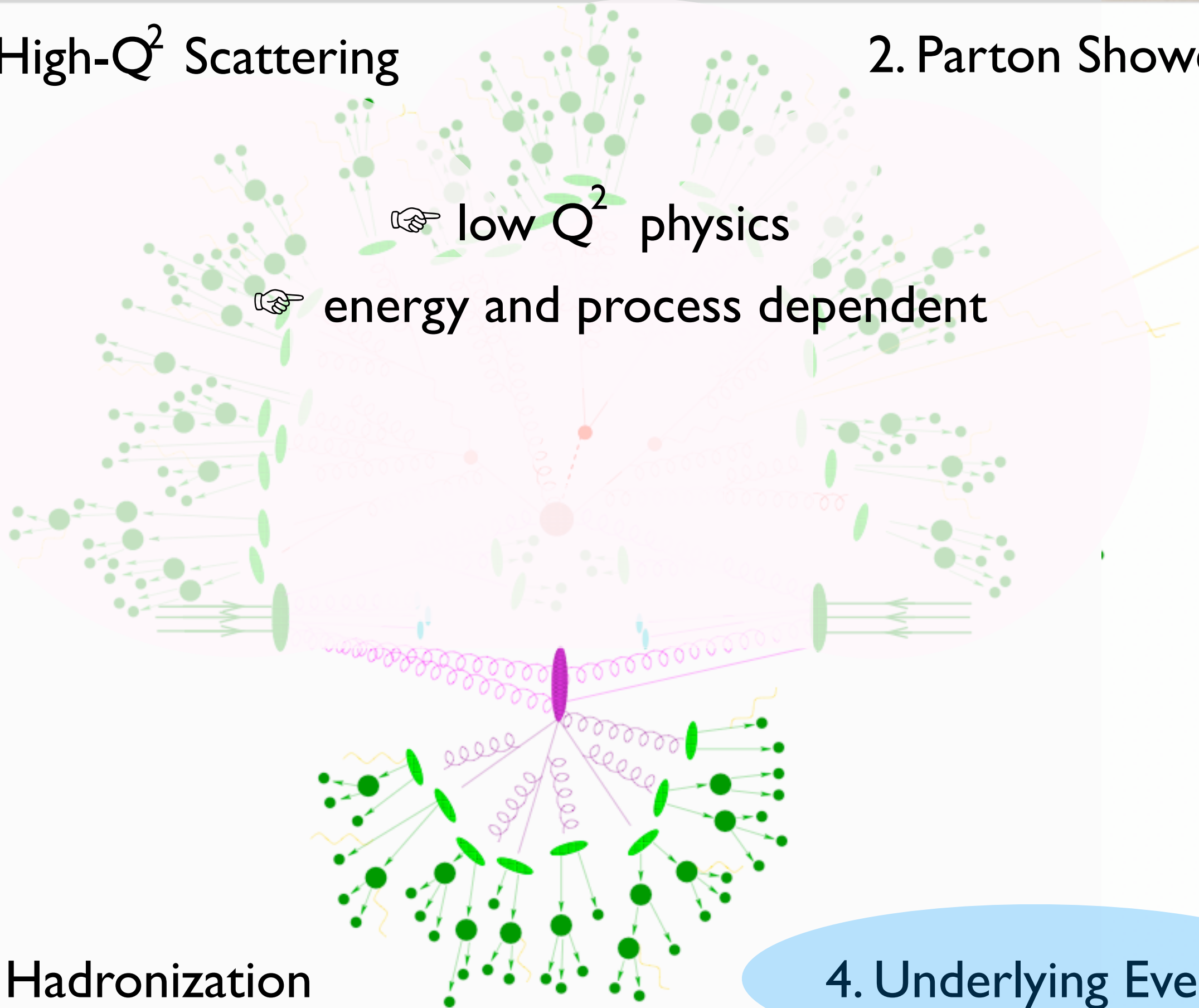


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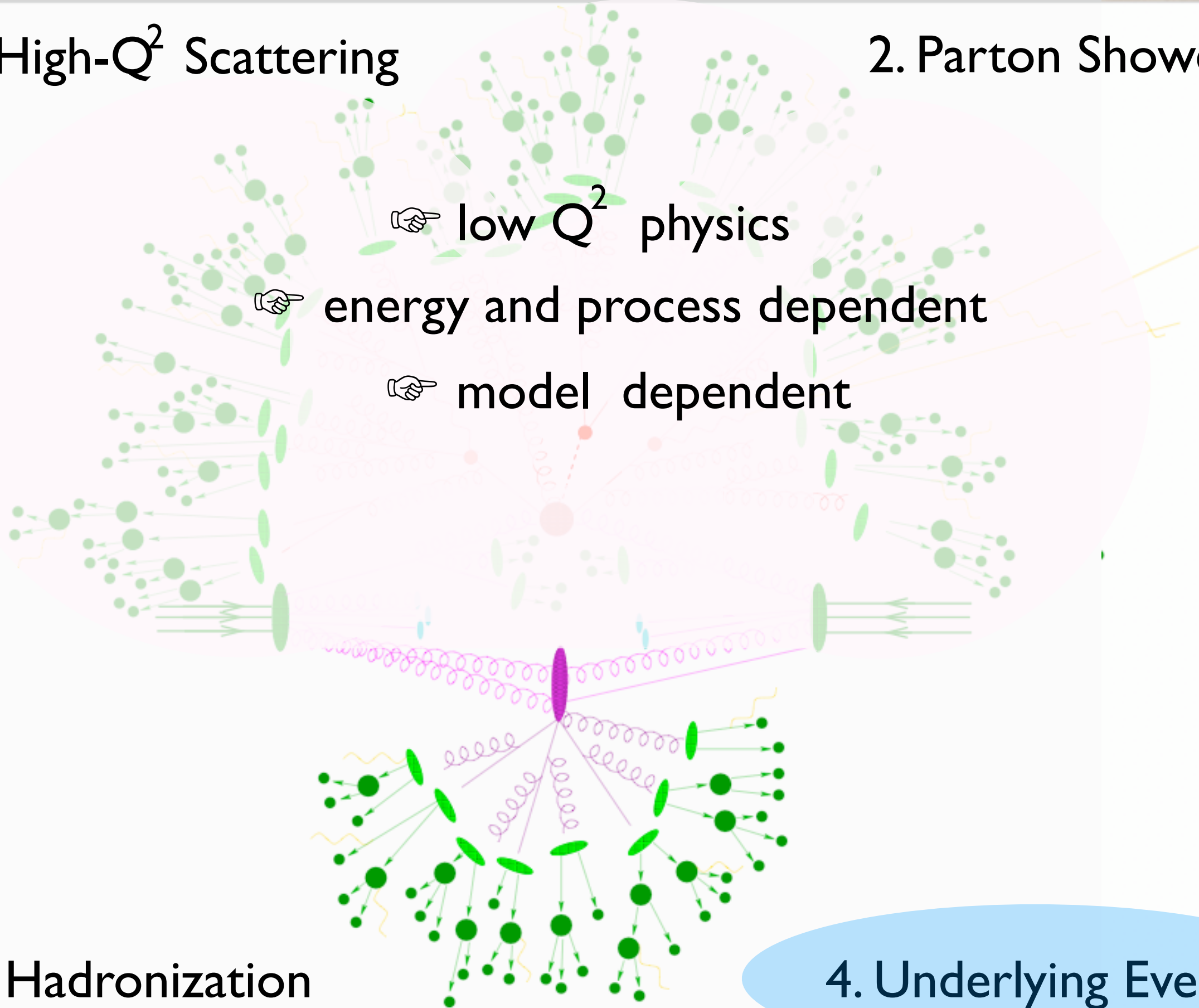


3. Hadronization

4. Underlying Event

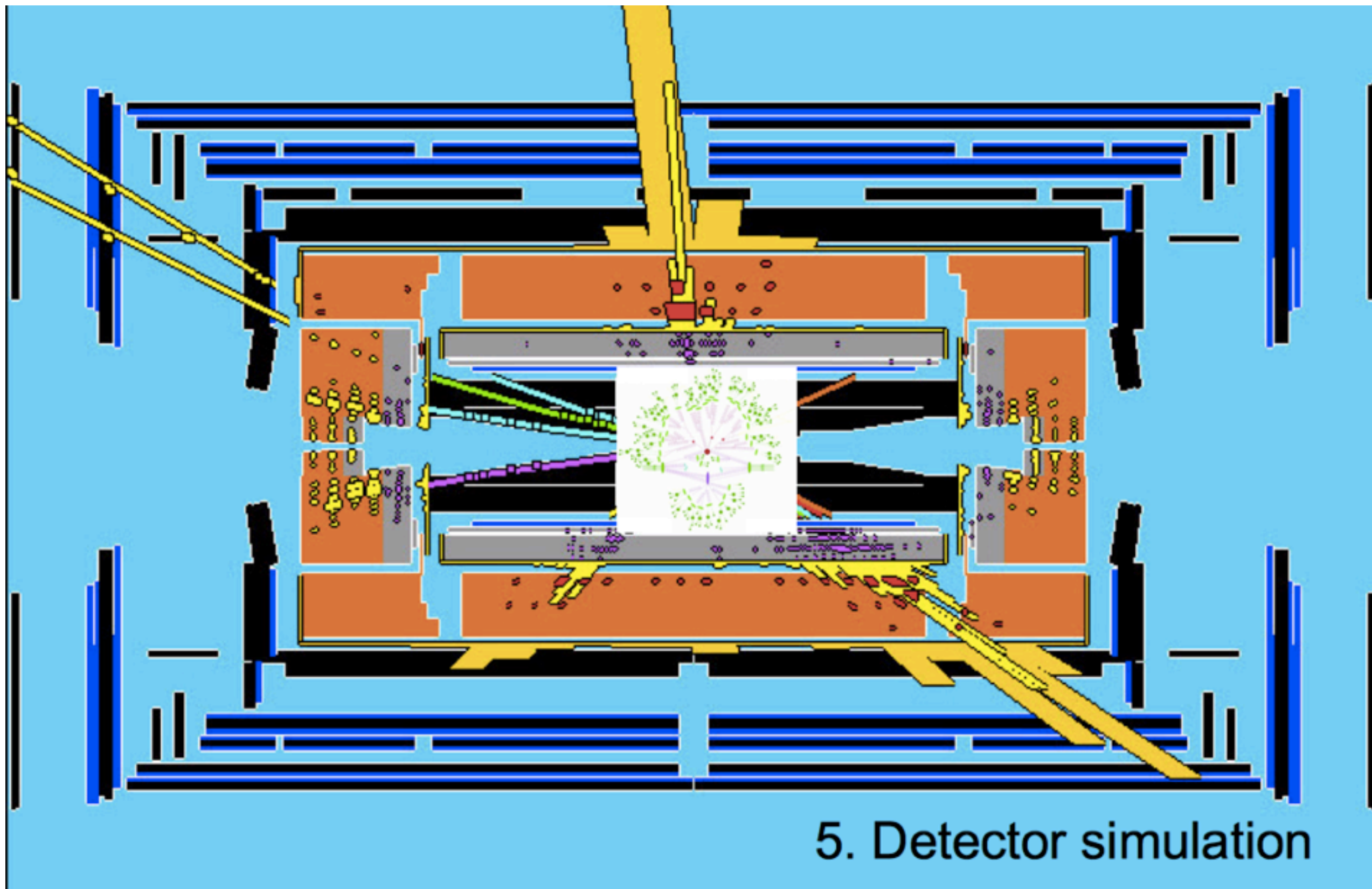
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3. Hadronization

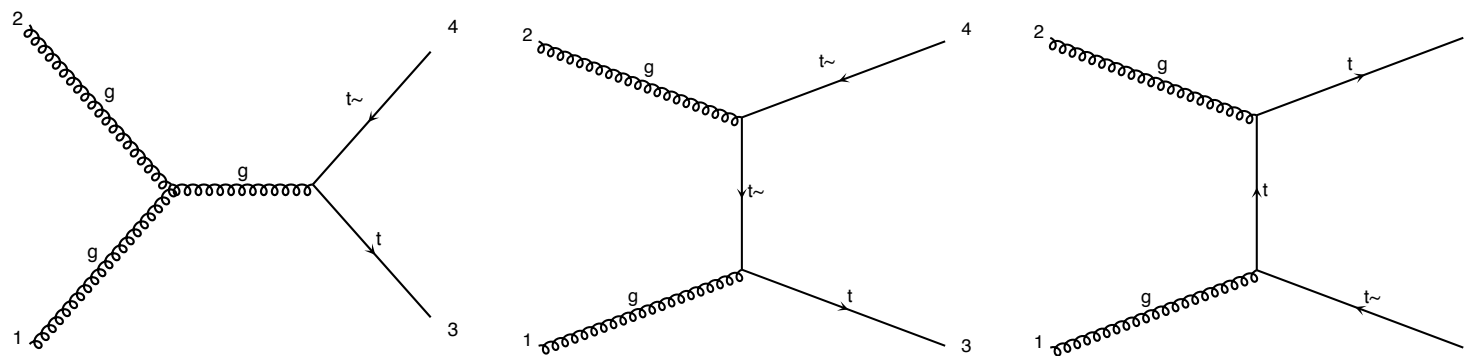
4. Underlying Event



Matrix Element calculation at Hadron Colliders

To calculate a given process (e.g., $p p \rightarrow t \bar{t}$)

- Determine contributing subprocesses
 $g g \rightarrow t \bar{t}$, $q \bar{q} \rightarrow t \bar{t}$, $q q \rightarrow t \bar{t}$ with $q = d, u, s, c, (b)$
- Determine matrix element for each subprocess



- Perform phase space integration for each subprocess

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

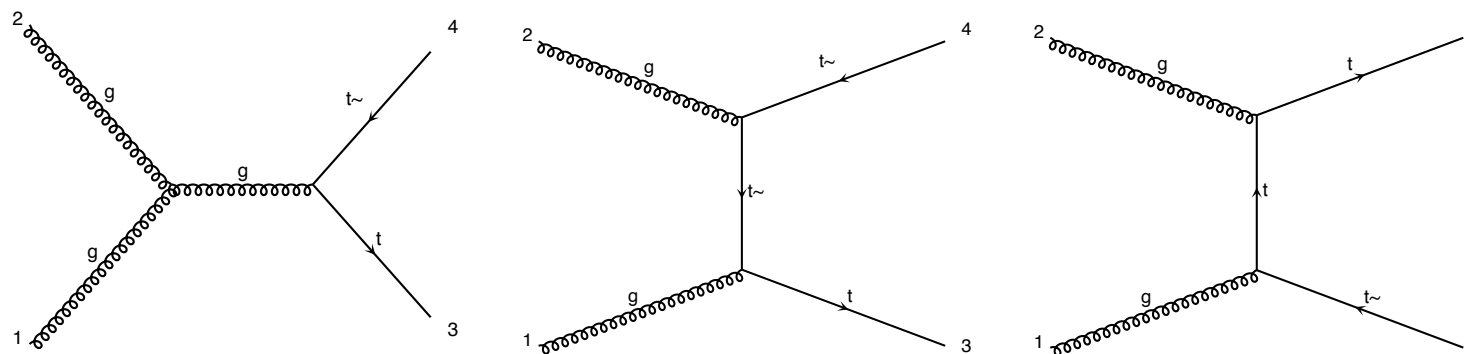
Matrix Element calculation at Hadron Colliders

To calculate a given process (e.g., $p p \rightarrow t \bar{t}$)

1. Determine contributing subprocesses

$g g \rightarrow t \bar{t}, q \bar{q} \rightarrow t \bar{t}, q \bar{q} \rightarrow t \bar{t}$ with $q = d, u, s, c, (b)$ ← Easy enough

2. Determine matrix element for each subprocess ← Hard



3. Perform phase space integration for each subprocess

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

← Very Hard (in general)

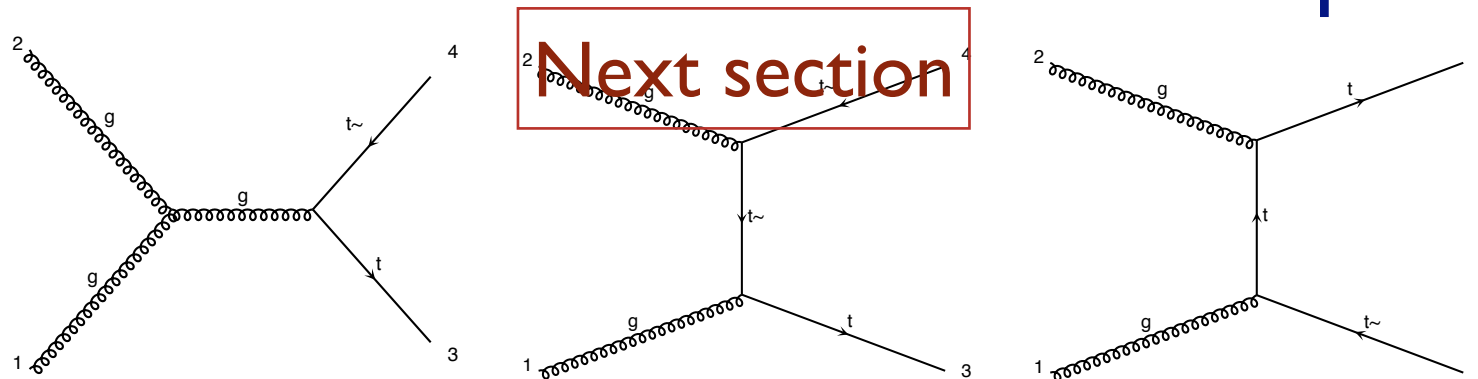
Matrix Element calculation at Hadron Colliders

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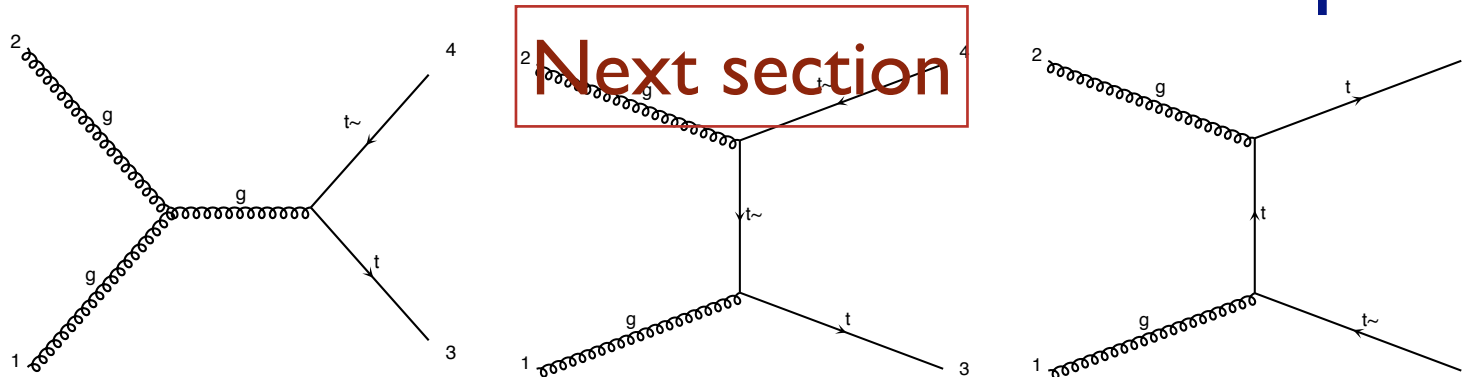
Matrix Element calculation at Hadron Colliders

To calculate a given process (e.g., $p p \rightarrow t \bar{t}$)

1. Determine contributing subprocesses

$g g \rightarrow t \bar{t}, q \bar{q} \rightarrow t \bar{t}, q q \rightarrow t \bar{t}$ with $q = d, u, s, c, (b)$ ← Easy enough

2. Determine matrix element for each subprocess ← Hard



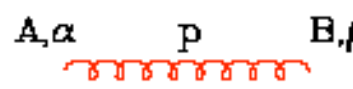
3. Perform phase space integration for each subprocess

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

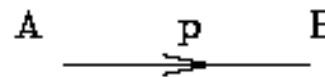
Not This times

← Very Hard (in general)

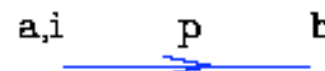
The Matrix Element



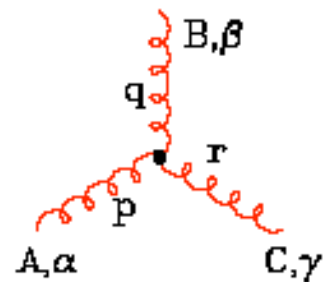
$$\delta^{AB} \left[-g^{\alpha\beta} + (1-\lambda) \frac{p^\alpha p^\beta}{p^2 + i\epsilon} \right] \frac{i}{p^2 + i\epsilon}$$



$$\delta^{AB} \frac{i}{(p^2 + i\epsilon)}$$

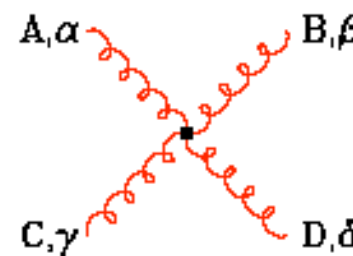


$$\delta^{ab} \frac{i}{(\not{p} - m + i\epsilon)_\mu}$$



$$-g f^{ABC} \left[(p-q)^\gamma g^{\alpha\beta} + (q-r)^\alpha g^{\beta\gamma} + (r-p)^\beta g^{\gamma\alpha} \right]$$

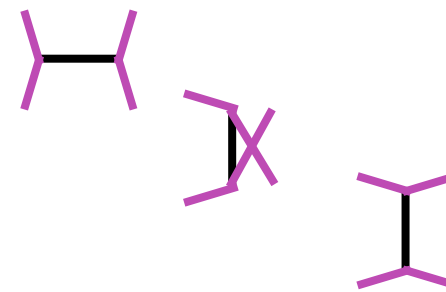
(all momenta incoming)

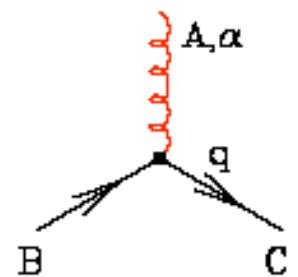


$$-ig^2 f^{XAC} f^{XBD} \left[g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\delta} g^{\beta\gamma} \right]$$

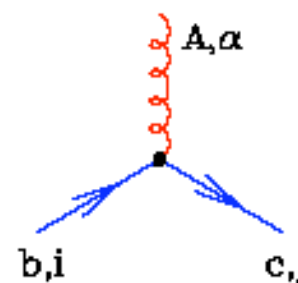
$$-ig^2 f^{XAD} f^{XBC} \left[g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\gamma} g^{\beta\delta} \right]$$

$$-ig^2 f^{XAB} f^{XCD} \left[g^{\alpha\gamma} g^{\beta\delta} - g^{\alpha\delta} g^{\beta\gamma} \right]$$



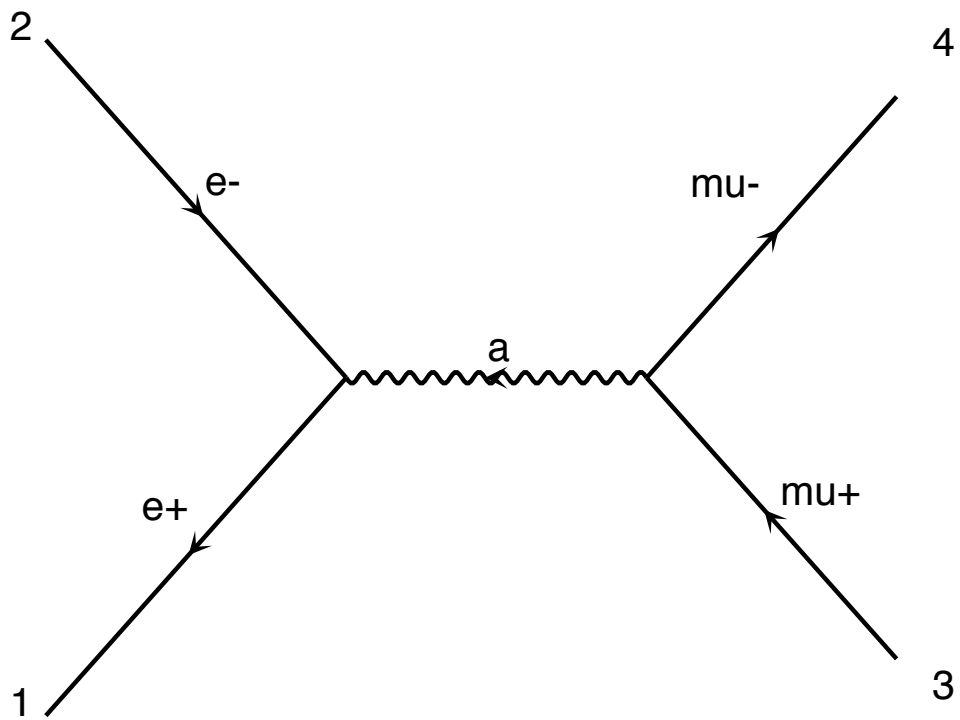


$$g f^{ABC} q^\alpha$$

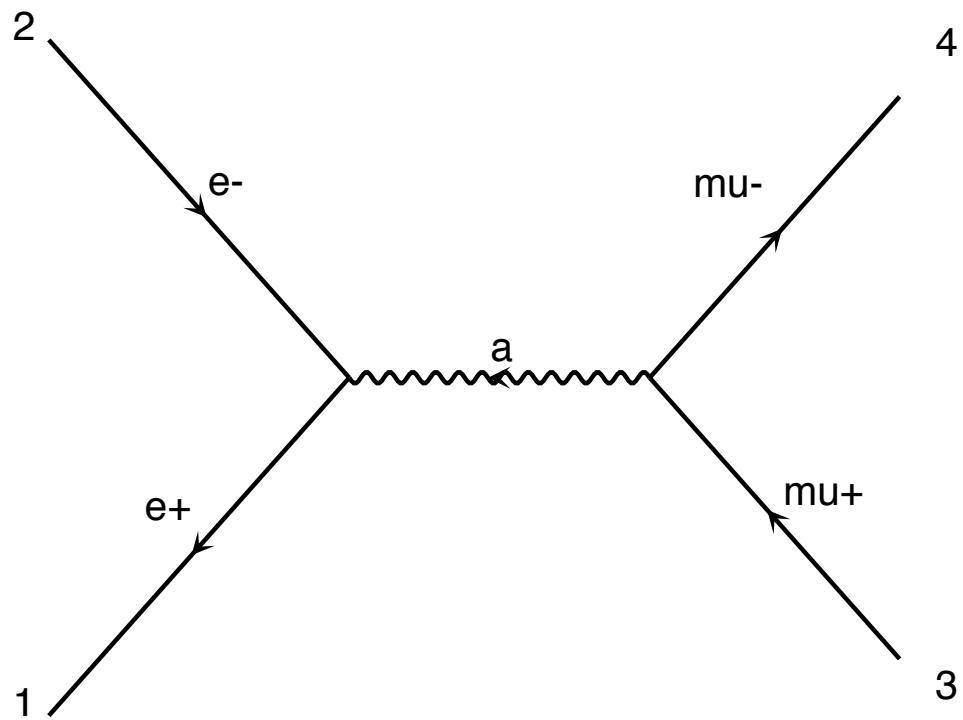


$$-ig (t^A)_{cb} (\gamma^\alpha)_{ij}$$

Evaluate a square Matrix Element

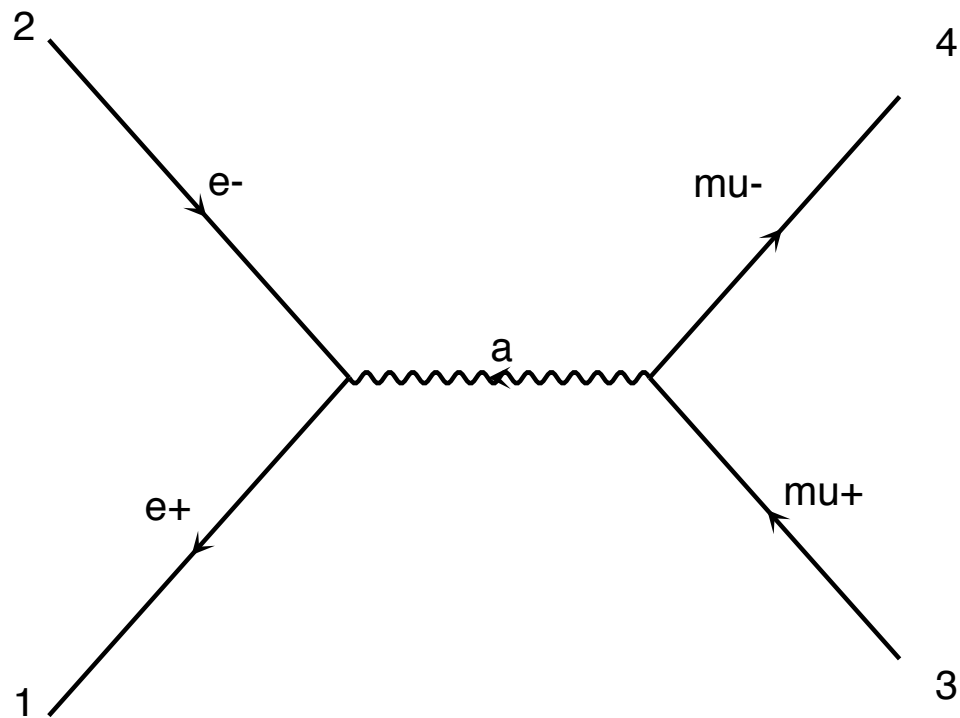


Evaluate a square Matrix Element



$$\mathcal{M} = e^2 (\bar{u} \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{u} \gamma^\nu v)$$

