Parton Shower Monte Carlos

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2012 FR/MG school

Outline

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- Why parton showers

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- Final-state showers

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- Sudakov form factor and unitarity

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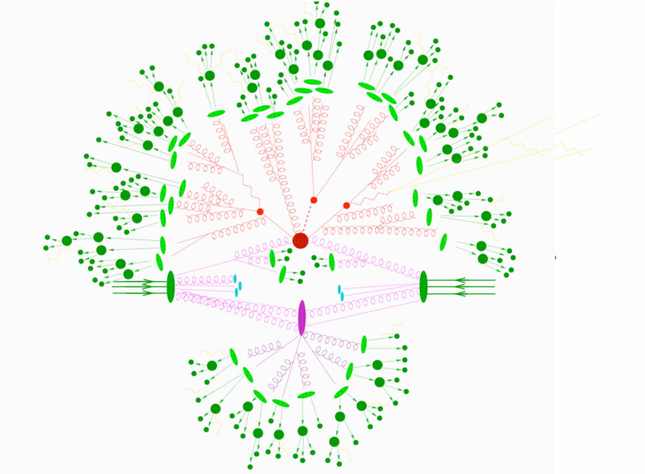

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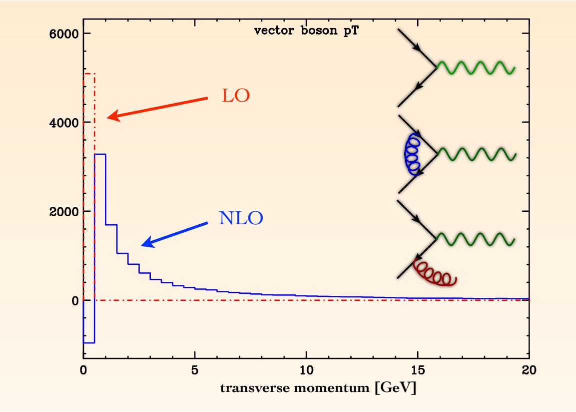
Parton showers offer a versatile tool to realise this.

Example of breakdown of fixed-order PT

$W\ p_T$ in $pp \to W$ @NLO



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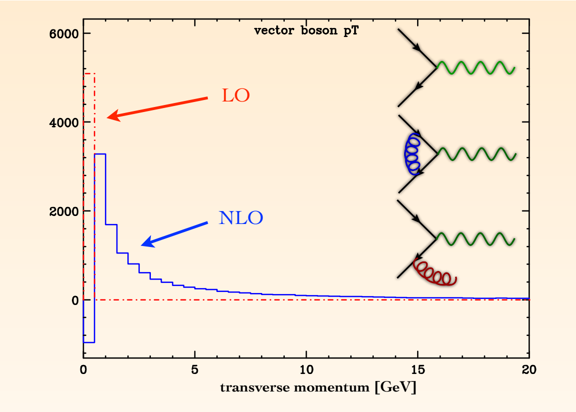

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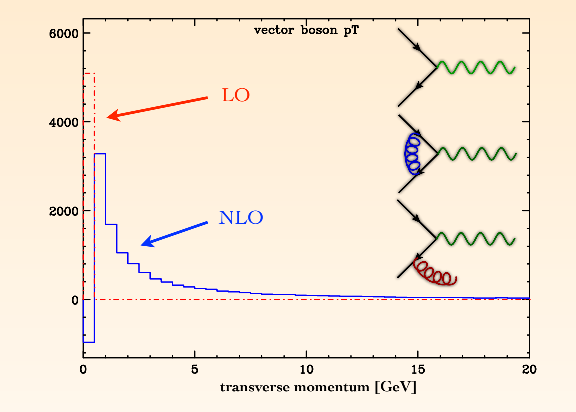

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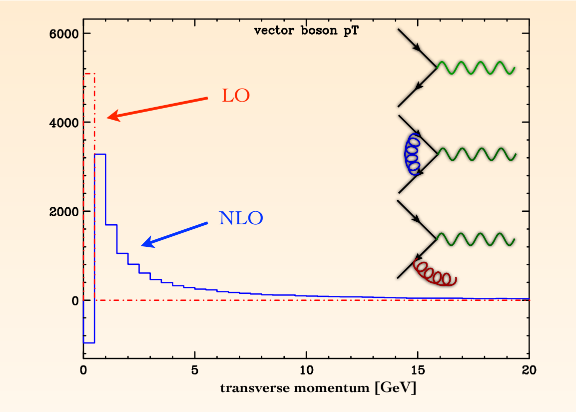

