Next-to-Leading Order and aMC@NLO

Rikkert Frederix

University of Zurich

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Contents

- Today:
  - Ingredients to a NLO calculations
    - A bit more detail on canceling divergences
  - Matching to the parton shower: MC@NLO and POWHEG
- Tomorrow (By Valentin Hirschi):
  - Computing loops efficiently

Master equation for hadron colliders

$$d\sigma = \sum_{a,b} \int dx_1 dx_2 \;\; f_a(x_1,\mu_F) f_b(x_2,\mu_F)\; d\hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Parton density functions

Parton-level (differential) cross section

- Parton-level cross section from matrix elements: model and process dependent
- Parton density (or distribution) functions: process independent
- Differences between colliders given by parton luminosities

Perturbative expansion

$d\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$ Parton-level cross section

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter, schematically:

$$\hat{\sigma} = \sigma^{\rm Born}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \ldots\right)$$

- Including higher corrections improves predictions and reduces theoretical uncertainties

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LO predictions | NLO corrections | NNLO corrections | NNNLO corrections

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LO predictions | NLO corrections | NNLO corrections | NNNLO corrections

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Improved predictions

$$d\sigma = \sum_{a,b} \int dx_1 dx_2 \;\; f_a(x_1,\mu_F) f_b(x_2,\mu_F)\; d\hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

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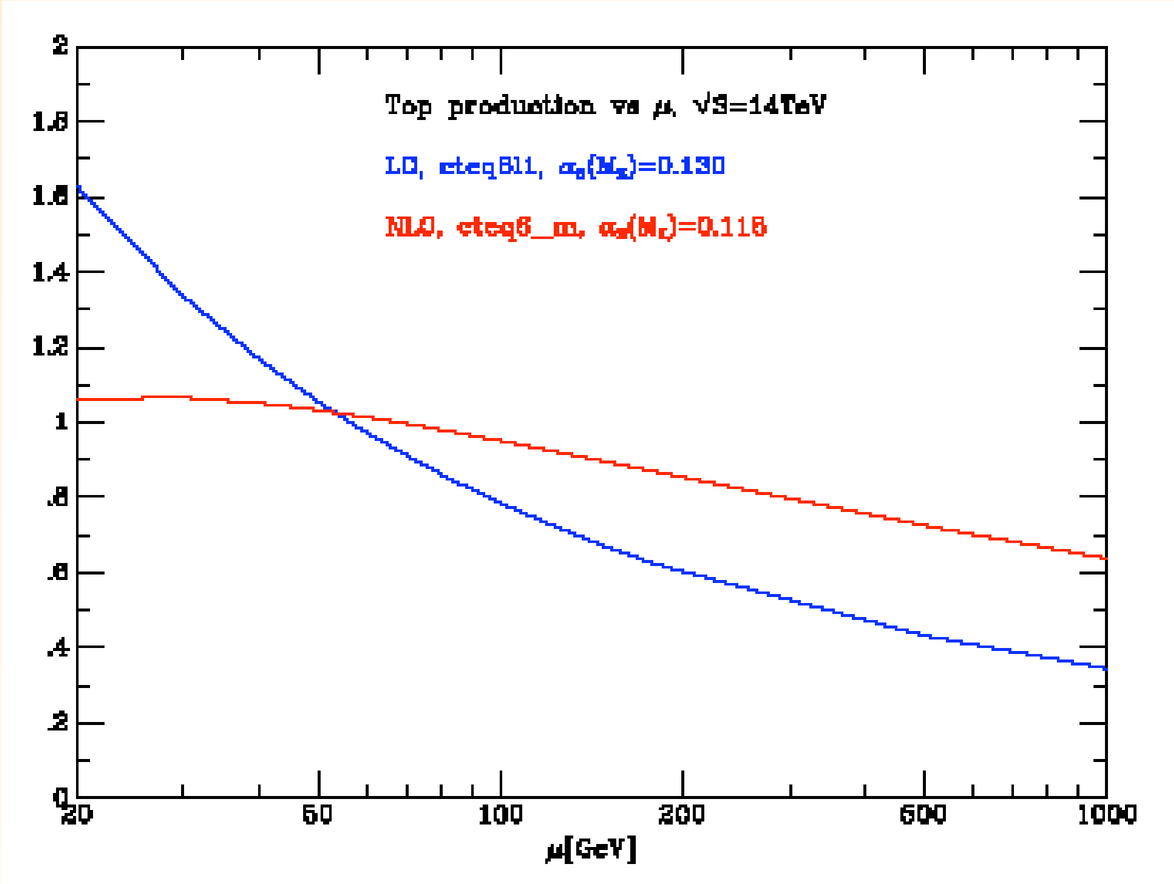
- Remember, **predictions are inclusive**: also at LO initial state radiation is included via the PDF; final state radiation by the definition of the parton, which represents all final state evolutions
- Due to these approximations, Leading Order predictions can depend strongly on the renormalization and factorization scales
- Including higher order corrections reduces the dependence on these scales

GOING NLO

- At NLO the dependence on the renormalization and factorization scales is reduced
  - First order where scale dependence in the running coupling and the PDFs is compensated for via the loop corrections: **first reliable estimate of the total cross section**
  - Better description of final state: impact of extra radiation included (e.g. jets can have substructure)
  - Opening of additional initial state partonic channels



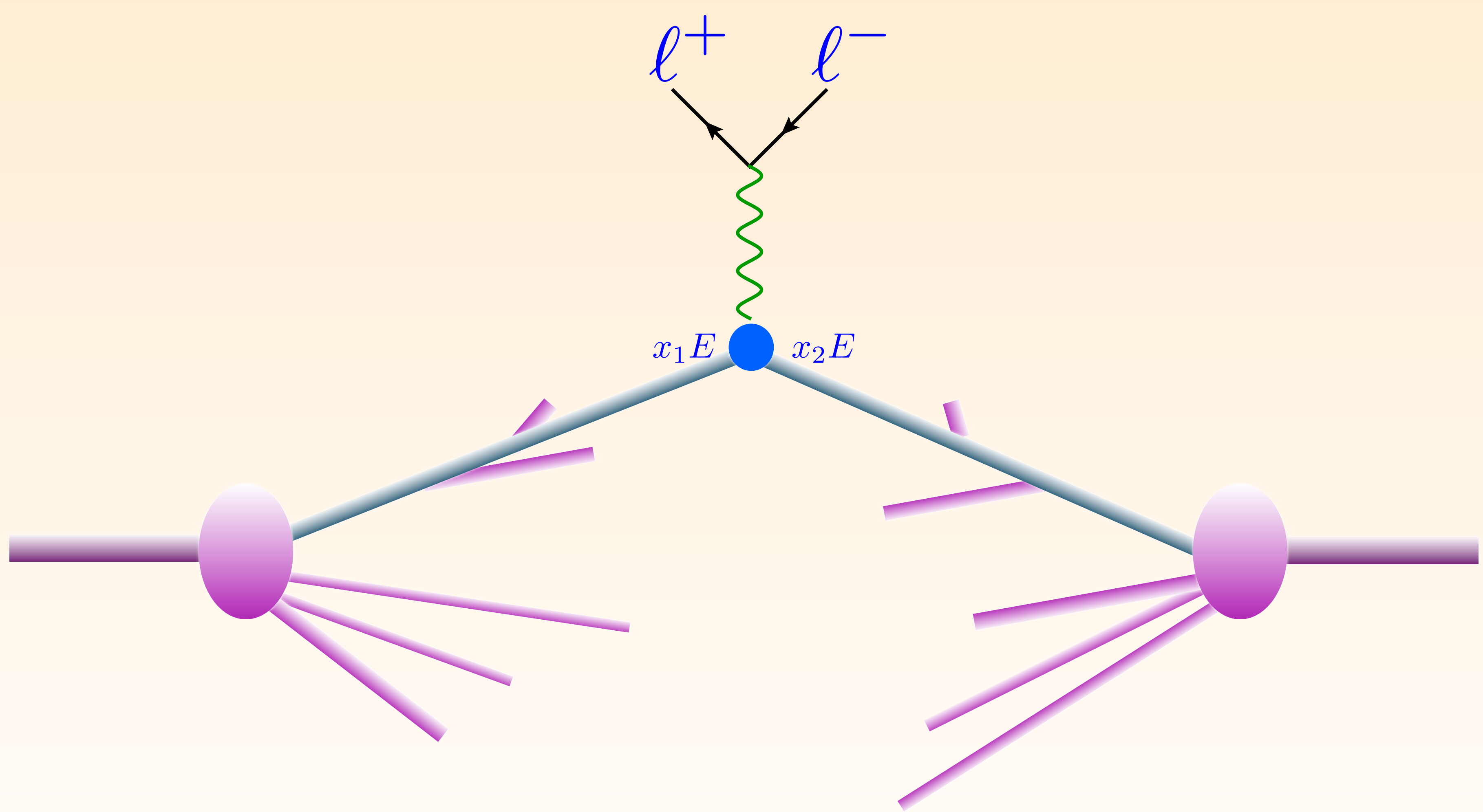
NLO corrections

- NLO corrections have three parts:
  - The Born contribution, i.e. the Leading order.
  - Virtual (or Loop) corrections: formed by an amplitude with a closed loop of particles interfered with the Born amplitudes
  - Real emission corrections: formed by amplitudes with one extra parton compared to the Born process
- Both Virtual and Real emission have one power of α_s extra compared to the Born process

$$\sigma^{\text{NLO}} = \int_m d\sigma^B + \int_m d\sigma^V + \int_{m+1} d\sigma^R$$

NLO predictions

- As an example, consider Drell-Yan Z/γ* production



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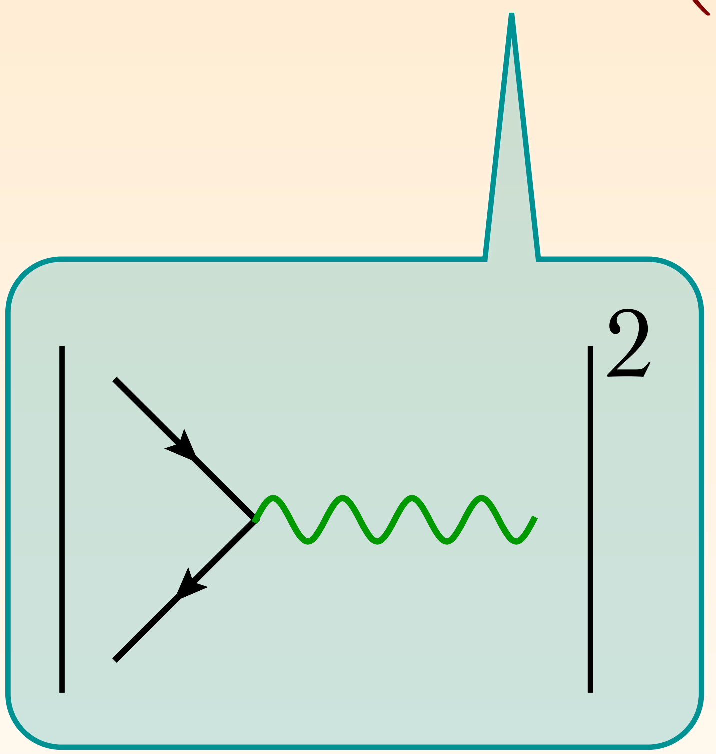

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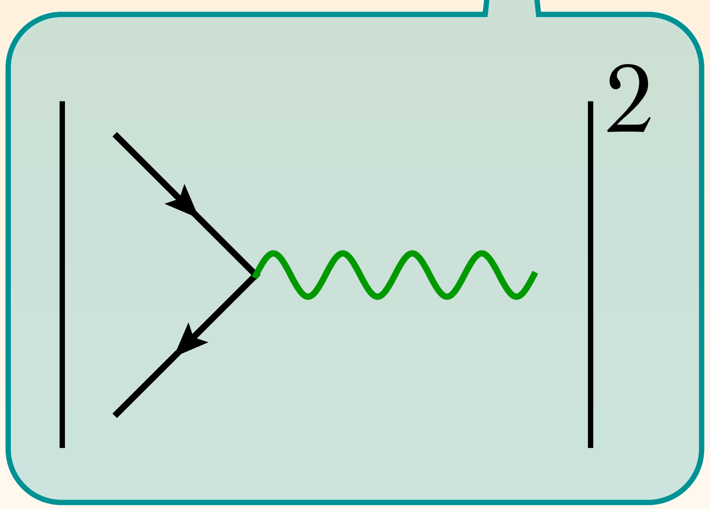

