Physics of Massive Neutrinos

Zhi-zhong Xing

(IHEP, Beijing)

A: Neutrino history and lepton family

B: Neutrino masses and flavor mixing

C: Description of neutrino oscillations

D: Implication of neutrino oscillations

E: θ_{13} hint and sterile neutrino puzzle

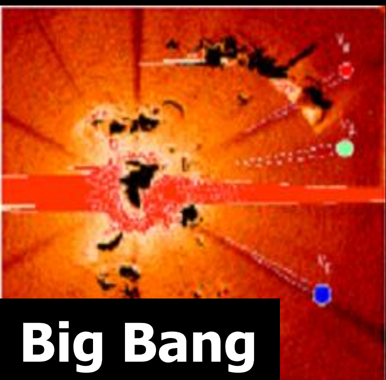
F: On TeV neutrino physics at the LHC

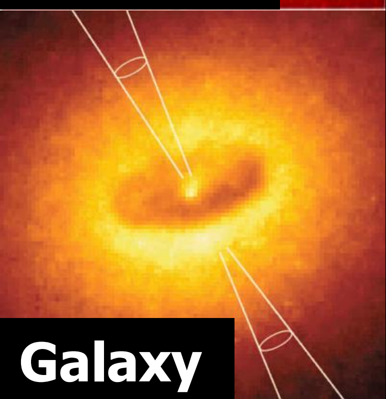
G: Extreme corners in the neutrino sky

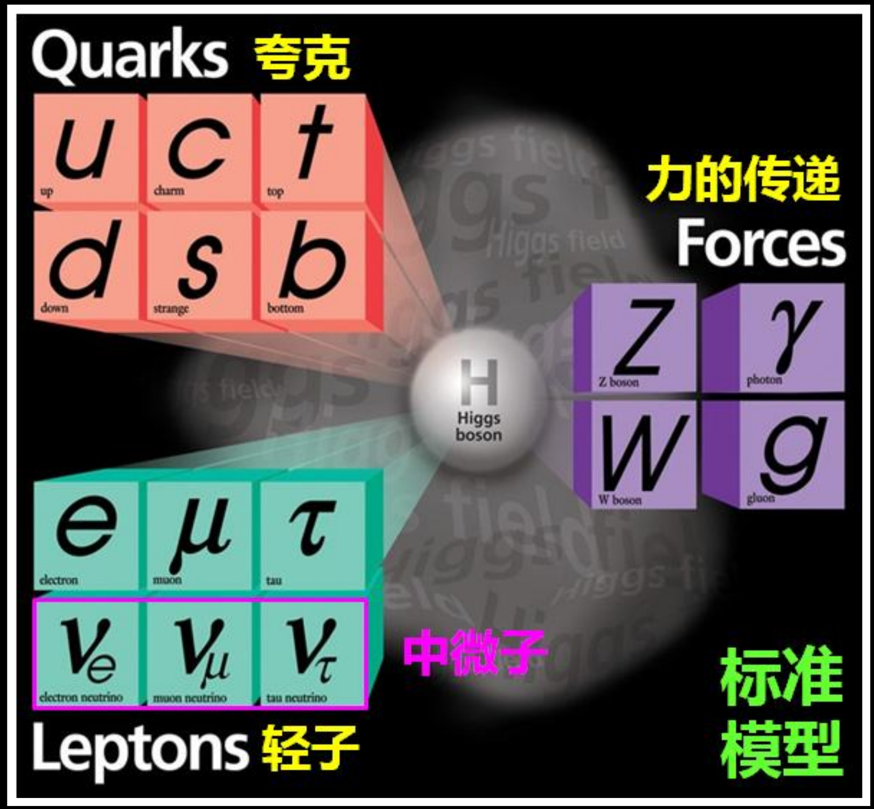
H: Cosmic baryon number asymmetry

@Lectures at CP3, UCL, Louvain-la-Neuve, Belgium, Jan 2012

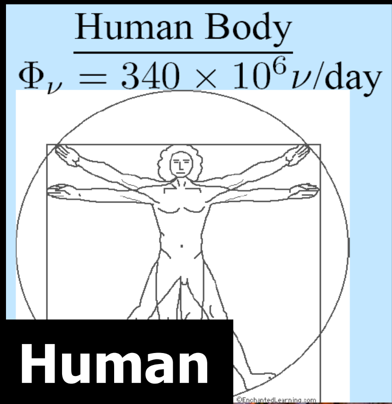
Neutrinos: how elusive they are?