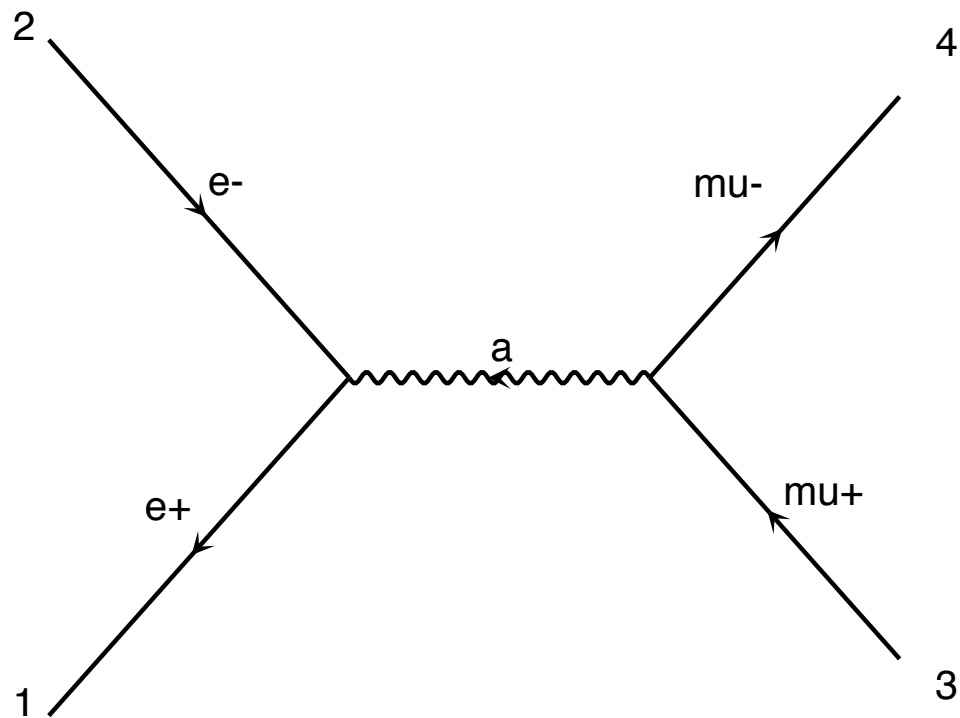
Evaluate a square Matrix Element



$$\mathcal{M} = e^2 (\bar{u} \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{u} \gamma^\nu v)$$

$$\frac{1}{4} \sum_{pol} |\mathcal{M}|^2 = \frac{1}{4} \sum_{pol} \mathcal{M}^* \mathcal{M}$$

Evaluate a square Matrix Element

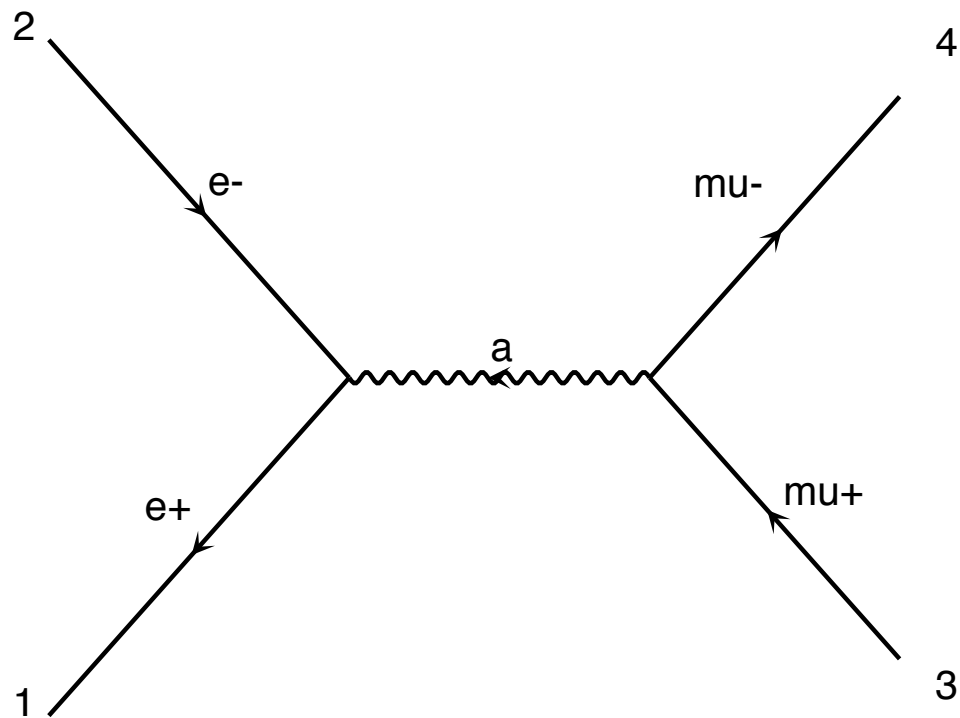


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$$\frac{1}{4} \sum_{pol} |\mathcal{M}|^2 = \frac{1}{4} \sum_{pol} \mathcal{M}^* \mathcal{M}$$

$$\sum_{pol} \bar{u} u = \not{p} + m$$

Evaluate a square Matrix Element



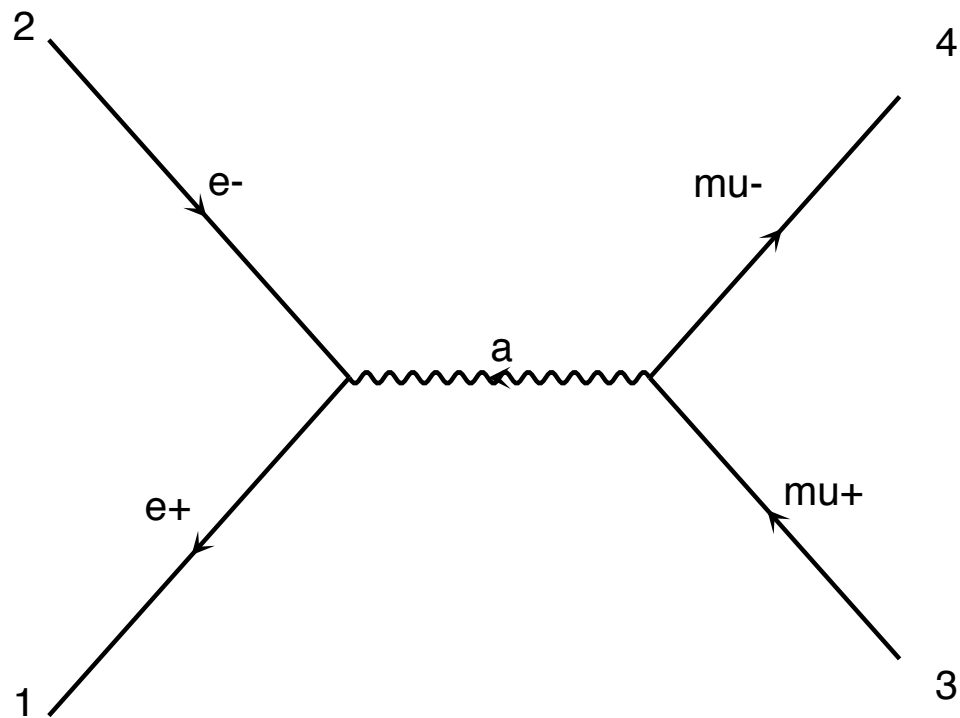
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$$\sum_{pol} \bar{u} u = \not{p} + m$$

$$\rightarrow \frac{e^4}{4q^4} \text{Tr}[\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu] \text{Tr}[\not{p}_3 \gamma_\mu \not{p}_4 \gamma_\nu]$$

Evaluate a square Matrix Element



$$\mathcal{M} = e^2 (\bar{u} \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{u} \gamma^\nu v)$$

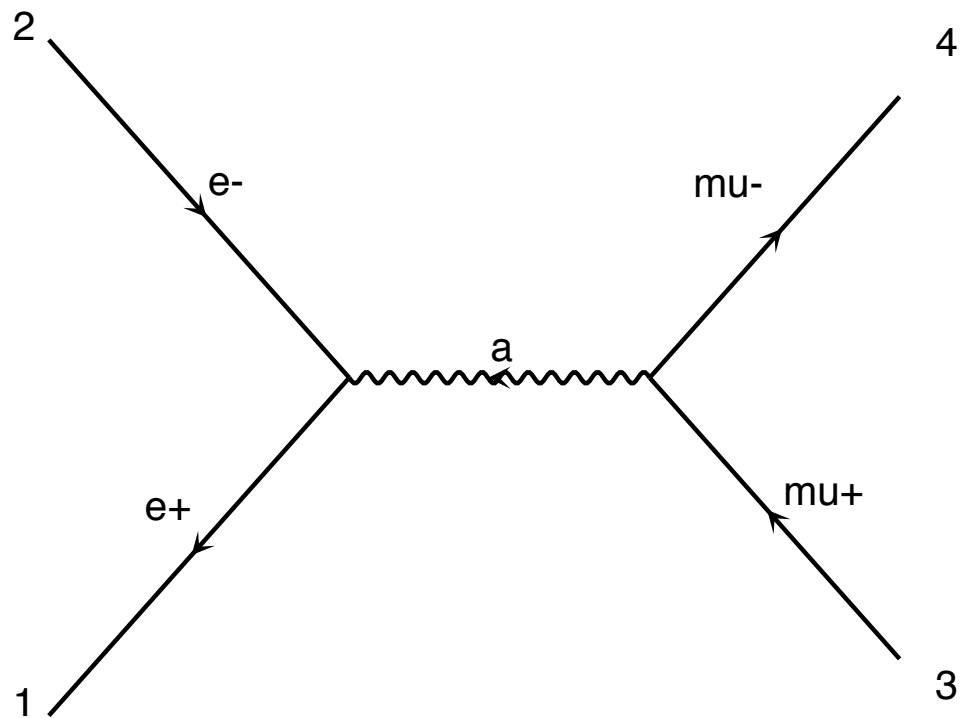
$$\frac{1}{4} \sum_{pol} |\mathcal{M}|^2 = \frac{1}{4} \sum_{pol} \mathcal{M}^* \mathcal{M}$$

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$$\rightarrow \frac{e^4}{4q^4} \text{Tr}[\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu] \text{Tr}[\not{p}_3 \gamma_\mu \not{p}_4 \gamma_\nu]$$

Very Efficient !!!

Evaluate a square Matrix Element



$$\mathcal{M} = e^2 (\bar{u} \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{u} \gamma^\nu v)$$

$$\frac{1}{4} \sum_{pol} |\mathcal{M}|^2 = \frac{1}{4} \sum_{pol} \mathcal{M}^* \mathcal{M}$$

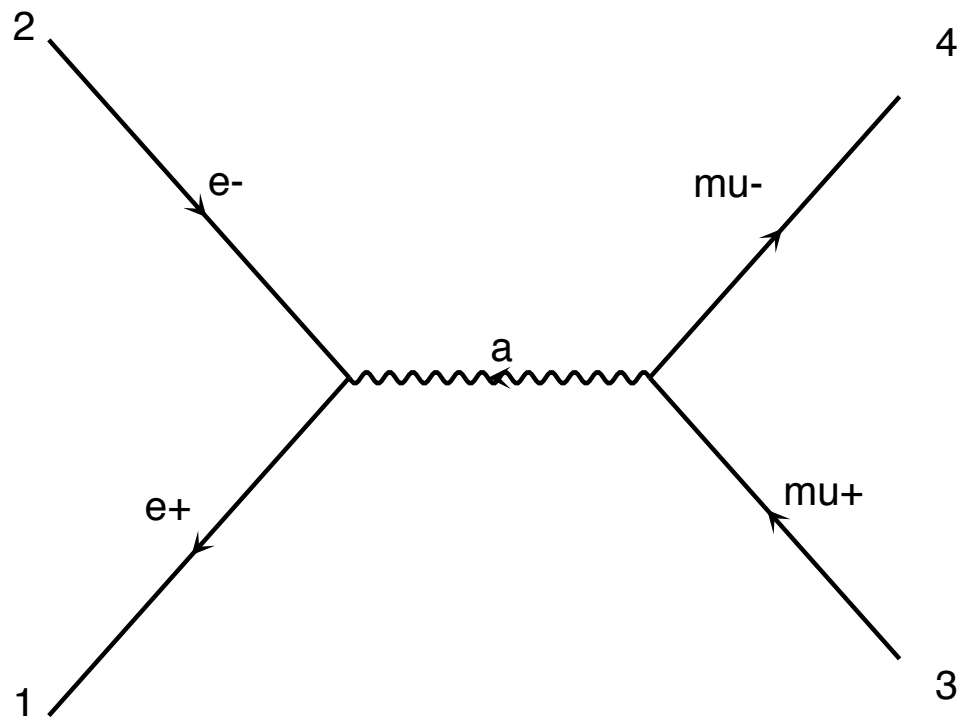
$$\sum_{pol} \bar{u} u = \not{p} + m$$

$$\rightarrow \frac{e^4}{4q^4} \text{Tr}[\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu] \text{Tr}[\not{p}_3 \gamma_\mu \not{p}_4 \gamma_\nu]$$

Very Efficient !!!

But The number of term raises as N^2

Evaluate a square Matrix Element



$$\mathcal{M} = e^2 (\bar{u} \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{u} \gamma^\nu v)$$

$$\frac{1}{4} \sum_{pol} |\mathcal{M}|^2 = \frac{1}{4} \sum_{pol} \mathcal{M}^* \mathcal{M}$$

$$\sum_{pol} \bar{u} u = \not{p} + m$$

$$\rightarrow \frac{e^4}{4q^4} \text{Tr}[\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu] \text{Tr}[\not{p}_3 \gamma_\mu \not{p}_4 \gamma_\nu]$$

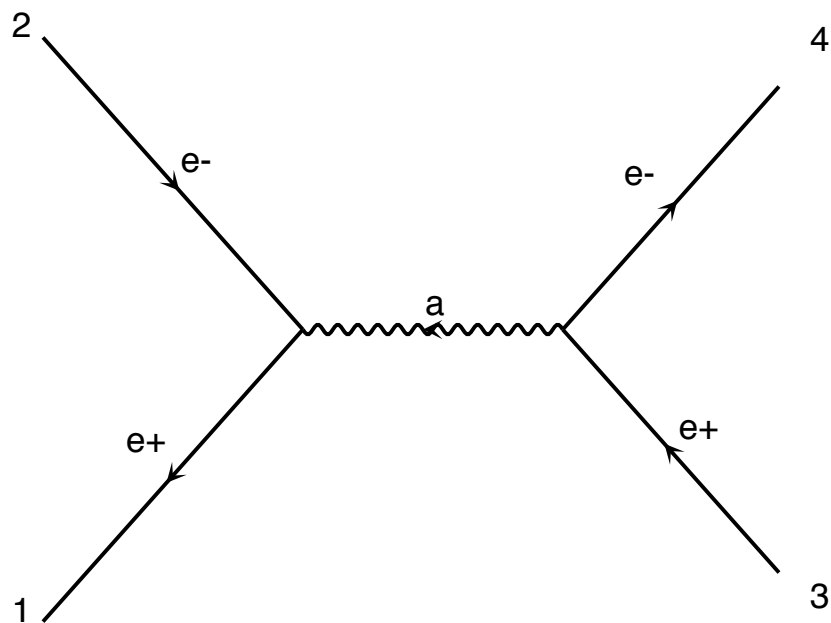
Very Efficient !!!

But The number of term raises as N^2

Only for $2 \rightarrow 2$ and $2 \rightarrow 3$

Basics: Helicity amplitudes

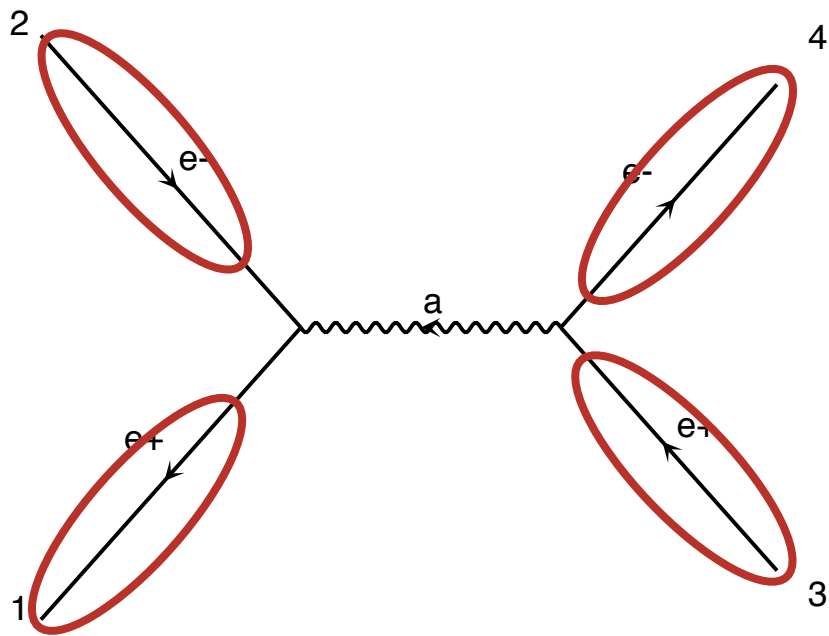
Idea: Evaluate \mathcal{M} for fixed helicity of external particles



$$\mathcal{M} = \bar{u} \gamma^\mu v P_{\mu\nu} \bar{u} \gamma^\nu v$$

Basics: Helicity amplitudes

Idea: Evaluate \mathcal{M} for fixed helicity of external particles

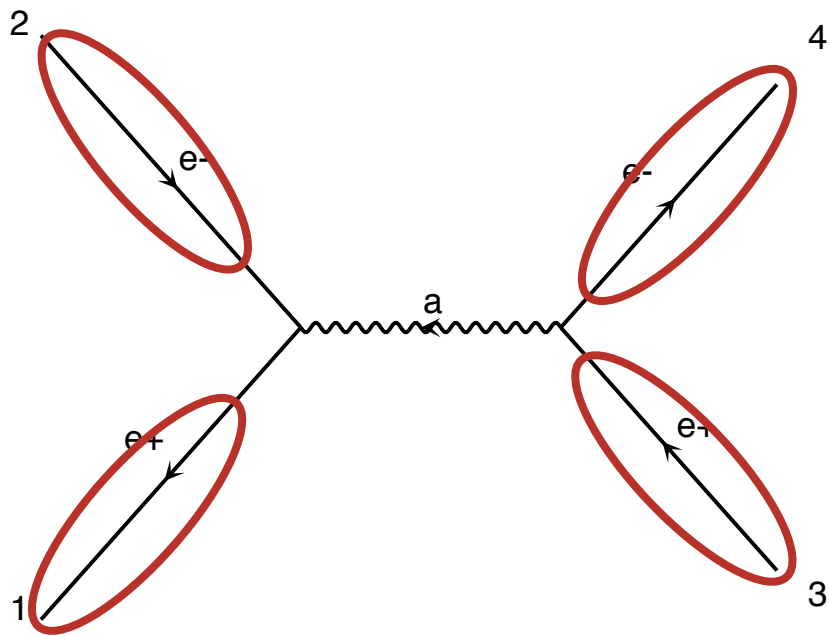


$$\mathcal{M} = \bar{u} \gamma^\mu v P_{\mu\nu} \bar{u} \gamma^\nu v$$

Numbers for given helicity and momenta

Basics: Helicity amplitudes

Idea: Evaluate \mathcal{M} for fixed helicity of external particles



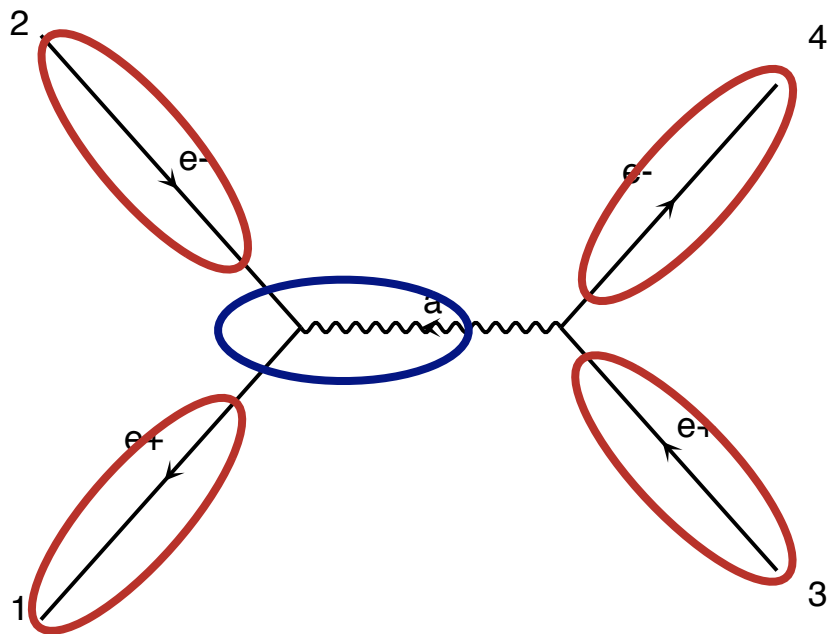
$$\mathcal{M} = \bar{u} \gamma^\mu v P_{\mu\nu} \bar{u} \gamma^\nu v$$

Numbers for given helicity and momenta

```
CALL OXXXXX (P (0 , 1) , ZERO , NHEL (1) , -1*IC (1) , W (1 , 1) )
CALL IXXXXX (P (0 , 2) , ZERO , NHEL (2) , +1*IC (2) , W (1 , 2) )
CALL IXXXXX (P (0 , 3) , ZERO , NHEL (3) , -1*IC (3) , W (1 , 3) )
CALL OXXXXX (P (0 , 4) , ZERO , NHEL (4) , +1*IC (4) , W (1 , 4) )
```

Basics: Helicity amplitudes

Idea: Evaluate \mathcal{M} for fixed helicity of external particles



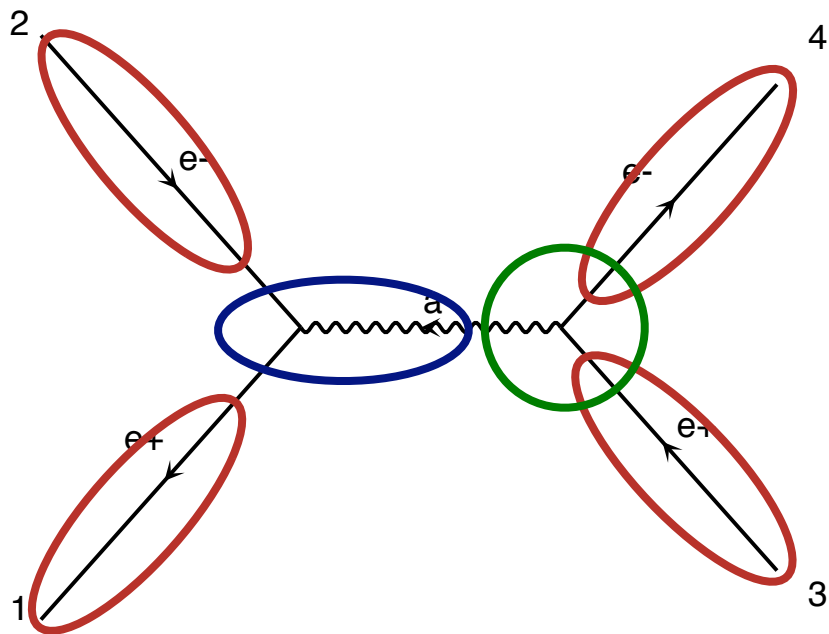
$$\mathcal{M} = \bar{u} \gamma^\mu v P_{\mu\nu} \bar{u} \gamma^\nu v$$

Numbers for given helicity and momenta
Calculate propagator wavefunctions

```
CALL OXXXXX (P (0 , 1) , ZERO , NHEL (1) , -1*IC (1) , W (1 , 1) )
CALL IXXXXX (P (0 , 2) , ZERO , NHEL (2) , +1*IC (2) , W (1 , 2) )
CALL IXXXXX (P (0 , 3) , ZERO , NHEL (3) , -1*IC (3) , W (1 , 3) )
CALL OXXXXX (P (0 , 4) , ZERO , NHEL (4) , +1*IC (4) , W (1 , 4) )
CALL JIOXXX (W (1 , 2) , W (1 , 1) , GAL , ZERO , ZERO , W (1 , 5) )
```

Basics: Helicity amplitudes

Idea: Evaluate \mathcal{M} for fixed helicity of external particles



$$\mathcal{M} = \bar{u} \gamma^\mu v P_{\mu\nu} \bar{u} \gamma^\nu v$$

Numbers for given helicity and momenta

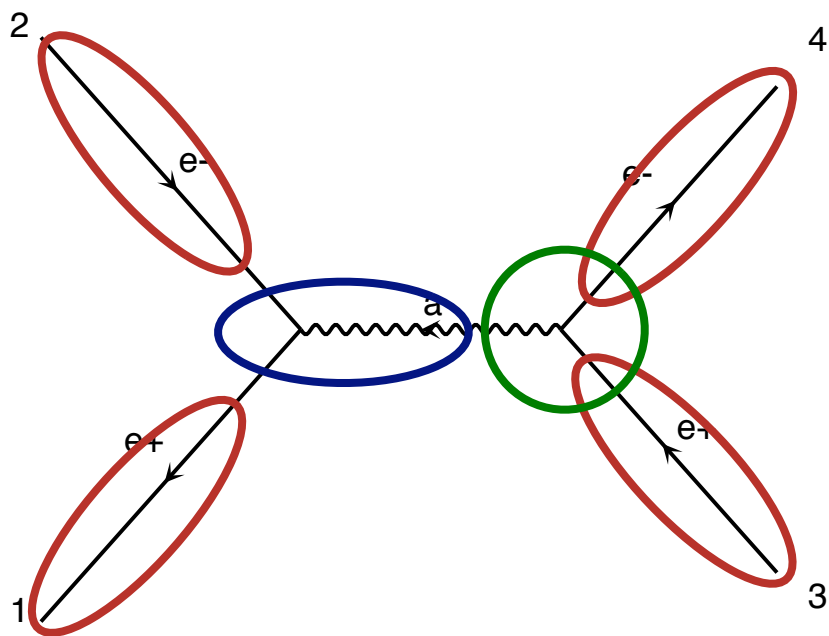
Calculate propagator wavefunctions

Finally evaluate amplitude (c-number)

```
CALL OXXXXX (P (0 , 1) , ZERO , NHEL (1) , -1*IC (1) , W (1 , 1) )
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CALL JIOXXX (W (1 , 2) , W (1 , 1) , GAL , ZERO , ZERO , W (1 , 5) )
CALL IOVXXX (W (1 , 3) , W (1 , 4) , W (1 , 5) , GAL , AMP (1) )
```