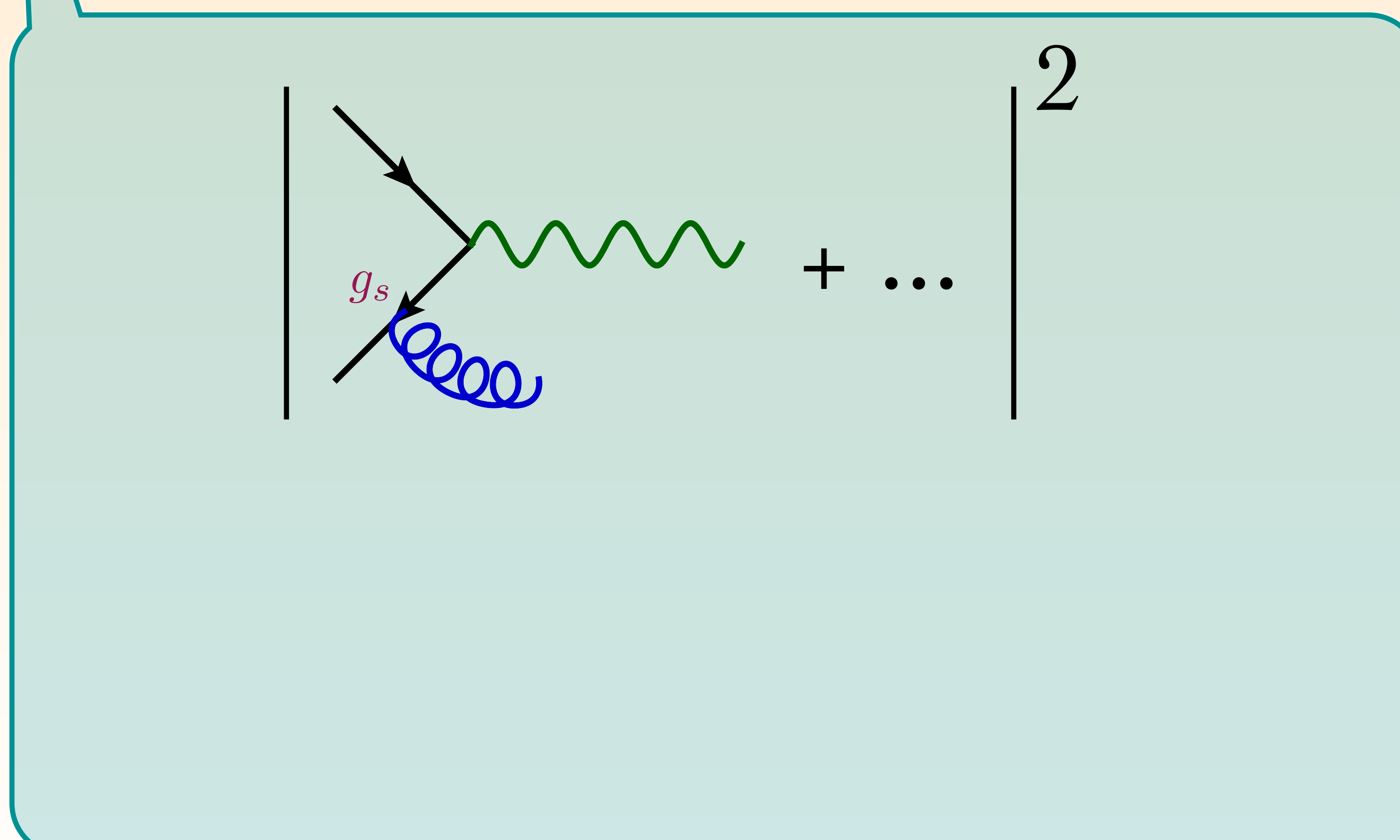

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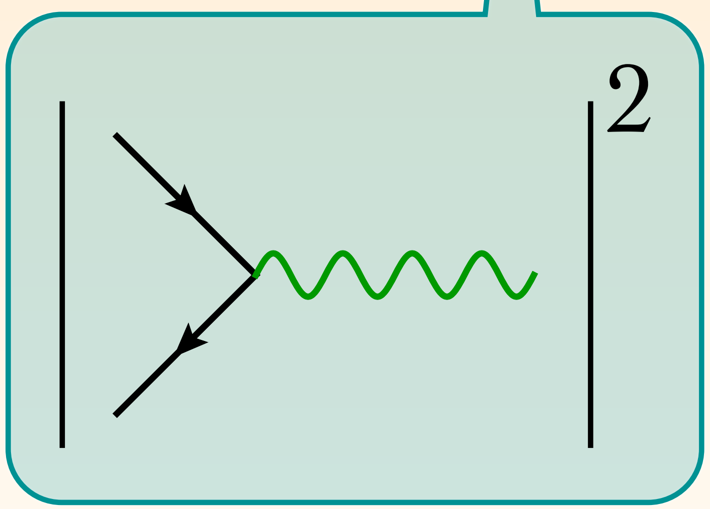
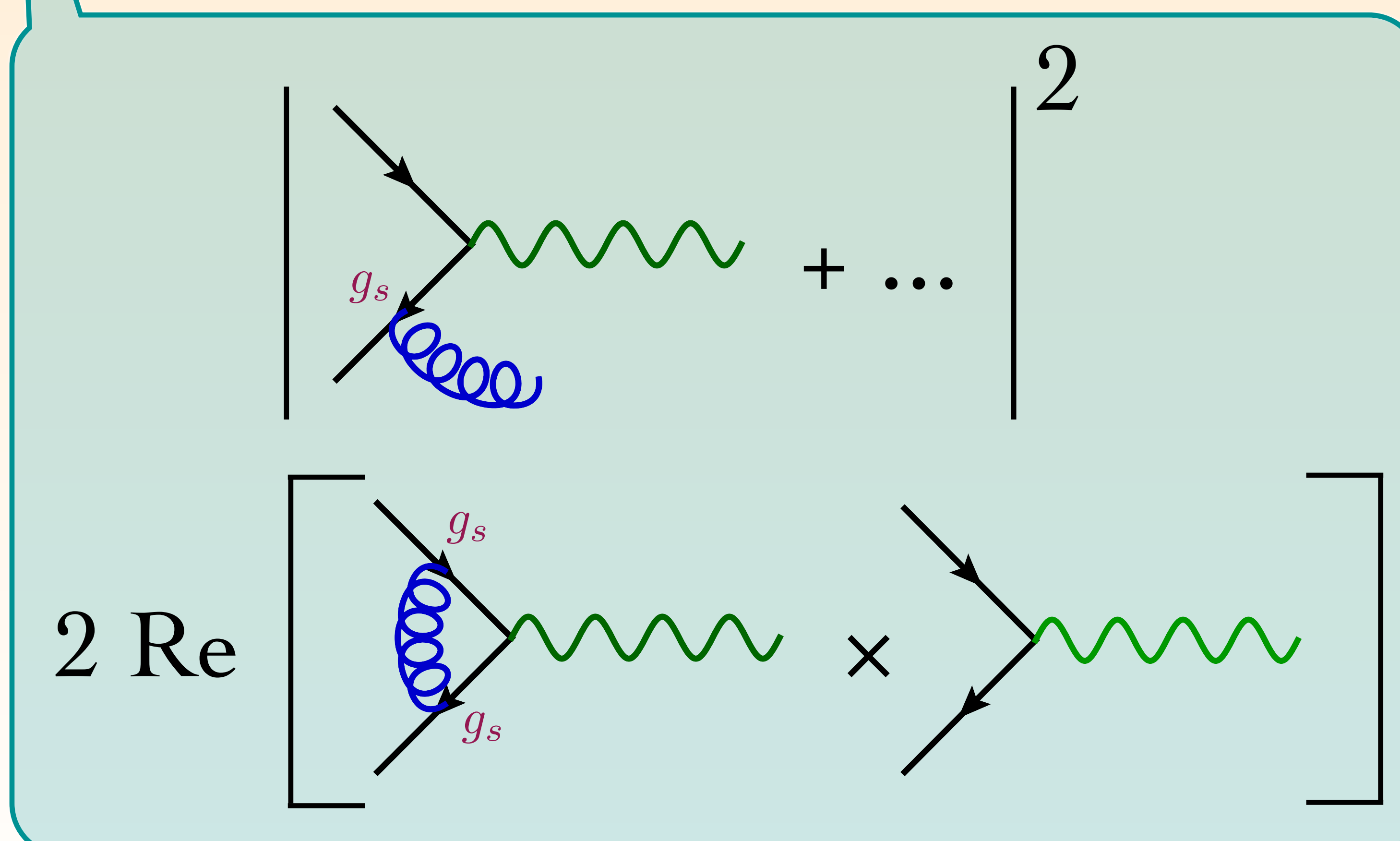
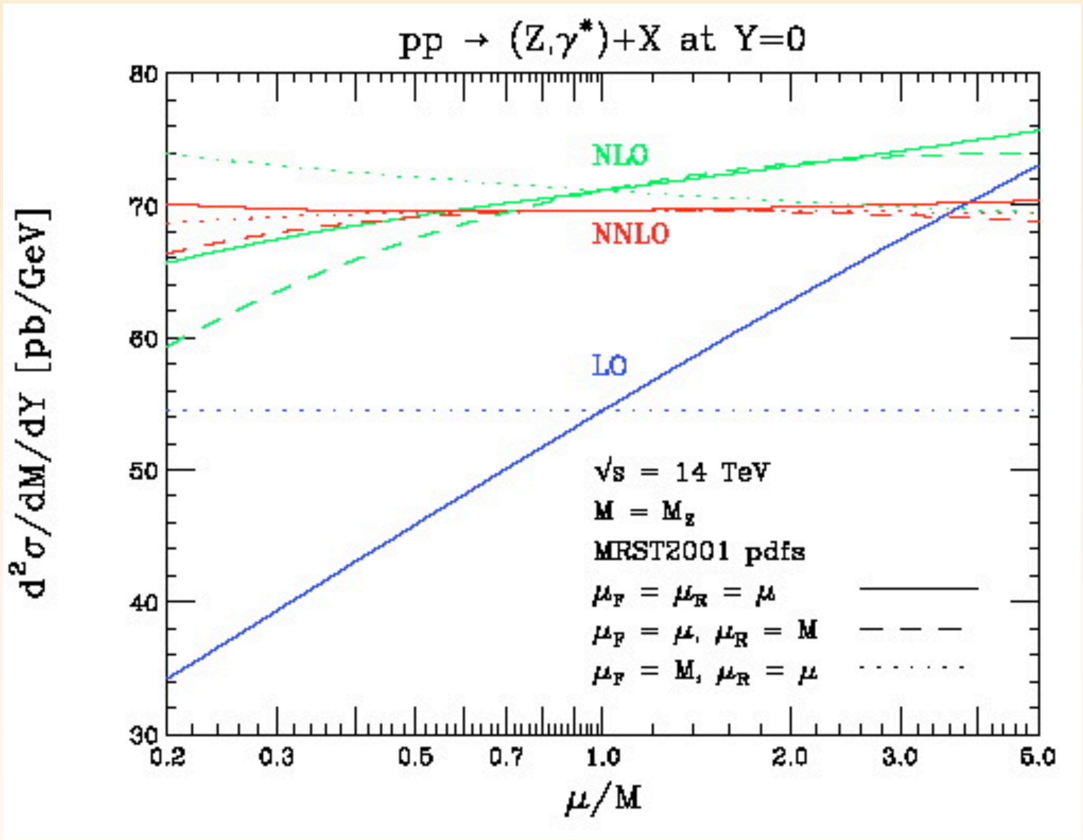


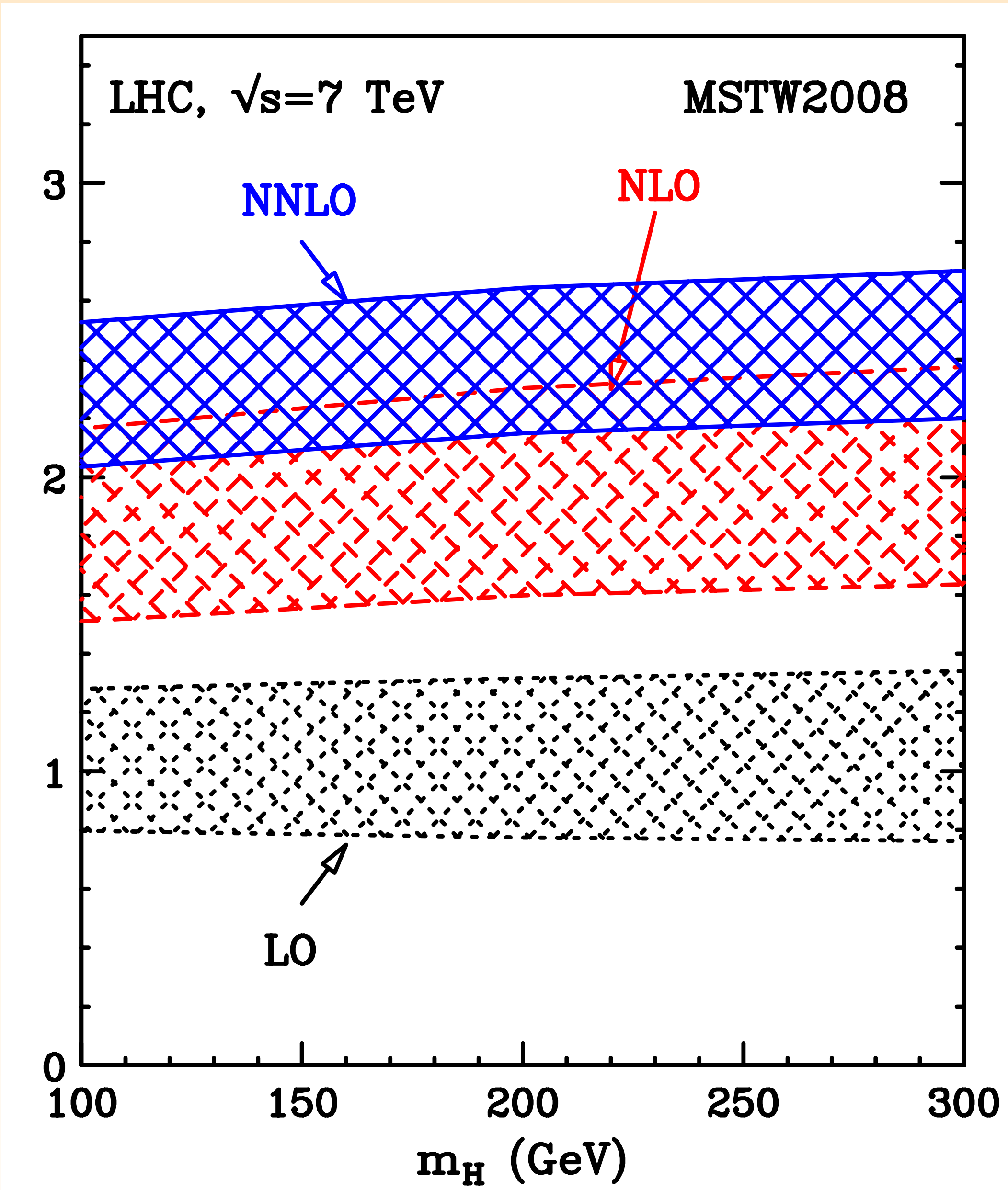


Going NNLO...?

- NNLO is the current state-of-the-art. There are only a few results available: Higgs, Drell-Yan, ttbar (qqbar induced only)
- Why do we need it?
  - An NNLO calculation gives control of the uncertainties in a calculation
  - It is "mandatory" if NLO corrections are very large to check the behavior of the perturbative series
  - It is the best we have! It is needed for Standard Candles and very precise tests of perturbation theory, exploiting all the available information, e.g. for determining NNLO PDF sets

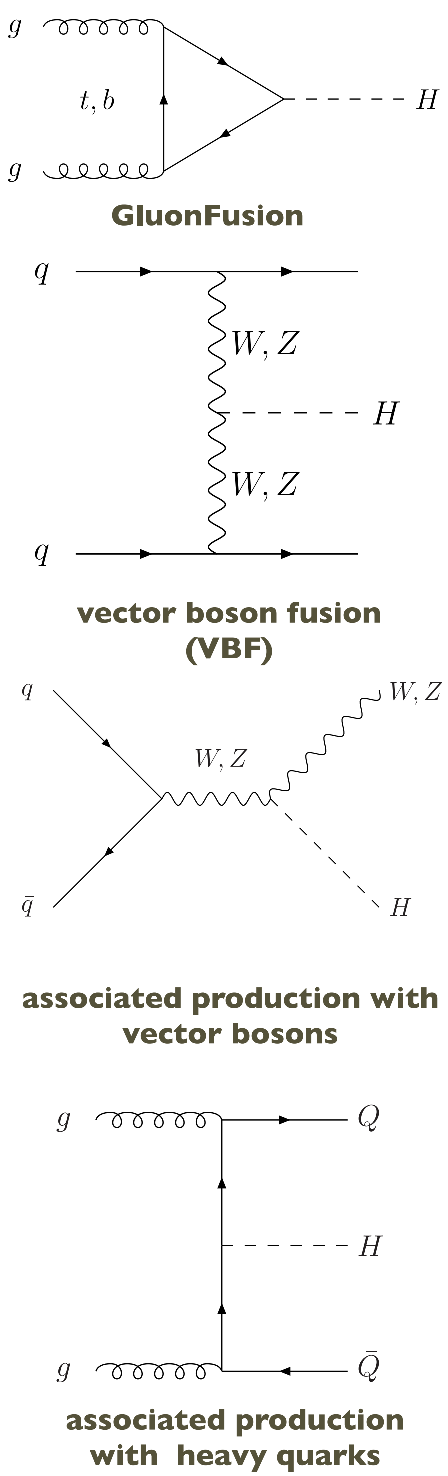


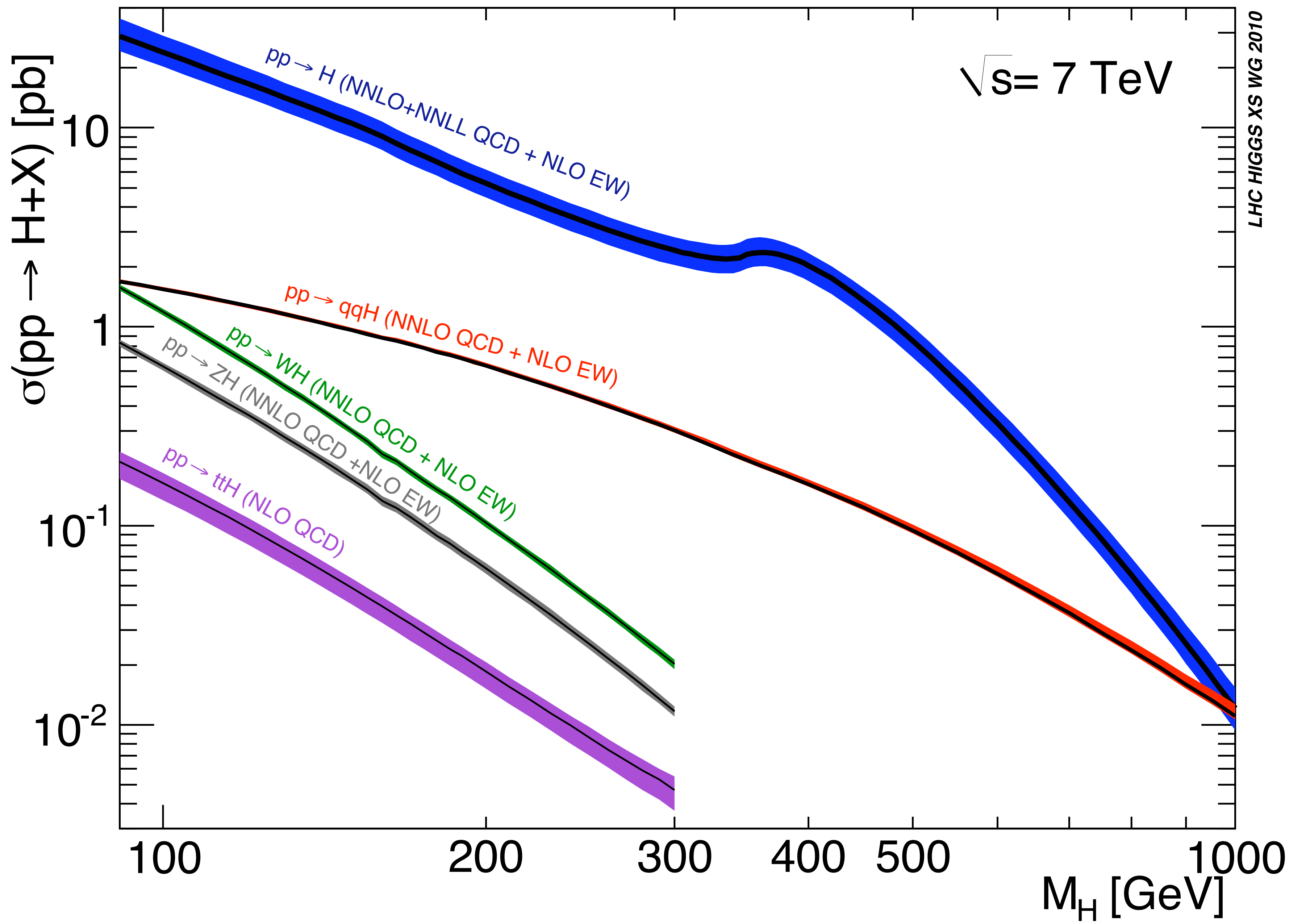
Higgs predictions at NNLO



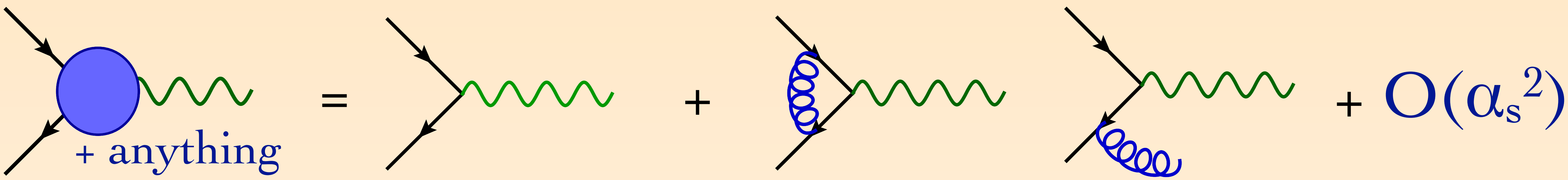
- LO calculation is not reliable,
- but the perturbative series stabilizes at NNLO
- NLO estimation of the uncertainties (by scale variation) works reasonably well

Higgs predictions at LHC





Difficulties



- Let us focus on NLO... there are already enough steps to be taken:
  - Virtual amplitudes: how to compute the loops automatically in a reasonable amount of time
  - How to deal with infra-red divergences: virtual corrections and real-emission corrections are separately divergent and only their sum is finite (for IR-safe observables) according to the KLN theorem
  - How to match these processes to a parton shower without double counting

Why an automatic tool?

- To save time

  Trade human time and expertise on computing one process at the time with time on physics and phenomenology.