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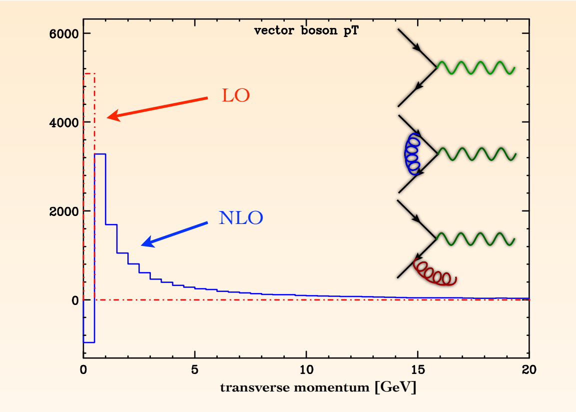

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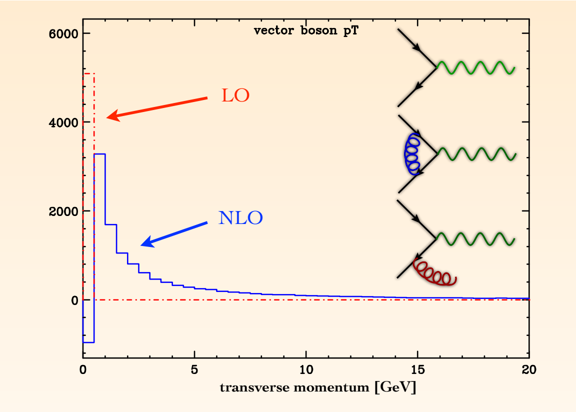

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- Small p_T of the extra radiation is the tricky region.
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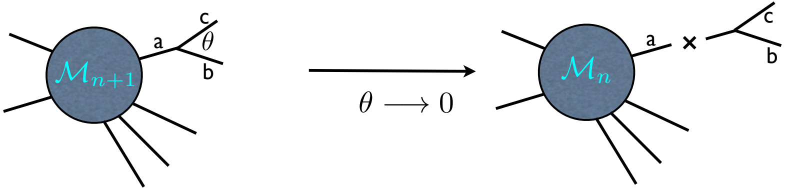
- This region is where to start formulating a complement to fixed-order PT.

Parton branching



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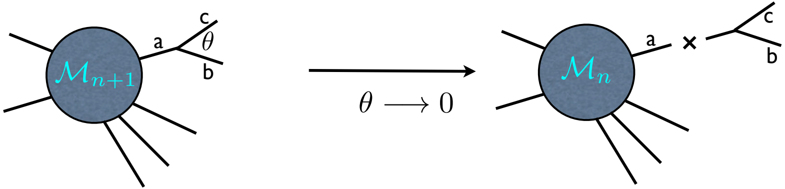

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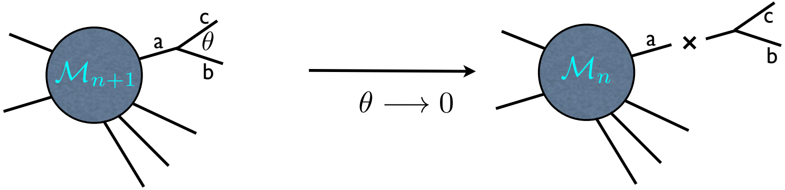

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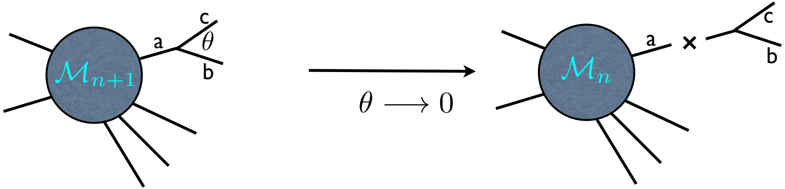

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- The whole process cross section should be writeable in this limit as the basic one times some branching probability.