Basics: Helicity amplitudes

Idea: Evaluate \mathcal{M} for fixed helicity of external particles



$$\mathcal{M} = \bar{u} \gamma^\mu v P_{\mu\nu} \bar{u} \gamma^\nu v$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

Finally evaluate amplitude (c-number)

Helicity amplitude calls
written by MadGraph

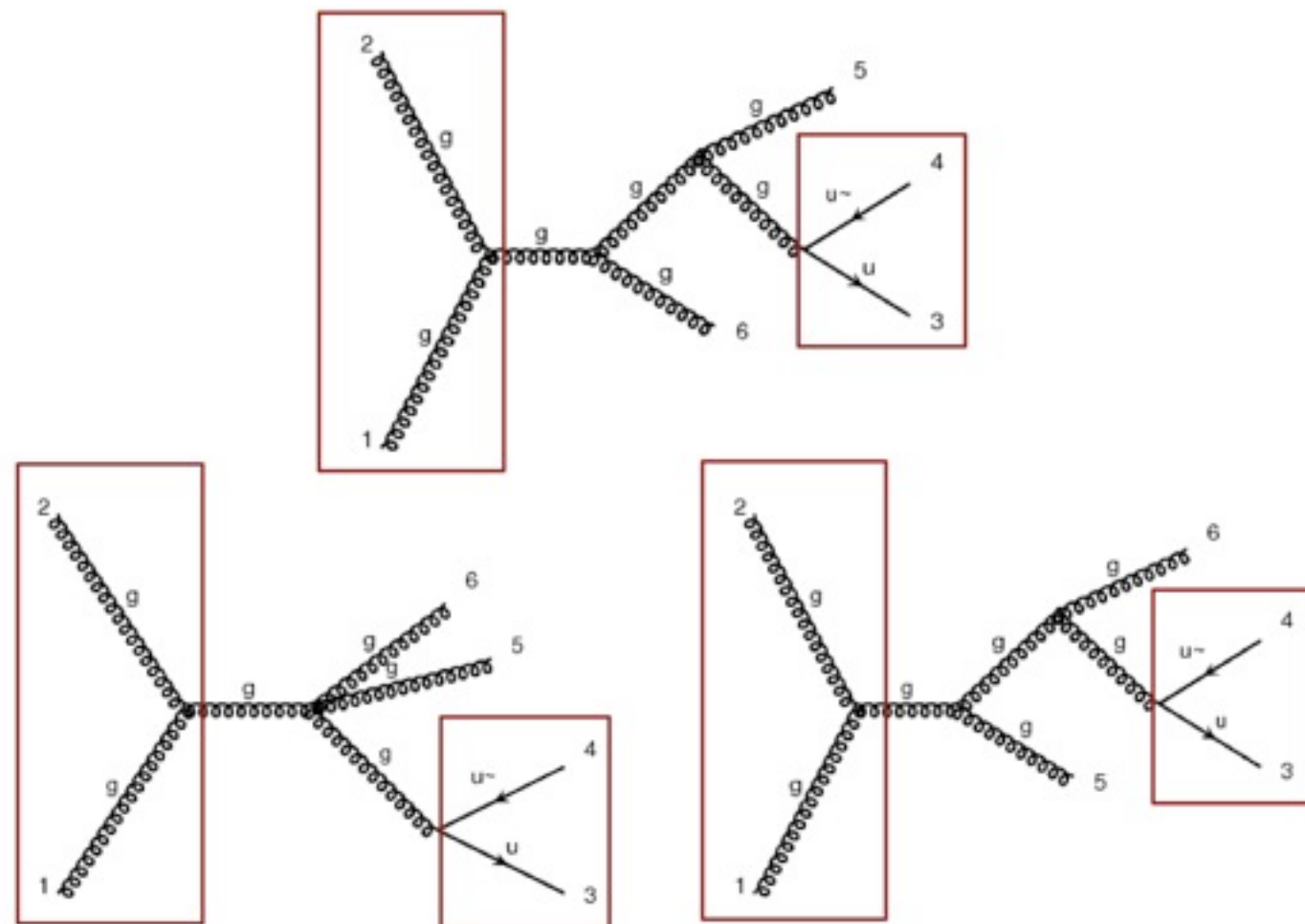
```
CALL OXXXXX (P (0 , 1) , ZERO , NHEL (1) , -1*IC (1) , W (1 , 1) )
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CALL OXXXXX (P (0 , 4) , ZERO , NHEL (4) , +1*IC (4) , W (1 , 4) )
CALL JIOXXX (W (1 , 2) , W (1 , 1) , GAL , ZERO , ZERO , W (1 , 5) )
CALL IOVXXX (W (1 , 3) , W (1 , 4) , W (1 , 5) , GAL , AMP (1) )
```


HELAS

- Original HELicity Amplitude Subroutine library
Murayama, Watanabe, Hagiwara (1991)
- All helicity amplitude routines needed for the Standard Model, MSSM and certain other applications in hand-written library
- Any new Lorentz structures or other refinements need addition by hand
- Introduced a severe restriction on types of models that could be implemented in MadGraph

Basics: Helicity amplitudes

- Allows for fast calculation of any tree-level matrix element through efficient reuse of previously calculated wavefunctions across diagrams



Basics: Helicity amplitudes

- Thanks to new diagram generation algorithm, wf recycling much more efficient in MG5 than MG4

Process	Amplitudes	Wavefunctions		Run time	
		MG 4	MG 5	MG 4	MG 5
$u\bar{u} \rightarrow e^+e^-$	2	6	6	$< 6\mu\text{s}$	$< 6\mu\text{s}$
$u\bar{u} \rightarrow e^+e^-e^+e^-$	48	62	32	0.22 ms	0.14 ms
$u\bar{u} \rightarrow e^+e^-e^+e^-e^+e^-$	3474	3194	301	46.5 ms	19.0 ms
$u\bar{u} \rightarrow d\bar{d}$	1	5	5	$< 4\mu\text{s}$	$< 4\mu\text{s}$
$u\bar{u} \rightarrow d\bar{d}g$	5	11	11	27 μs	27 μs
$u\bar{u} \rightarrow d\bar{d}gg$	38	47	29	0.42 ms	0.31 ms
$u\bar{u} \rightarrow d\bar{d}ggg$	393	355	122	10.8 ms	6.75 ms
$u\bar{u} \rightarrow u\bar{u}gg$	76	84	40	1.24 ms	0.80 ms
$u\bar{u} \rightarrow u\bar{u}ggg$	786	682	174	35.7 ms	17.2 ms
$u\bar{u} \rightarrow d\bar{d}d\bar{d}$	14	28	19	84 μs	83 μs
$u\bar{u} \rightarrow d\bar{d}d\bar{d}g$	132	178	65	1.88 ms	1.15 ms
$u\bar{u} \rightarrow d\bar{d}d\bar{d}gg$	1590	1782	286	141 ms	34.4 ms
$u\bar{u} \rightarrow d\bar{d}d\bar{d}d\bar{d}$	612	758	141	42.5 ms	6.6 ms

Time for matrix element evaluation on a Sony Vaio TZ laptop

MadGraph 5

MadGraph 5

Alwall, Herquet, Maltoni, OM, Stelzer, arXiv:1106.0522

- MadGraph 5 is the successor of the well-known MadGraph matrix element generator
- Original MadGraph by Tim Stelzer was written in Fortran, first version from 1994 [hep-ph/9401258](https://arxiv.org/abs/hep-ph/9401258)
- Event generation by MadEvent using the single diagram enhanced multichannel integration technique in 2002 (Stelzer, Maltoni) [hep-ph/0208156](https://arxiv.org/abs/hep-ph/0208156)
- Full support for BSM (and many other improvements) in MG/ME 4 (2006-2008) [arXiv:0706.2334](https://arxiv.org/abs/0706.2334), [arXiv:0809.2410](https://arxiv.org/abs/0809.2410)

Master formula

Master formula

$$\hat{\sigma}_{ab \rightarrow X}(\hat{s}, \dots)$$

Parton level
cross section

- Parton level cross section from matrix element

Master formula

$$\hat{\sigma}_{ab \rightarrow X}(\hat{s}, \dots) f_a(x_1) f_b(x_2)$$

Parton level
cross section

Parton density
functions

- Parton level cross section from matrix element
- Parton density (or distribution) functions:
Process independent, determined by particle type

Master formula

$$\int \hat{\sigma}_{ab \rightarrow X}(\hat{s}, \dots) f_a(x_1) f_b(x_2) dx_1 dx_2 d\Phi_{FS}$$

Parton level cross section
Parton density functions
Phase space integral

- Parton level cross section from matrix element
- Parton density (or distribution) functions:
Process independent, determined by particle type

Demo

Using MadGraph on the Web!

To generate matrix elements using MadGraph:

- Go to <http://madgraph.hep.uiuc.edu/>
(or google for MadGraph)
- Register
- Write your process
- Press
- Download the tar file or
generate events directly online on our clusters!

file:///Users/omatt/Documents/eclipse/madgraph5/PROC_sm_0/HTML/run_01/results.html

$s = 0.00053102 \pm 2.91e-07$ (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
P0 sum	0.000531025				
P0 ll ll	0.000531	2.91e-07	358013.0	19088.0	0

[P0 ll ll](#)

$s = 0.00053102 \pm 2.91e-07$ (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1	0.0004673	2.62e-07	330011.0	17468.0	3.74e+07
G2	6.368e-05	1.27e-07	28002.0	1620.0	2.55e+07

Not the single diagram contribution but:

$$f_i = \frac{|A_i|^2}{\sum_i |A_i|^2} |A_{\text{tot}}|^2$$

Using MadGraph on your computer!

To generate matrix elements and events:

- Download MadGraph 5 from <https://launchpad.net/madgraph5>
- Untar and run bin/mg5
- Write “generate *process*”
- Write “output”
- Write “launch”

Sounds easy? It is! Let me show you!

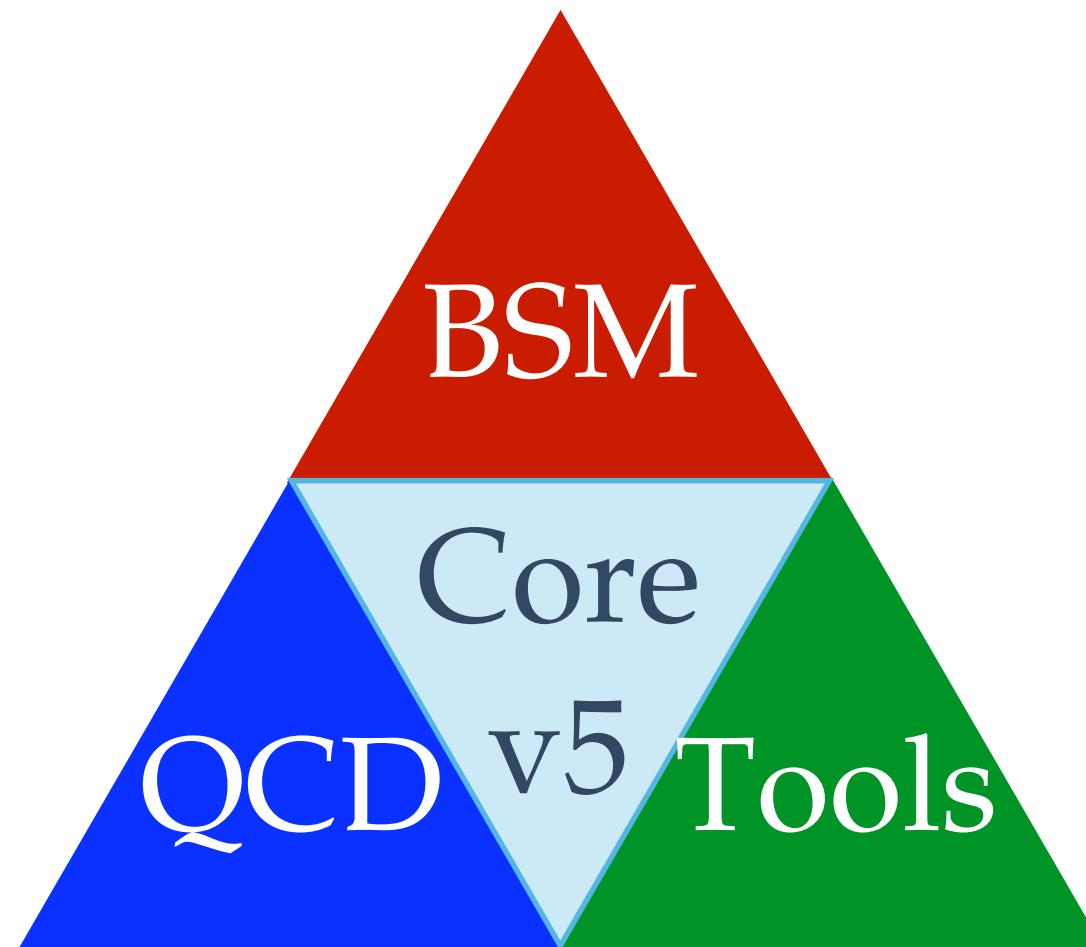
Examples shown

- $p p \rightarrow t \bar{t}$
This gives only (the dominant) QCD vertices, and ignores (the negligible) QED vertices.
- $p p \rightarrow t \bar{t}$ QED=2
This gives both QED and QCD vertices.
- $p p \rightarrow w^+ j j, w^+ \rightarrow l^+ \nu_l$
More complicated example.

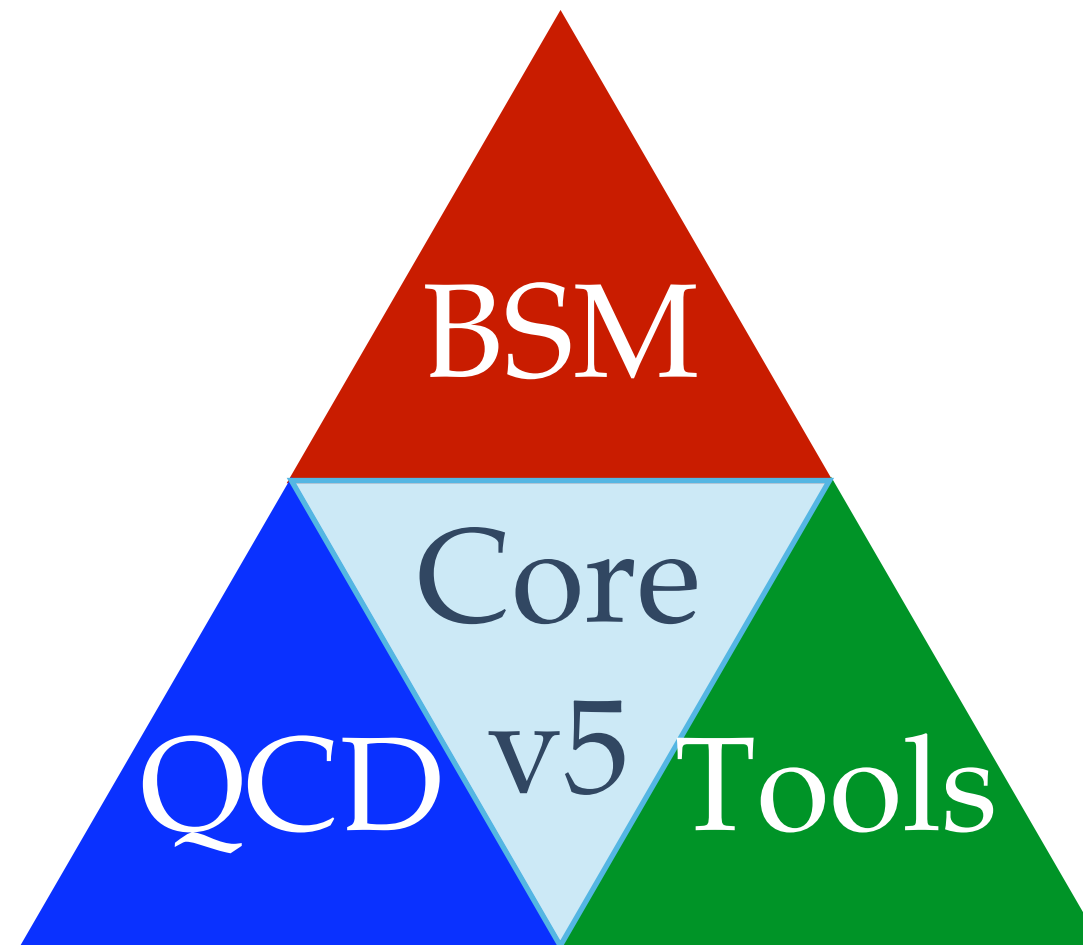
More syntax examples

- $p p \rightarrow t \bar{t} j$ QED=2: Generate all combinations of processes for particles defined in multiparticle labels p / j , including up to two QED vertices (and unlimited QCD vertices)
- $p p \rightarrow t \bar{t}, (t \rightarrow b W^+, W^+ \rightarrow l^+ \nu_l), \bar{t} \rightarrow b \bar{j} j$:
 - Only diagrams compatible with given decay
 - Only t / \bar{t} and W^+ close to mass shell in event generation
- $p p \rightarrow W^+ W^- / h$: Exclude any diagrams with h
- $p p \rightarrow W^+ W^- \& h$: Exclude on-shell h in event generation (but retain interference effects)

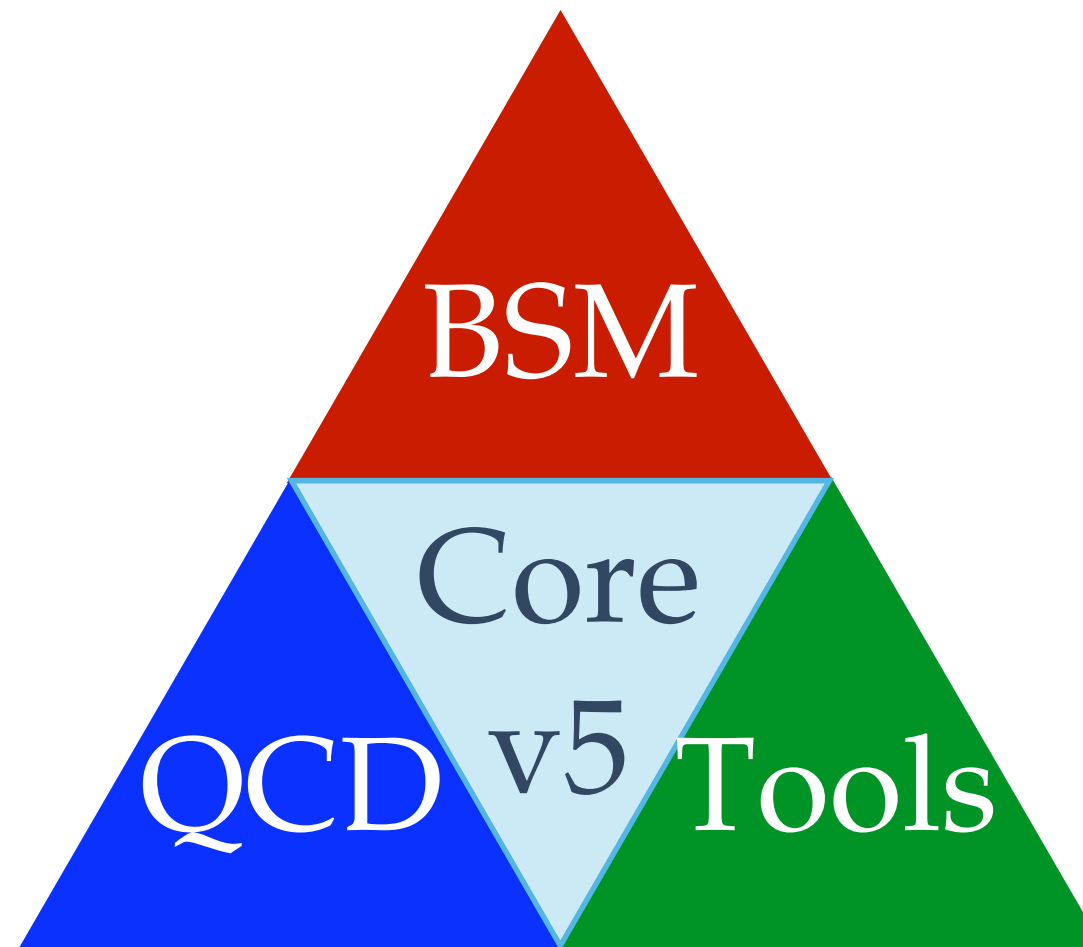
MadGraph 5 feature



UFO / ALOHA

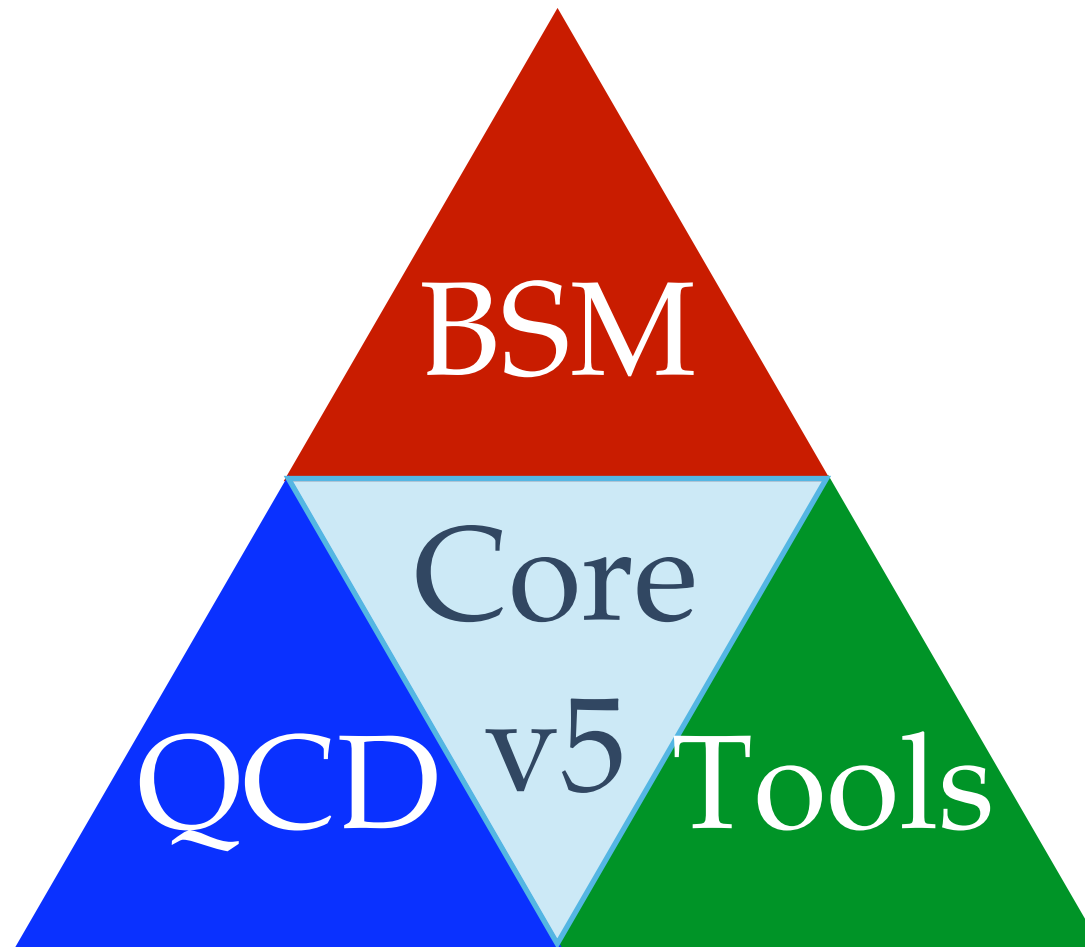


UFO / ALOHA



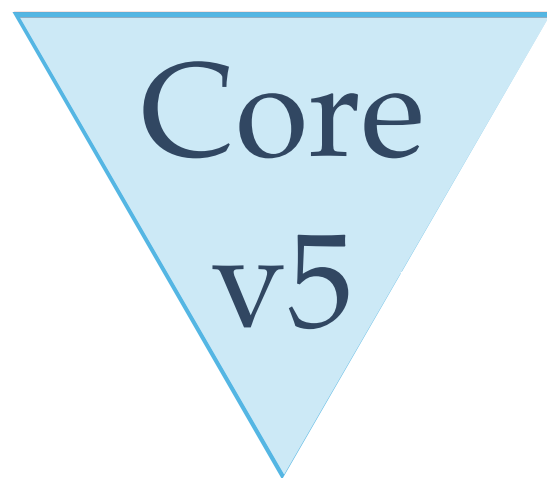
DECAY
MADWEIGHT
MadAnalysis5

UFO / ALOHA



MADLOOP
MADFKS
MADGOLEM

DECAY
MADWEIGHT
MadAnalysis5



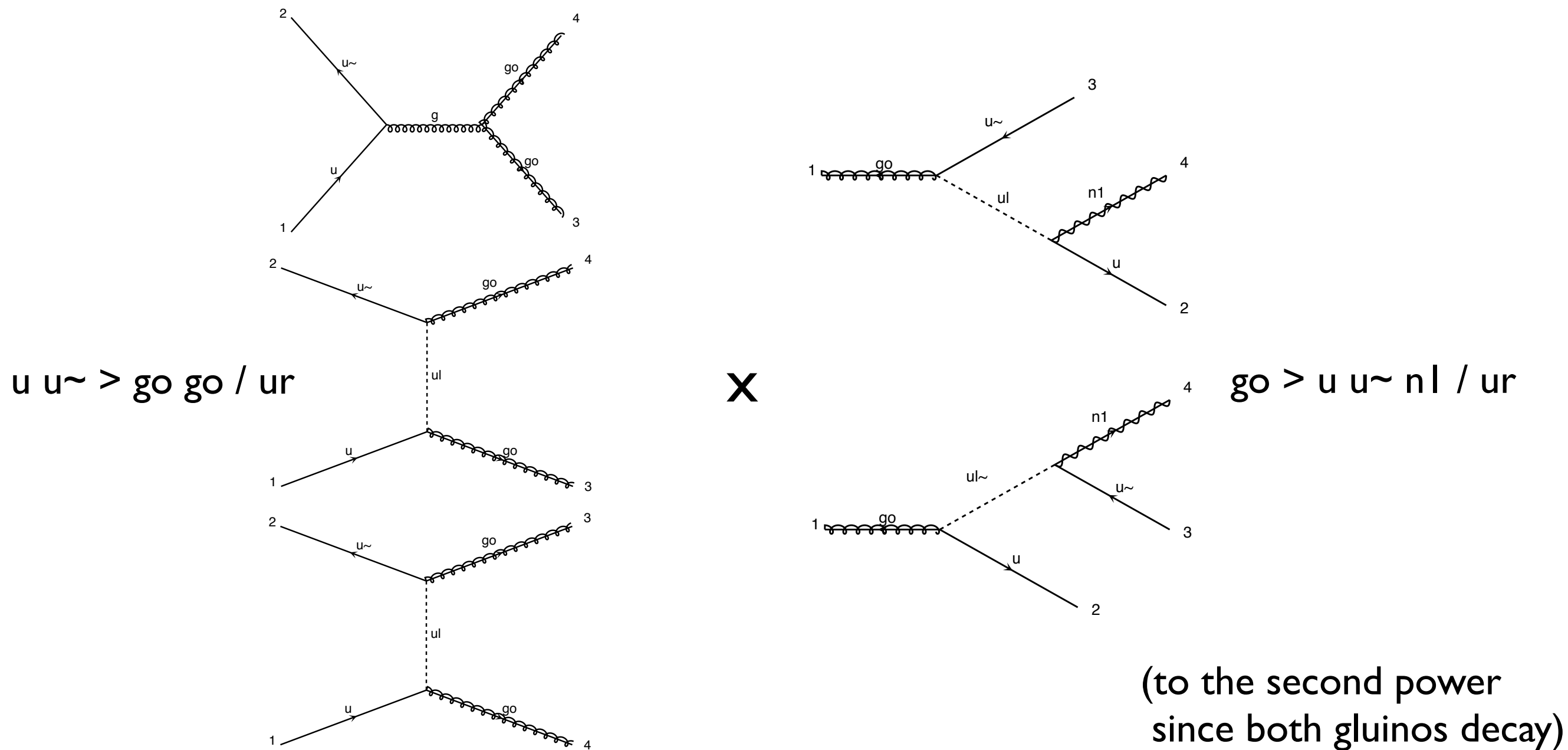
- Diagram Generation
- Cross-section computation
- Generation of Events