- Robustness

  Modular code structure means that elements can be checked systematically and extensively once and for all. Trust can easily be build.

- Wide accessibility

  One framework for all. Available to everybody for an unlimited set of applications. Suitable for Experimental collaborations.

Canceling infrared divergences: FKS subtraction

NLO predictions

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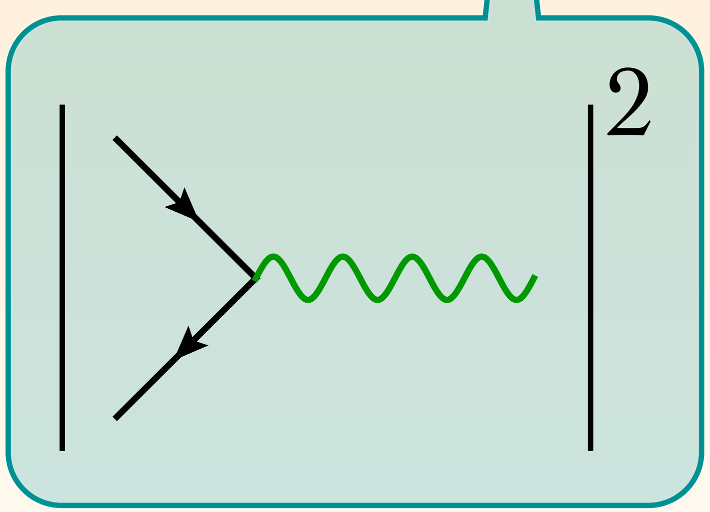

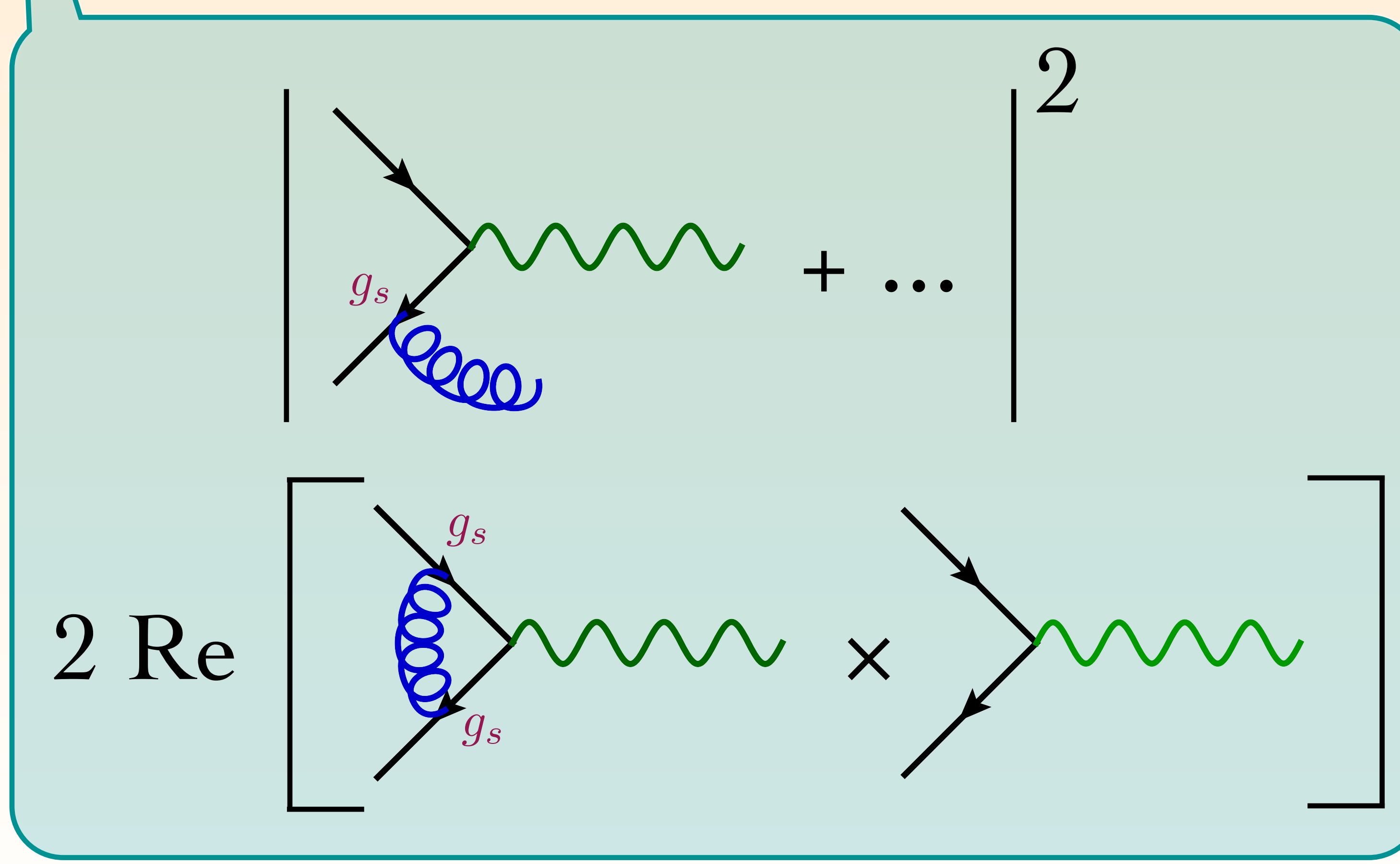

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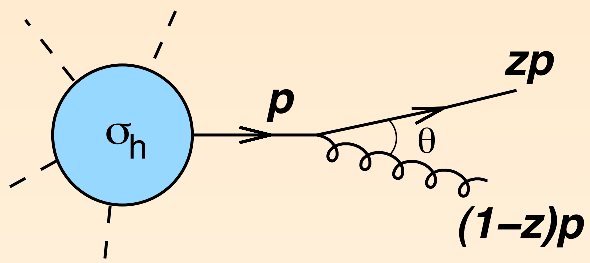
Branching

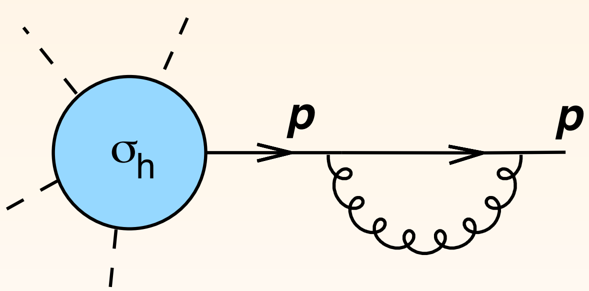
- In the soft and collinear region, the branching of a gluon from a quark can be written as



$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

where k_t is the transverse momentum of the gluon, $k_t = E \sin\theta$.

- The singularities cancel against the singularities in the virtual corrections, which result from the integral over the loop momentum of the function



$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

- The sum is finite for observables that cannot distinguish between two collinear partons ($k_t \longrightarrow 0$); a hard and a soft parton ($z \longrightarrow 1$); and a single parton (in the virtual contributions)

Infrared cancellation

$$\sigma^{\rm NLO} \sim \int d^4\Phi_m\, B(\Phi_m) + \int d^4\Phi_m \int_{\rm loop} d^d l\, V(\Phi_m) + \int d^d\Phi_{m+1}\, R(\Phi_{m+1})$$

- The KLN theorem tells us that divergences from virtual and real-emission corrections cancel in the sum for observables insensitive to soft and collinear radiation ("IR-safe observables")
- When doing an analytic calculation in dimensional regularization this can be explicitly seen in the cancellation of the $1/\epsilon$ and $1/\epsilon^2$ terms (with ϵ the regulator, $\epsilon \rightarrow 0$)
- In the real emission corrections, the explicit poles enter after the phase-space integration (in d dimensions)

Infrared safe observables

- For an observable to be calculable in fixed-order perturbation theory, the observable should be infrared safe, i.e., it should be insensitive to the emission of soft or collinear partons.

- In particular, if p_i is a momentum occurring in the definition of an observable, it most be invariant under the branching

  $$p_i \longrightarrow p_j + p_k,$$

  whenever p_j and p_k are collinear or one of them is soft.

- Examples

  - "The number of gluons" produced in a collision is not an infrared safe observable

  - "The number of hard jets defined using the k_T algorithm with a transverse momentum above 40 GeV," produced in a collision is an infrared safe observable

PHASE-SPACE INTEGRATION

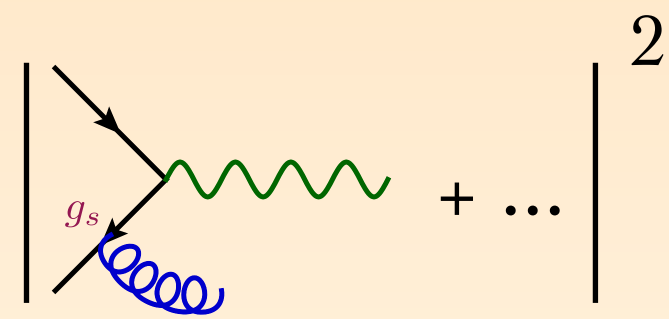
$$\sigma^{\rm NLO} \sim \int d^4\Phi_m\, B(\Phi_m) + \int d^4\Phi_m \int_{\rm loop} d^d l\, V(\Phi_m) + \int d^d\Phi_{m+1}\, R(\Phi_{m+1})$$

- For complicated processes we have to result to numerical phase-space integration techniques ("Monte Carlo integration"), which can only be performed in an integer number of dimensions
  - Cannot use a finite value for the dimensional regulator and take the limit to zero in a numerical code
- But we still have to cancel the divergences explicitly
- Use a subtraction method to explicitly factor out the divergences from the phase-space integrals

Example

- Suppose we want to compute the integral ("real emission radiation", where the 1-particle phase-space is referred to as the 1-dimensional x)

$$\int_0^1 dx\, f(x)$$



where $f(x) = \dfrac{g(x)}{x}$ and $g(x)$ is finite everywhere

- Let's introduce a regulator

$$\lim_{\epsilon \to 0} \int_0^1 dx\, \frac{g(x)}{x^{1+\epsilon}} = \lim_{\epsilon \to 0} \int_0^1 dx\, x^{-\epsilon} f(x)$$

for any non-integer non-zero value for ϵ this integral is finite

- We would like to factor out the explicit poles in ϵ so that they can be canceled explicitly against the virtual corrections

Subtraction method

$$\lim_{\epsilon \to 0} \int_0^1 dx\, x^{-\epsilon} f(x) \qquad\qquad f(x) = \frac{g(x)}{x}$$

- Add and subtract the same term

$$\lim_{\epsilon \to 0} \int_0^1 dx\, x^{-\epsilon} f(x) = \lim_{\epsilon \to 0} \int_0^1 dx\, x^{-\epsilon} \left[\frac{g(0)}{x} + f(x) - \frac{g(0)}{x} \right]$$

$$= \lim_{\epsilon \to 0} \int_0^1 dx \left[g(0) \frac{x^{-\epsilon}}{x} + \frac{g(x) - g(0)}{x^{1+\epsilon}} \right]$$

$$= \lim_{\epsilon \to 0} \frac{-1}{\epsilon} g(0) + \int_0^1 dx\, \frac{g(x) - g(0)}{x}$$

- We have factored out the $1/\epsilon$ divergence and are left with a finite integral
- According to the KLN theorem the divergence cancels against the virtual corrections

Limitations

"Plus distribution"

Subtraction: $\int_0^1 dx\, \frac{g(x) - g(0)}{x}$

- Even though the divergence is factored, there are cancellations between large numbers: if for an observable O, if $\lim_{x\to 0} O(x) \neq O(0)$ or we choose the bin-size too small, instabilities render the computation useless
  - We already knew that! KLN is sufficient; one must have infra-red safe observables and cannot ask for infinite resolution (need a finite bin-size)
- Subtraction method is very flexible -> method of choice in automation

NLO with Subtraction

$$\sigma^{\rm NLO} \sim \int d^4\Phi_m\, B(\Phi_m) + \int d^4\Phi_m \int_{\rm loop} d^d l\, V(\Phi_m) + \int d^d\Phi_{m+1}\, R(\Phi_{m+1})$$

- With the subtraction method this is replace by

$$\begin{aligned}\sigma^{\rm NLO} \sim & \int d^4\Phi_m\, B(\Phi_m)\\ & + \int d^4\Phi_m \left[\int_{\rm loop} d^d l\, V(\Phi_m) + \int d^d\Phi_1 G(\overline{\Phi}_{m+1})\right]_{\epsilon\to 0}\\ & + \int d^4\Phi_{m+1}\left[R(\Phi_{m+1}) - G(\overline{\Phi}_{m+1})\right]\end{aligned}$$

- Terms between the brackets are finite. Can integrate them numerically and independent from one another in 4 dimensions

Subtraction methods

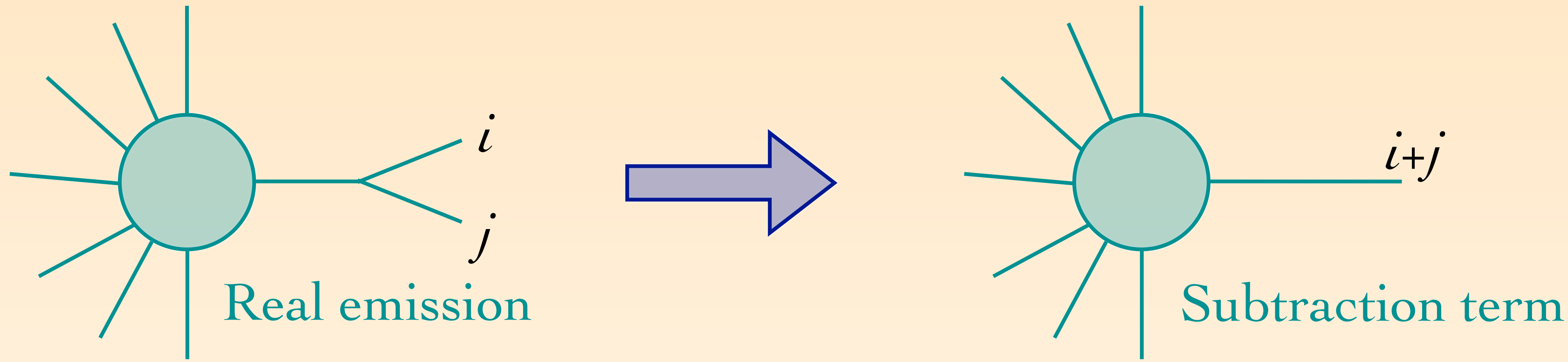
- $G(\overline{\Phi}_{m+1})$ should be defined such that
  1) it exactly matches the singular behavior of $R(\Phi_{m+1})$
  2) its form is convenient for numerical integration techniques
  3) it is exactly integrable in d dimensions over the one-particle subspace $\int d^d\Phi_1 G(\overline{\Phi}_{m+1})$, leading to soft and/or collinear divergences as explicit poles in the dimensional regulator
  4) it is universal, i.e. process independent
     → overall factor times the Born process

Two methods

- Catani-Seymour dipole subtraction
  - ☑ Most used method
  - ☑ Clear written paper on how to use this method in practice
  - ☑ Method evolved from cancellation of the soft divergence
  - ☑ Proven to work for simple as well as complicated processes
  - ☑ Automation in publicly available packages: MadDipole, AutoDipole, Helac-Dipoles, Sherpa

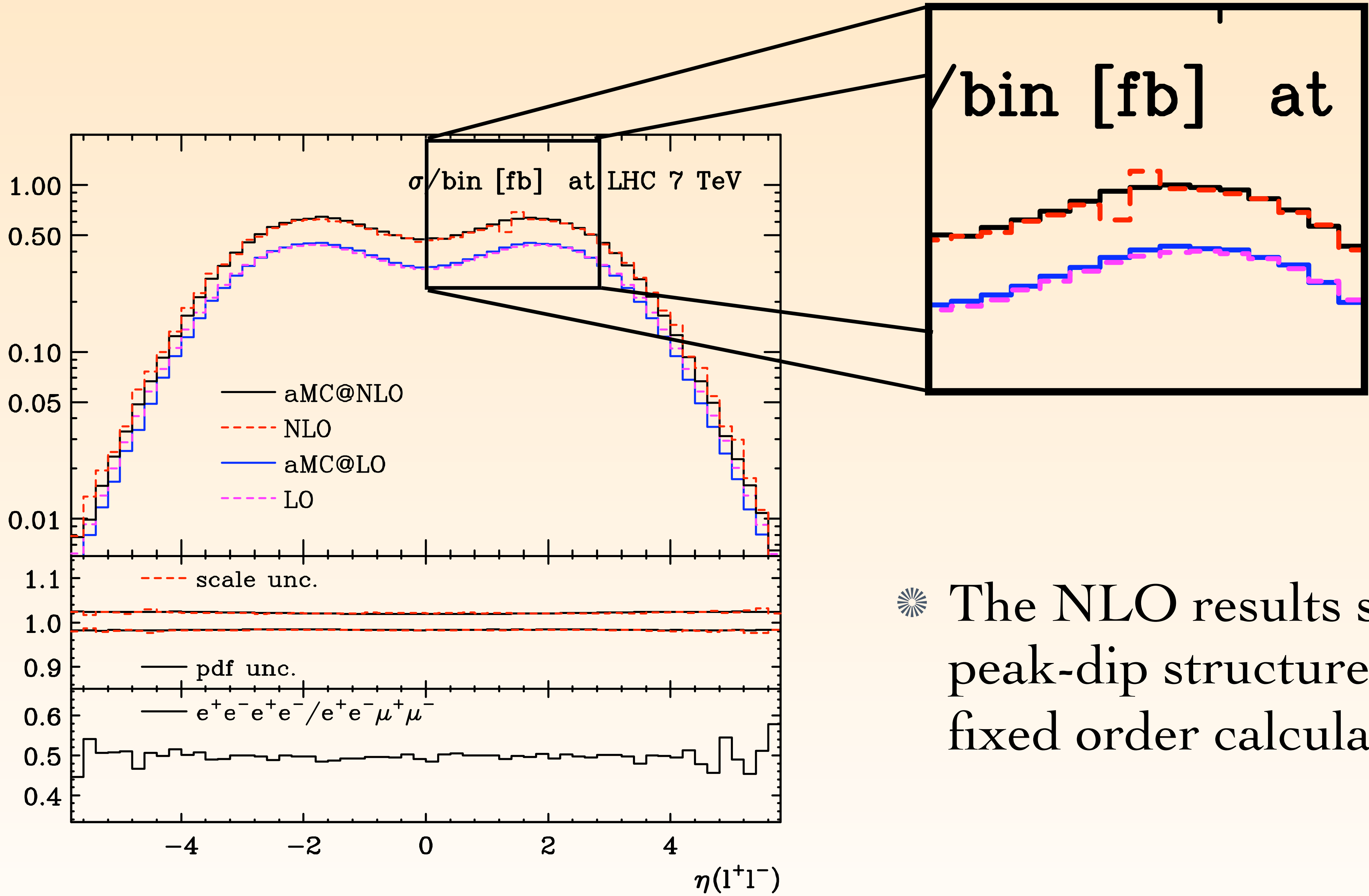
- FKS subtraction
  - ☑ Not so well-known
  - ☑ (Probably) more efficient, because less subtraction terms are needed
  - ☑ Collinear divergences as a starting point
  - ☑ Proven to work for simple as well as complicated processes
  - ☑ Preferred method when interfacing NLO to a parton shower
  - ☑ Automated in aMC@NLO & POWHEG BOX