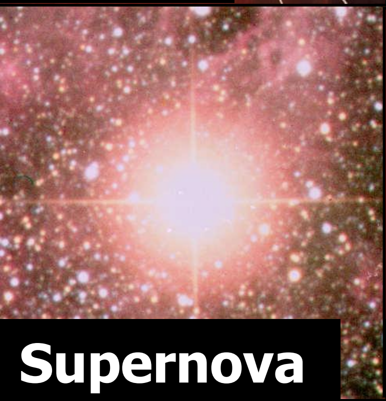




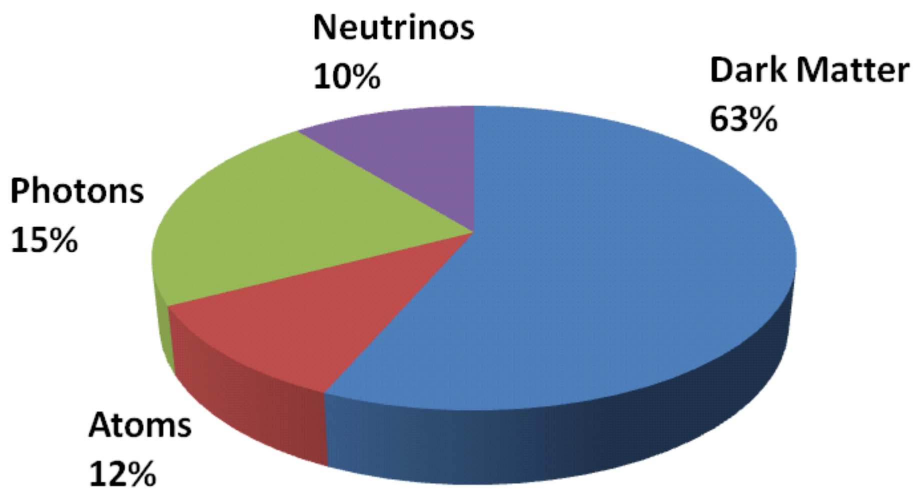


charge = 0
spin = ½
mass = 0
speed = c

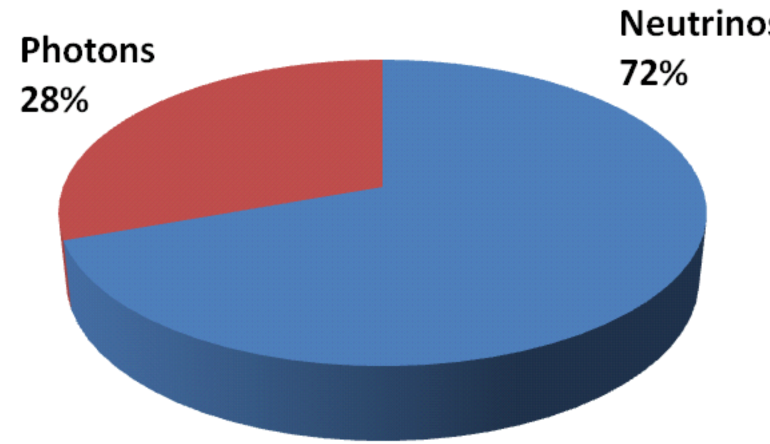




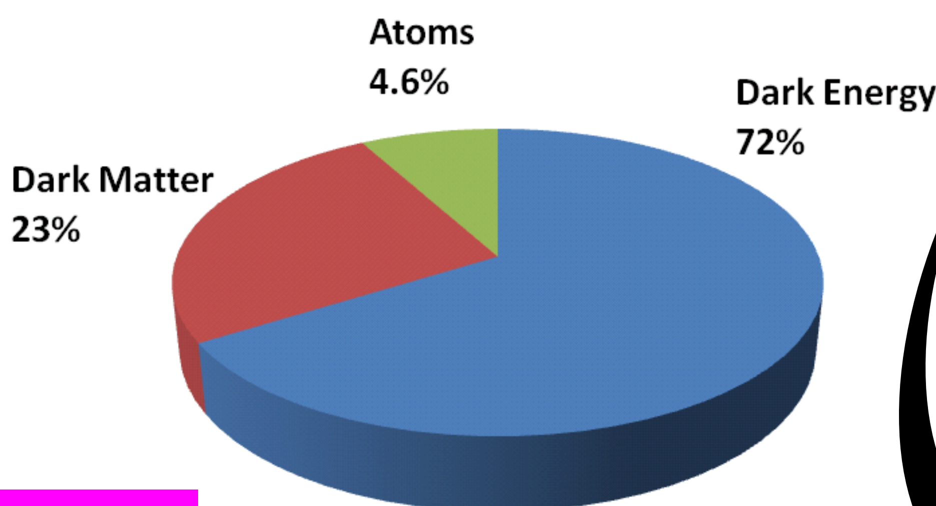
NEUTRINOS



photon decoupling
t = 380 000 years



neutrino decoupling
t = 1 second



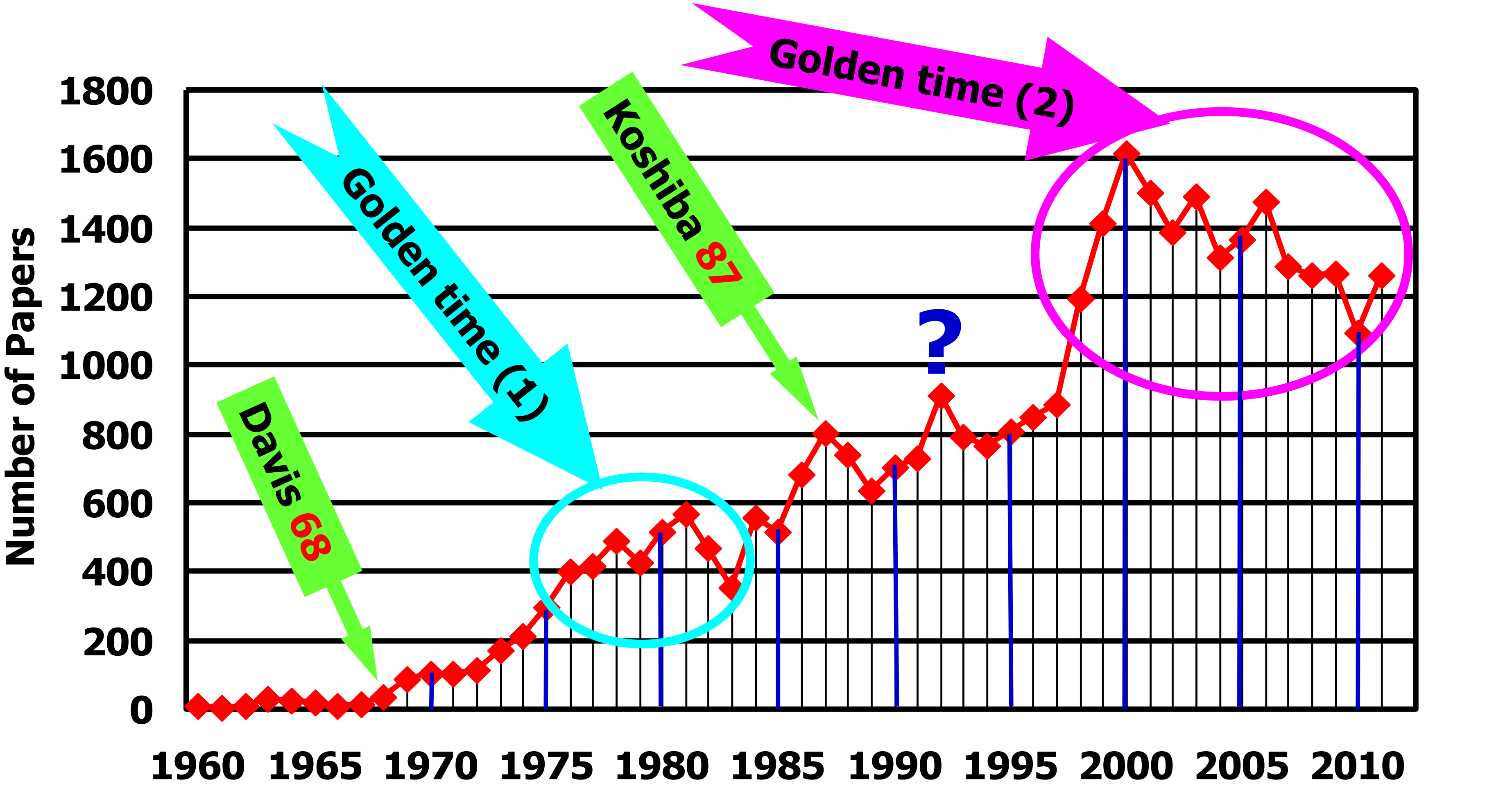
Today
t = 13.7 billion years

< 1%

A very important role in the evolution of the Universe

ν ν ν

Annual papers on neutrinos >1000



Open Questions

the absolute mass scale?

the mass hierarchy problem?

the flavor desert problem?

how small is θ_{13}?

leptonic CP violation?

the Majorana nature?

how many species? ...

cosmic ν background?