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Case $a = q$, $b = q$ with relative energy z, $c = g$ with relative energy $1 - z$.

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Polarizations for small θ:

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u_a^+ &= \sqrt{E_a}(1,0,1,0), \qquad u_a^- = \sqrt{E_a}(0,1,0,-1),\\
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Amplitudes in the $t \to 0$ limit ($t = p_a^2 = 2E_bE_c(1-\cos\theta) \sim z(1-z)E_a^2\theta^2$):

$$\mathcal{M}_{n+1}(\pm,\pm,in) \sim \mathcal{M}_n \frac{g_S t^c}{t} \bar{u}_b^\pm \gamma^\mu u_a^\pm \epsilon_c^{in} \sim -i\mathcal{M}_n \frac{g_S t^c}{\sqrt{t}} \frac{1-z}{\sqrt{1-z}},$$

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Phase space: $d\Phi_{n+1} = d\Phi_n \frac{dz\, dt\, d\phi}{4(2\pi)^3}$.

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Unpolarized cross section (up to terms regular as $t \to 0$):

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d\sigma_{n+1} \sim d\sigma_n \frac{dt}{t} dz \frac{\alpha_S}{2\pi} C_F \frac{1+z^2}{1-z}.
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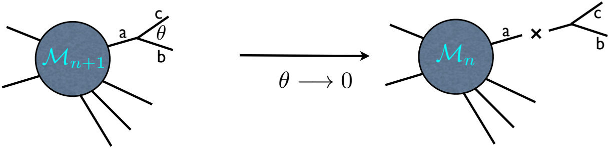
Collinear factorization



Analogously happens for $g \to gg$ and $g \to q\bar{q}$: cross section factorization in the collinear limit:

$$d\sigma_{n+1} \sim d\sigma_n \frac{dt}{t} dz \frac{\alpha_S}{2\pi} P_{a\to bc}(z).$$

Universal: $P_{a\to bc}(z)$ just depends on parton identities and energy fraction, not on $\mathcal{M}_n$. It is a sort of "branching probability".

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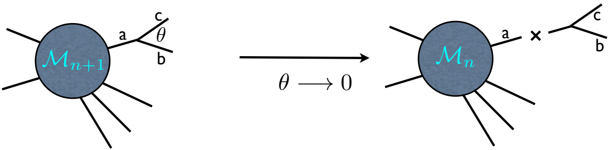

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$$P_{g\to qq}(z) \quad = \quad T_R\left[z^2 + (1-z)^2\right], \qquad P_{q\to qg}(z) = C_F\left[\frac{1+z^2}{1-z}\right],$$

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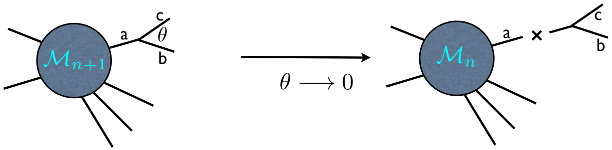

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Comments. 1) Soft singularity as emitted gluon goes soft.
2) Gluons radiate the most.

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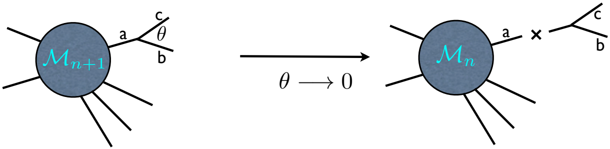

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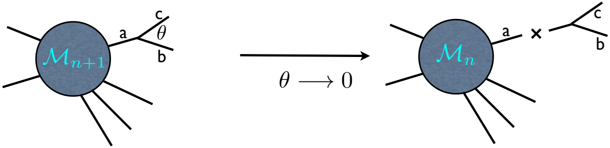

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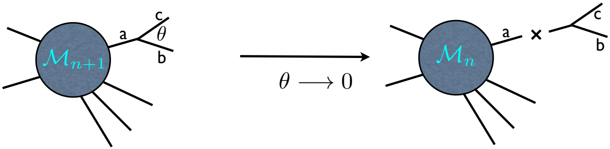

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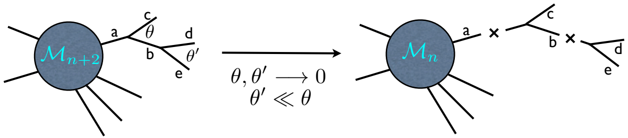
  It represents the momentum sharing between b and c and tends to 1 in as c goes soft.