Output Format

- StandAlone (Fortran / C++)
- Event Generation (Fortran)
- Pythia 8 (C++)

Decay chains

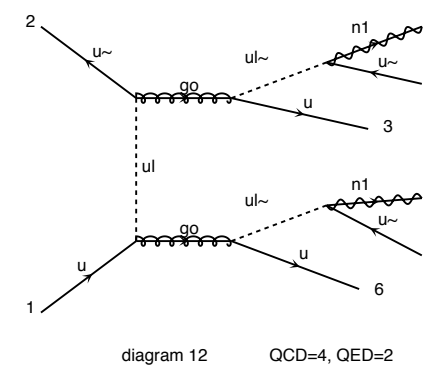
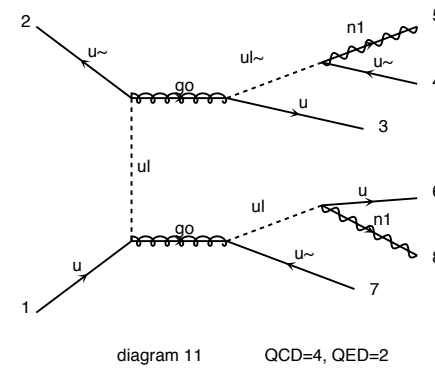
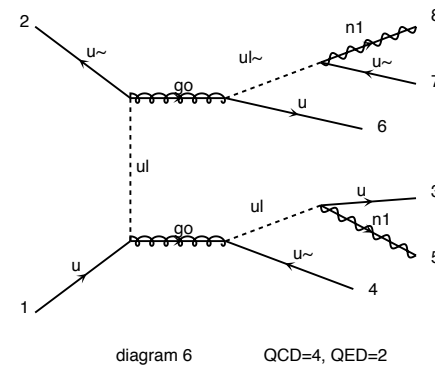
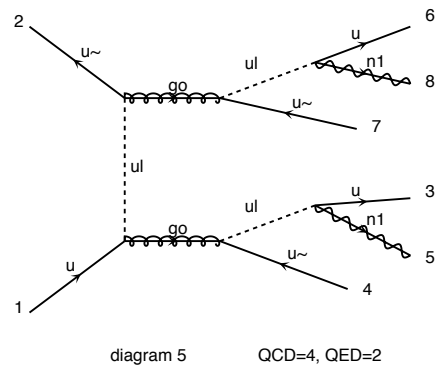
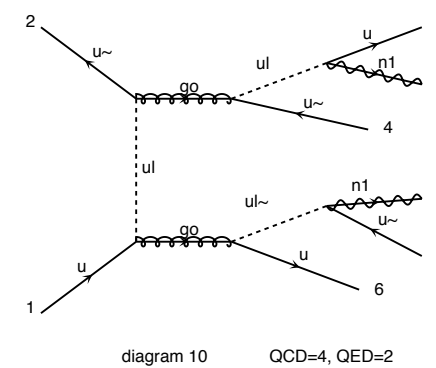
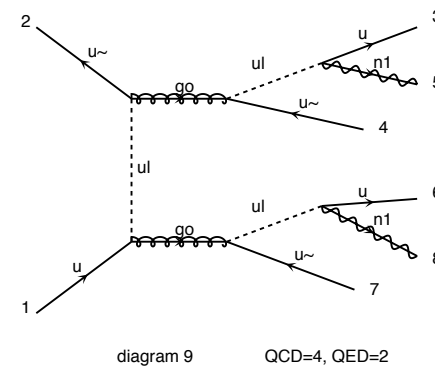
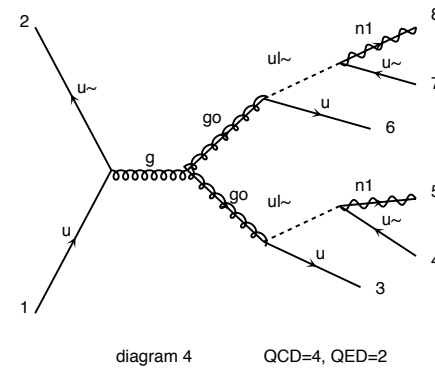
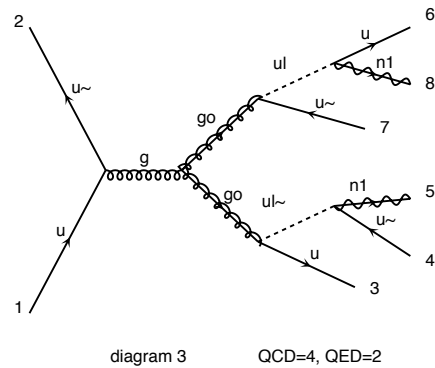
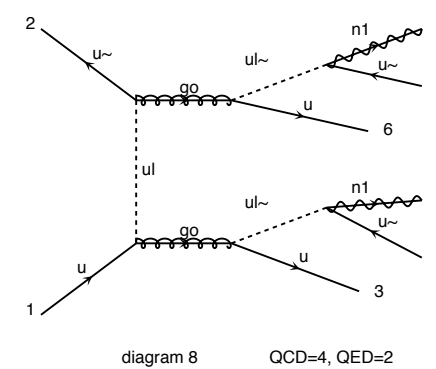
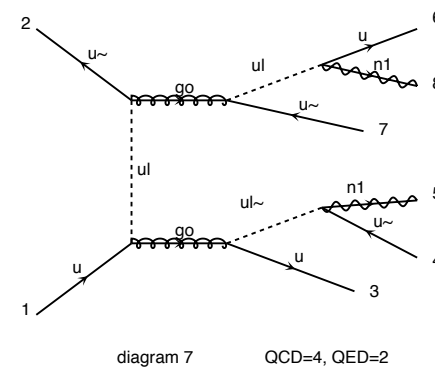
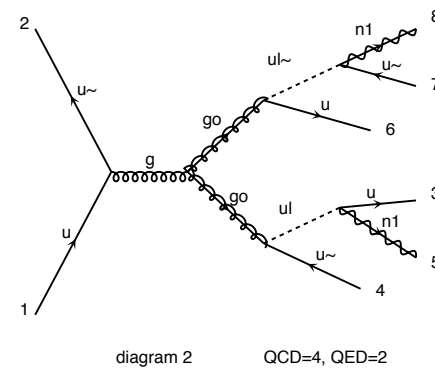
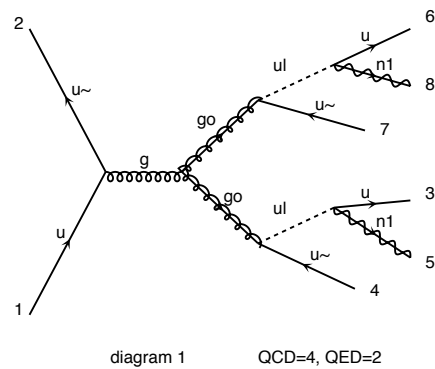
- If multiple diagrams in decays, need to multiply together core process and decay diagrams:



Decay chains

- If multiple diagrams in decays, need to multiply together core process and decay diagrams:

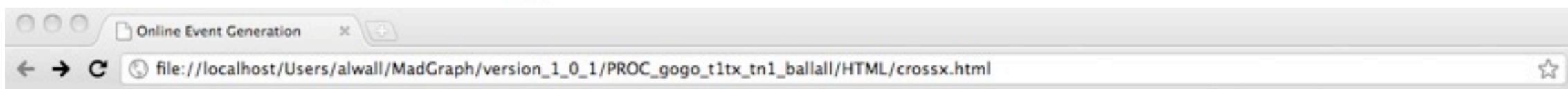
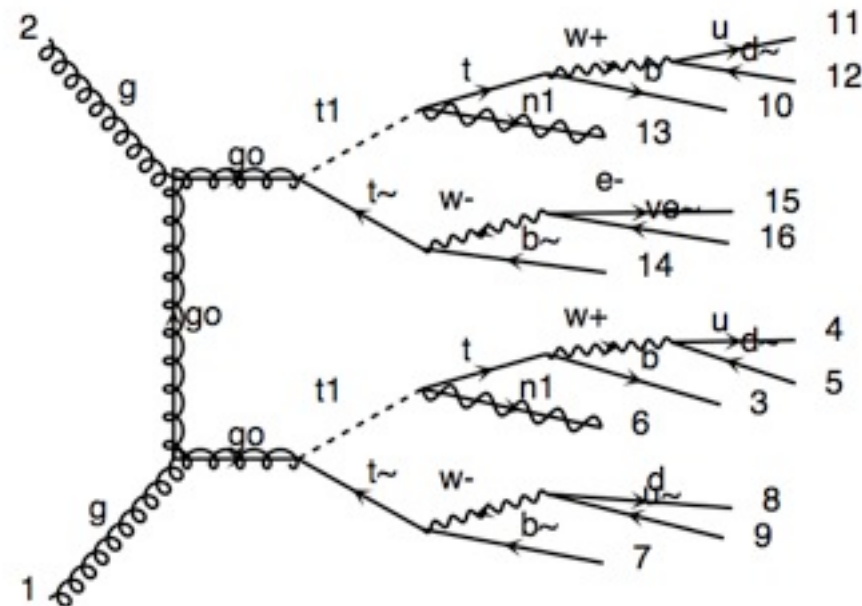
$$u u^{\sim} \rightarrow g o \ g o / u r, g o \rightarrow u u^{\sim} \ n 1 / u r$$



Decay chains

- Decay chains retain **full matrix element** for the diagrams compatible with the decay
- Full spin correlations (within and between decays)
- Full width effects
- However, no interference with non-resonant diagrams
 - ➔ Description only valid close to pole mass
 - ➔ Cutoff at $|m \pm n\Gamma|$ where n is set in `run_card`.

Decay chains



Results for $g g \rightarrow g_0 g_0$, ($g_0 \rightarrow t \bar{t}$, $t \rightarrow b \bar{b}$ all all / h+ , ($t \rightarrow t n_1$, $t \rightarrow b$ all all / h+)) in the mssm

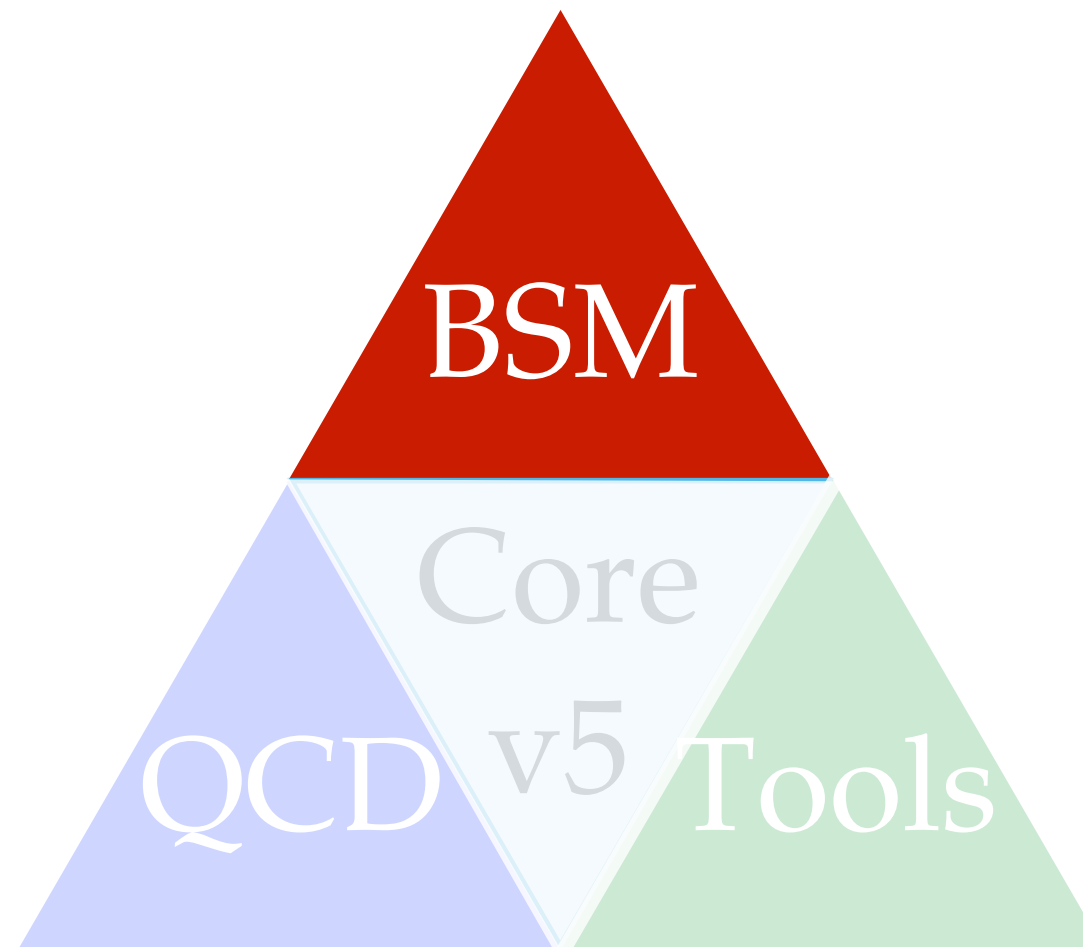
Available Results

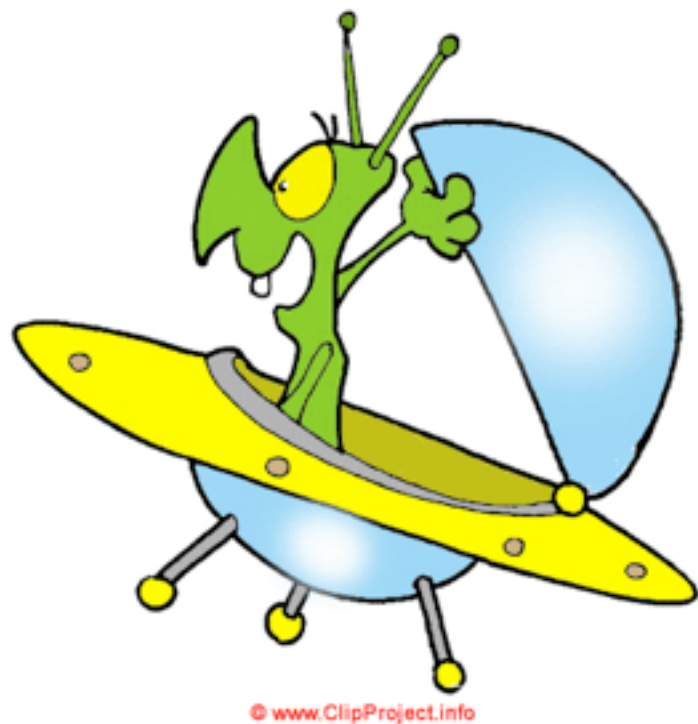
Links	Events	Tag	Run	Collider	Cross section (pb)	Events
results banner	Parton-level LHE	fermi	test	pp 7000 x 7000 GeV	.33857E-03	10000

[Main Page](#)

Thanks to developments in MadEvent, also (very) long decay chains possible to simulate directly in MadGraph!

UFO / ALOHA

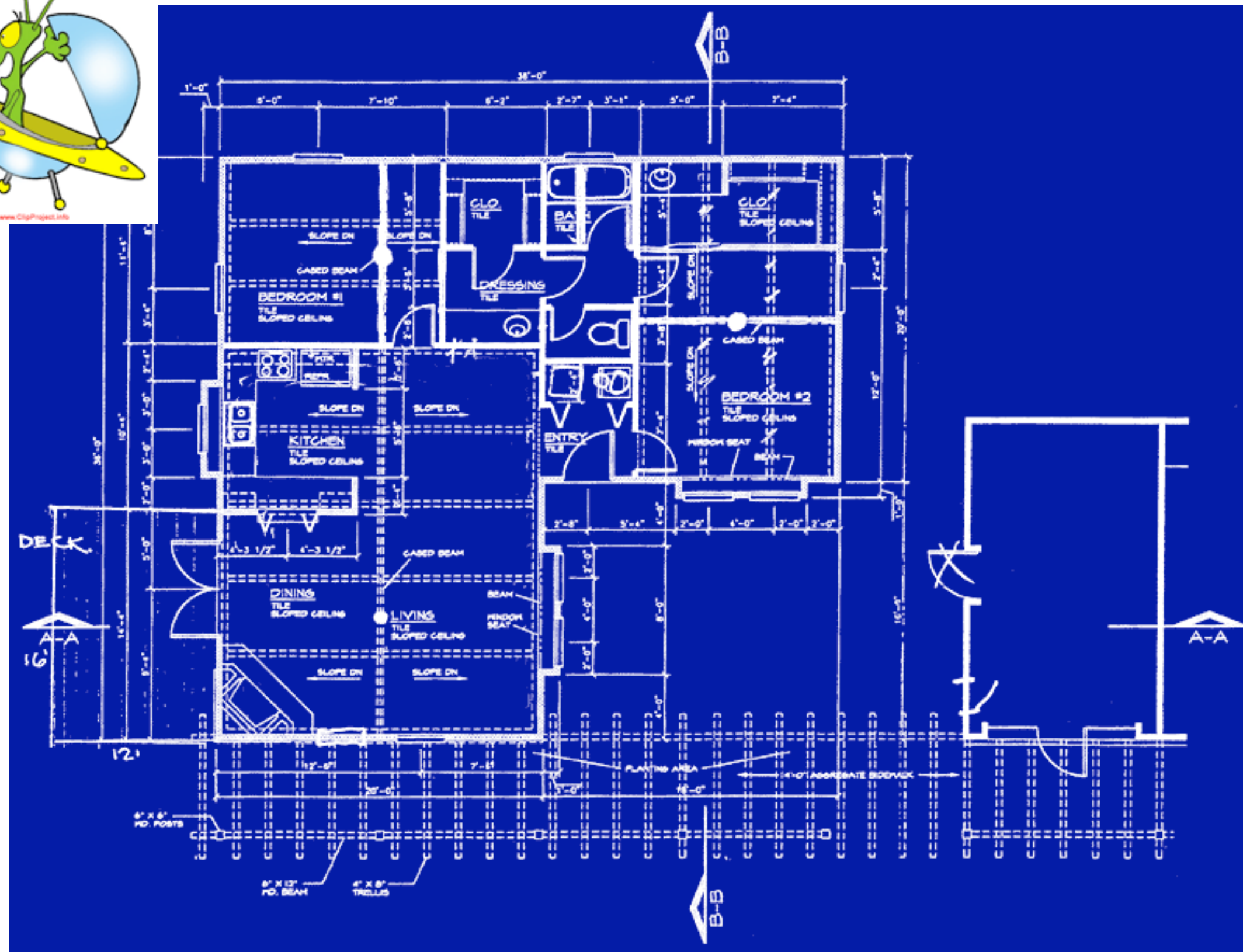




- New Model Format
- Gosam/ Herwig++/ MG5
- Fully generic color/Lorentz/...

- Automatic Creation of HELAS routine for ANY BSM theory
- Fortran / C++ / Python





Universal FeynRules Output (UFO)

`vertices.py:`

```
V_2 = Vertex(name = 'V_2',  
             particles = [ P.G, P.G, P.G ],  
             color = [ 'f(1,2,3)' ],  
             lorentz = [ L.VVV1 ],  
             couplings = {(0,0):C.GC_4})
```

Universal FeynRules Output (UFO)

particles.py:

```
G = Particle(pdg_code = 21,  
            name = 'G',  
            antiname = 'G',  
            spin = 3,  
            color = 8,  
            mass = 'ZERO',  
            width = 'ZERO',  
            texname = 'G',  
            antitexname = 'G',  
            line = 'curly',  
            charge = 0,  
            LeptonNumber = 0,  
            GhostNumber = 0)
```

vertices.py:

```
V_2 = Vertex(name = 'V_2',  
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            charge = 0,  
            LeptonNumber = 0,  
            GhostNumber = 0)
```

lorentz.py:

```
VVV1 = Lorentz(name = 'VVV1',  
              spins = [ 3, 3, 3 ],  
              Structure =  
                'P(3,1)*Metric(1,2) -  
                P(3,2)*Metric(1,2) -  
                P(2,1)*Metric(1,3) +  
                P(2,3)*Metric(1,3) +  
                P(1,2)*Metric(2,3) -  
                P(1,3)*Metric(2,3)')
```

vertices.py:

```
V_2 = Vertex(name = 'V_2',  
            particles = [ P.G, P.G, P.G ],  
            color = [ 'f(1,2,3)' ],  
            lorentz = [ L.VVV1 ],  
            couplings = {(0,0):C.GC_4})
```


Universal FeynRules Output (UFO)

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            antiname = 'G',  
            spin = 3,  
            color = 8,  
            mass = 'ZERO',  
            width = 'ZERO',  
            texname = 'G',  
            antitexname = 'G',  
            line = 'curly',  
            charge = 0,  
            LeptonNumber = 0,  
            GhostNumber = 0)
```

lorentz.py:

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VVV1 = Lorentz(name = 'VVV1',  
              spins = [ 3, 3, 3 ],  
              Structure =  
                'P(3,1)*Metric(1,2) -  
                P(3,2)*Metric(1,2) -  
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                P(2,3)*Metric(1,3) +  
                P(1,2)*Metric(2,3) -  
                P(1,3)*Metric(2,3)')
```

couplings.py:

```
GC_4 = Coupling(name = 'GC_4',  
               value = '-G',  
               order = {'QCD':1})
```

vertices.py:

```
V_2 = Vertex(name = 'V_2',  
            particles = [ P.G, P.G, P.G ],  
            color = [ 'f(1,2,3)' ],  
            lorentz = [ L.VVV1 ],  
            couplings = {(0,0):C.GC_4})
```

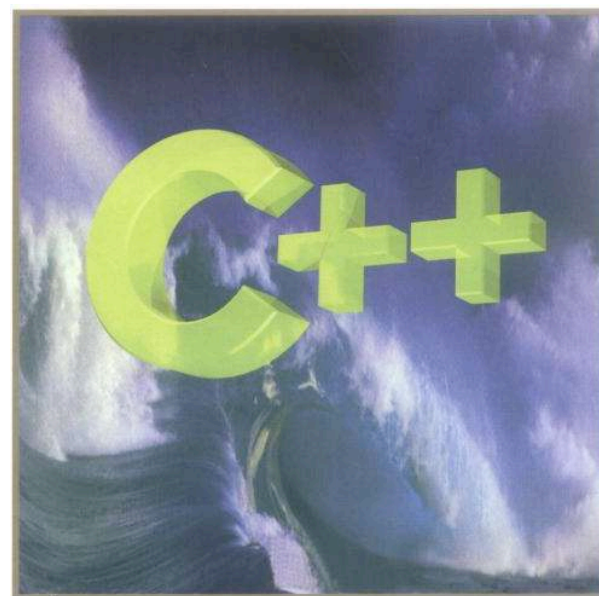
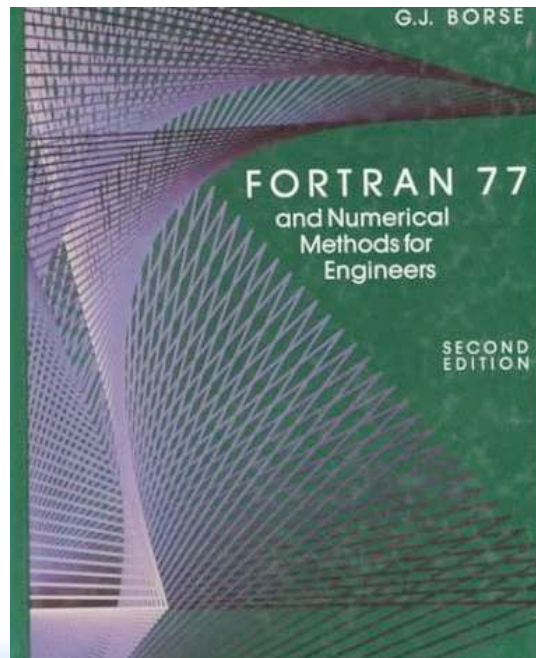



ALOHA

~~ALOHA Google translate~~

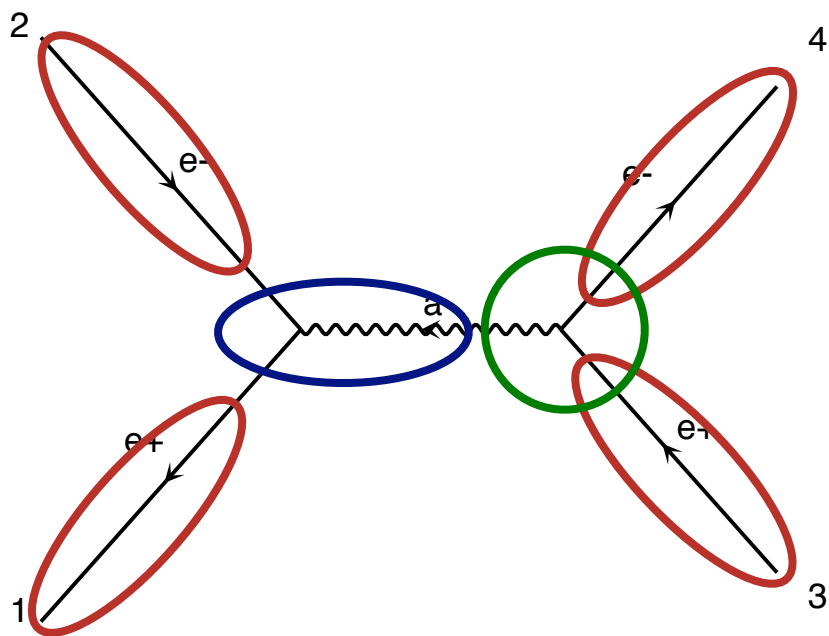
From: [UFO] To: Helicity [Translate]

Type text or a website address or translate a document.