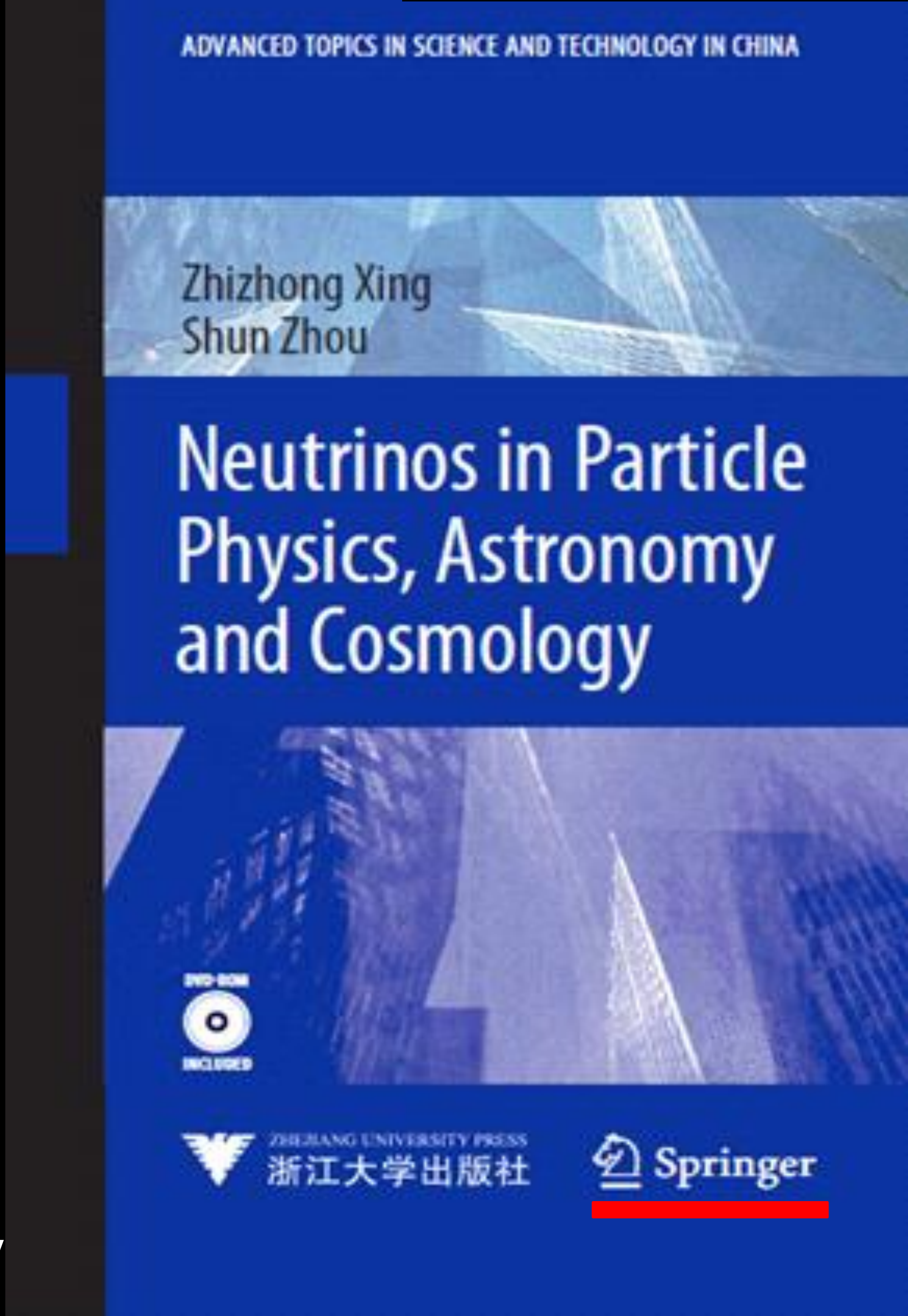
supernova & stellar ν's?

UHE cosmic ν's?

warm dark matter?

matter-antimatter asymmetry

2011 (440 pages)

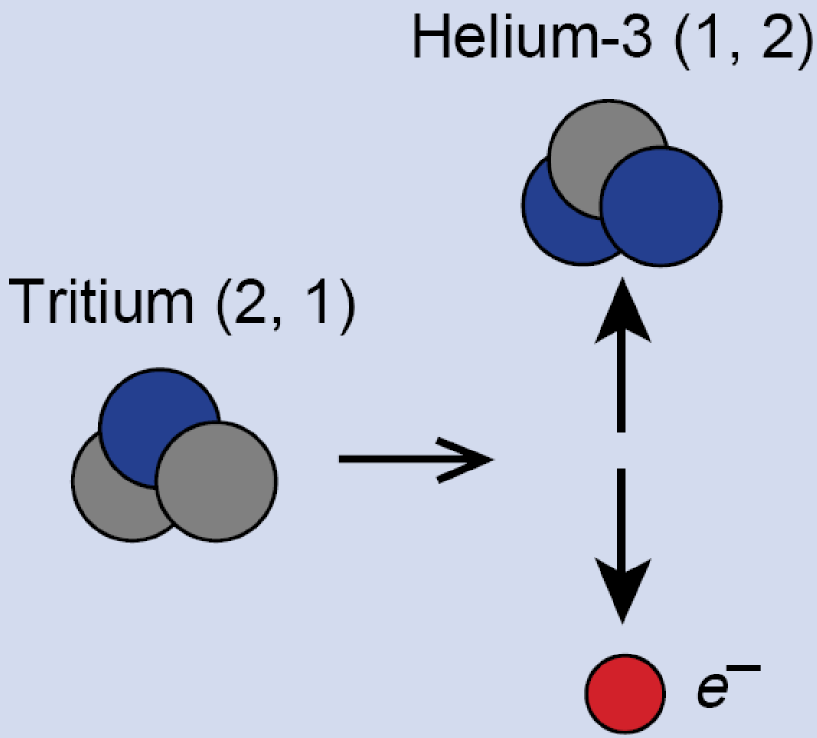


Lecture A

- ★ Neutrino: Concept and Discovery
- ★ Parity, Helicity and Handedness
- ★ Lepton Flavors and Families