Multiple emission



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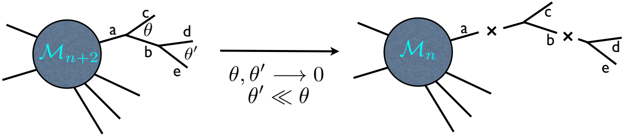

Now consider $\mathcal{M}_{n+1}$ as the new core process and use the same recipe to get the dominant collinear contribution to the $n+2$-body cross section: add a new branching at angle $\theta' \ll \theta$:

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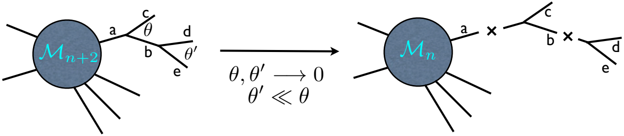

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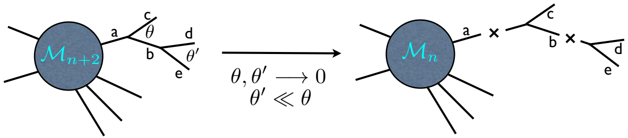

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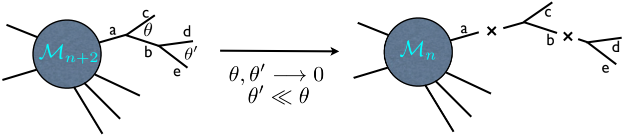
- Can be iterated for an arbitrary number of emissions.
- The recipe to get the leading collinear singularity is an iterative sequence of emissions with no memory of the past history of the system, so a Markov chain.

Multiple emission



Now consider $\mathcal{M}_{n+1}$ as the new core process and use the same recipe to get the dominant collinear contribution to the $n+2$-body cross section: add a new branching at angle $\theta' \ll \theta$:

$$\begin{aligned} d\sigma_{n+2} &\sim d\sigma_{n+1}\frac{dt'}{t'}dz'\frac{\alpha_S}{2\pi}P_{b\to de}(z') \\ &\sim d\sigma_n\frac{dt}{t}dz\frac{\alpha_S}{2\pi}P_{a\to bc}(z)\frac{dt'}{t'}dz'\frac{\alpha_S}{2\pi}P_{b\to de}(z'). \end{aligned}$$

- Can be iterated for an arbitrary number of emissions.
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- Process independence (no reference to $d\sigma_n$).

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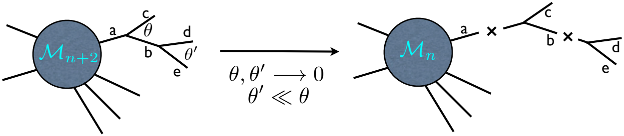

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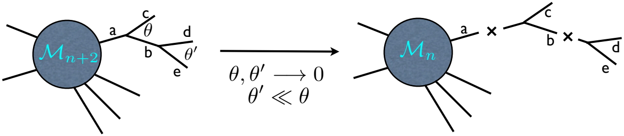

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$$\sigma_{n+k} \propto \sigma_n \, \alpha_S^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^{t} \frac{dt'}{t'} \ldots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left(\frac{\alpha_S}{2\pi}\right)^k \log^k(Q^2/Q_0^2),$$

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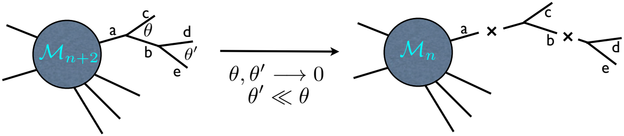

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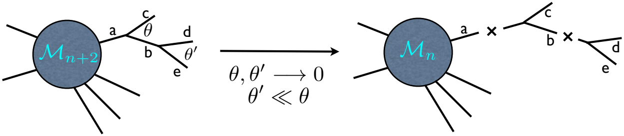

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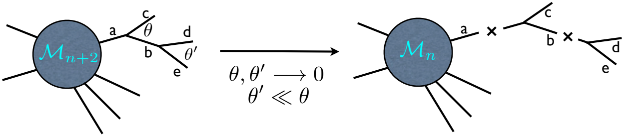

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- Now clear why fixed-order PT breaks down at small p_T: effective coupling is $\alpha_S \log(Q^2/Q_0^2)$, not just α_S.