Basics: Helicity amplitudes

Idea: Evaluate \mathcal{M} for fixed helicity of external particles



$$\mathcal{M} = \bar{u} \gamma^\mu v P_{\mu\nu} \bar{u} \gamma^\nu v$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

Finally evaluate amplitude (c-number)

Helicity amplitude calls
written by MadGraph

```
CALL OXXXXX (P (0 , 1) , ZERO , NHEL (1) , -1*IC (1) , W (1 , 1) )
CALL IXXXXX (P (0 , 2) , ZERO , NHEL (2) , +1*IC (2) , W (1 , 2) )
CALL IXXXXX (P (0 , 3) , ZERO , NHEL (3) , -1*IC (3) , W (1 , 3) )
CALL OXXXXX (P (0 , 4) , ZERO , NHEL (4) , +1*IC (4) , W (1 , 4) )
CALL JIOXXX (W (1 , 2) , W (1 , 1) , GAL , ZERO , ZERO , W (1 , 5) )
CALL IOVXXX (W (1 , 3) , W (1 , 4) , W (1 , 5) , GAL , AMP (1) )
```

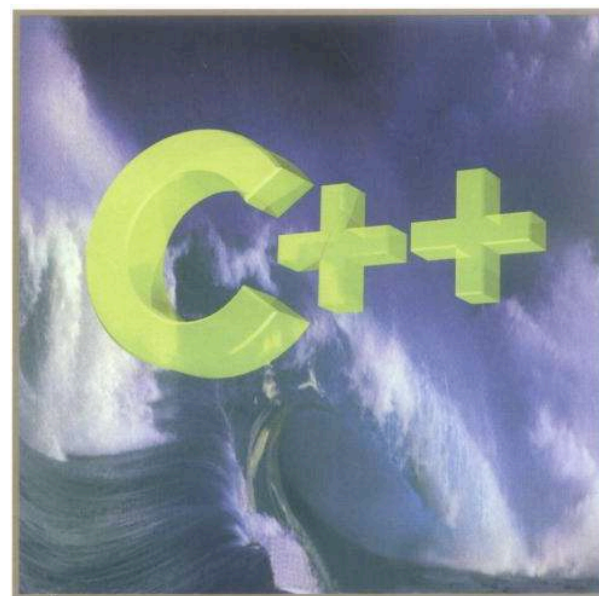
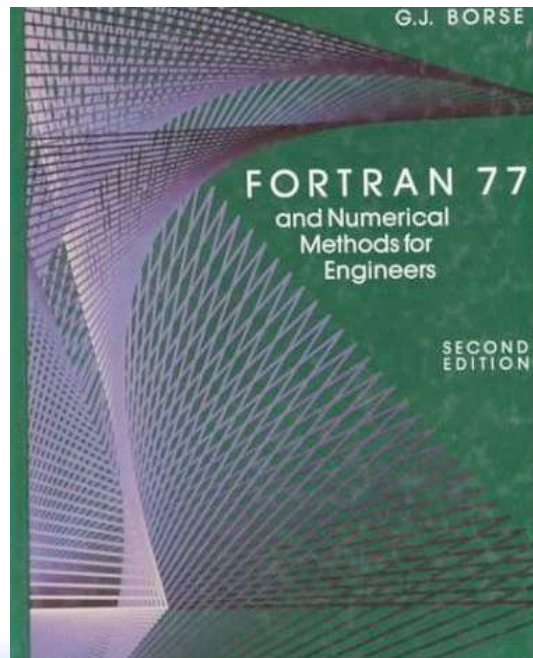


ALOHA

~~ALOHA
Google translate~~

From: [UFO] To: Helicity [Translate]

Type text or a website address or translate a document.



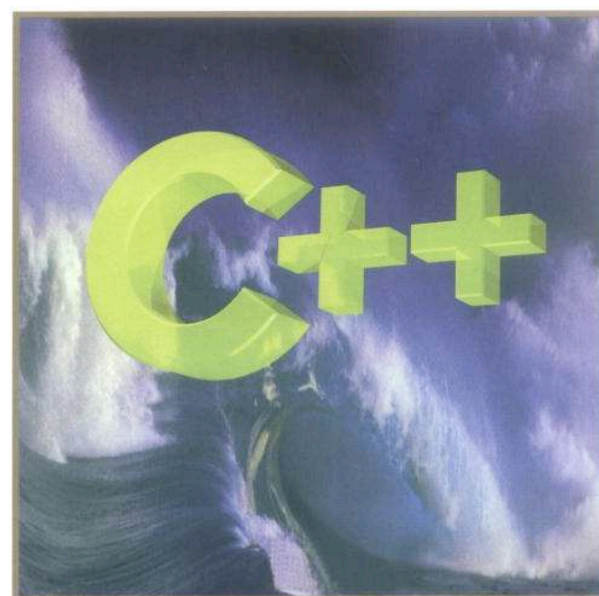
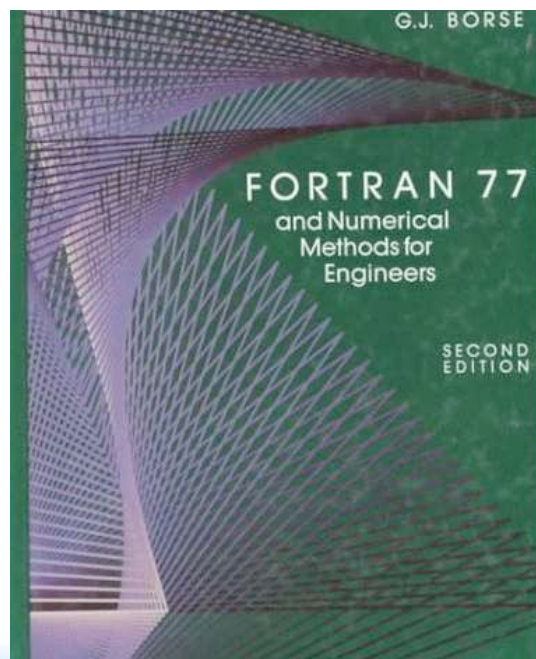


ALOHA



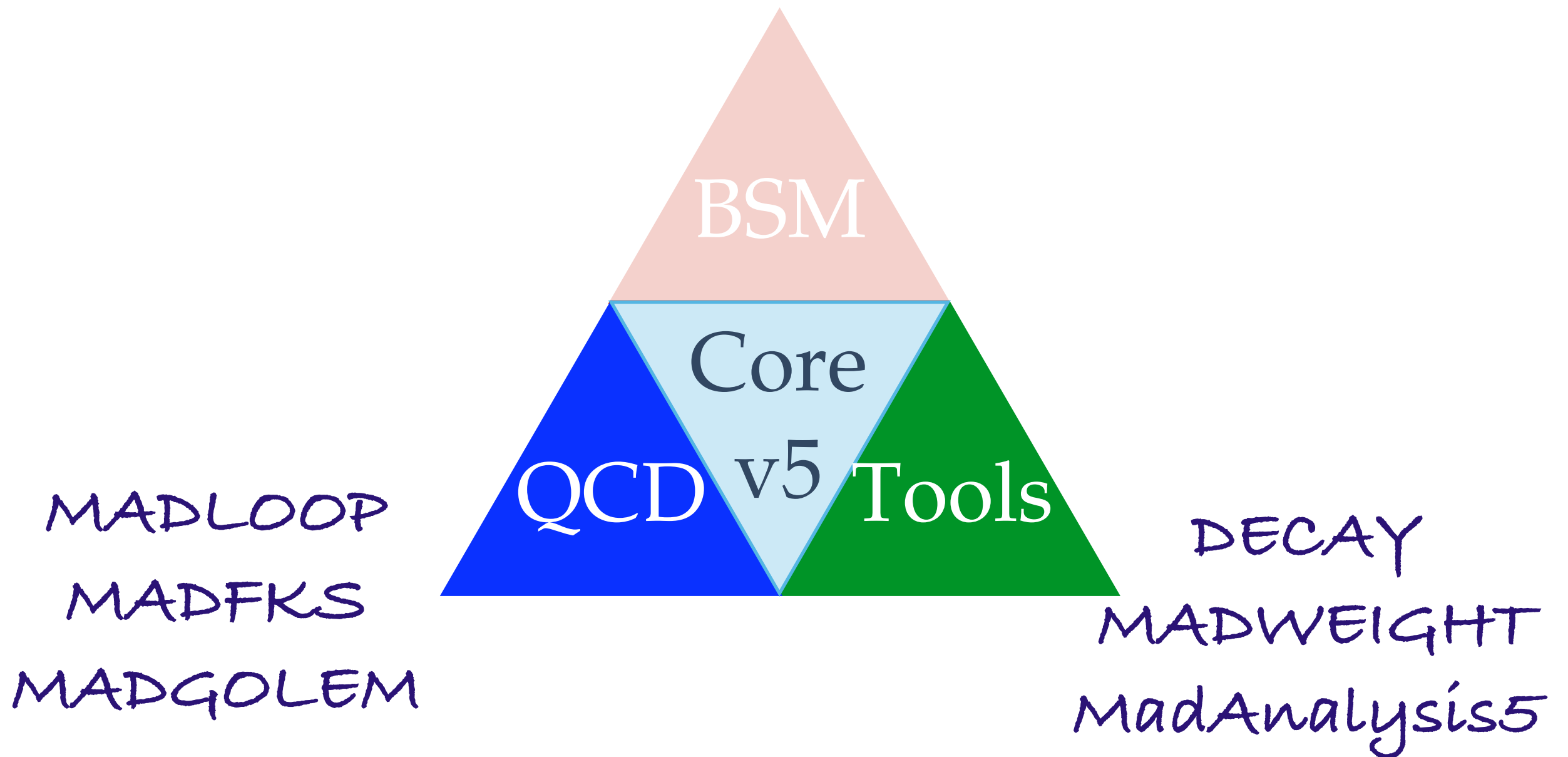
From: [UFO] [⇄] To: Helicity [Translate] Options: Standard (HELAS)
 Feynman gauge
 Complex-mass scheme
 Loop

Type text or a website address or translate a document.



UFO + ALOHA + MG5 =

Any BSM should be possible in a fully automatic and efficient way!



See the others Lectures !!

It's your turn to play with it



The Cross Section

Master formula

Master formula

$$\hat{\sigma}_{ab \rightarrow X}(\hat{s}, \dots)$$

Parton level
cross section

- Parton level cross section from matrix element

Master formula

$$\hat{\sigma}_{ab \rightarrow X}(\hat{s}, \dots) f_a(x_1) f_b(x_2)$$

Parton level
cross section

Parton density
functions

- Parton level cross section from matrix element
- Parton density (or distribution) functions:
Process independent, determined by particle type

Master formula

$$\int \hat{\sigma}_{ab \rightarrow X}(\hat{s}, \dots) f_a(x_1) f_b(x_2) dx_1 dx_2 d\Phi_{FS}$$

Parton level
cross section
Parton density
functions
Phase space
integral

- Parton level cross section from matrix element
- Parton density (or distribution) functions:
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Master formula

$$\int \hat{\sigma}_{ab \rightarrow X}(\hat{s}, \dots) f_a(x_1) f_b(x_2) dx_1 dx_2 d\Phi_{FS}$$

Parton level cross section
Parton density functions
Phase space integral

- Parton level cross section from matrix element
- Parton density (or distribution) functions:
Process independent, determined by particle type
- $\hat{s} = x_1 x_2 s$ (s = collision energy of the collider)

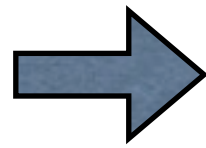
Integrals as averages



Integrals as averages

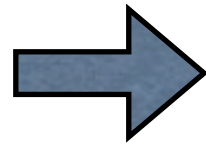


$$I = \int_{x_1}^{x_2} f(x) dx$$



$$I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^N f(x_i)$$

$$V = (x_2 - x_1) \int_{x_1}^{x_2} [f(x)]^2 dx - I^2$$



$$V_N = (x_2 - x_1)^2 \frac{1}{N} \sum_{i=1}^N [f(x_i)]^2 - I_N^2$$

Integrals as averages



$$I = \int_{x_1}^{x_2} f(x) dx \quad \longrightarrow \quad I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^N f(x_i)$$

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$$I = I_N \pm \sqrt{V_N/N}$$

Integrals as averages



$$I = \int_{x_1}^{x_2} f(x) dx \quad \longrightarrow \quad I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^N f(x_i)$$

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👉 Convergence is slow but it can be easily estimated

Integrals as averages



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$$I = I_N \pm \sqrt{V_N / N}$$

- ☞ Convergence is slow but it can be easily estimated
- ☞ Improvement by minimizing V_N .

Integrals as averages



$$I = \int_{x_1}^{x_2} f(x) dx \quad \longrightarrow \quad I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^N f(x_i)$$

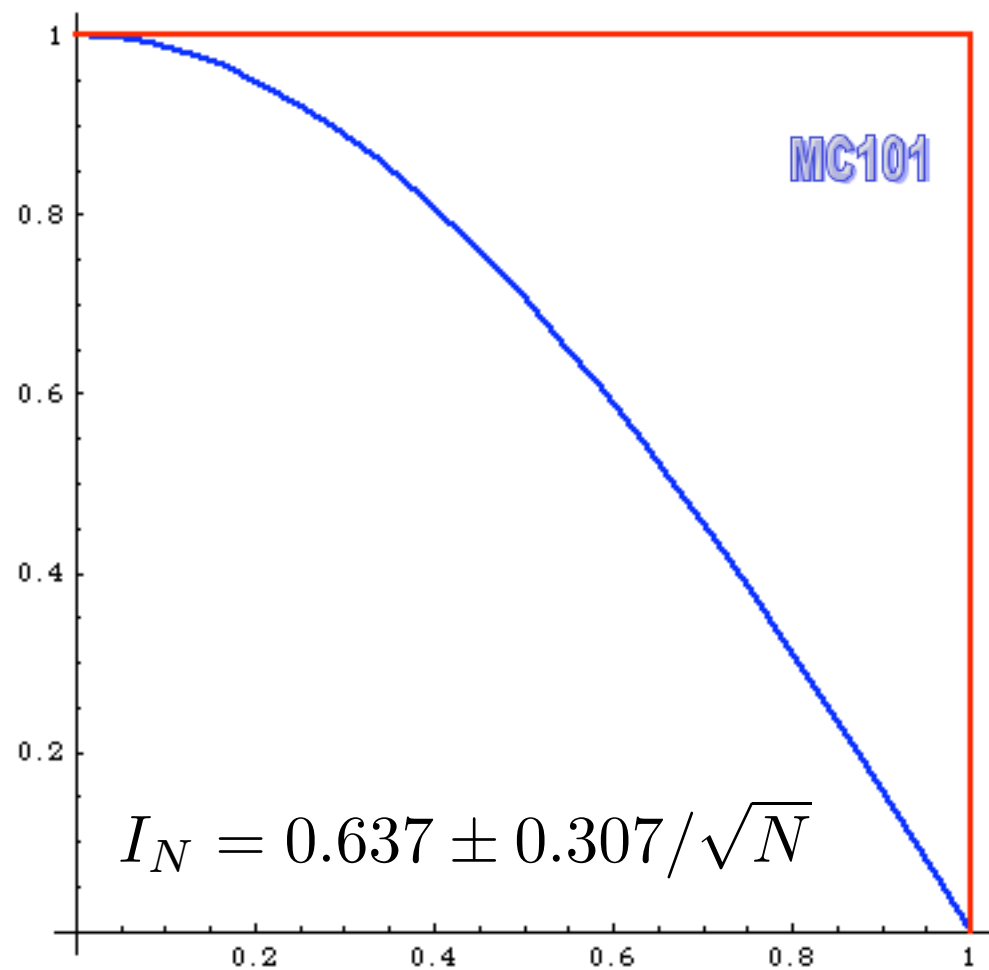
$$V = (x_2 - x_1) \int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \longrightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N} \sum_{i=1}^N [f(x_i)]^2 - I_N^2$$

$$I = I_N \pm \sqrt{V_N/N}$$

- ☞ Convergence is slow but it can be easily estimated
- ☞ Improvement by minimizing V_N .
- ☞ Optimal/Ideal case: $f(x)=C \Rightarrow V_N=0$

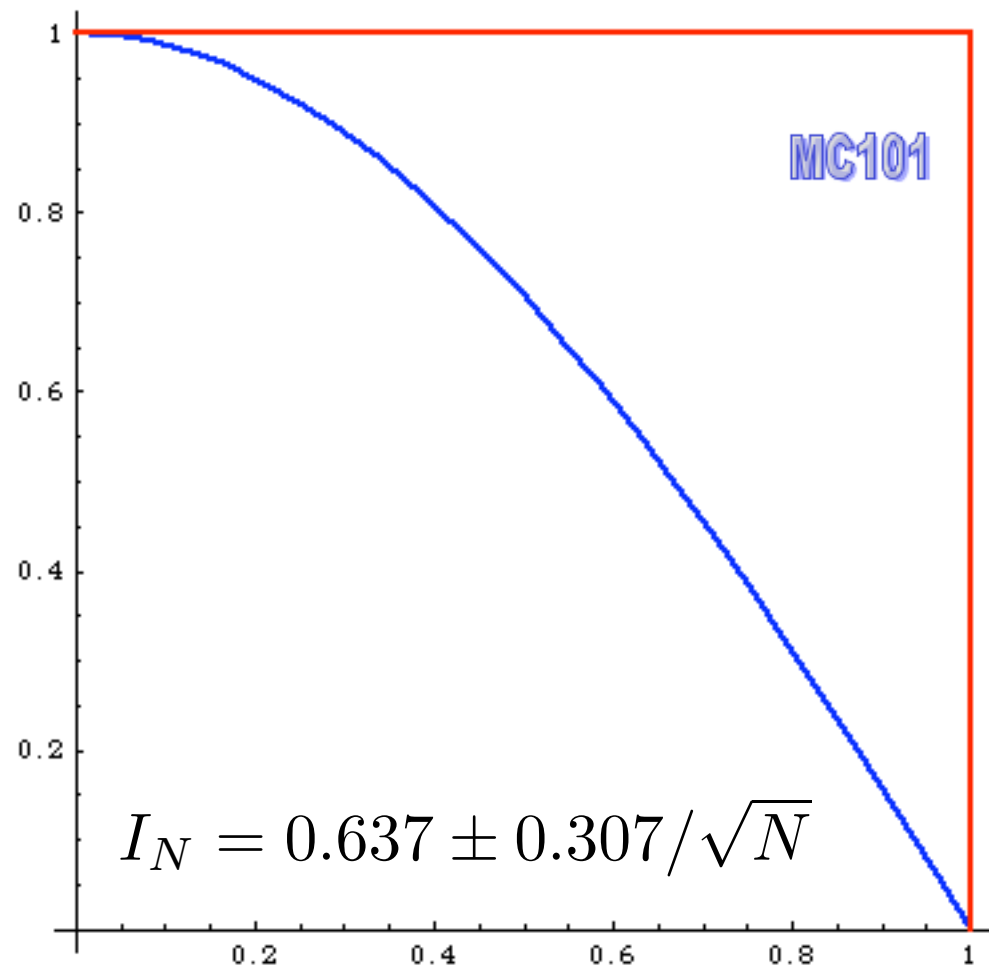
Importance Sampling

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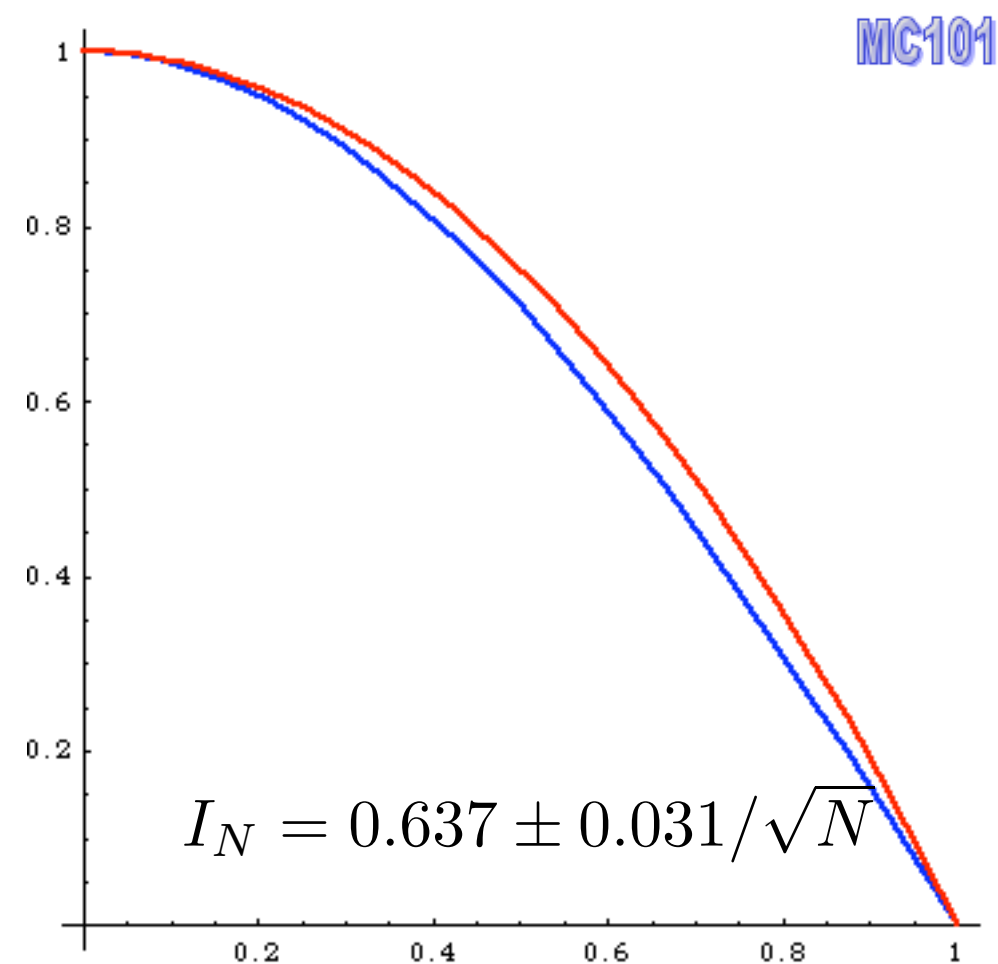


$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

Importance Sampling

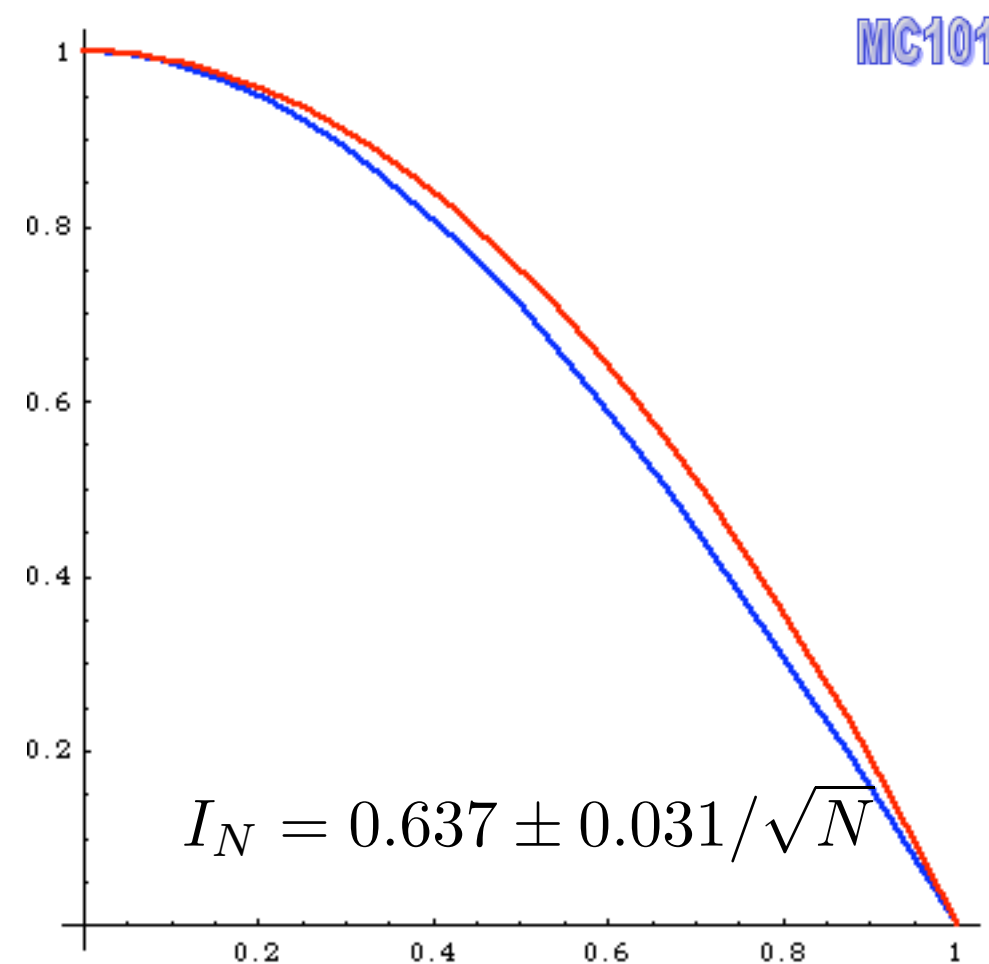
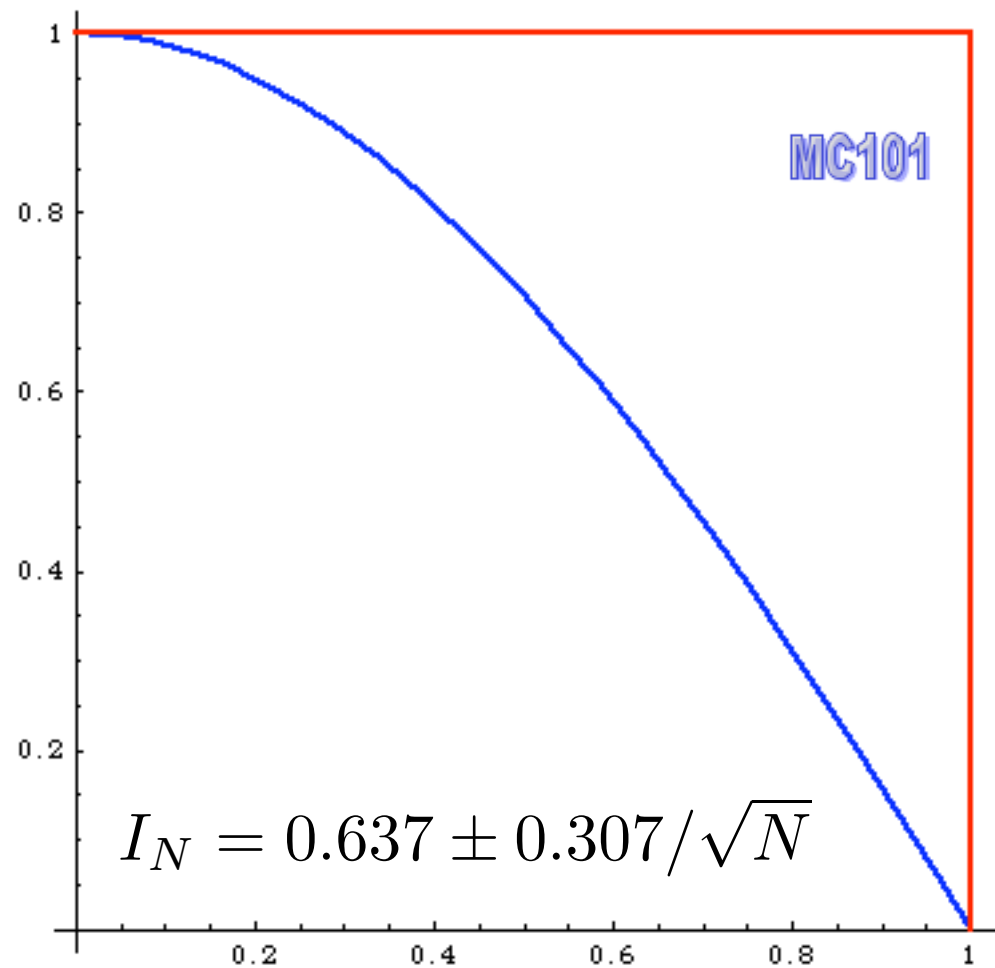