Kinematics of counter events



- If i and j are two on-shell particles that are present in a splitting that leads to an singularity, for the counter events we need to combine their momenta to a new on-shell parton that's the sum of $i+j$
- This is not possible without changing any of the other momenta in the process
- When applying cuts or making plots, events and counter events might end-up in different bins
  - Use IR-safe observables and don't ask for infinite resolution! (KLN theorem)

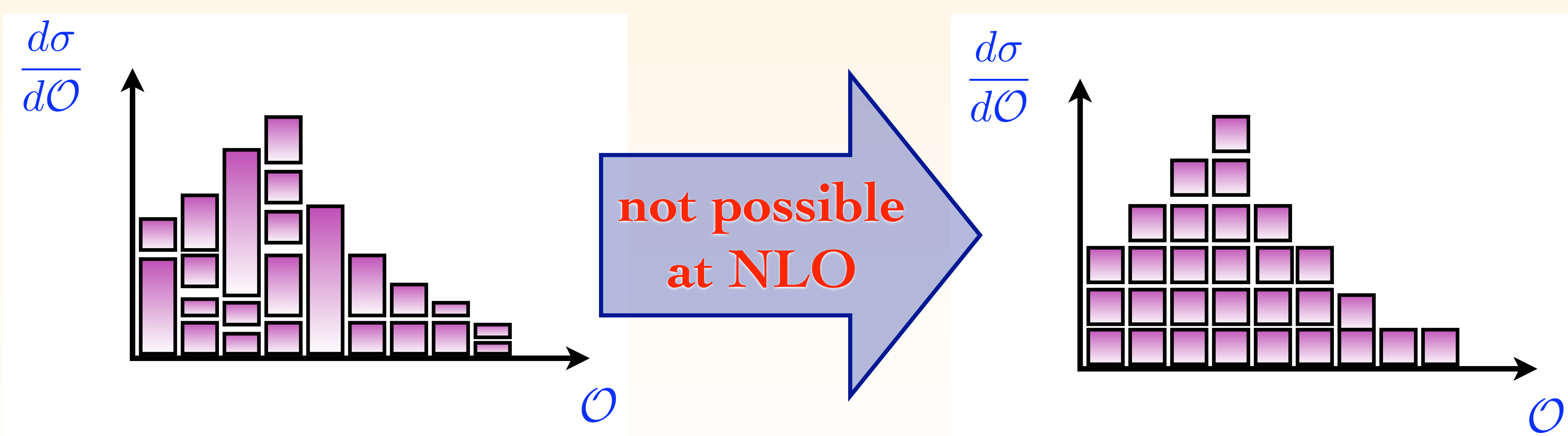
Example in 4 charged lepton production



- The NLO results shows a typical peak-dip structure that hampers fixed order calculations

Event unweighting?

- Another consequence of this kinematic mismatch is that we cannot generate events at fixed order NLO
  - Even though the integrals are finite, they are not bounded (compare with $\int_0^1 dx \, \frac{1}{\sqrt{x}}$), so there is no maximum to unweight against: a single event can have an arbitrarily large weight
  - Furthermore, event and counter event have different kinematics: which one to use for the unweighted event?



Filling histograms on-the-fly

$$\sigma^{\rm NLO} \sim \int d^4\Phi_m\, B(\Phi_m)$$
$$+ \int d^4\Phi_m \left[\int_{\rm loop} d^d l\, V(\Phi_m) + \int d^d\Phi_1 G(\overline{\Phi}_{m+1}) \right]_{\epsilon\to 0}$$
$$+ \int d^4\Phi_{m+1} \left[R(\Phi_{m+1}) - G(\overline{\Phi}_{m+1}) \right]$$

- In practice, when we do the MC integration we generate 2 sets of momenta

  1. An m-body set (for the Born, virtual and integrated counter terms)
  2. An m+1-body (for the NLO) which we map to the counter term momenta (for the counter terms)

- We compute the above formula; and apply cuts and fill histograms using the momenta corresponding to each term with the weight of that corresponding term

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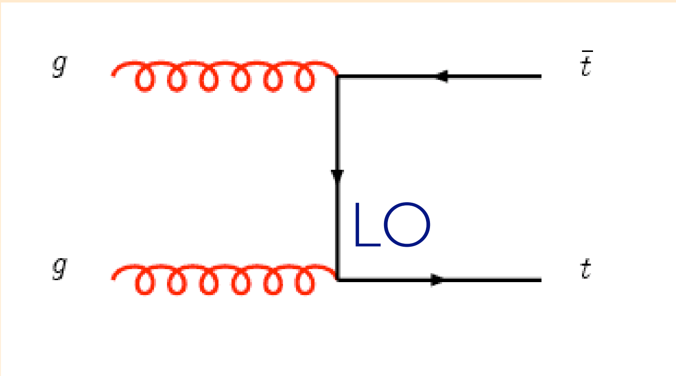
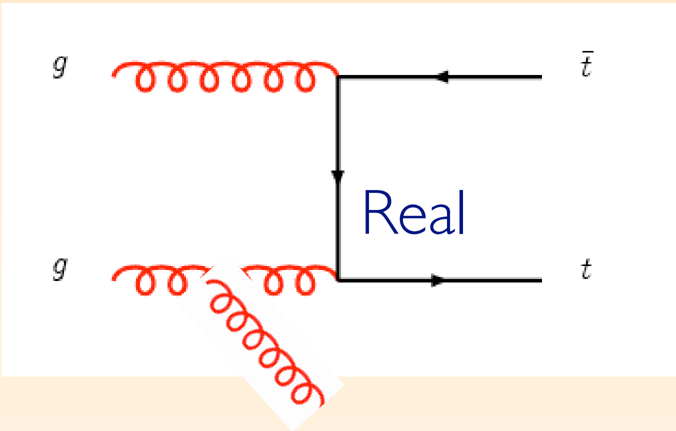
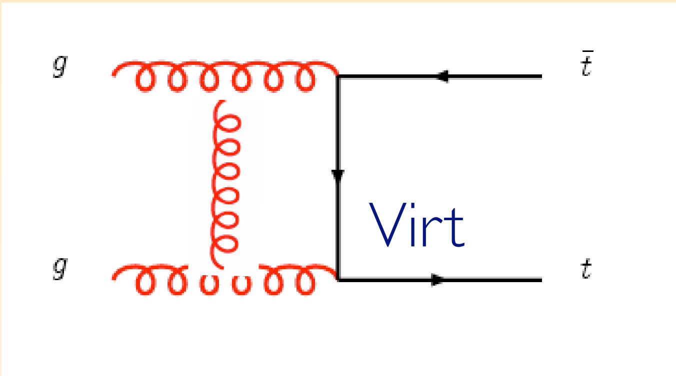
NLO...?

- Are all (IR-safe) observables that we can compute using a NLO code correctly described at NLO? Suppose we have a NLO code for pp ⟶ ttbar







- Total cross section
- Transverse momentum of the top quark
- Transverse momentum of the top-antitop pair
- Transverse momentum of the jet
- Top-antitop invariant mass
- Azimuthal distance between the top and anti-top

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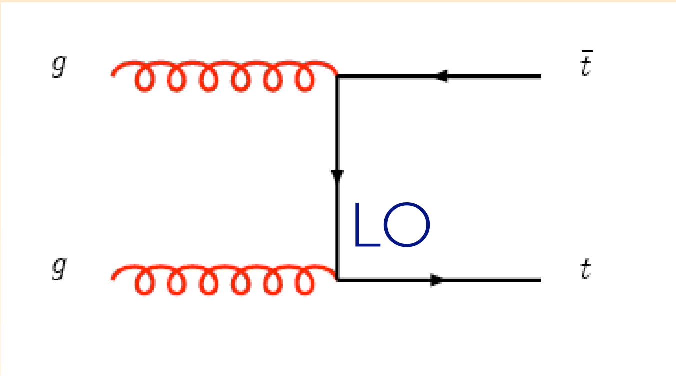

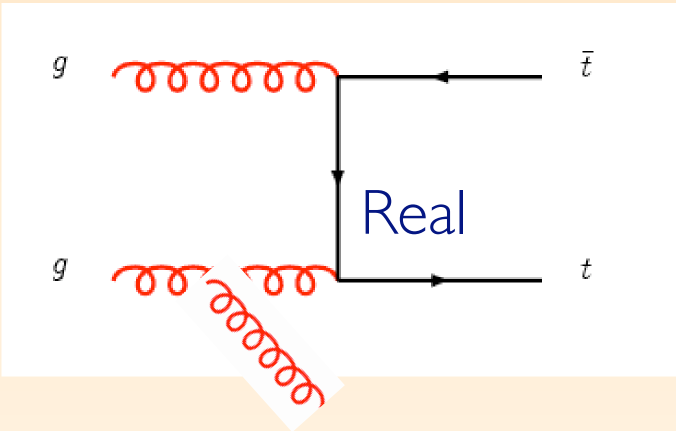

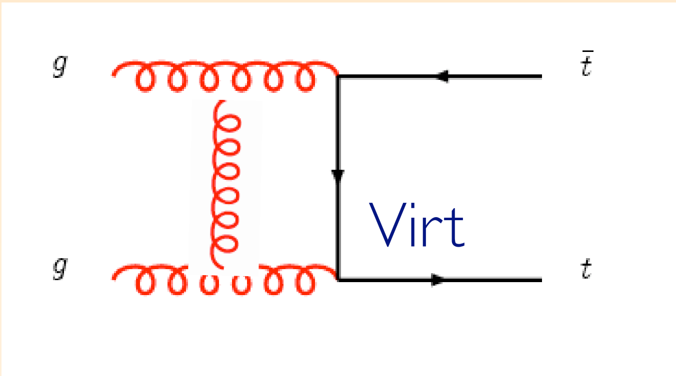

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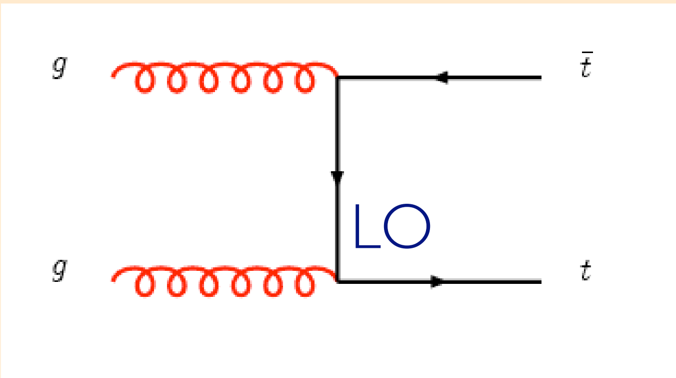

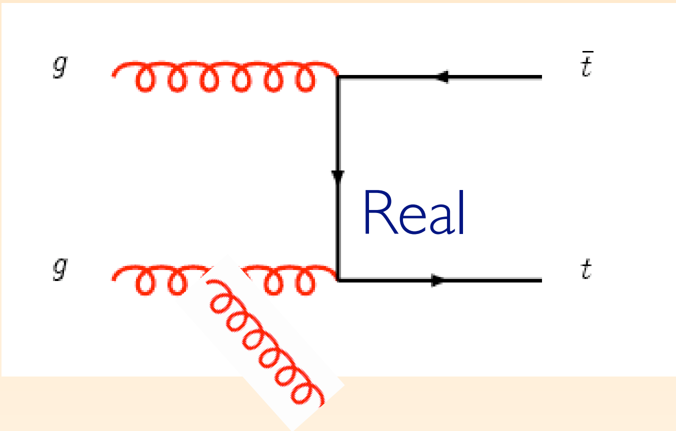

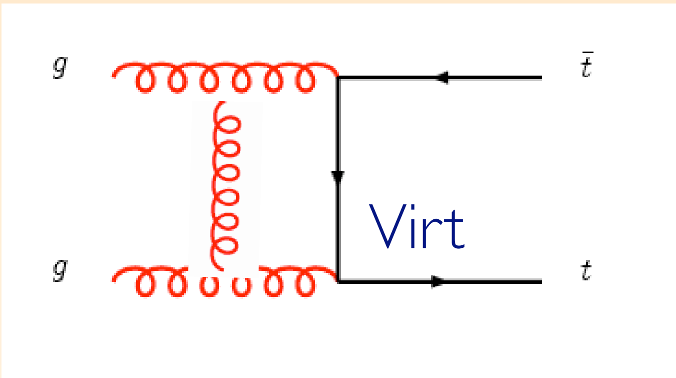

NLO?

- Total cross section ✔
- Transverse momentum of the top quark
- Transverse momentum of the top-antitop pair
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- Top-antitop invariant mass
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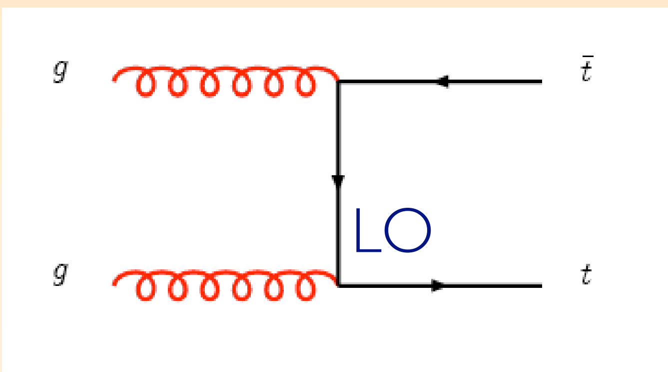

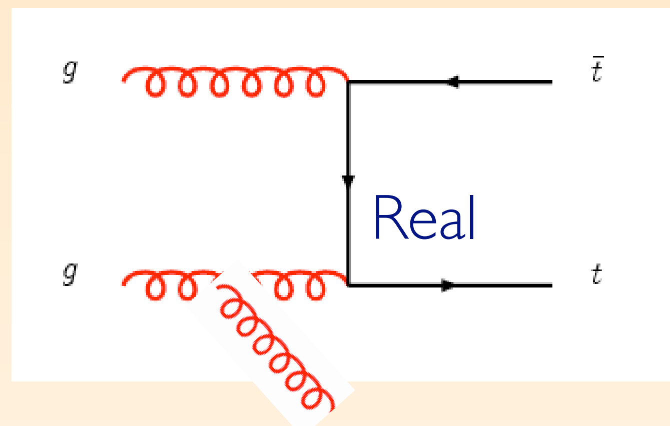

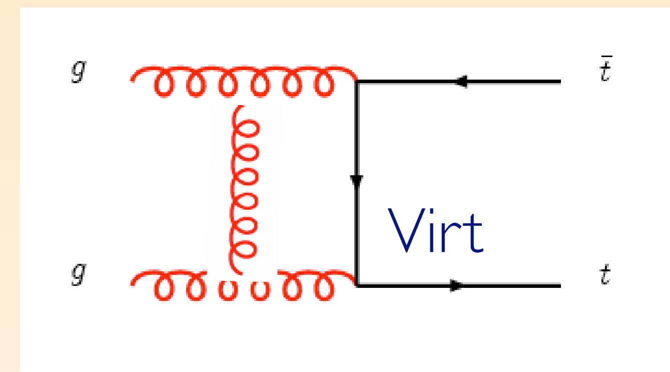

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Total cross section	✔
Transverse momentum of the top quark	✔
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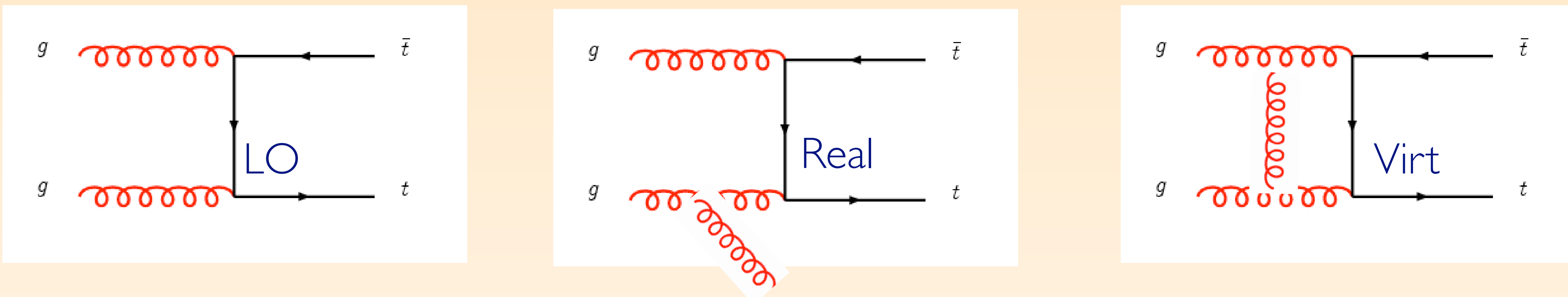