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- Nevertheless, it captures the leading singularities, so it gives the amazing possibility of describing an arbitrary number of emissions.

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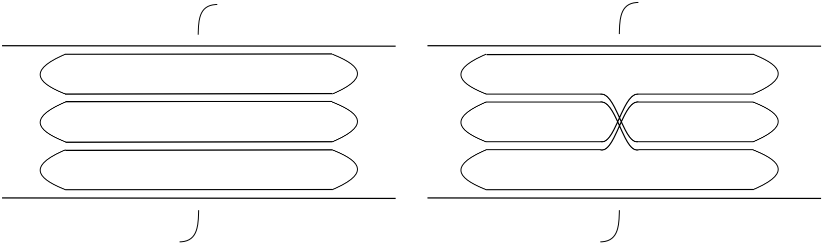

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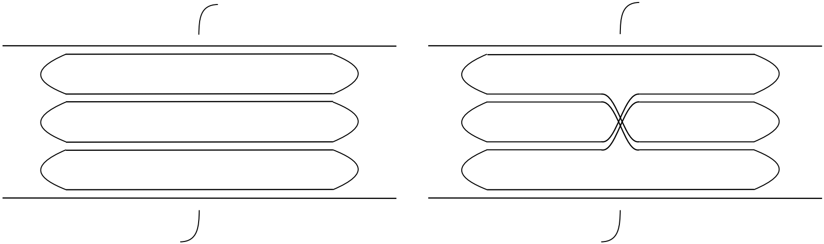

- In the picture: interference (left) suppressed by N_c^2 wrt amplitude squared (right).
- Absence of interference in the emission chain implies that the colour flow in the parton shower is correct only for $N_c \to \infty$.

Emission probability and Sudakov form factor

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- Properties

$$\frac{d\Delta(Q^2, t)}{dt} = \frac{dp(t)}{dt} \Delta(Q^2, t),$$

$$\Delta(t_a, t_b) = \Delta(t_a, t_c)\Delta(t_c, t_b) = \frac{\Delta(t_a, t_c)}{\Delta(t_b, t_c)}.$$

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- Analogy: in $e^+e^- \to$ jets the jet separation plays the role of the resolution scale Q_0. Unitarity is implemented by $\sigma_{\text{NLO}} = \sigma_2 + \sigma_3 =$ finite, and one can define probabilities for jet multiplicity m as $\sigma_m/\sigma_{\text{NLO}}$.

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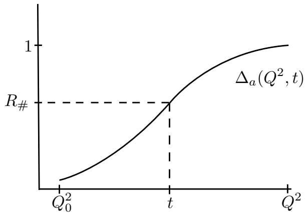

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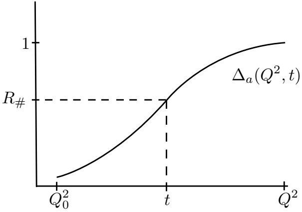

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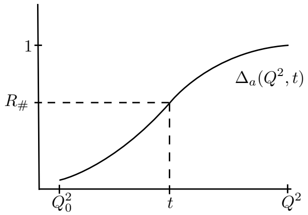

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  If only one possible branching, define

$$I(z) \equiv \int^z dz' \frac{\alpha_S}{2\pi} P_{a\to bc}(z')$$

  and solve $I(z)/I(z_{max}) = R'_\#$.
  For many possible branchings, pick one at random according to $I_i(z_{max})/\sum_j I_j(z_{max})$ and then extract z.

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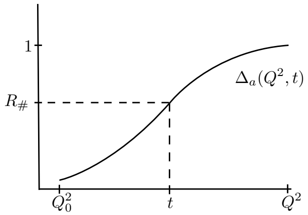

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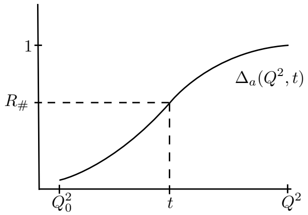

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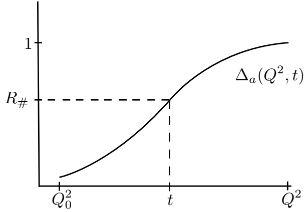

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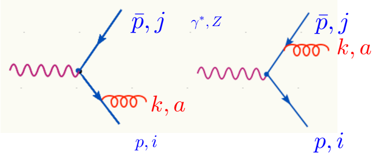
- Put partons on shell and hadronize (see later).