Beta Decay in 1930

Two-Body Final State

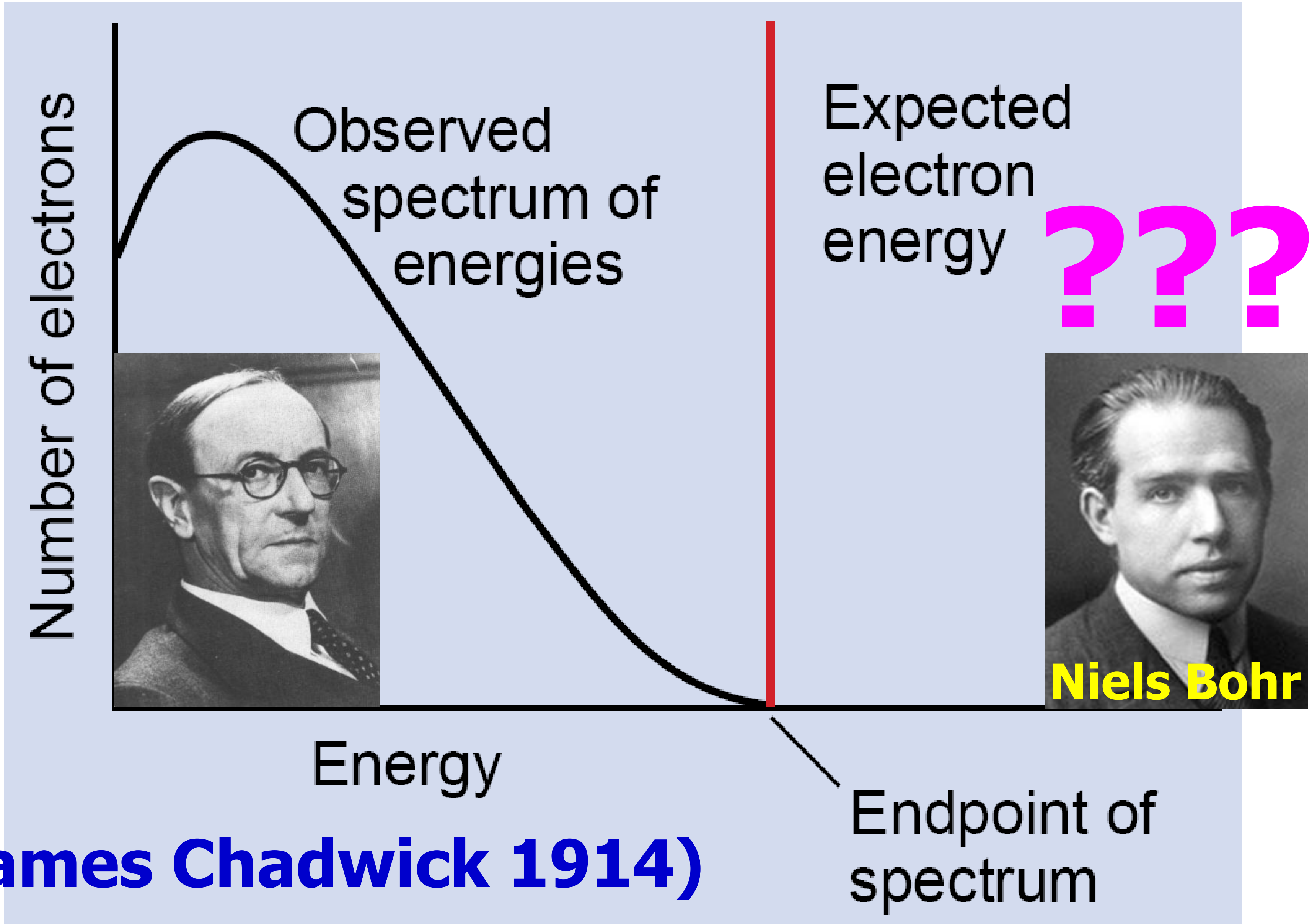


Recoil nucleus and electron separate with equal and opposite momentum.

$$(N, Z) \longrightarrow (N-1, Z+1) + e^- ,$$

where N = number of neutrons, and
Z = number of protons.