Observable	NLO?
Total cross section	✔
Transverse momentum of the top quark	✔
Transverse momentum of the top-antitop pair	✘
Transverse momentum of the jet	
Top-antitop invariant mass	
Azimuthal distance between the top and anti-top	

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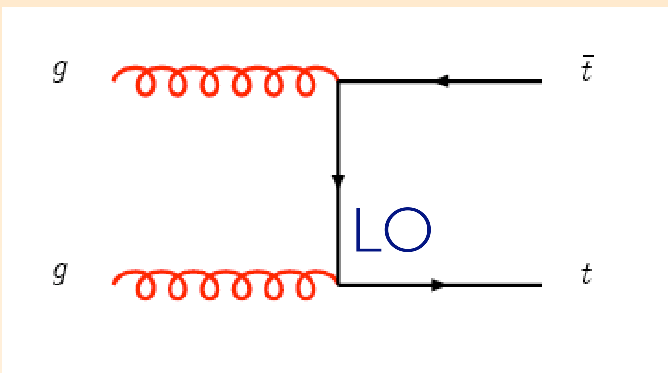

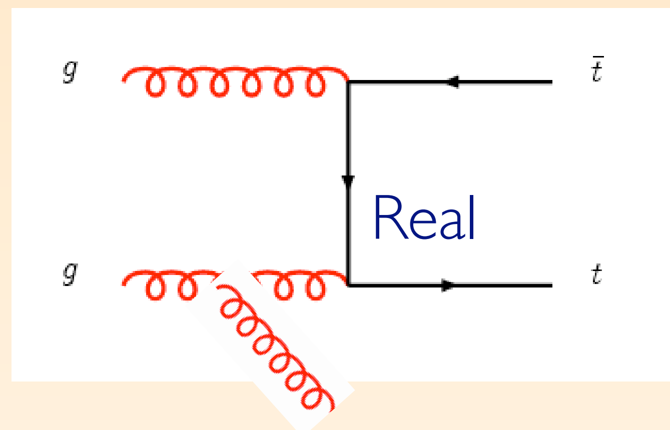

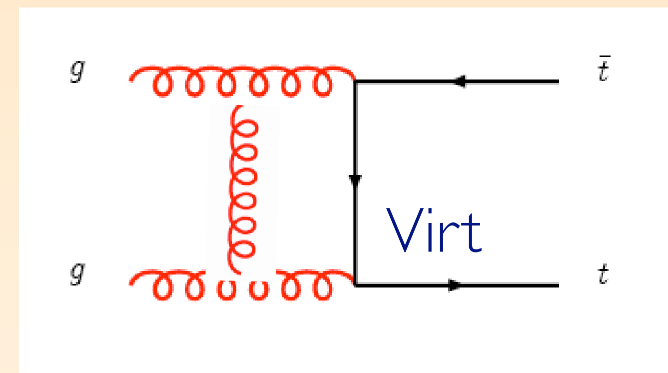

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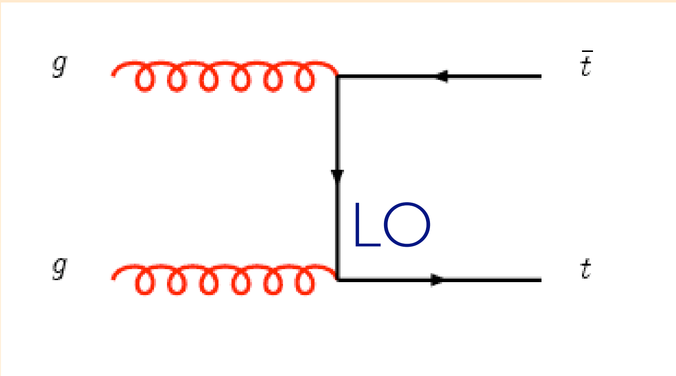

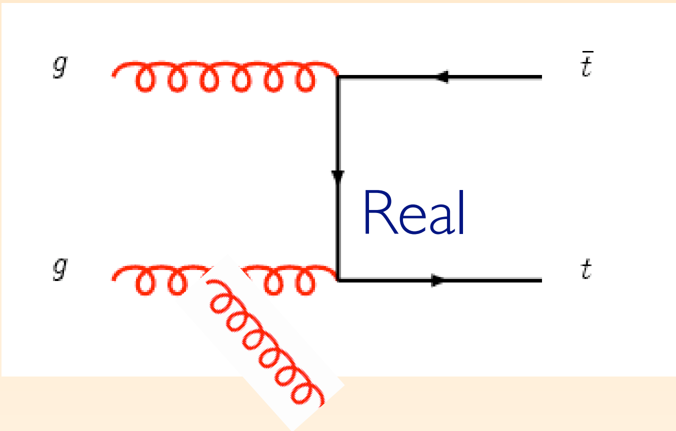

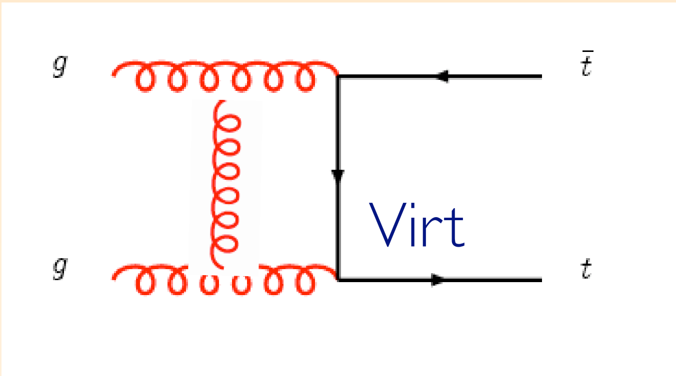

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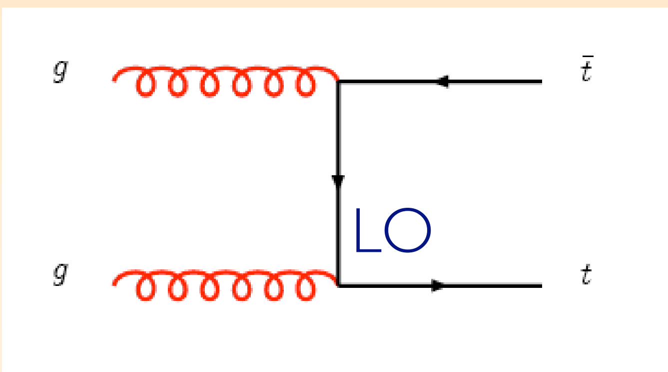

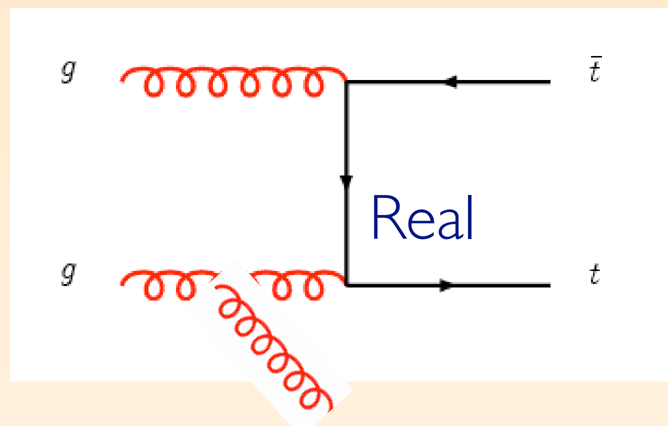

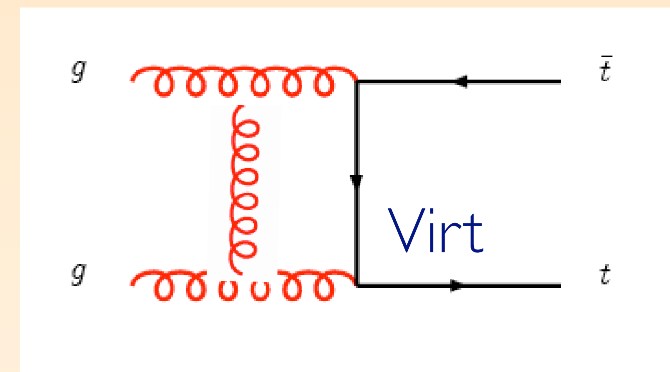

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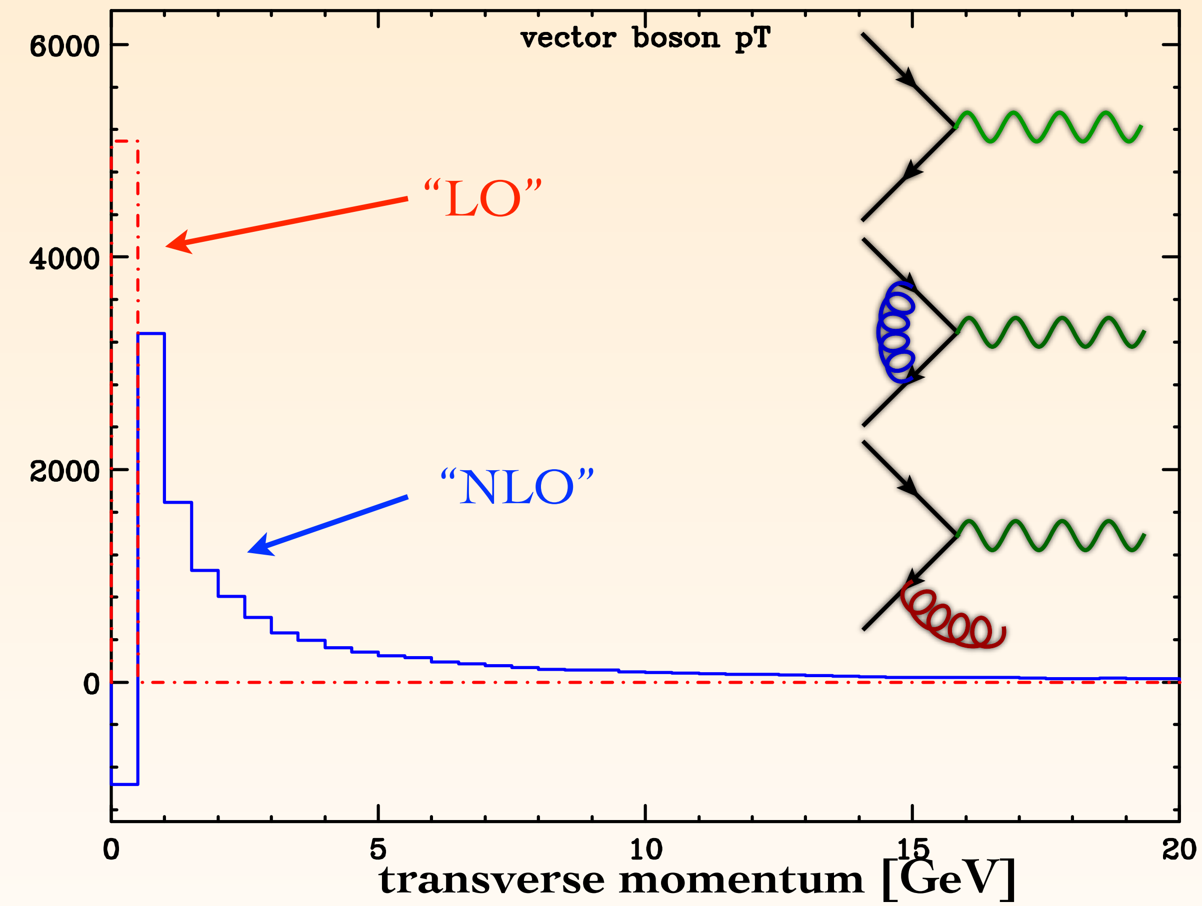
Limitations of Fixed Order calculations

- In fact, for the observables that are not described at NLO accuracy, the situation is actually a bit worse:



- In the small transverse momentum region, this calculation breaks down (it's even negative in the first bin!), and anywhere else it is purely a LO calculation for V+1j

Summary

- Both the virtual and real-emission corrections are IR divergent, but their sum is finite: We can use a subtraction methods to factor the divergences in the real-emission phase-space integration and cancel them explicitly against the terms in the virtual corrections
- This generates events and counter events with slightly different kinematics. This means we cannot generate unweighed events (integrals are not bounded), but we can fill plots with weighted events: MC integrator (not an MC event generator)
- When making plots or applying cuts, use only IR safe observables with finite resolution
- Phase-space integrals are finite, but not bounded: cannot unweight the events

NLO+PS matching

IMPROVING MC's

- There are two ways to improve a Parton Shower Monte Carlo event generator with matrix elements:
  - ME+PS merging: Include matrix elements with more final state partons to describe hard, well-separated radiation better (already discussed by Fabio)
  - NLO+PS matching: Include full NLO corrections to the matrix elements to reduce theoretical uncertainties in the matrix elements. The real-emission matrix elements will describe the hard radiation

Limitations of Fixed Order calculations

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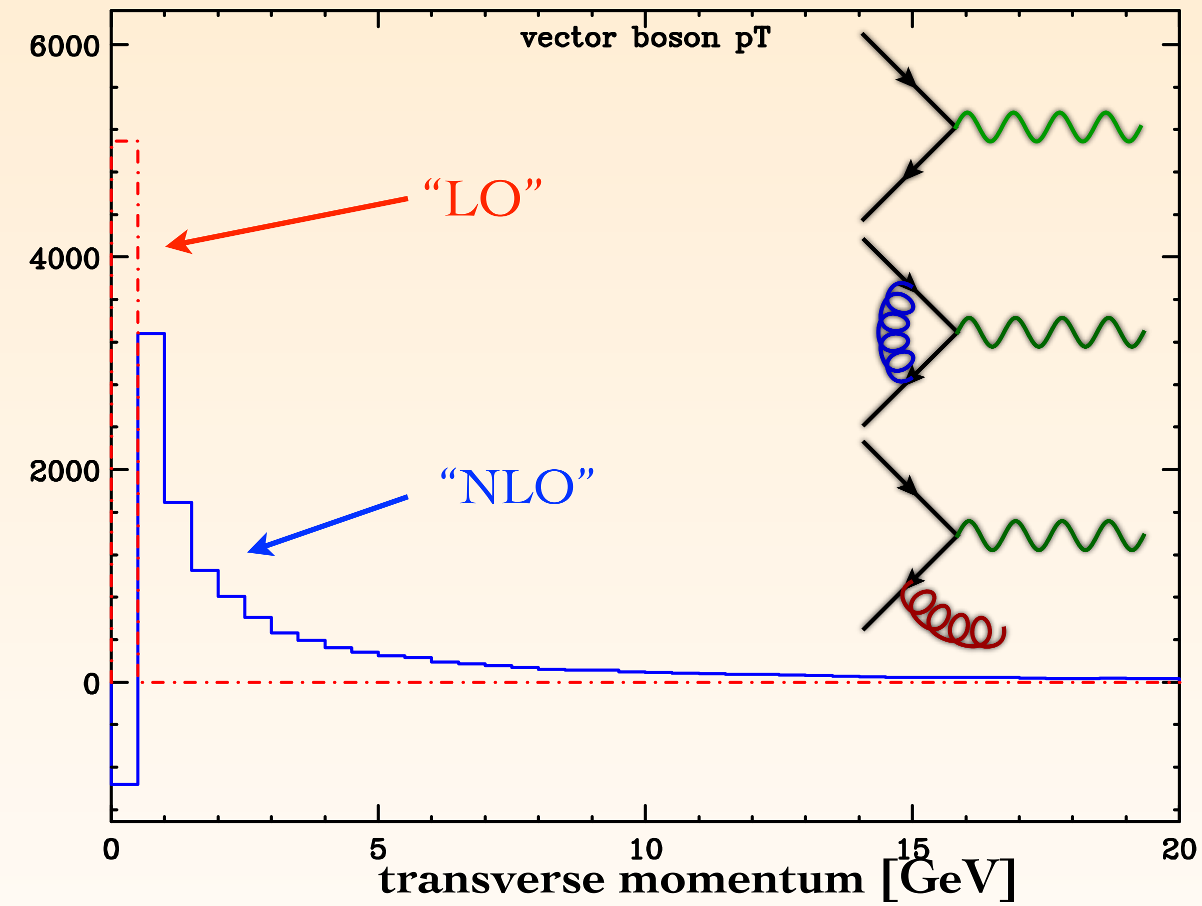

Matrix elements vs. Parton showers

ME

1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

Shower MC

1. Resums logs to all orders
2. Computationally cheap
3. No limit on particle multiplicity
4. Valid when partons are collinear and/or soft
5. Partial interference through angular ordering
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Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

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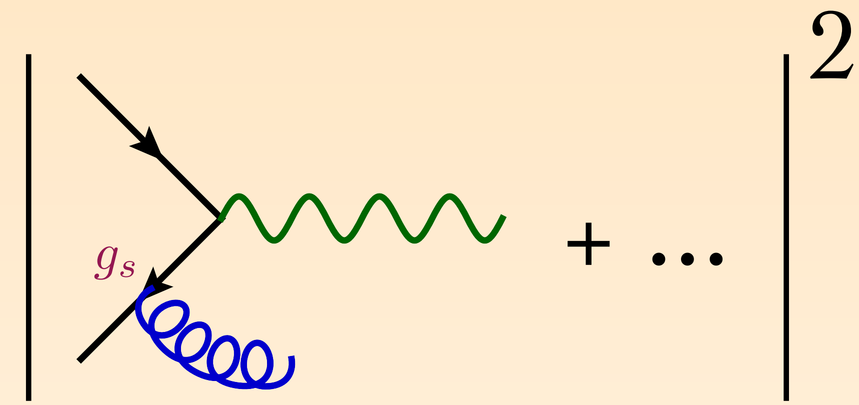
No longer true at NLO!

