- Introduction: Gauge invariant unstable particle
- Complex mass scheme at tree level
- CMS at NLO
- Generalization through EFT approach

Gauge invariant unstable particles

- Diagrams with unstable particles present in general an imaginary part in the Dyson-ressumed propagator:

$$P(p) = [p^2 - m_0^2 + Pi(p^2)]^{-1}$$

- The self energy, $\Pi(s)$, develops an imaginary part according to its virtuality; , in particular $\Pi(t < 0) = 0$.
- Mixing of different perturbative orders breaks gauge invariance. Fine cancellations spoiled, leading to enhanced violation of unitarity;
- *fixed width scheme*: $P(p) = [p^2 - M^2 + iM\Gamma]^{-1}$, also for $p^2 < 0$. Restores $U(1)_{em}$ current conservation but does not respect $SU(2) \times U(1)$ WI, not OK for VV scattering for example;
- **Complex mass scheme**, $M \to \sqrt{M^2 - iM\Gamma}$, completely restores gauge invariance at the Lagrangian level, at the cost of incorporating spurious imaginary part in other parameters, like the Weinberg angle: $c_w^2 = \frac{M_W^2 - iM_W\Gamma_W}{M_W^2 - iM_W\Gamma_W}$ and the Yukawas (besides the usual fixed width in propagators).

CMS at tree level

- For each non-zero width related to mass M, create a new complex variable $CM = \sqrt{M^2 - iM\Gamma}$ and compute all internal variables with the new parameters.
- in the terminal: mg5> set complex_mass_scheme True;
- **Parameters**
  - in order to maintain the precision of the calculation, it is recomended that the width is computed at one order further than the accuracy of the computation;
  - MG5 normally uses M_Z, G_F and α_{ew} as input parameter. By promoting M_Z to complex and computing $M_W(M_Z, G_F, \alpha_{ew})$, the resulting width is meaningless. *It is necessary to use the masses of the unstable particles as IP*.
  - At the moment there is a model, sm_mw, which does this (not desirable) $\rightarrow$ implementing a method to promote α_{ew} to complex, and inverting the equation to compute it from the value of M_W computed and Γ_W given as parameter.

- **Checking gauge invariance**
  - Usual $k_\mu M^\mu = 0$ check with processes with photons or gluons;
  - Feynman gauge implemented. In the terminal: mg5> set gauge Feynman
  - compare unitary and Feynman gauge automatically called when user does: mg5> check gauge <process>.

$\|A\|^2$ - |Feynman-unitary|/unitary	complex mass	fixed width
$e^+e^- \to u\bar{u}d\bar{d}$	1.5334067678e-15	1.2312200197e-09
$u\bar{u} \to u\bar{u}d\bar{d}$	2.0862057616e-16	2.7696013365e-10
$u\bar{u} \to b\bar{b}e^+\nu_e\mu^-\nu_\mu$ (real Yuk)	1.7934842084e-06	2.2832833007e-05
"(complex Yuk)	8.5986902303e-16	2.2832833007e-05

$\sigma(pb)$ for $gg \to b\bar{b}e^+\nu_e\mu^-\bar{\nu_\mu}$			
gauge - scheme	complex-mass	fix width	no width
feynman	1.796e-05 $\pm$ 2.3e-08	1.787e-05 $\pm$ 2.5e-08	
unitary	1.792e-05 $\pm$ 2.1e-08	1.778e-05 $\pm$ 2.4e-08	1.810e-05 $\pm$ 2.4e-08

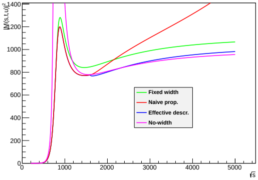
CMS at NLO

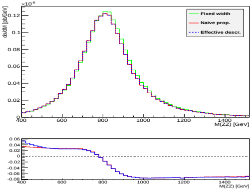
- Renormalization in the pole scheme, e.g. $s_H = \mu^2 - i\mu\gamma$
- counter terms:
  $\Pi^R_{HH}(s_H) = 0$, $\Pi'^R_{HH}(s_H) = 0$
  $\Pi^R_{HH}(s) = \Pi_{HH}(s) - \delta s_H + (s - s_H)\delta Z$
  $\delta s_H = \Pi_{HH}(s_H)$, $\delta Z = -\Pi'_{HH}(s_H)$.
- IPS must be set correctly (use mass as IP). γ must be given computed with accuracy $\mathcal{O}(\alpha^2)$
- Simple check works. In specific PS point and process, in CMS the IR poles cancel, while in fixed width this is not the case (needs more robust test);
- For eventual EW loops: ghosts are implemented in the Feynman gauge at tree level, but at the moment they are "turned off" (since we are at tree level).

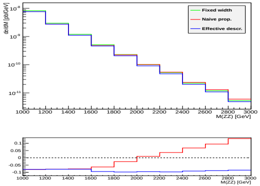
Generalization of CMS through EFT (with Cen Zhang and Fabio)

- Ideally one should have, $P(p^2) = [p^2 - m_R^2 + i\Pi(p^2)]^{-1}$. Resonance region better described, spurious term in the CMS is of order, $\mathcal{O}(\Gamma/M)$;
- For a heavy and broad resonance this is important, e.g 800 GeV scalar, $\Gamma \sim 300\ \mathrm{GeV}$;
- it is possible to include the running behavior of the self energy in the propagator through an EFT approach. Adding gauge invariant operators that reproduce the self energy:
  $O_{\Pi_1} = \phi^\dagger \Pi_1(-D^2)\phi \qquad \bar{O}_{\Pi_2} = \frac{1}{2v^2}\left(\phi^\dagger\phi - v^2\right)\Pi_2(-\partial^2)\left(\phi^\dagger\phi - v^2\right)$
  $\Pi(s) = \Pi_1(s) + \Pi_2(s)$

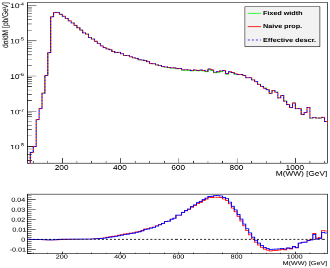
Vector Boson Scattering (e.g. $uc \to uczz$, $ZZ \to ZZ$)

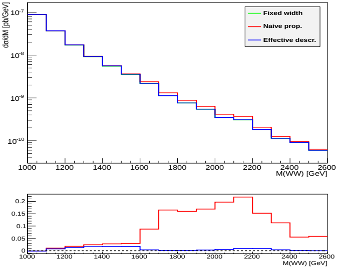






Similarly for $gg \to H$ and $H \to t\bar{t}$ through VBF





- equivalent to the CMS if $\Pi(s) = iM\Gamma$ constant;
- in principle, the method can be applied at NLO in analogy with the CMS, with appropriate renormalization.

Prospects

- Adjust correctly the parameters, both for LO and NLO - and complex renormalization;
- Have a robust set of checks also for NLO (cancelation of poles, gauge checks with photons, integral level validation);
- Produce results for $pp \to t\bar{t} \to W^+W^-b\bar{b} \to e^+\nu_e\mu^-\bar{\nu_\mu}$ at NLO, for massive bottoms in the complete spectrum. Important in particular when $t\bar{t}$ is off-shell, as a background.