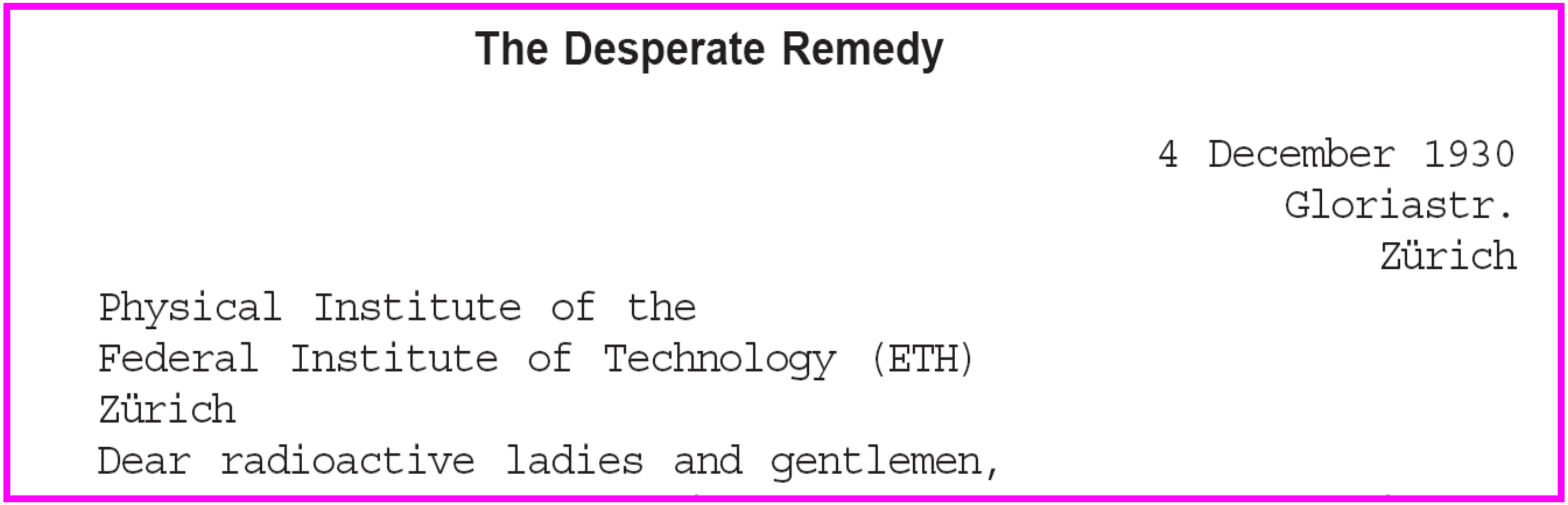
Enegy Crisis?



(James Chadwick 1914)

Desperate Remedy

Wolfgang Pauli (1930)



The Desperate Remedy

4 December 1930
Gloriastr.
Zürich

Physical Institute of the
Federal Institute of Technology (ETH)
Zürich

Dear radioactive ladies and gentlemen,

Pauli's Letter

As the bearer of these lines, to whom I ask you to listen graciously, will explain more exactly, considering the 'false' statistics of N-14 and Li-6 nuclei, as well as the continuous β-spectrum, I have hit upon a desperate remedy to save the "exchange theorem"* of statistics and the energy theorem. Namely [there is] the possibility that there could exist in the nuclei electrically neutral particles that I wish to call neutrons,** which have spin 1/2 and obey the exclusion principle, and additionally differ from light quanta in that they do not travel with the velocity of light: The mass of the neutron must be of the same order of magnitude as the electron mass and, in any case, not larger than 0.01 proton mass. The continuous β-spectrum would then become understandable by the assumption that in β decay a neutron is emitted together with the electron, in such a way that the sum of the energies of neutron and electron is constant.