Approaches are complementary: merge them!

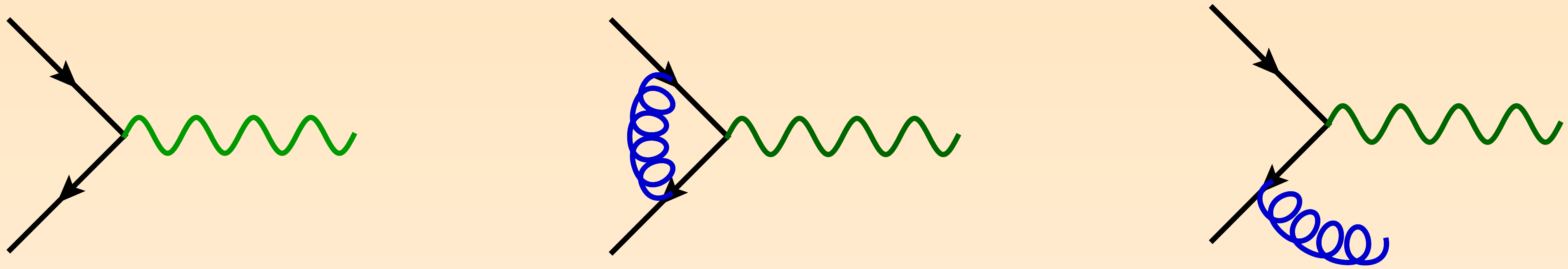
Difficulty: avoid double counting, ensure smooth distributions

At NLO



- We have to integrate the real emission over the **complete** phase-space of the one particle that can go soft or collinear to obtain the infra-red poles that will cancel against the virtual corrections
- We cannot use the same matching procedure: requiring that all partons should produce separate jets is not infrared safe
- We have to invent a new procedure to match NLO matrix elements with parton showers

NAIVE (WRONG) APPROACH



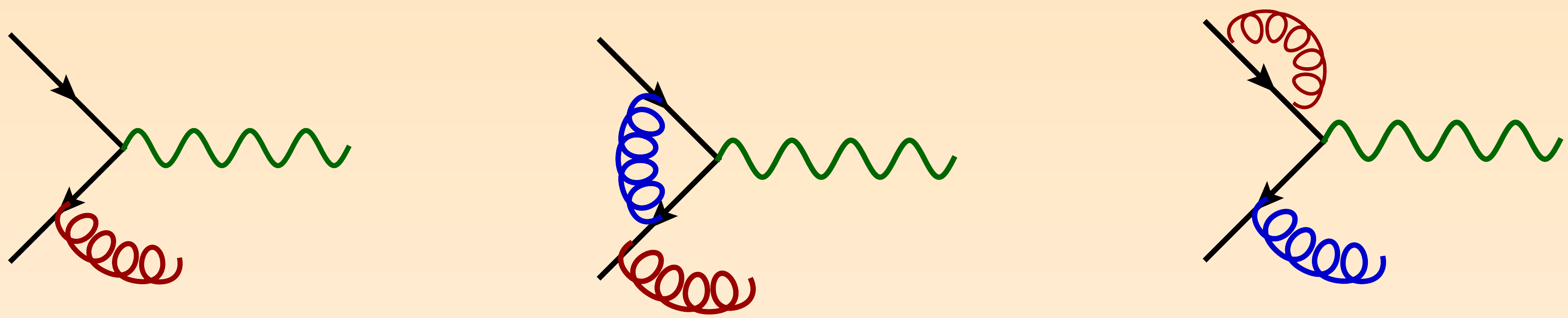
- In a fixed order calculation we have contributions with m final state particles and with m+1 final state particles

$$\sigma^{\rm NLO} \sim \int d^4\Phi_m\, B(\Phi_m) + \int d^4\Phi_m \int_{\rm loop} d^d l\, V(\Phi_m) + \int d^d\Phi_{m+1}\, R(\Phi_{m+1})$$

- We could try to shower them independently
- Let $I_{\rm MC}^{(k)}(O)$ be the parton shower spectrum for an observable O, showering from a k-body initial condition
- We can then try to shower the m and m+1 final states independently

$$\frac{d\sigma_{\rm NLOwPS}}{dO} = \left[d\Phi_m(B + \int_{\rm loop} V)\right] I_{\rm MC}^{(m)}(O) + \left[d\Phi_{m+1}R\right] I_{\rm MC}^{(m+1)}(O)$$

Naive (wrong) approach



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Double counting

$$\frac{d\sigma_{\text{NLOwPS}}}{dO} = \left[d\Phi_m(B + \int_{\text{loop}} V)\right] I_{\text{MC}}^{(m)}(O) + \left[d\Phi_{m+1} R\right] I_{\text{MC}}^{(m+1)}(O)$$

- But this is wrong!
- If you expand this equation out up to NLO, there are more terms then there should be and the total rate does not come out correctly
- Schematically $I_{\text{MC}}^{(k)}(O)$ for 0 and 1 emission is given by

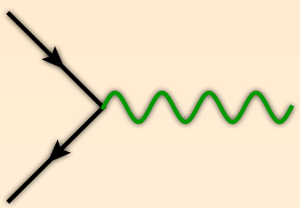
$$I_{\text{MC}}^{(k)}(O) \sim \Delta_a(Q^2, Q_0^2) + \Delta_a(Q^2, t) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_s(t)}{2\pi} P_{a\to bc}(z)$$

- And Δ is the Sudakov factor

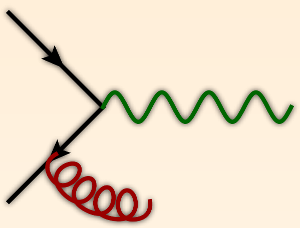
$$\Delta_a(Q^2, t) = \exp\left[-\sum_{bc} \int_t^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_s(t')}{2\pi} P_{a\to bc}\right]$$

Sources of double counting

Born+Virtual:



Real emission:



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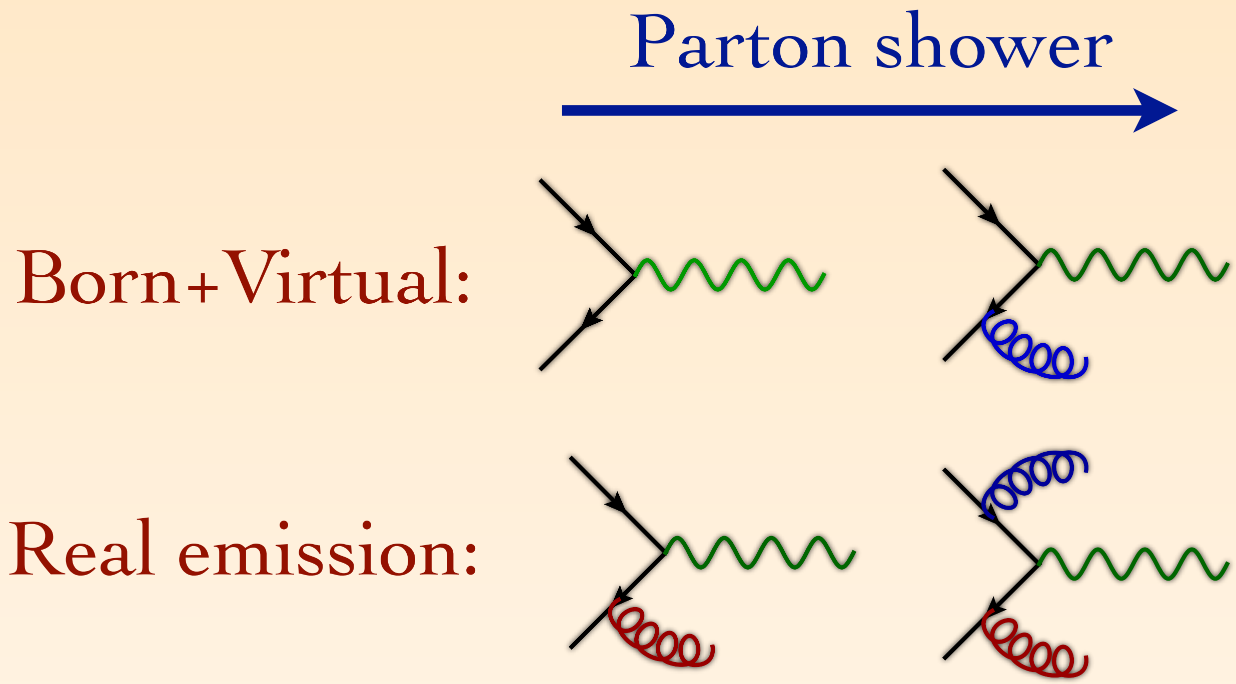

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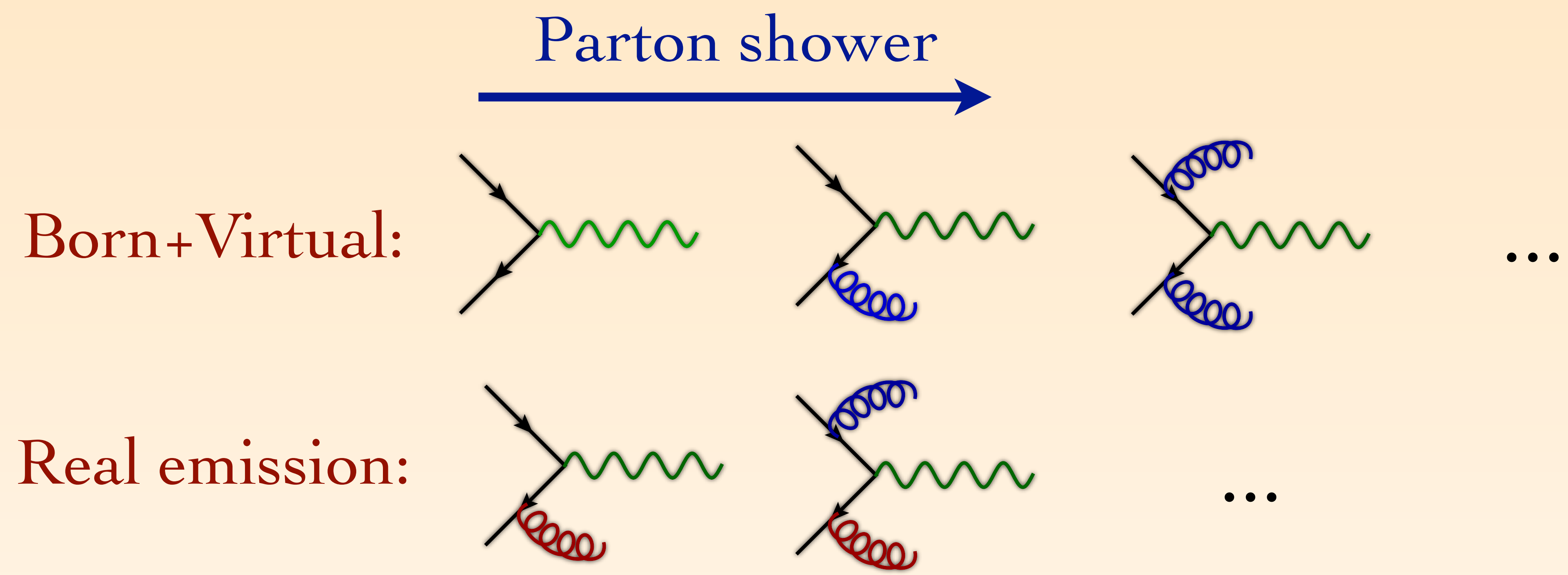

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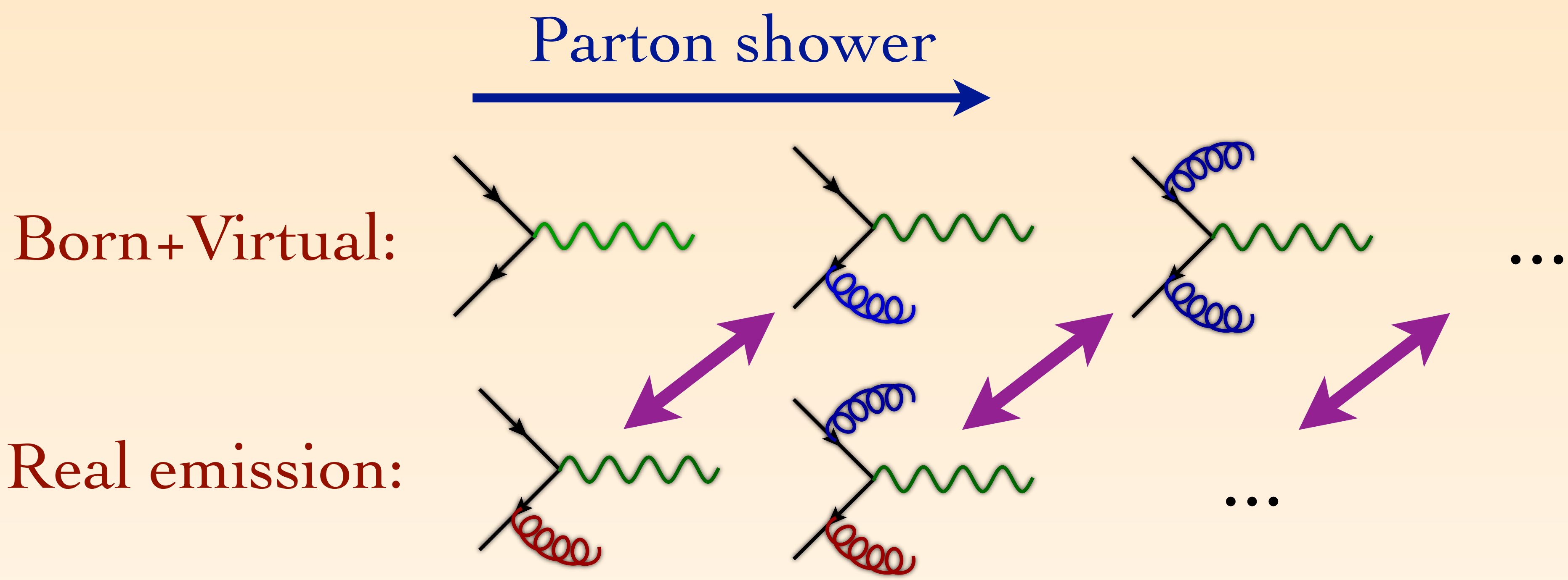

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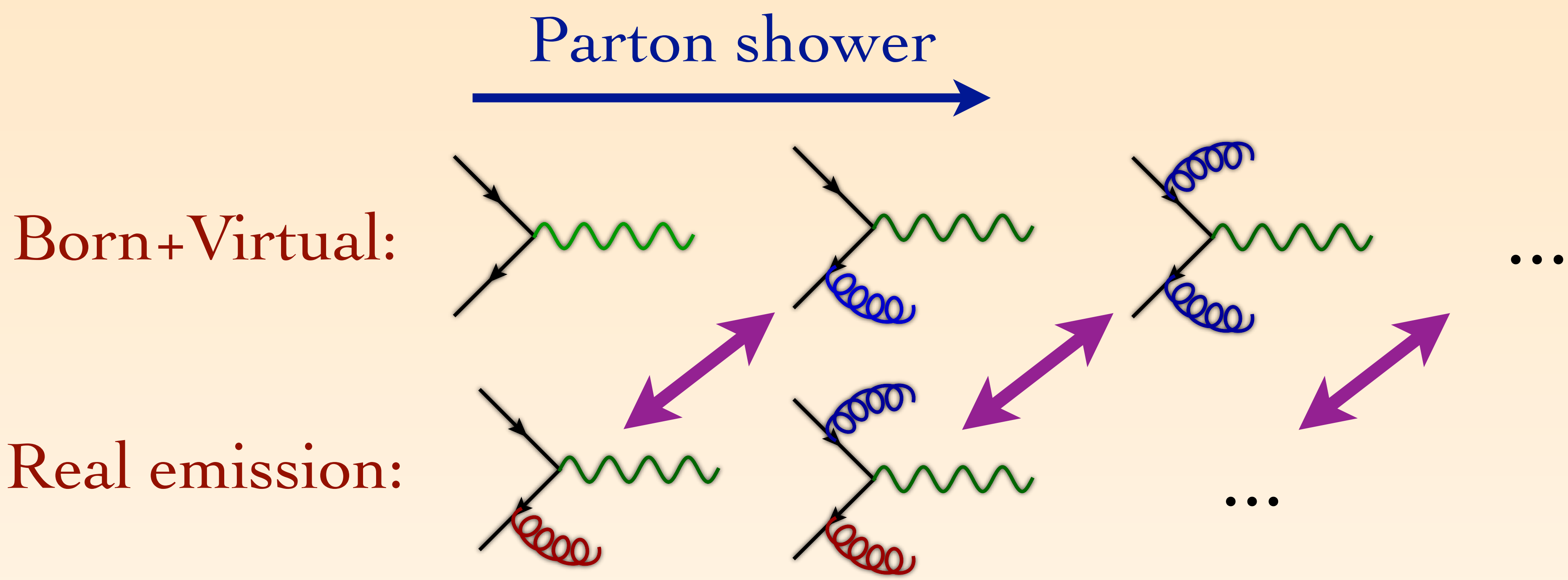


- There is double counting between the real emission matrix elements and the parton shower: the extra radiation can come from the matrix elements or the parton shower
- There is also an overlap between the virtual corrections and the Sudakov suppression in the zero-emission probability

Double counting in virtual/Sudakov

- The Sudakov factor Δ (which is responsible for the resummation of all the radiation in the shower) is the no-emission probability
- It's defined to be Δ = 1 - *P*, where *P* is the probability for a branching to occur
- By using this conservation of probability in this way, Δ contains contributions from the virtual corrections implicitly
- Because at NLO the virtual corrections are already included via explicit matrix elements, Δ is double counting with the virtual corrections
- In fact, because the shower is unitary, what we are double counting in the real emission corrections is exactly equal to what we are double counting in the virtual corrections (but with opposite sign)!