Including subleading logs: angular ordering

Soft gluon limit:



$$d\sigma_{n+1} = d\sigma_n C_F \frac{\alpha_S}{2\pi} \frac{dz}{z} \frac{d\phi}{2\pi} d\cos\theta \frac{\zeta_{ij}}{\zeta_{ik}\zeta_{jk}},$$

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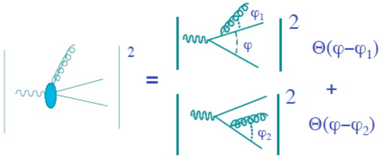

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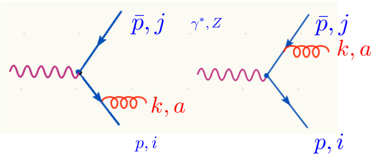

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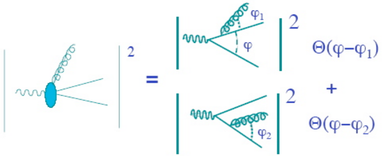

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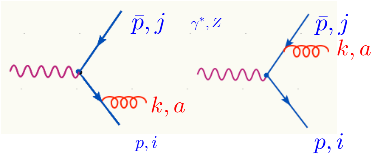

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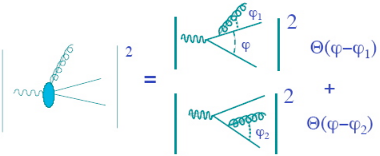

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- This can be reiterated to further gluon radiation: emission angle gets smaller and smaller.

Angular ordering in a parton shower

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- Indeed one can show that the angular-ordered algorithm reproduces the leading and next-to leading collinear logarithms in the soft gluon limit.

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- For initial-state radiation adopt instead backwards evolution: start from the hard subprocess, and evolve back to the incoming colliding hadrons.
- Use DGLAP equation to determine the parton evolution backwards in time.

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- In formulae:

$$
\begin{aligned}
df_b(z,t) &= \frac{dt}{t}\sum_{ac}\int_z^1 dz' \int_0^1 dw \frac{\alpha_S}{2\pi} f_a(z',t) P_{a\to bc}(w)\delta(z-wz') \\
&= \frac{dt}{t}\sum_{ac}\int_0^1 \frac{dw}{w}\frac{\alpha_S}{2\pi} f_a\left(\frac{z}{w},t\right) P_{a\to bc}(w).
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- Thus, initial-state radiation, from the hard process backwards in time, can be described in a way similar to final-state radiation, but with a different dp.
- Consequently the Sudakov form factor for initial-state radiation is

$$\hat{\Delta}(z, Q^2, t) = \exp\left[-\int_{|t|}^{Q^2} d\hat{p}(z,t')\right].$$

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- Many initial-state emissions evolve the scale t backwards in time, until the true parton inside the hadron is reached.

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- However the formulation of such models can be guided by some phenomenological considerations.

Hadronization: cluster model

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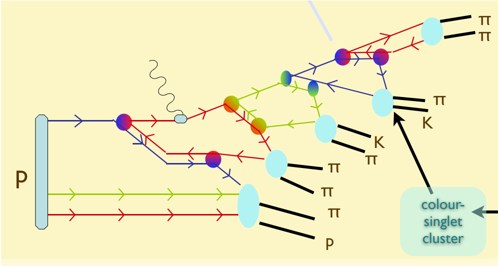
- Expecially in an angular-ordered shower colour partners are close in phase space: colour "preconfinement".
- Formation of small-mass colourless clusters to be decayed into physical hadrons.