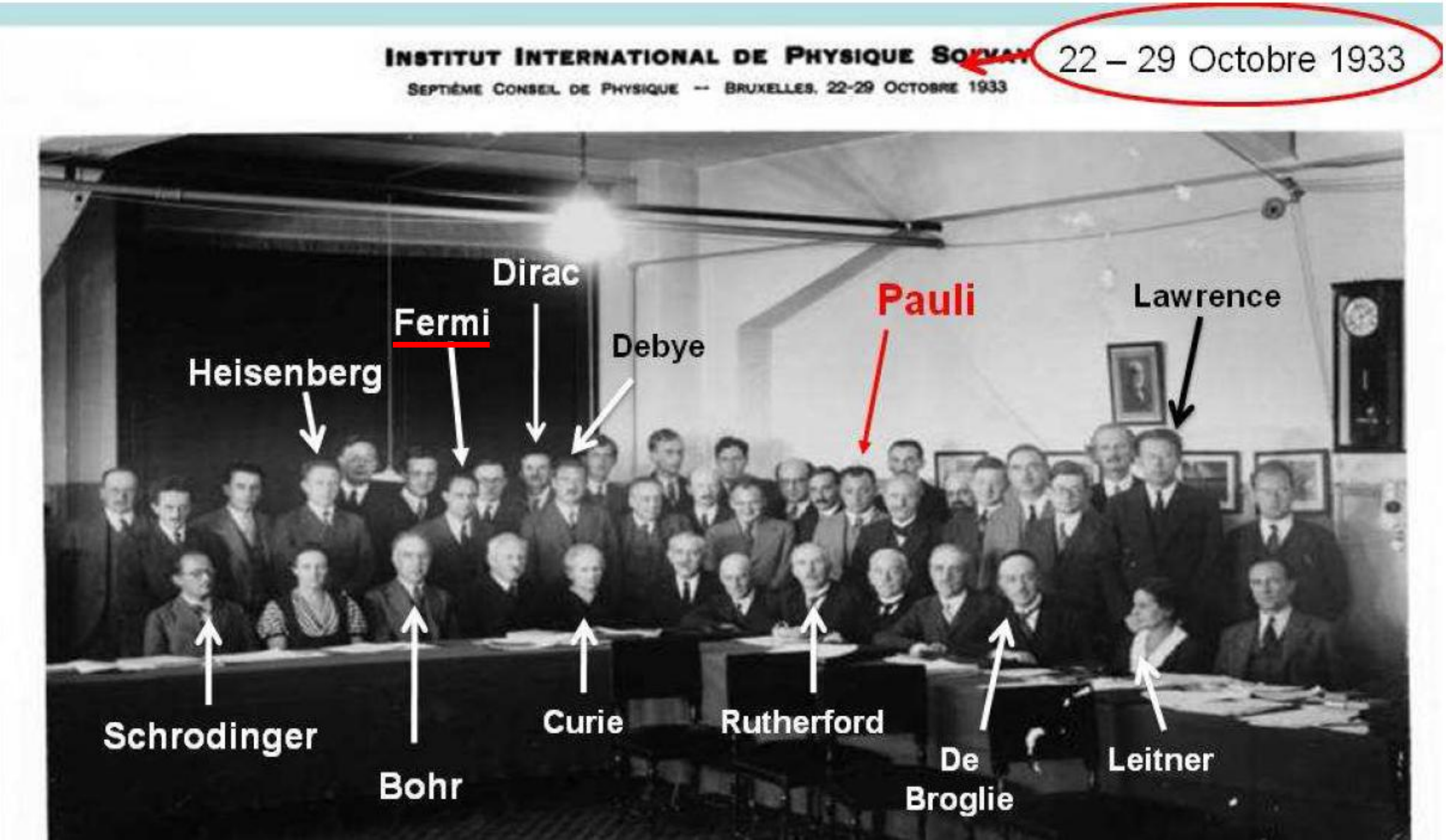
But I don't feel secure enough to publish anything about this idea, so I first turn confidently to you, dear radioactives, with a question as to the situation concerning experimental proof of such a neutron, if it has something like about 10 times the penetrating capacity of a γ ray.

I admit that my remedy may appear to have a small *a priori* probability because neutrons, if they exist, would probably have long ago been seen. However, only those who wager can win, and the seriousness of the situation of the continuous β-spectrum can be made clear by the saying of my honored predecessor in office, Mr. Debye, who told me a short while ago in Brussels, "One does best not to think about that at all, like the new taxes." Thus one should earnestly discuss every way of salvation.—So, dear radioactives, put it to test and set it right.—Unfortunately, I cannot personally appear in Tübingen, since I am indispensable here on account of a ball taking place in Zürich in the night from 6 to 7 of December.—With many greetings to you, also to Mr. Back, your devoted servant,