Avoiding double counting

- There are two methods to circumvent this double counting
  - MC@NLO (Frixione & Webber)
  - POWHEG (Nason)

MC@NLO procedure

Frixione & Webber (2002)

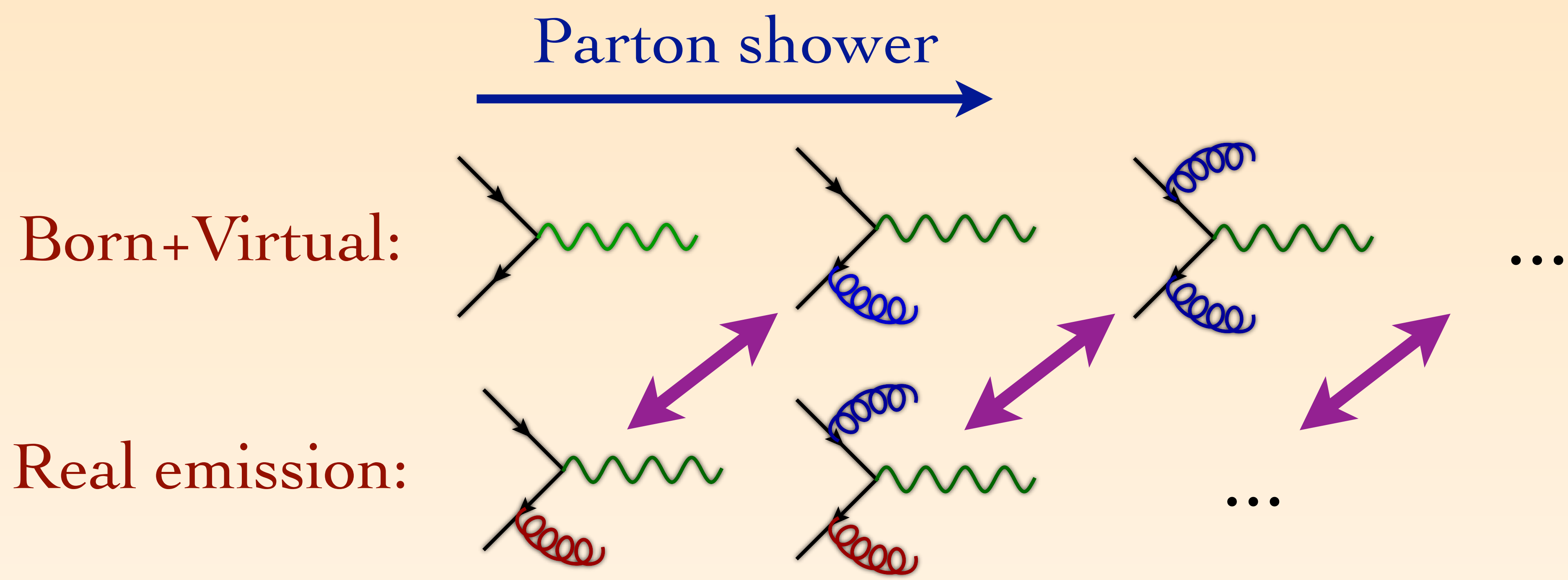
- To remove the double counting, we can add and subtract the same term to the m and m+1 body configurations

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m \left(B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC\right)\right] I_{\mathrm{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1} (R - MC)\right] I_{\mathrm{MC}}^{(m+1)}(O)$$

Where the *MC* are defined to be the contribution of the parton shower to get from the m body Born final state to the m+1 body real emission final state

MC@NLO PROCEDURE



$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m \left(B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC \right) \right] I_{\mathrm{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1} \left(R - MC \right) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

- Double counting is explicitly removed by including the "shower subtraction terms"

MC@NLO properties

- Good features of including the subtraction counter terms

  1. **Double counting avoided**: The rate expanded at NLO coincides with the total NLO cross section
  2. **Smooth matching**: MC@NLO coincides (in shape) with the parton shower in the soft/collinear region, while it agrees with the NLO in the hard region
  3. **Stability**: weights associated to different multiplicities are separately finite. The *MC* term has the same infrared behavior as the real emission (there is a subtlety for the soft divergence)

- Not so nice feature (for the developer):

  1. **Parton shower dependence**: the form of the *MC* terms depends on what the parton shower does exactly. Need special subtraction terms for each parton shower to which we want to match

Double counting avoided

$$\frac{d\sigma_{\text{NLOwPS}}}{dO} = \left[d\Phi_m (B + \int_{\text{loop}} V + \int d\Phi_1 MC) \right] I_{\text{MC}}^{(m)}(O)$$
$$+ \left[d\Phi_{m+1} (R - MC) \right] I_{\text{MC}}^{(m+1)}(O)$$

- Expanded at NLO

$$I_{\text{MC}}^{(m)}(O) dO = 1 - \int d\Phi_1 \frac{MC}{B} + d\Phi_1 \frac{MC}{B} + \ldots$$

$$d\sigma_{\text{NLOwPS}} = \left[d\Phi_m (B + \int_{\text{loop}} V + \int d\Phi_1 MC) \right] I_{\text{MC}}^{(m)}(O) dO$$
$$+ \left[d\Phi_{m+1} (R - MC) \right]$$
$$\simeq d\Phi_m (B + \int_{\text{loop}} V) + d\Phi_{m+1} R = d\sigma_{\text{NLO}}$$

Smooth matching

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m (B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC) \right] I_{\mathrm{MC}}^{(m)}(O)$$
$$+ \left[d\Phi_{m+1} (R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

- Smooth matching:
  - Soft/collinear region: $R \simeq MC \Rightarrow d\sigma_{\mathrm{MC@NLO}} \sim I_{\mathrm{MC}}^{(m)}(O)dO$
  - Hard region (shower effects suppressed), ie.

$$MC \simeq 0 \quad I_{\mathrm{MC}}^{(m)}(O) \simeq 0 \quad I_{\mathrm{MC}}^{(m+1)}(O) \simeq 1$$
$$\Rightarrow d\sigma_{\mathrm{MC@NLO}} \sim d\Phi_{m+1} R$$

Stability & unweighting

$$\frac{d\sigma_{\text{NLOwPS}}}{dO} = \left[d\Phi_m(B + \int_{\text{loop}} V + \int d\Phi_1 MC)\right] I_{\text{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1}(R - MC)\right] I_{\text{MC}}^{(m+1)}(O)$$

- The *MC* subtraction terms are defined to be what the shower does to get from the *m* to the *m*+1 body matrix elements. Therefore the cancellation of singularities is exact in the (*R* - *MC*) term: there is no mapping of the phase-space in going from events to counter events as we saw in the FKS subtraction

- The integral is bounded all over phase-space; we can therefore generate **unweighted events**!
  - "S-events" (which have *m* body kinematics)
  - "H-events" (which have *m*+1 body kinematics)

Negative weights

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m \left(B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC\right)\right] I_{\mathrm{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1} (R - MC)\right] I_{\mathrm{MC}}^{(m+1)}(O)$$

- We generate events for the two terms between the square brackets (S- and H-events) separately
- There is no guarantee that these contributions are separately positive (even though predictions for infra-red safe observables should always be positive!)
- Therefore, when we do event unweighting we can only unweight the events **up to a sign**. These signs should be taken into account when doing a physics analysis (i.e. making plots etc.)
- The events are only physical when they are showered

POWHEG

Nason (2004)

- Consider the probability of the first emission of a leg (inclusive over later emissions)

$$d\sigma = d\sigma_m d\Phi_m \Big[\Delta(Q^2,Q_0^2) + \Delta(Q^2,t)\sum_{bc} dz \frac{dt}{t}\frac{d\phi}{2\pi}\frac{\alpha_{\rm S}}{2\pi} P_{a\to bc}(z)\Big]$$

- In the notation used here, this is equivalent to

$$d\sigma = d\Phi_m\, B\Big[\Delta(Q^2,Q_0^2) + \Delta(Q^2,t) d\Phi_{(+1)}\frac{MC}{B}\Big]$$

- One could try to get NLO accuracy by replacing B with the NLO rate (integrated over the extra phase-space)

$$B \to B + V + \int d\Phi_{(+1)}\, R$$

- This naive definition is not correct: the radiation is still described only at leading logarithmic accuracy, which is not correct for hard emissions.

POWHEG

- This is double counting.
  To see this, expand the equation up to the first emission
  $$d\Phi_B \left[B + V + \int d\Phi_{(+1)} R\right] \left[1 - \int d\Phi_{(+1)} \frac{MC}{B} + d\Phi_{(+1)} \frac{MC}{B}\right]$$
  which is not equal to the NLO

- In order to avoid double counting, one should replace the definition of the Sudakov form factor with the following:
  $$\Delta(Q^2, Q_0^2) = \exp\left[-\int_{Q_0^2}^{Q^2} d\Phi_{(+1)} \frac{MC}{B}\right] \to \tilde{\Delta}(Q^2, Q_0^2) = \exp\left[-\int_{Q_0^2}^{Q^2} d\Phi_{(+1)} \frac{R}{B}\right]$$
  corresponding to a modified differential branching probability
  $$d\tilde{p} = d\Phi_{(+1)} R/B$$

- Therefore we find for the POWHEG differential cross section
  $$d\sigma_{\text{POWHEG}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R\right] \left[\tilde{\Delta}(Q^2, Q_0^2) + \tilde{\Delta}(Q^2, t)\; d\Phi_{(+1)} \frac{R}{B}\right]$$

Properties

$$d\sigma_{\text{POWHEG}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R\right] \left[\tilde{\Delta}(Q^2, Q_0^2) + \tilde{\Delta}(Q^2, t)\ d\Phi_{(+1)} \frac{R}{B}\right]$$

- The term in the square brackets integrates to one (integrated over the extra parton phase-space between scales Q_0^2 and Q^2)
  (this can also be understood as unitarity of the shower below scale t)
  POWHEG cross section is normalized to the NLO

- Expand up to the first-emission level:

$$d\sigma_{\text{POWHEG}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R\right] \left[1 - \int d\Phi_{(+1)} \frac{R}{B} + d\Phi_{(+1)} \frac{R}{B}\right] = d\sigma_{\text{NLO}}$$

  so double counting is avoided

- Its structure is identical an ordinary shower, with normalization rescaled by a global K-factor and a different Sudakov for the first emission: no negative weights are involved.

MC@NLO/POWHEG

The MC@NLO and POWHEG procedures can be cast in a single formula:

$$d\sigma_{\mathrm{NLO+PS}} = d\Phi_B \bar{B}^s(\Phi_B) \left[\Delta^s(p_\perp^{\mathrm{min}}) + d\Phi_{R|B} \frac{R^s(\Phi_R)}{B(\Phi_B)} \Delta^s(p_T(\Phi)) \right] + d\Phi_R R^f(\Phi_R)$$

where

$$\bar{B}^s(\Phi_B) = B(\Phi_B) + \left[V(\Phi_B) + \int d\Phi_{R|B} R^s(\Phi_{R|B}) \right]$$

and we have split the Real emission matrix elements in a singular and finite part:

$$R(\Phi_R) = R^s(\Phi_R) + R^f(\Phi_R)$$

The difference between MC@NLO and POWHEG is in the way the real matrix elements are split:

MC@NLO $\quad R^s(\Phi) = P(\Phi_{R|B})B(\Phi_B) = MC$

POWHEG $\quad R^s(\Phi) = F\,R(\Phi), \quad R^f(\Phi) = (1-F)R(\Phi)$

Need exact mapping $(\Phi_R, \Phi_B) \Rightarrow \Phi$ in MC subtraction term R^s

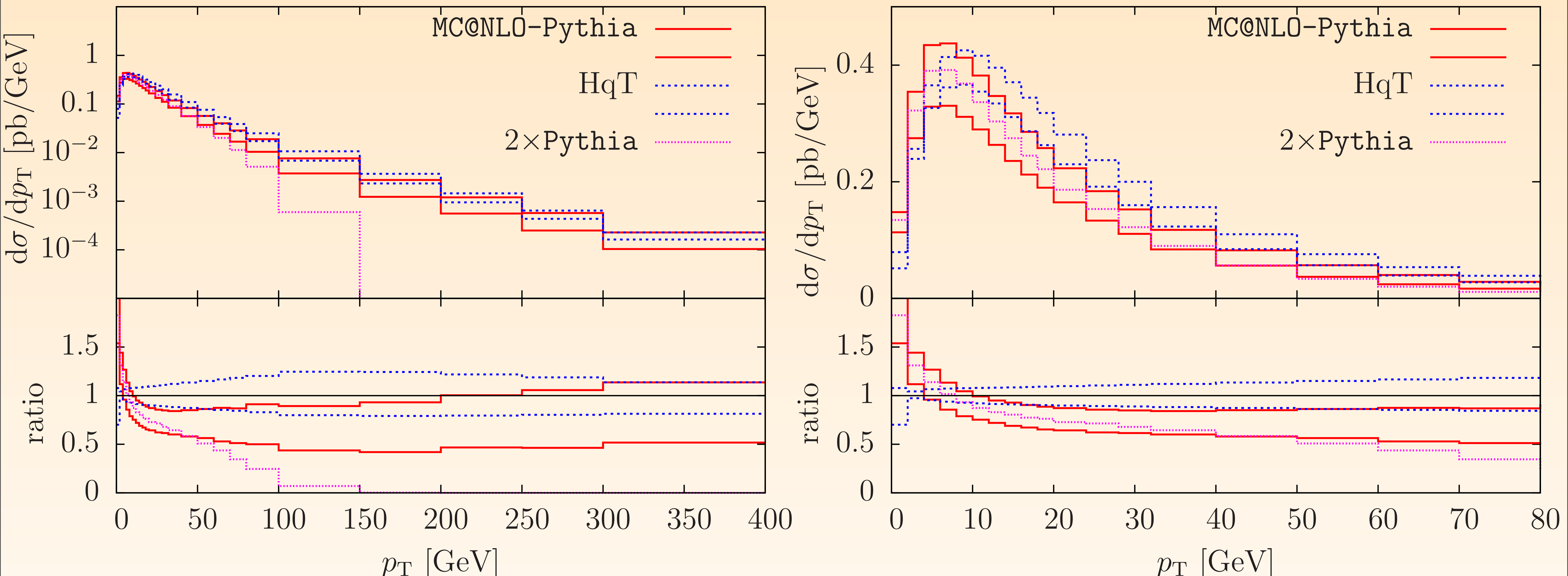
Default is $F = 1$: exponentiate the full real; it can be damped by hand

MC@NLO vs POWHEG

	MC@NLO	POWHEG
Parton showers are (usually) not exact in the soft limit: MC@NLO needs an artificial smoothing	☹	☺
MC@NLO does not exponentiate the non-singular part of the real emission amplitudes	☺	☹
MC@NLO does not require any tricks for treating Born zeros	☺	☹
POWHEG is independent from the parton shower (although, in general the shower should be a truncated vetoed)	☹	☺
POWHEG is (almost) no negative weighted events	☹	☺
Automation of the methods: http://amcatnlo.cern.ch http://powhegbox.mib.infn.it/ http://www.sherpa-mc.de	☺	☺

Results: Higgs production

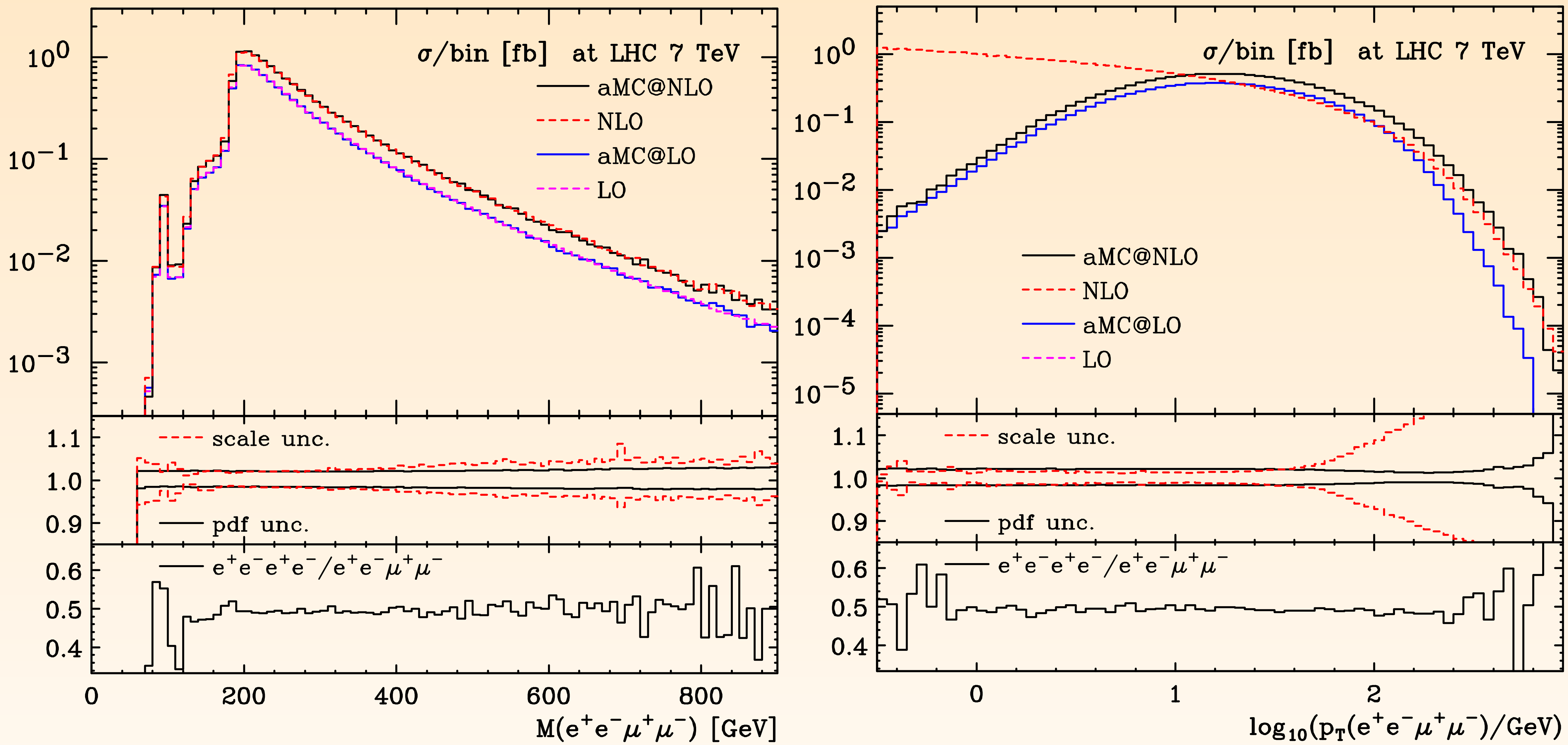
Nason & Webber, 2012



- Higgs transverse momentum. MC@NLO (with pythia) is in agreement with HqT (which is NNLO+NNLL) within uncertainty
- Pythia agrees with MC@NLO at low p_T (in shape, not in normalization), but does not describe the hard tail at all (without CKKW/MLM merging!)