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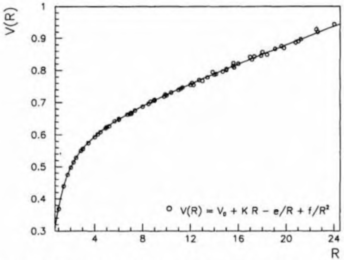

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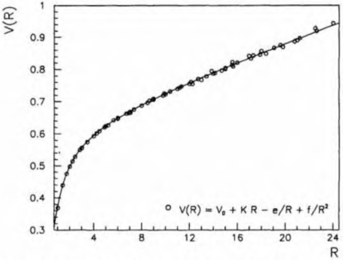

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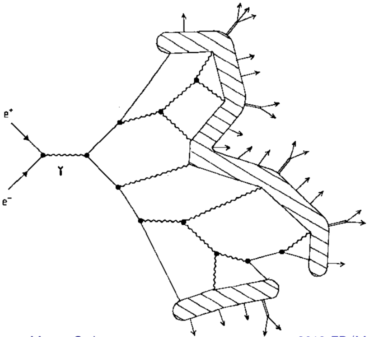
- This is modeled with a string with uniform tension (energy per unit length) k stretched between the q and the $\bar{q}$
- At a certain point it becomes energetically favorable to break the string in two by creating a new $q\bar{q}$ pair in the middle of the string.



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- The shower is developed from a Sudakov form factor

$$\Delta = \exp\left(- \int \frac{dt}{t} \int dz\ \alpha_S\ K_{mn,k} \right).$$

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Main Monte Carlos available on the market: SHERPA

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- Hadronization: cluster model.

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- For some simple $2 \to 2$ processes, the real emission matrix element ($d\sigma^1_{ME}$) is computed and compared with the first-emission parton shower cross section ($d\sigma^1_{MC}$).
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HERWIG: filling the dead-zones.

- The allowed region for the parton shower is kept limited, but in the dead zones radiation is generated according to the correct real-emission matrix-element distribution.

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- Matrix-element shower corrections available only for few very simple $2 \to 2$ processes.
- It corrects only for the first extra emission (superseded by MLM or CKKW merging).
- It is a tree-level method (superseded by MC@NLO).

Next lecturers will explain in detail!

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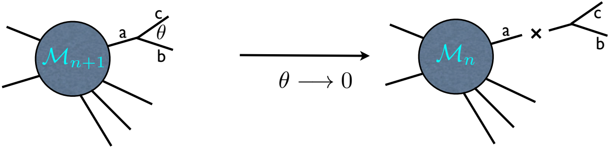
The nicest feature is that parton showers can be combined with PT.

Backup slides

Extra 1: collinear factorization



Cross section factorization in the collinear limit:

$$d\sigma_{n+1} \sim d\sigma_n \frac{dt}{t} dz \frac{\alpha_S}{2\pi} P_{a\to bc}(z).$$

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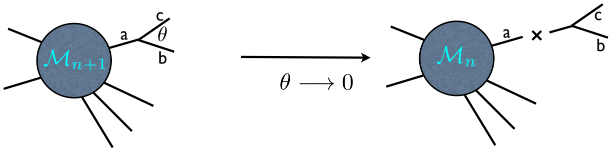

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- Example of $q \to qg$: quark helicity conserved, so $|\text{final spin} - \text{initial spin}| = 1$. The scattering happens in a p-wave, so it is suppressed as $t \to 0$.
- Indeed a factor $p_b \cdot p_c$ appears for all splittings at the numerator upon explicit computation.

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- Scale t has thus the role of evolution variable (as time in decays).

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- Azimuthal terms to be kept in mind if one wants $|\mathcal{M}_{n+1}|^2 d\Phi_{n+1}$ to represent the collinear limit of the real amplitude point by point.

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Each choice of argument for α_S equally acceptable at the leading-log. Remember

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- Take this into account by choosing $z(1-z)t \sim p_\perp^2$ as argument of the coupling. Indeed, the kernel $\alpha_S P_{a\to bc}(z)$ becomes

$$\begin{aligned} \alpha_S[z(1-z)t] P_{a\to bc}(z) &\sim \alpha_S(t)\left(1 - \alpha_S(t) b \log z(1-z)\right) P_{a\to bc}(z) \\ &= \alpha_S(t)\left(P_{a\to bc}(z) + \alpha_S(t) P'_{a\to bc}\right). \end{aligned}$$