$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$



$$I = \int_0^1 dx (1 - x^2) \frac{\cos \frac{\pi}{2} x}{1 - x^2}$$

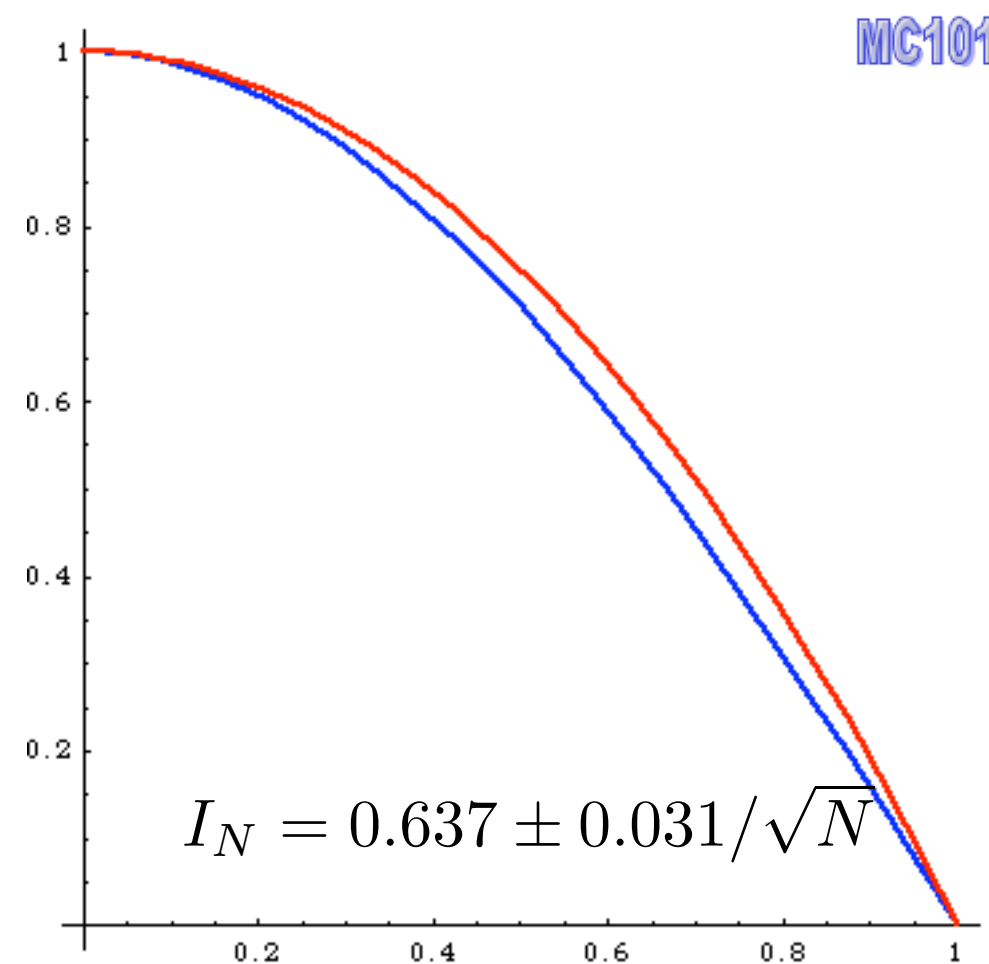
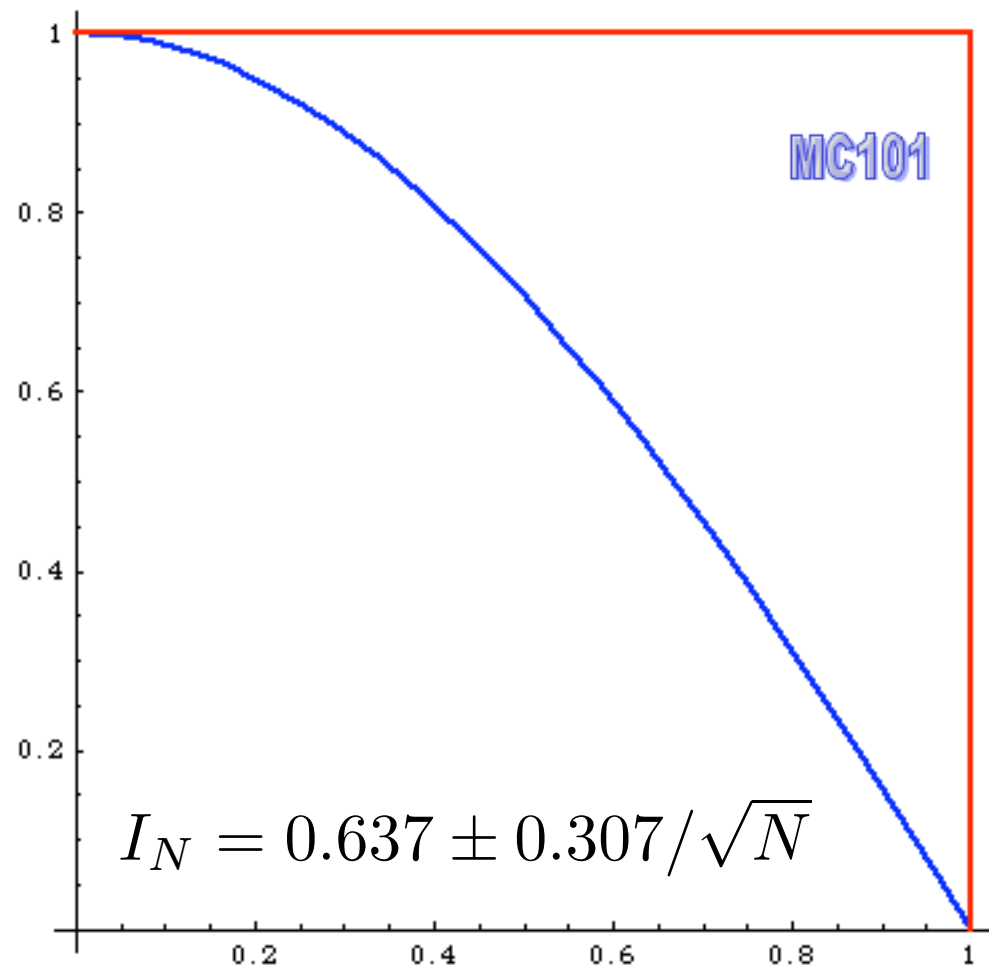
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$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$\begin{aligned}
 I &= \int_0^1 dx (1 - x^2) \frac{\cos \frac{\pi}{2} x}{1 - x^2} \\
 &= \int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1 - x[\xi]^2}
 \end{aligned}$$

Importance Sampling

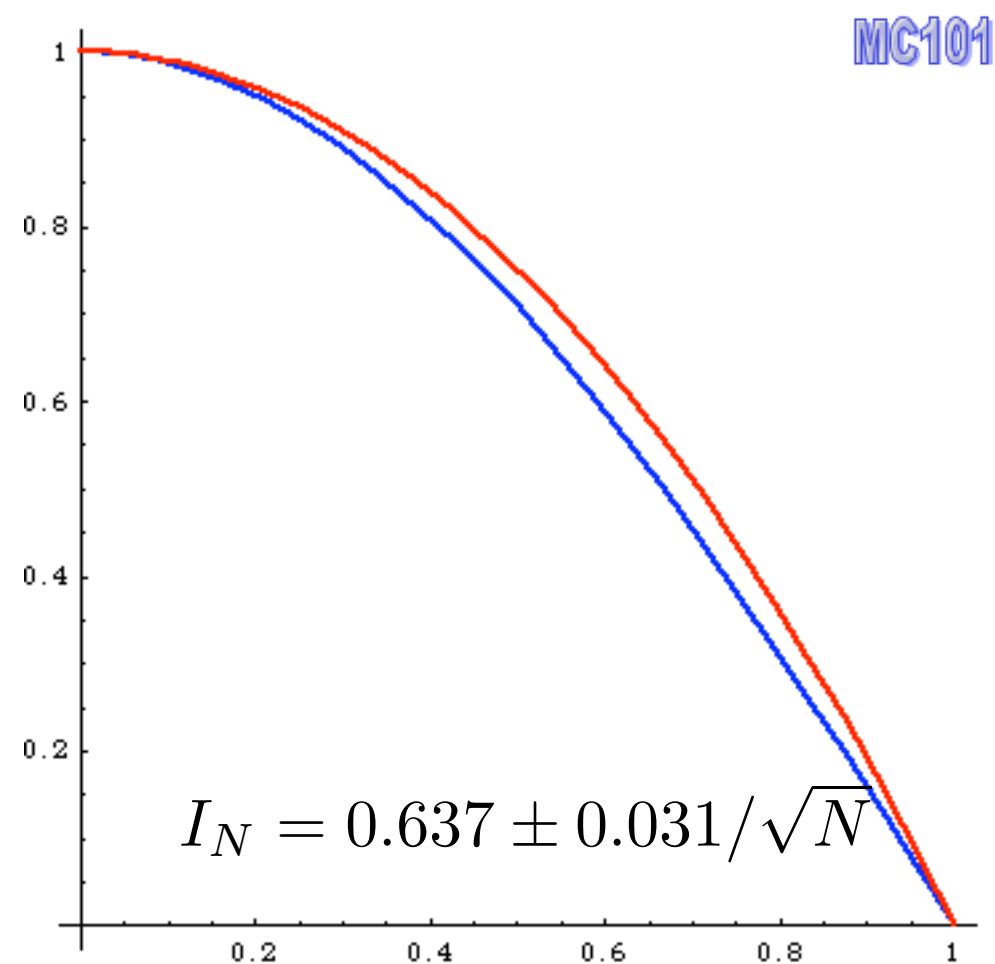
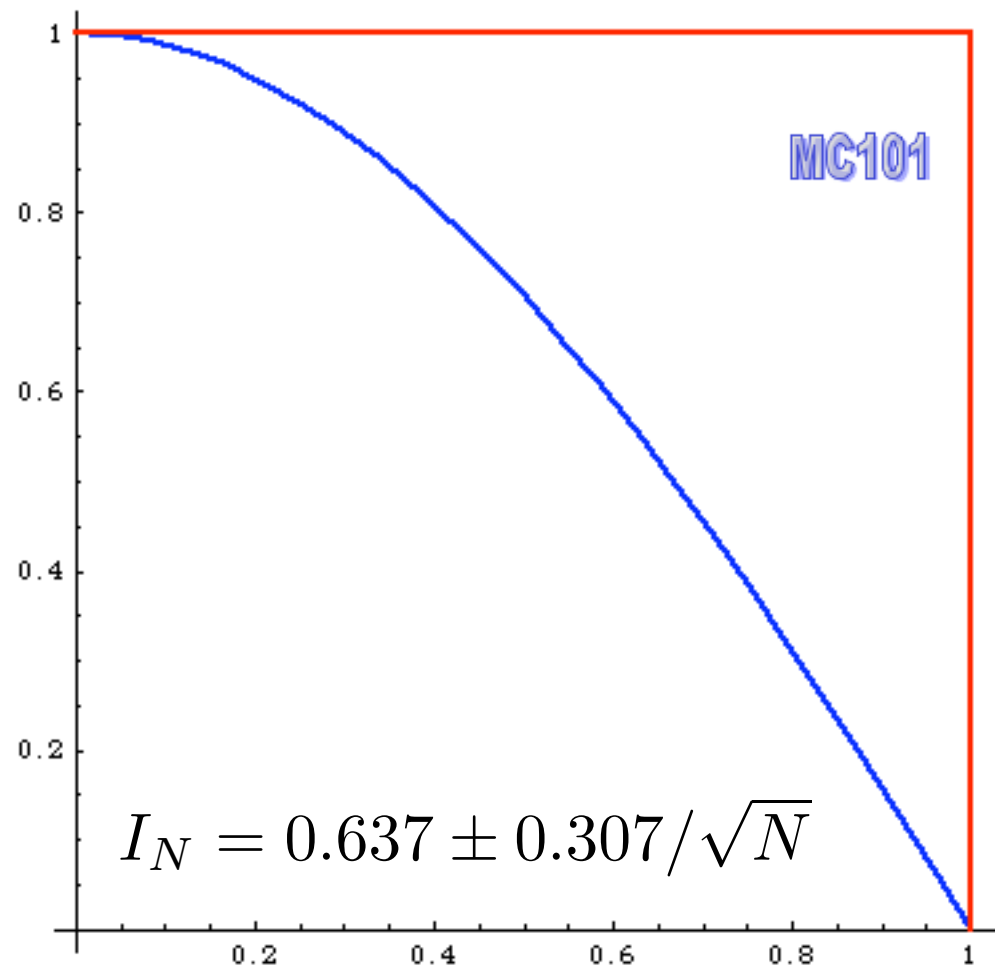


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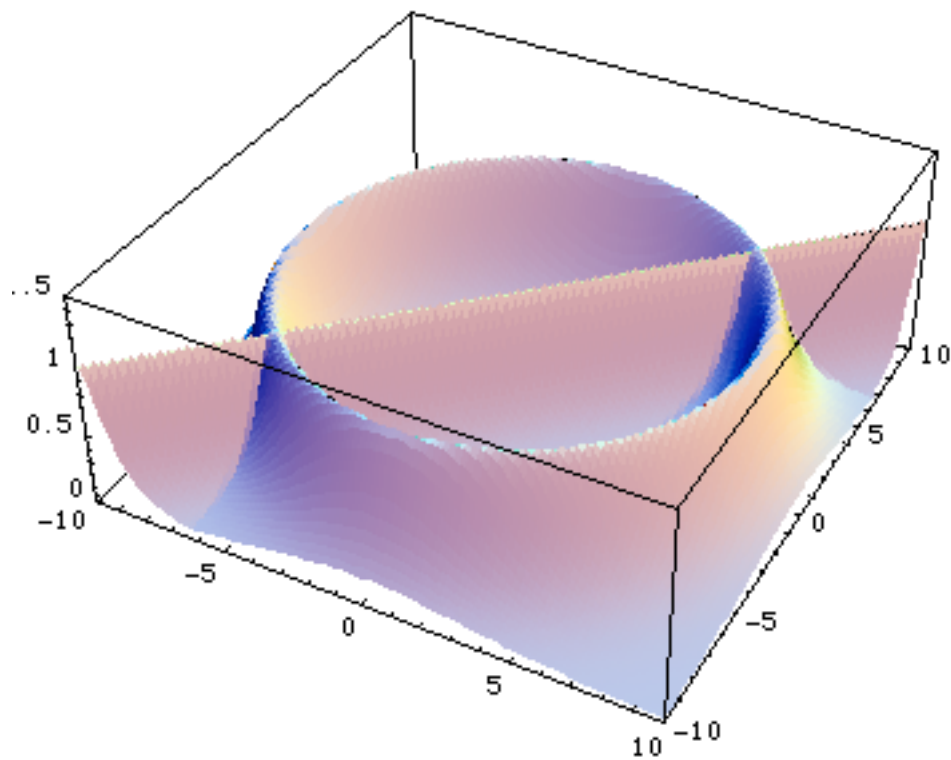


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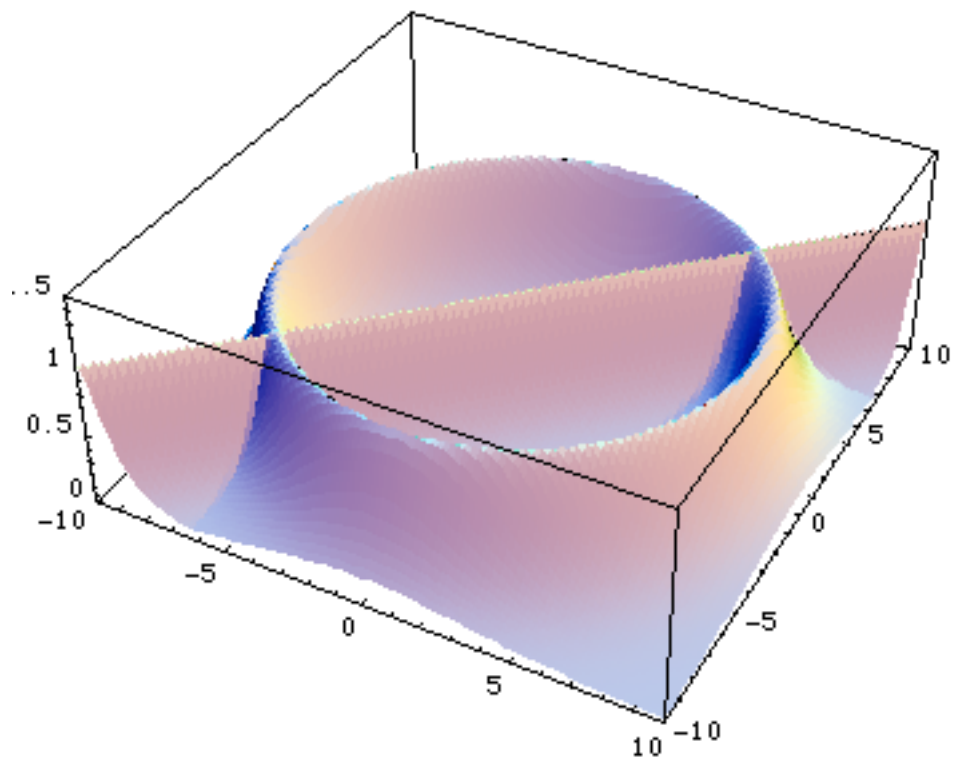
$$= \int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1-x[\xi]^2} \rightarrow \simeq 1$$

Multi-channel

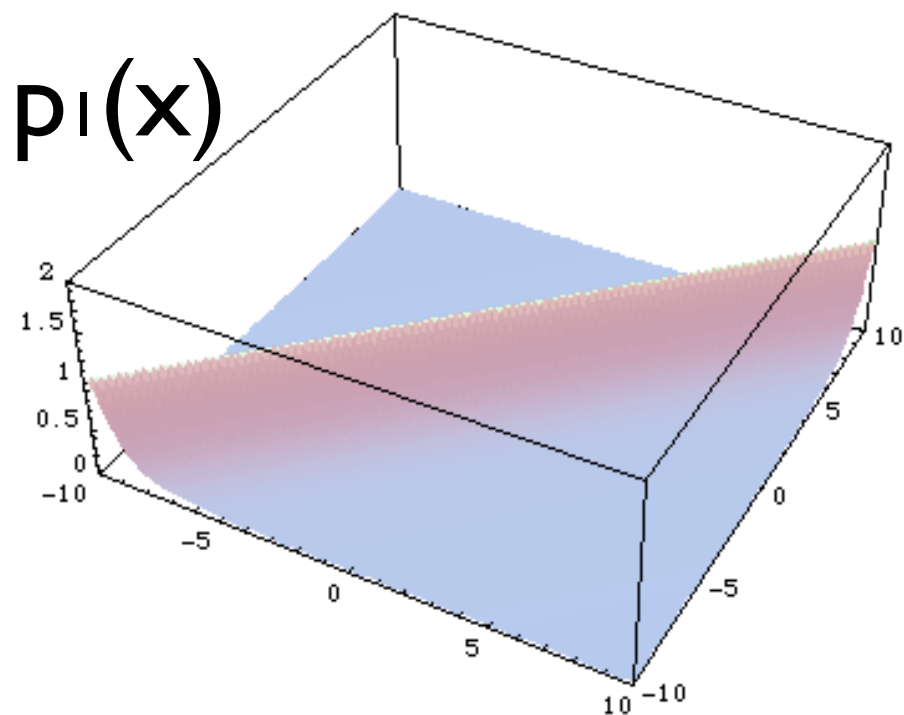


In this case there is no unique transformation

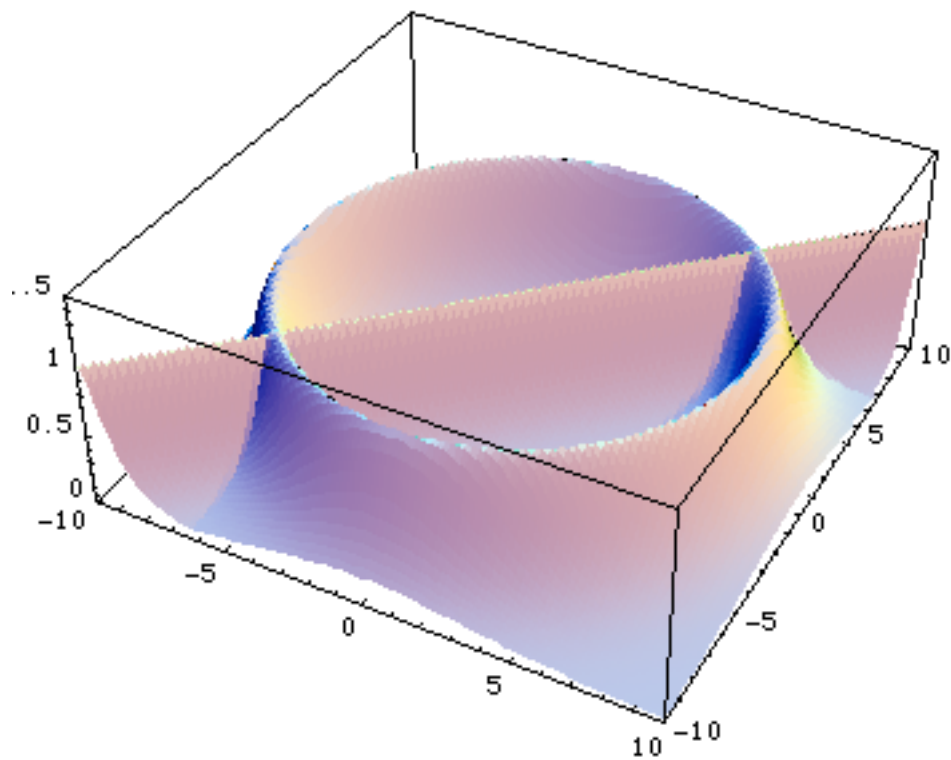
Multi-channel



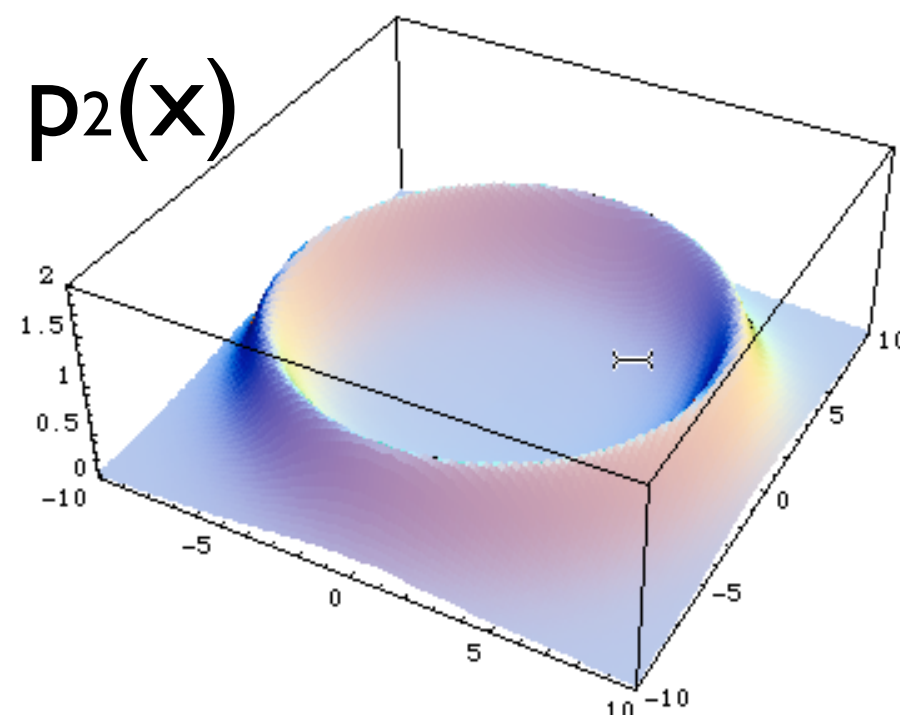
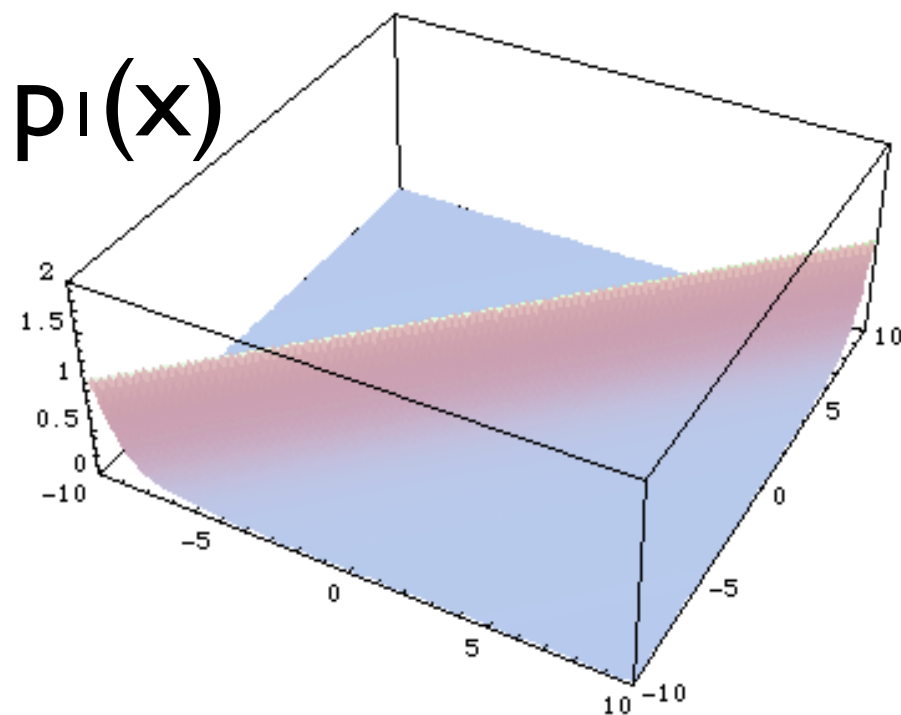
In this case there is no unique transformation



Multi-channel



In this case there is no unique transformation



Multi-channel based on single diagrams*

$$f_i = \frac{|A_i|^2}{\sum_i |A_i|^2} |A_{\text{tot}}|^2$$

- Key Idea
 - Any single diagram is “easy” to integrate (pole structures known based on propagators)
 - Divide integration into pieces, based on diagrams
- Get N independent integrals
 - Errors add in quadrature so no extra cost
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*Method used in MadGraph