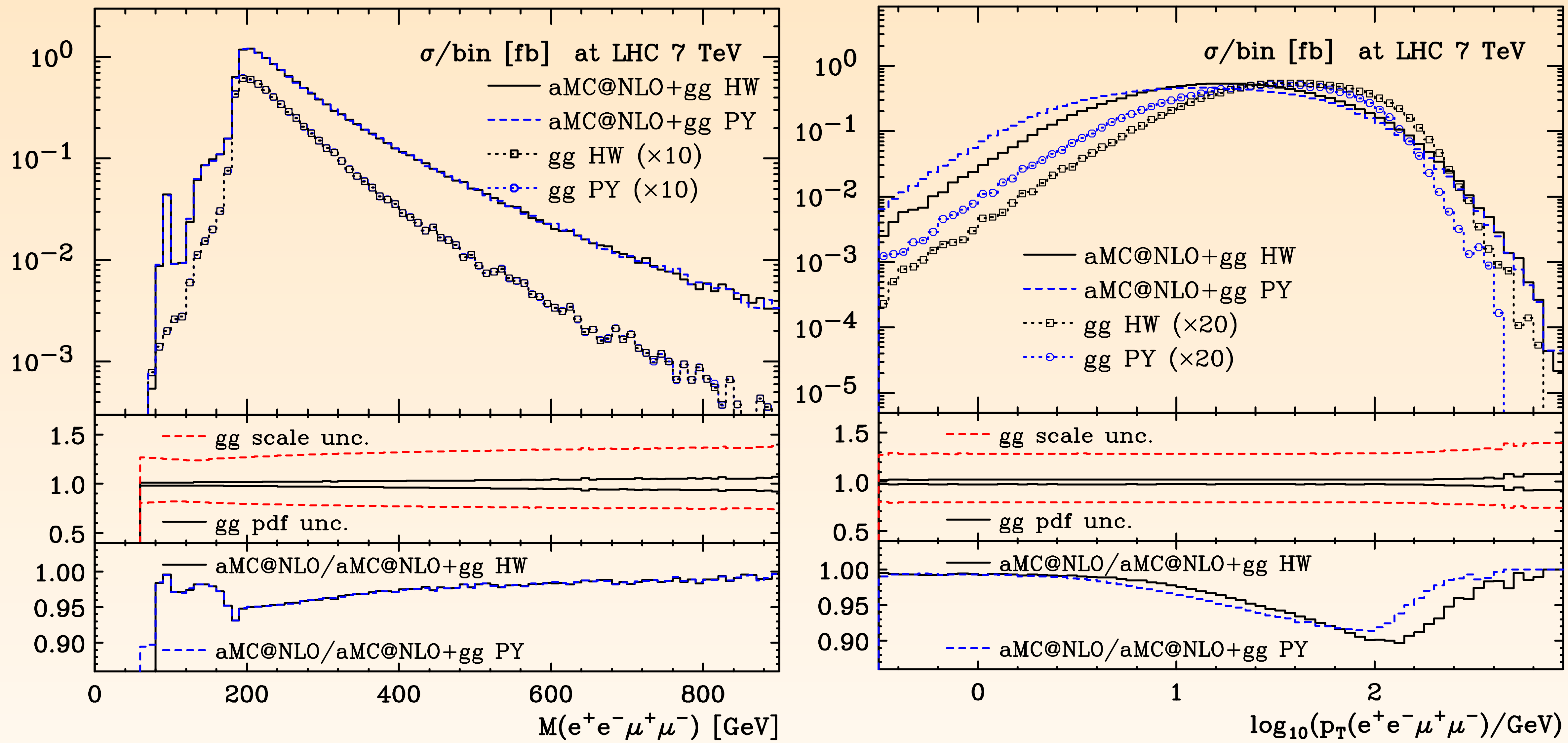
Four-lepton production

RF, Frixione, Hirschi, maltoni, Pittau & Torrielli (2011)



- 4-lepton invariant mass is almost insensitive to parton shower effects. 4-lepton transverse moment is extremely sensitive
- Including scale uncertainties

Four-lepton production

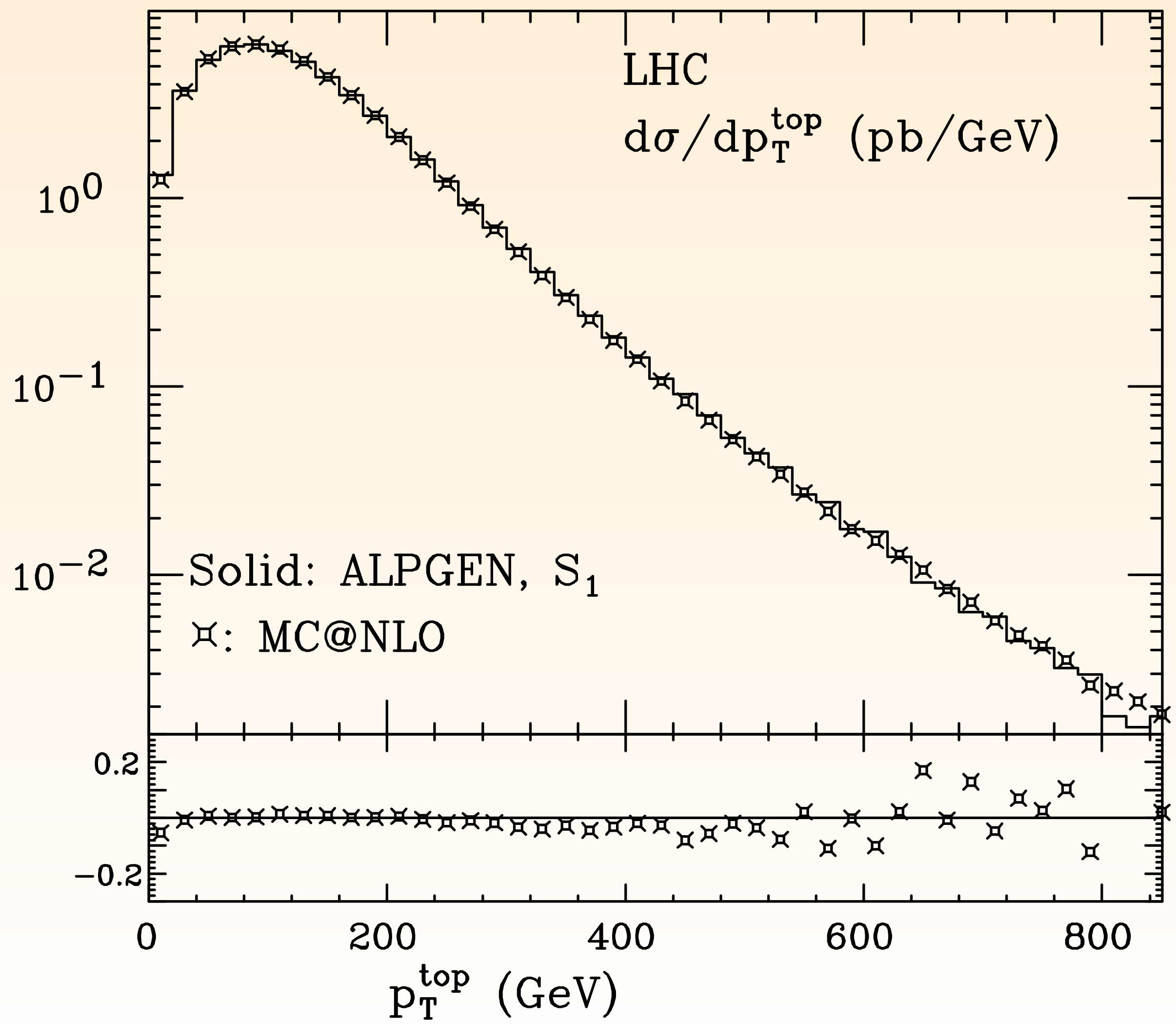


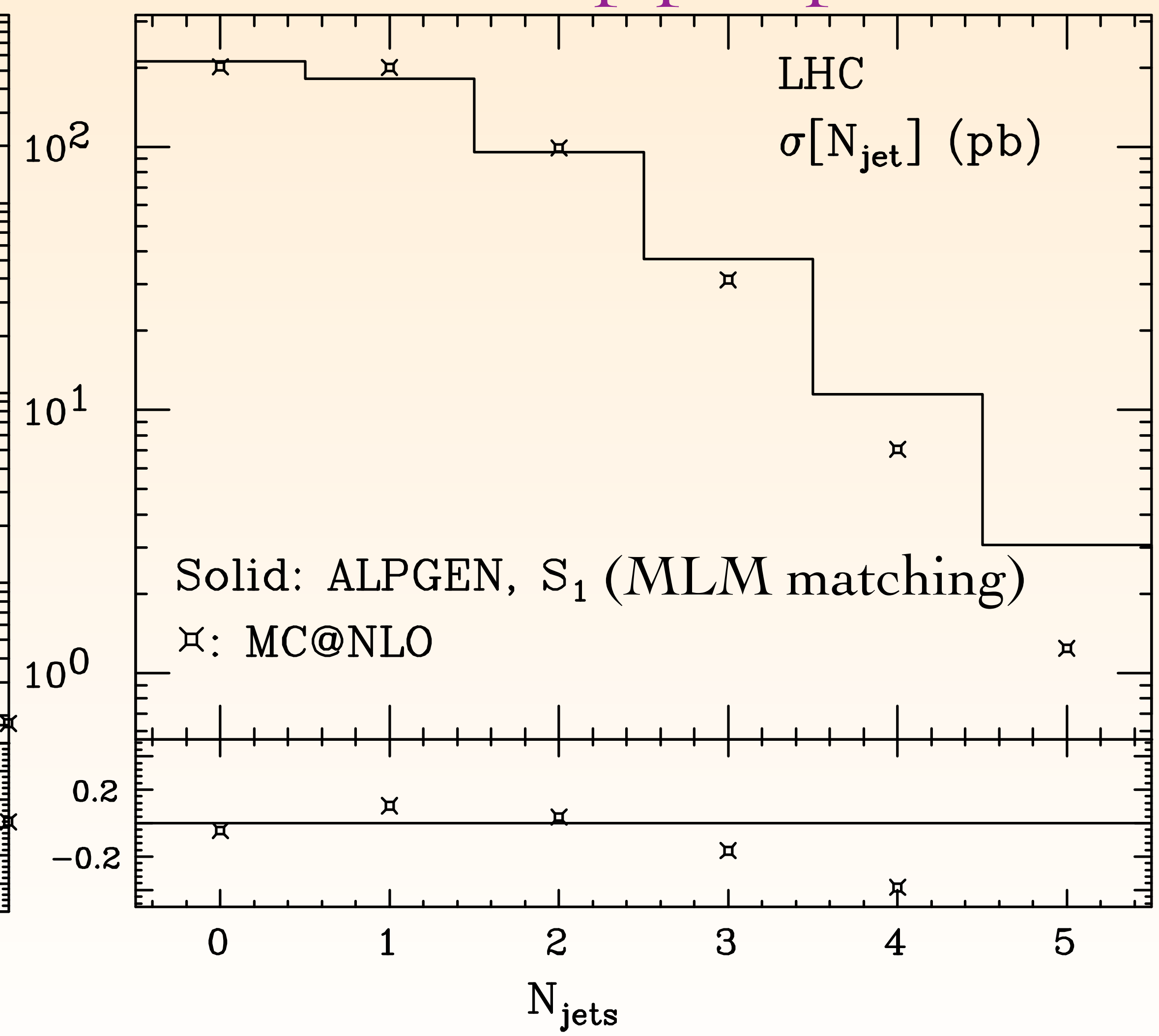
- Differences between Herwig (black) and Pythia (blue) showers large in the Sudakov suppressed region (much larger than the scale uncertainties)
- Contributions from gg initial state (formally NNLO) are of 5-10%

Is NLO+PS always the preferred method?

- It is the preferred method if the observable is described at NLO accuracy
- But there are many observables for which a given NLO+PS code has only zeroth order accuracy.

top pair production



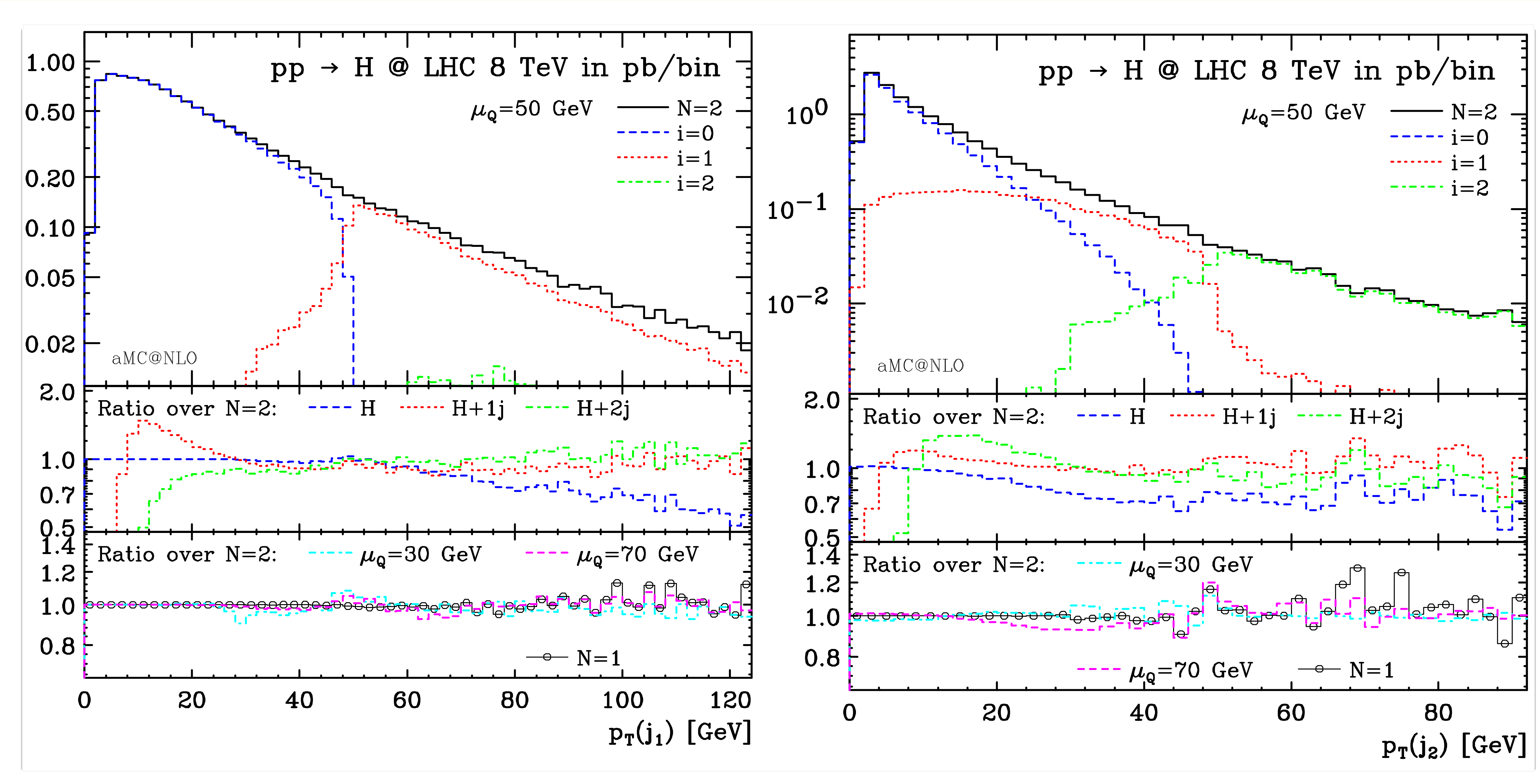


Summary

- We want to match NLO computations to parton showers to keep the good features of both approximations
  - In the MC@NLO method:
    by including the shower subtraction terms in our process we avoid double counting between NLO processes and parton showers
  - In the POWHEG method:
    apply an overall K-factor, and modify the (Sudakov of the) first emission to fill the hard region of phase-space according to the real-emission matrix elements
- First studies to combine NLO+PS matching with ME+PS merging have been made and result look very promising...

RF & Frixione, 2012

ME+PS MERGING AT NLO



- Hardest and 2nd hardest jets in Higgs production by gluon fusion
- Merged sample agrees with NLO in the regions of phase-space where it should; smooth in between; and nearly no dependence on the matching scale
- Not yet automated... work in progress