Multi-channel based on single diagrams*

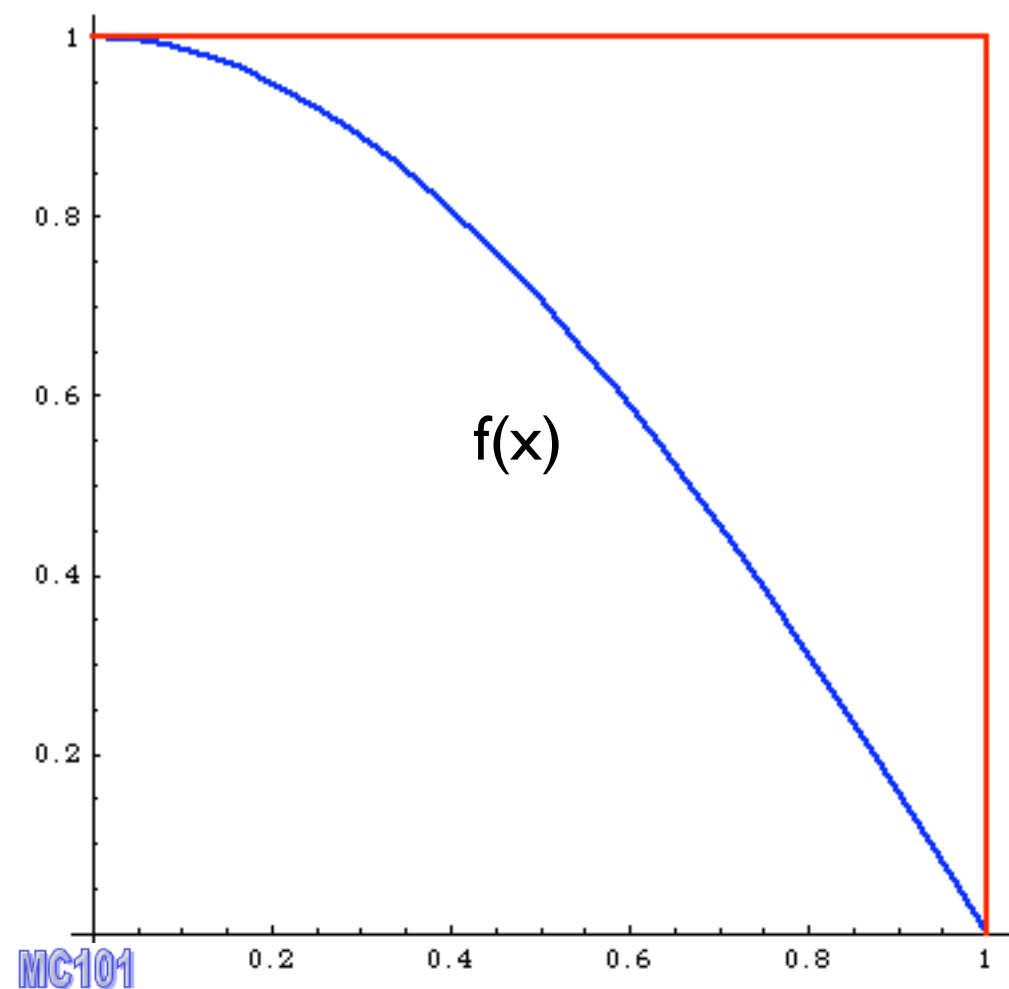
$$f_i = \frac{|A_i|^2}{\sum_i |A_i|^2} \simeq 1$$

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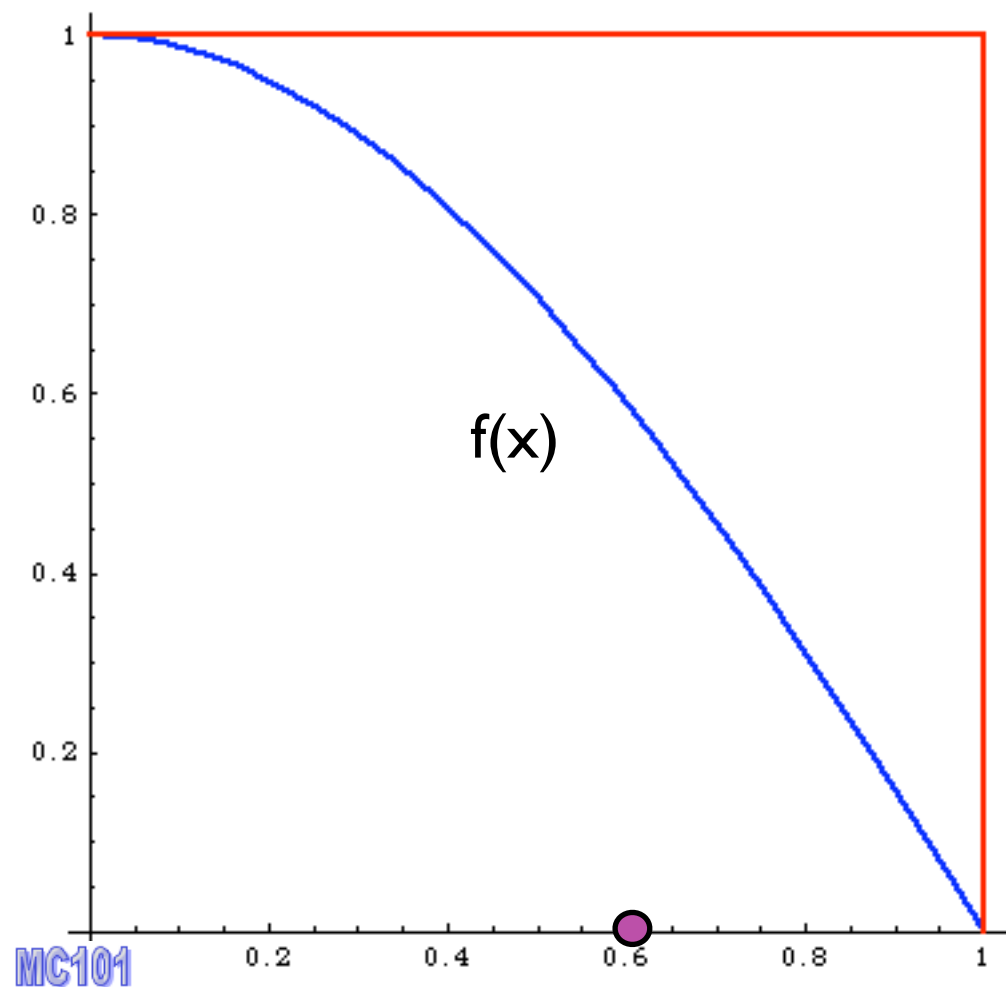
Event Generation

Event generation



Alternative way

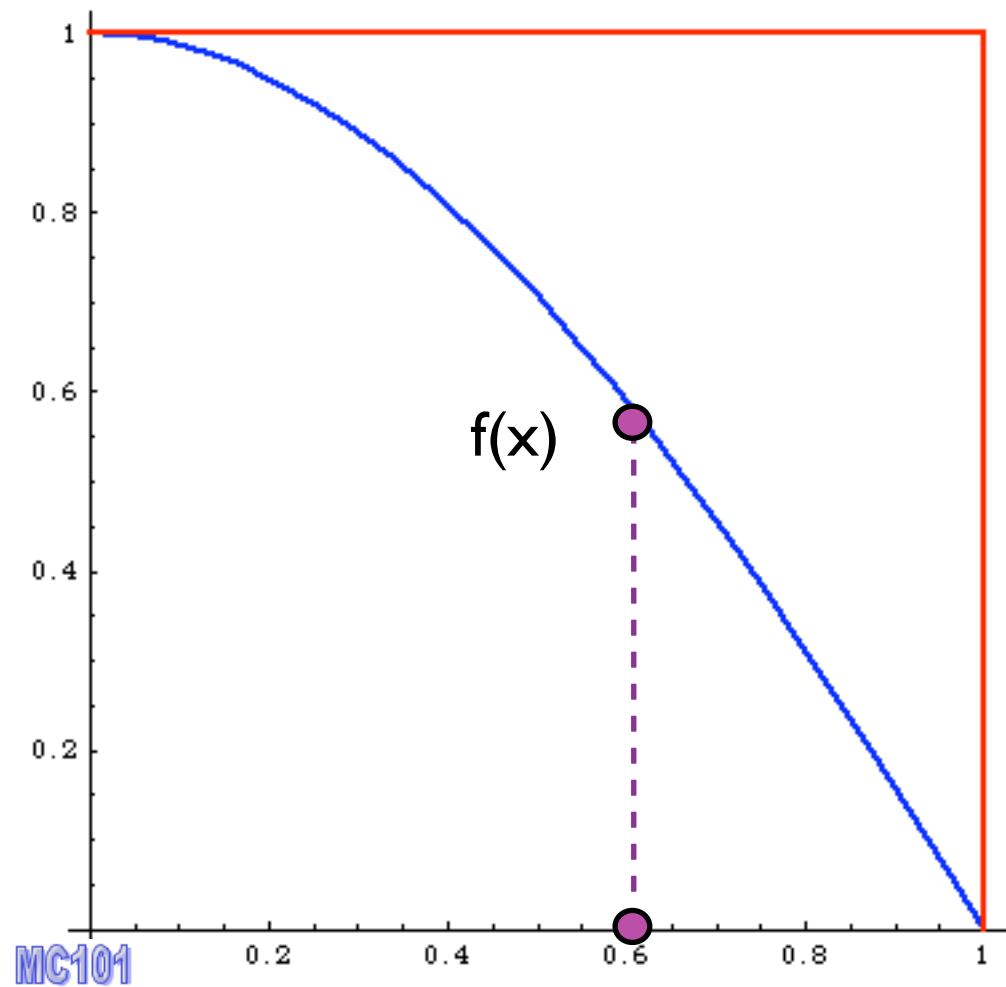
Event generation



Alternative way

1. pick x

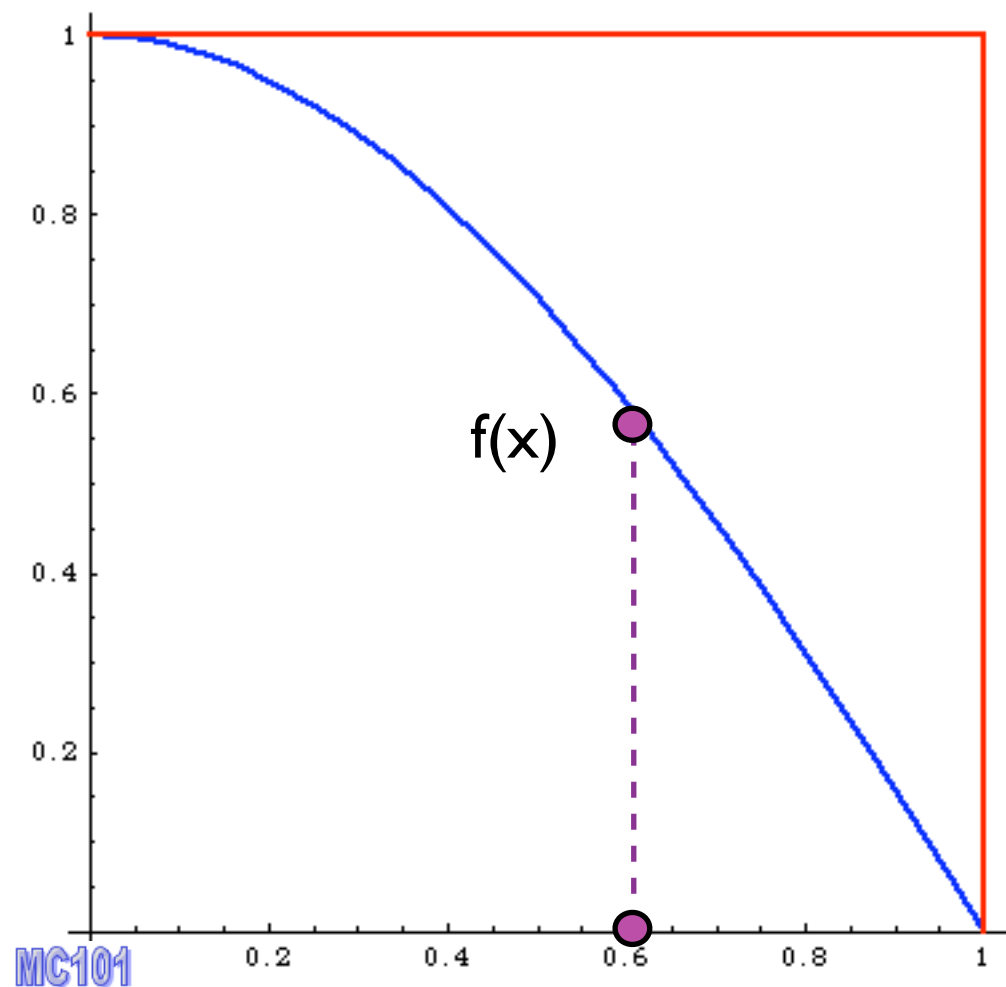
Event generation



Alternative way

1. pick x
2. calculate $f(x)$

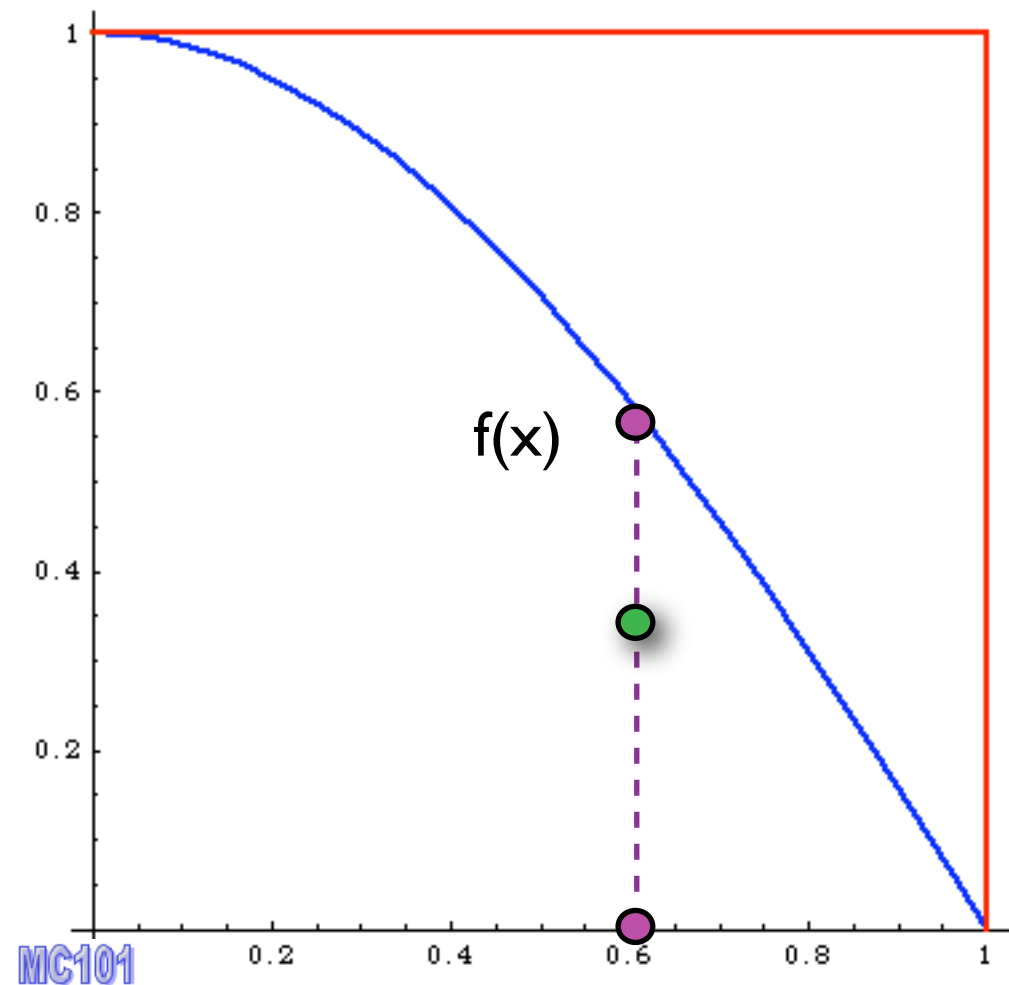
Event generation



Alternative way

1. pick x
2. calculate $f(x)$
3. pick $0 < y < f_{\max}$

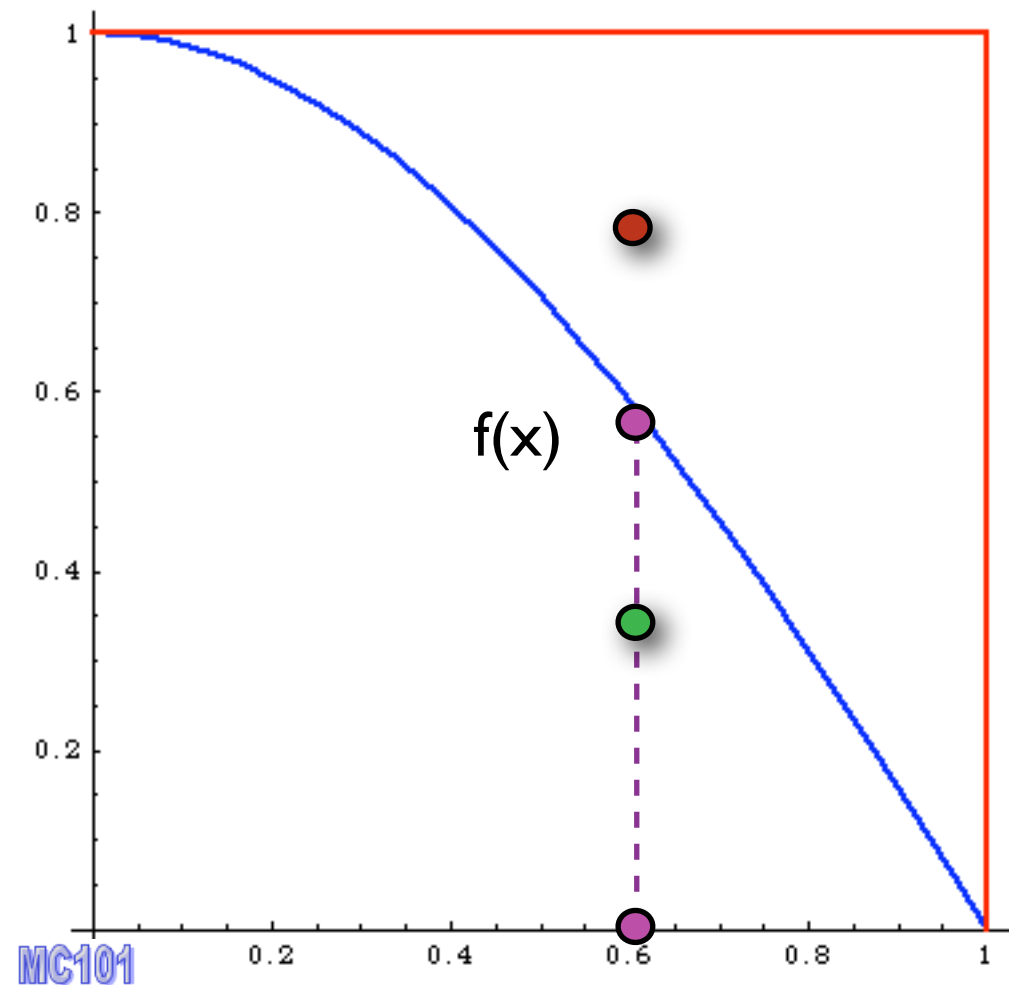
Event generation



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if $f(x) > y$ accept event,

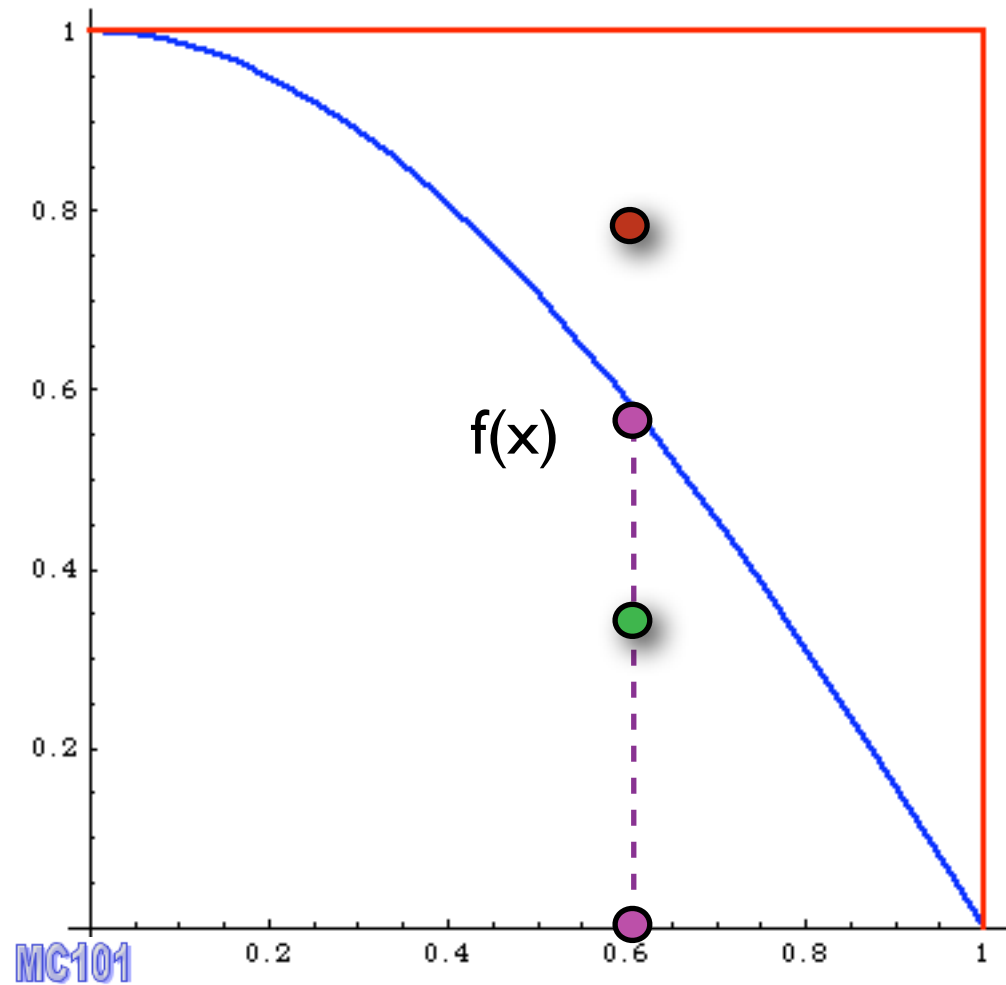
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Event generation

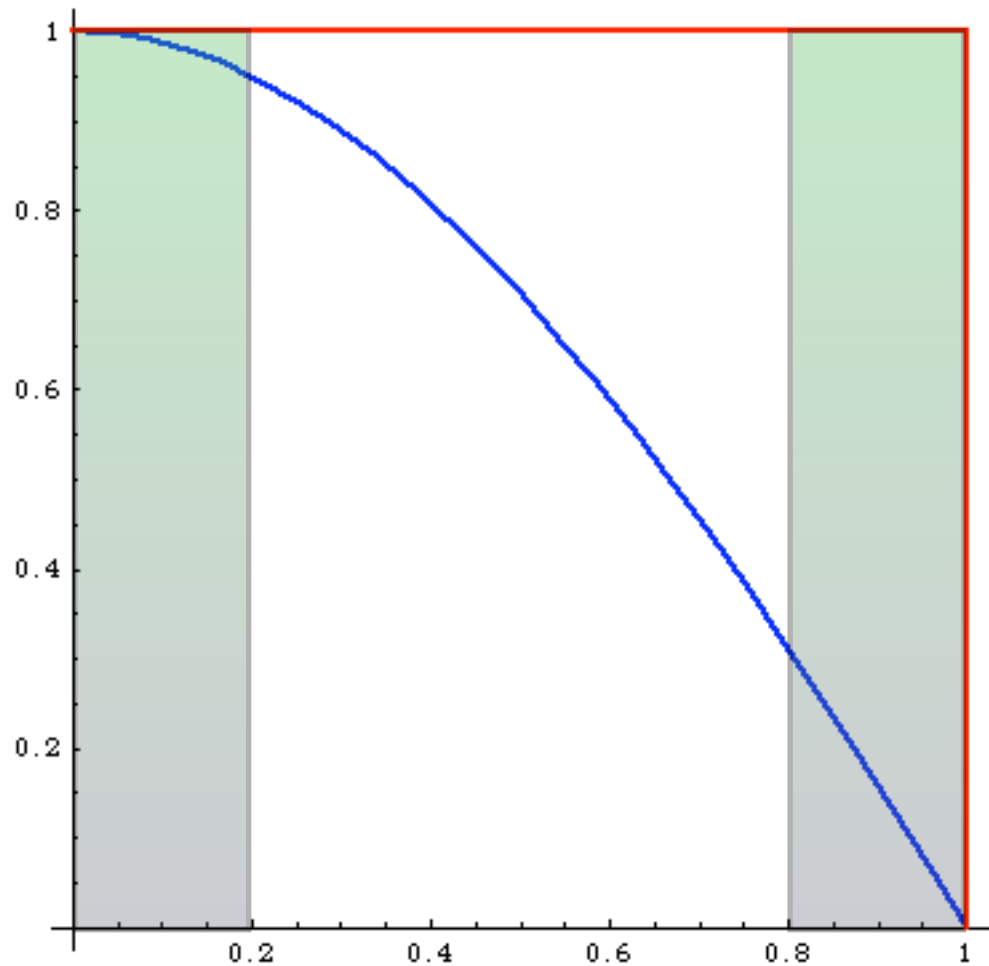


Alternative way

1. pick x
2. calculate $f(x)$
3. pick $0 < y < f_{\max}$
4. Compare:
if $f(x) > y$ accept event,
else reject it.

$$|\text{= } \frac{\text{accepted}}{\text{total tries}} = \text{efficiency}$$

Event generation

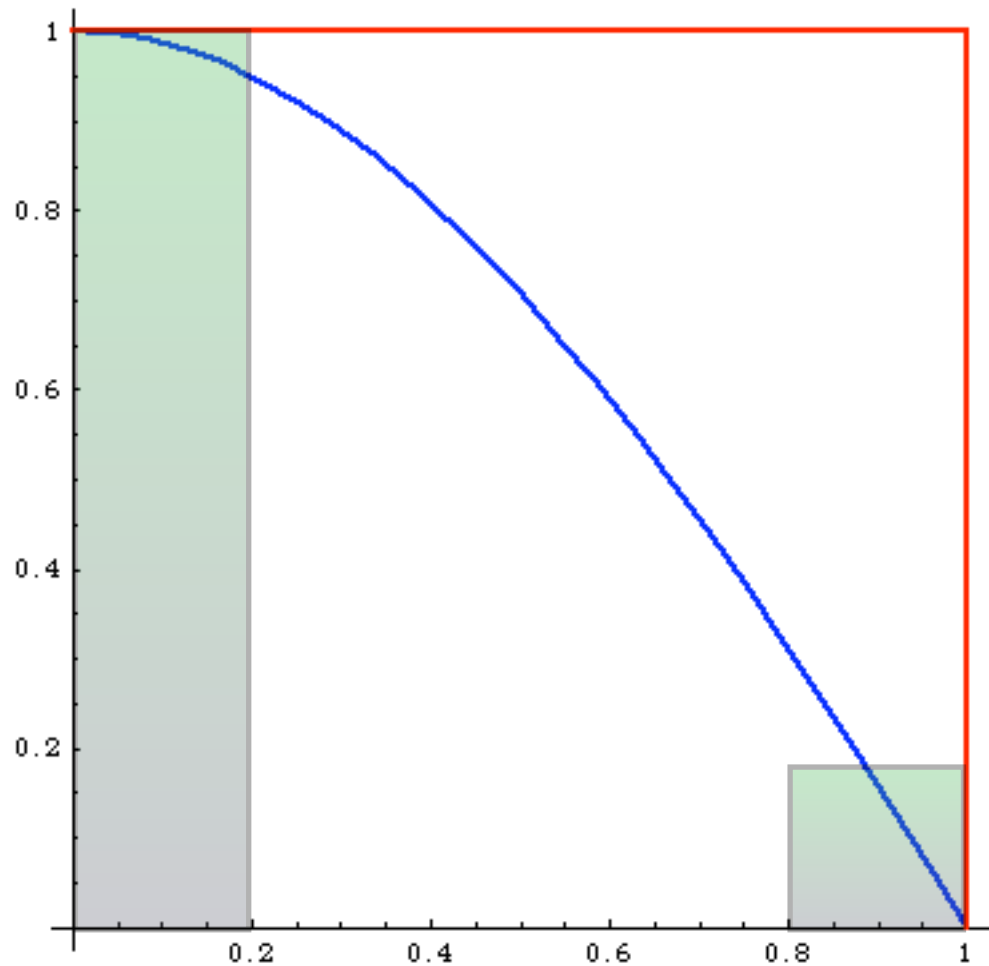


What's the difference?

before:

same # of events in areas of phase space with very different probabilities: events must have different weights

Event generation



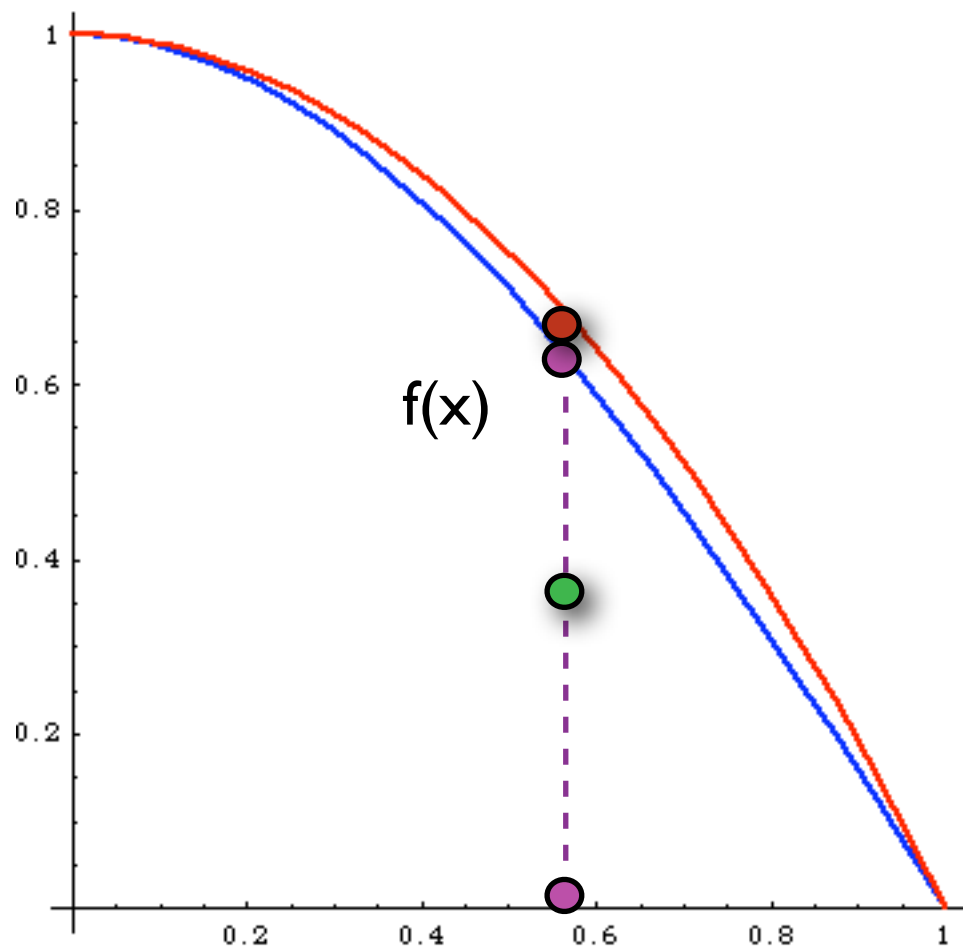
What's the difference?

after:

events is proportional to the probability of areas of phase space:
events have all the same weight ("unweighted")

Events distributed as in Nature

Event generation



Improved

1. pick x distributed as $p(x)$
2. calculate $f(x)$ and $p(x)$
3. pick $0 < y < 1$
4. Compare:
if $f(x) > y$ $p(x)$ accept event,
else reject it.

much better efficiency!!!

Event generation

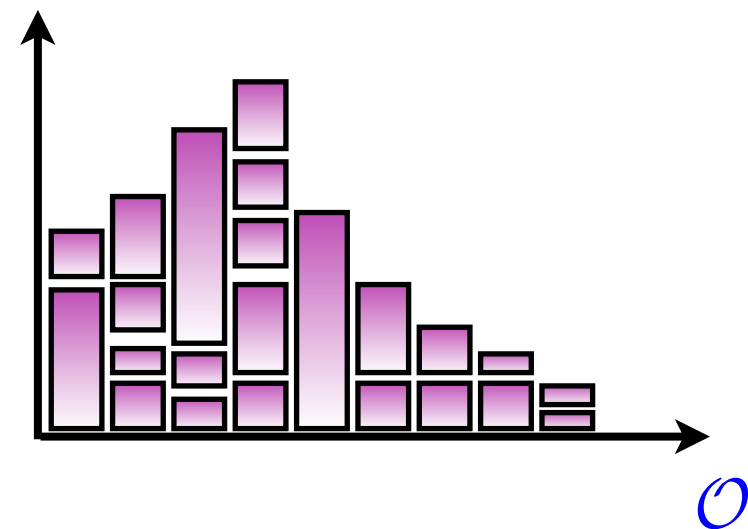
Event generation

MC integrator

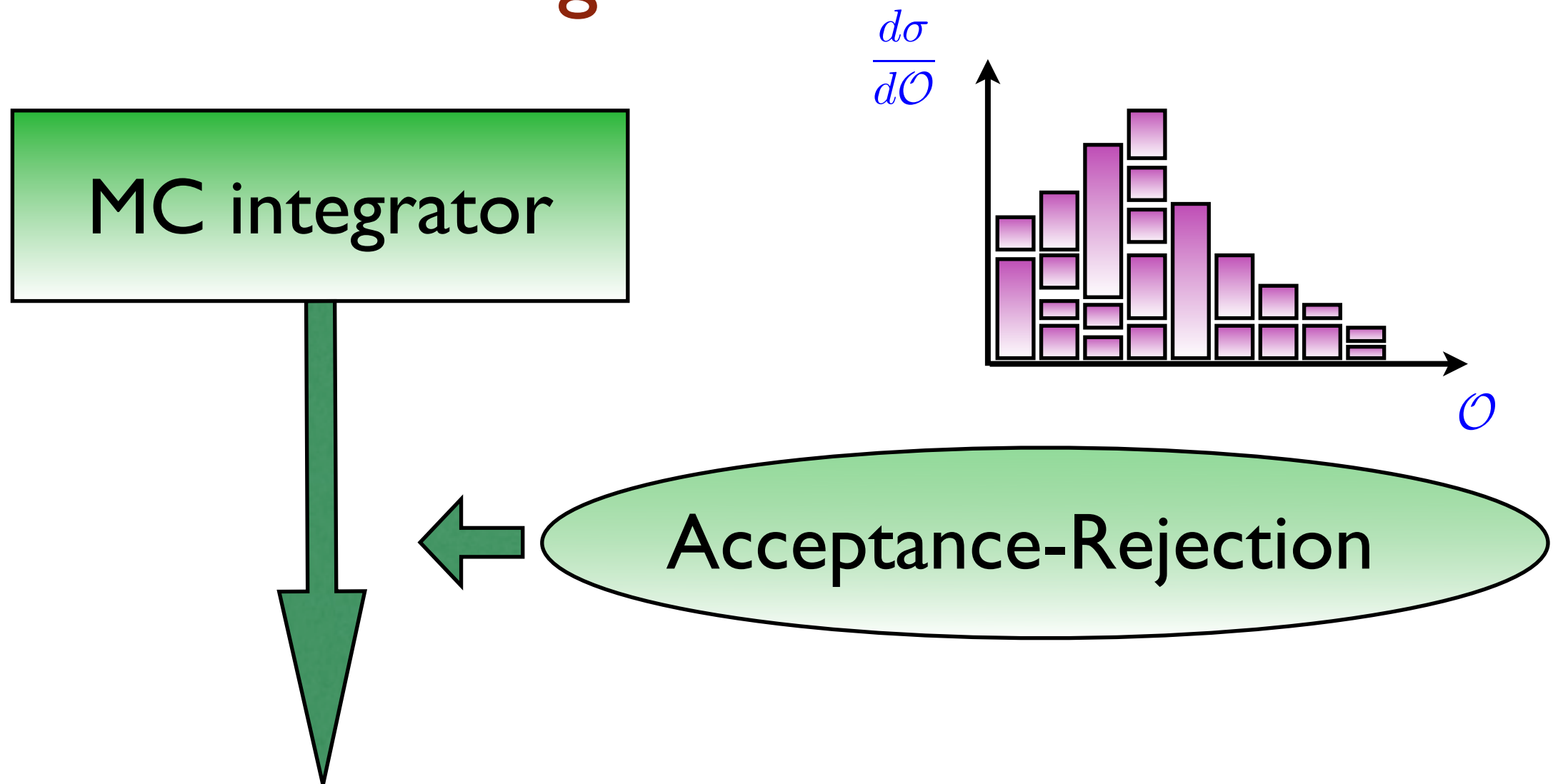
Event generation

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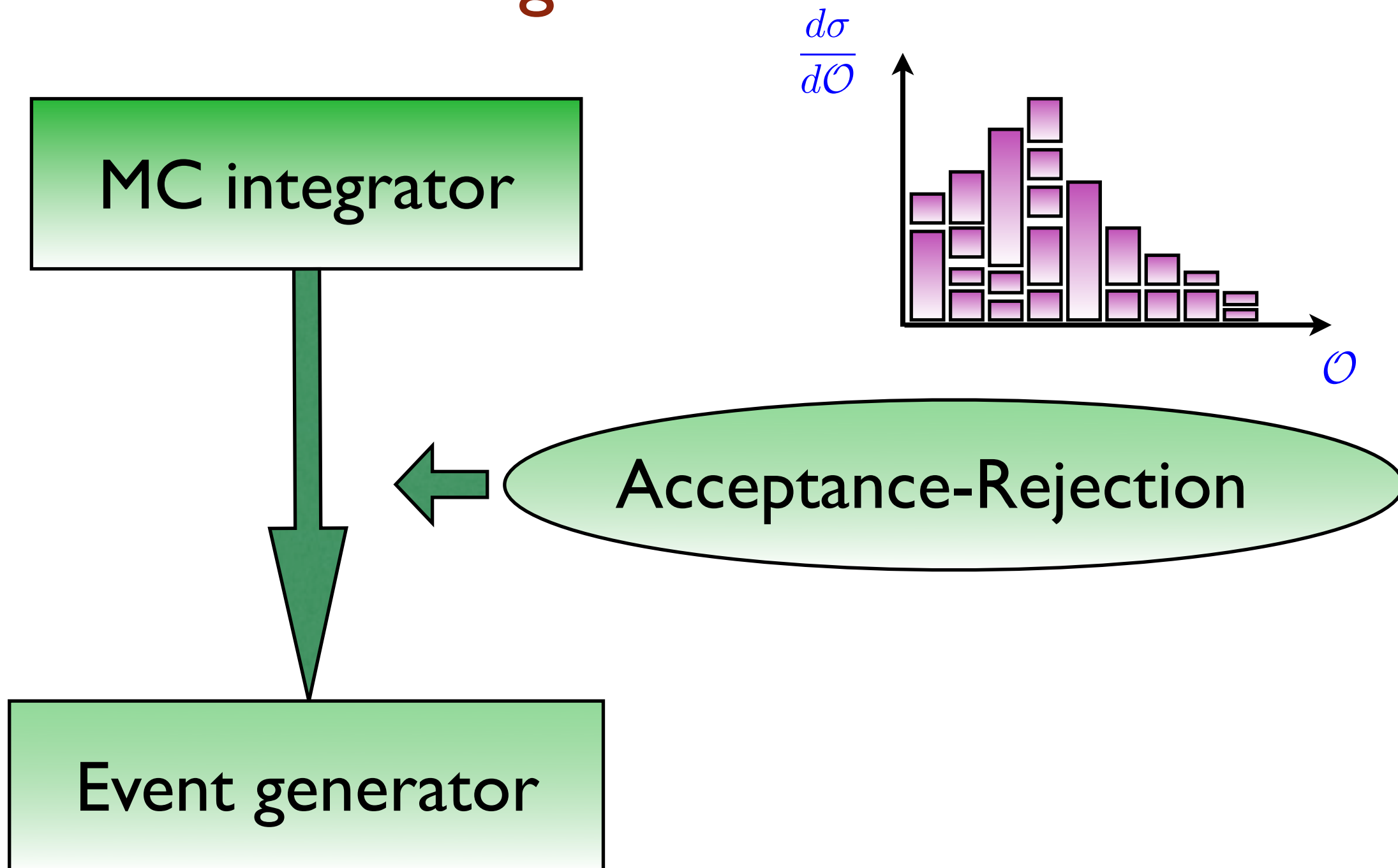
$$\frac{d\sigma}{d\mathcal{O}}$$



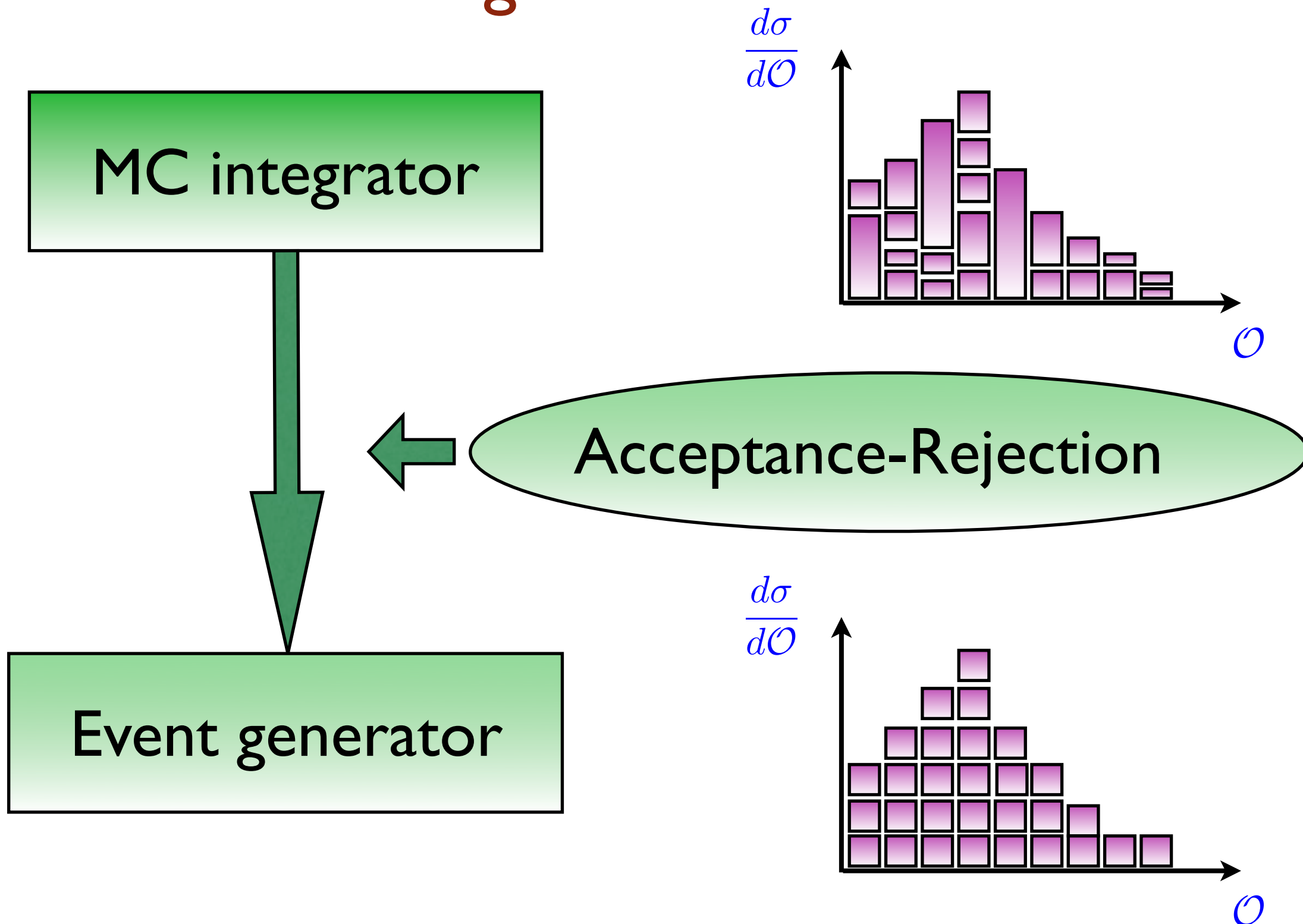
Event generation



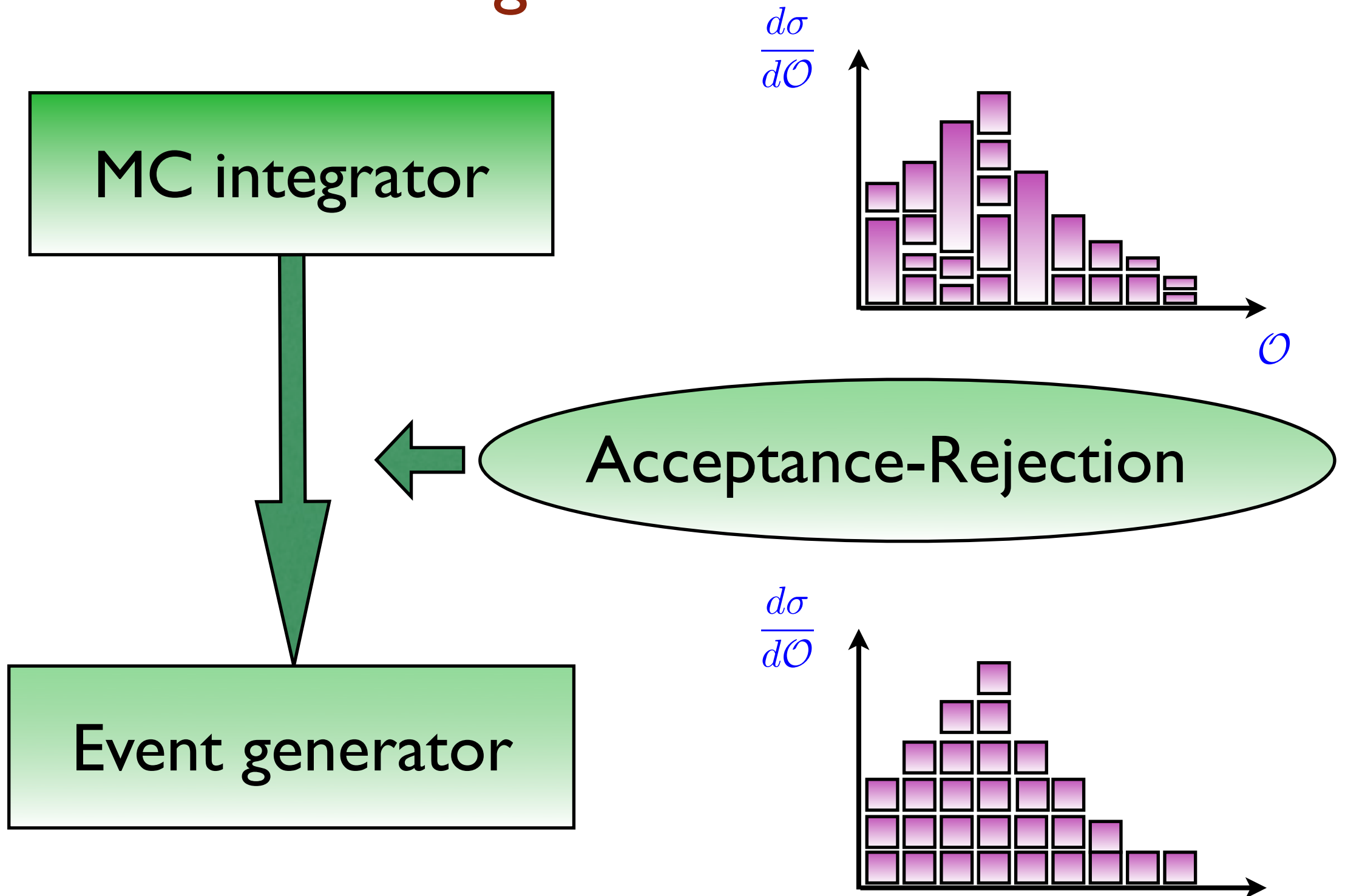
Event generation



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Event generation



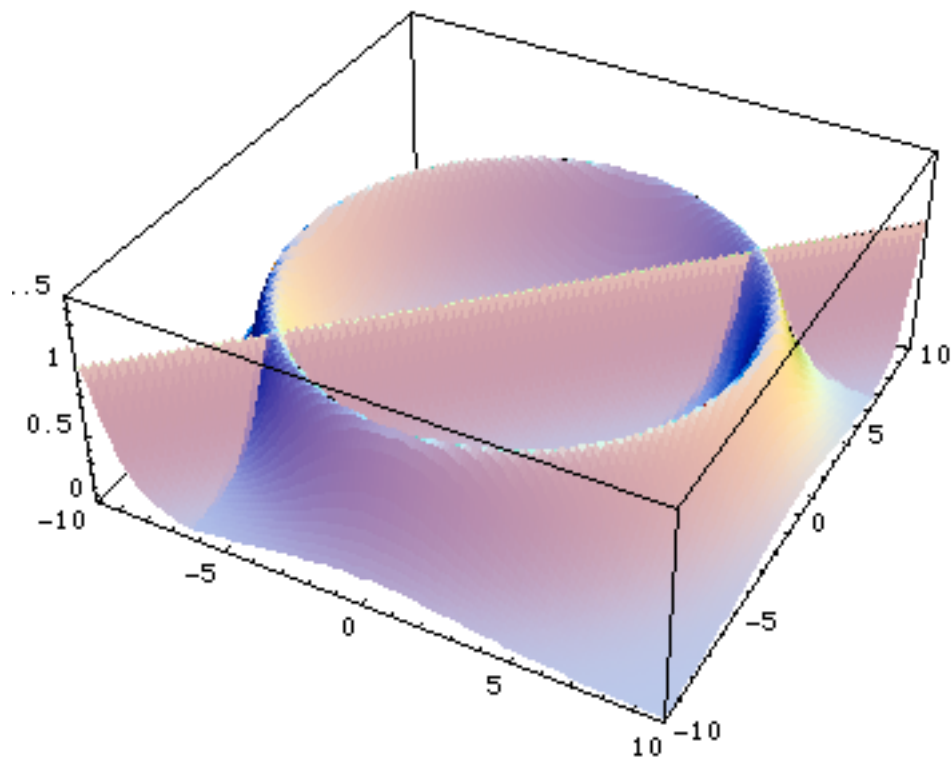
👉 This is possible only if $f(x) < \infty$ AND has definite sign!

Monte Carlo Event Generator: Definiton

At the most basic level a Monte Carlo event generator is a program which produces particle physics events with the same probability as they occur in nature (virtual collider).

In practice it performs a large number of (sometimes very difficult) integrals and then unweights to give the four momenta of the particles that interact with the detector (simulation).

Multi-channel



In this case there is no unique transformation:
Vegas is bound to fail!

If you know where the peaks are (=in which variables) we can use different transformations = channels:

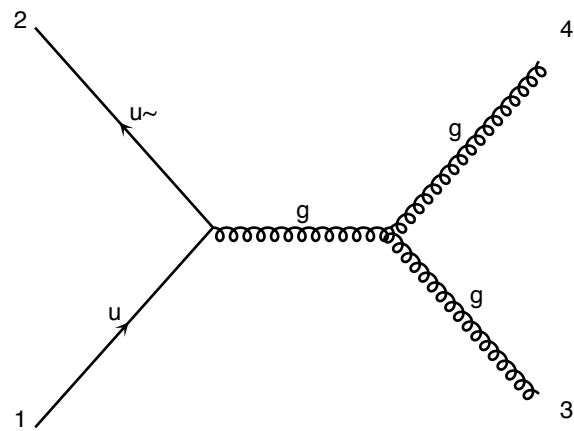
$$p(x) = \sum_{i=1}^n \alpha_i p_i(x) \quad \text{with} \quad \sum_{i=1}^n \alpha_i = 1$$

$$I = \int f(x) dx = \sum_{i=1}^n \alpha_i \int \frac{f(x)}{p_i(x)} p_i(x) dx$$

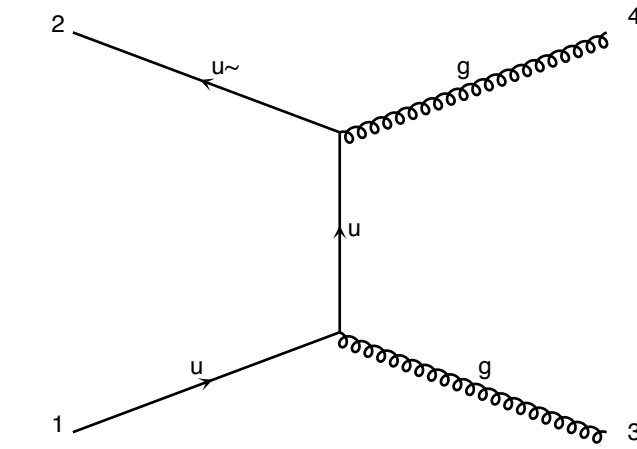
Multi-channel

- **Advantages**
 - The integral does not depend on the α_i but the variance does and can be minimized by a careful choice
- **Drawbacks**
 - Need to calculate all g_i values for each point
 - Each phase space channel must be invertible
 - N coupled equations for α_i so it might only work for small number of channels

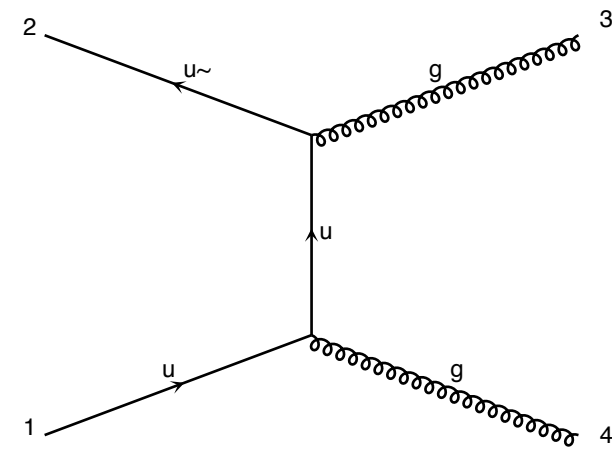
Example: QCD $2 \rightarrow 2$ production



$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1 + p_2)^2}$$



$$\propto \frac{1}{\hat{t}} = \frac{1}{(p_1 - p_3)^2}$$



$$\propto \frac{1}{\hat{u}} = \frac{1}{(p_1 - p_4)^2}$$

Three very different pole structures contributing to the same matrix element.

Multi-channel based on single diagrams

Consider the integration of an amplitude $|M|^2$ at tree level with many contributing diagrams. If there were a basis of functions,

$$f = \sum_{i=1}^n f_i \quad \text{with} \quad f_i \geq 0, \quad \forall i,$$

such that:

1. we know how to integrate each one of them,
2. they describe all possible peaks,

then the problem would be solved:

$$I = \int d\vec{\Phi} f(\vec{\Phi}) = \sum_{i=1}^n \int d\vec{\Phi} g_i(\vec{\Phi}) \frac{f_i(\vec{\Phi})}{g_i(\vec{\Phi})} = \sum_{i=1}^n I_i,$$

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Does such a basis exist?

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- What about interference?
 - Never creates “new” peaks, so we’re OK!

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YES!
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