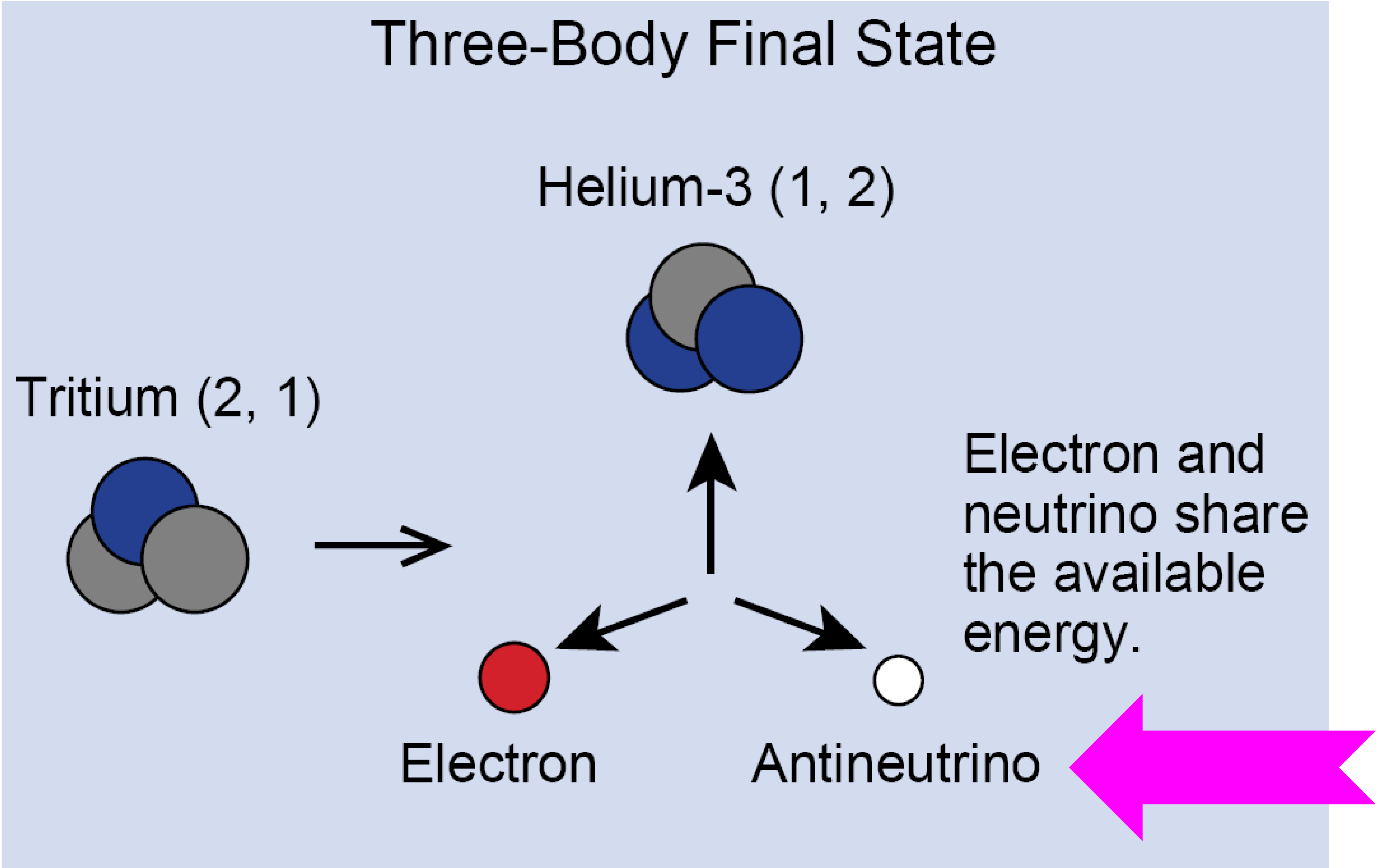
W. Pauli

Solvay 1933

Pauli gave a talk on his neutrino proposal in this congress.



True Picture of β Decay



$$(N, Z) \longrightarrow (N-1, Z+1) + e^{-} + \overline{\nu} \quad .$$

Fermi's Paper

<u>E. Fermi's publications on the Weak Interaction</u>

REJECTED

E. Fermi, "Tentative Theory of Beta Rays"
Letter Submitted to Nature (1933)

31 Dec, 1933

ANNO IV - VOL. II - N. 12 QUINDICINALE 31 DICEMBRE 1933 - XII

LA RICERCA SCIENTIFICA

ED IL PROGRESSO TECNICO NELL'ECONOMIA NAZIONALE

Tentativo di una teoria dell'emissione dei raggi "beta"

Nota del prof. ENRICO FERMI

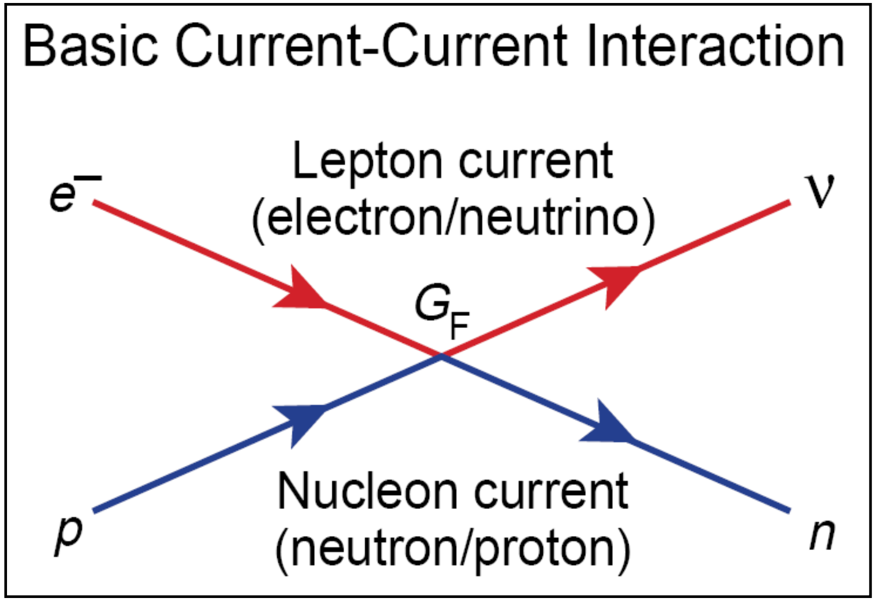
Riassunto: Teoria della emissione dei raggi β delle sostanze radioattive, fondata sull'ipotesi che gli elettroni emessi dai nuclei non esistano prima della disintegrazione ma vengano formati, insieme ad un neutrino, in modo analogo alla formazione di un quanto di luce che accompagna un salto quantico di un atomo. Confronto della teoria con l'esperienza.

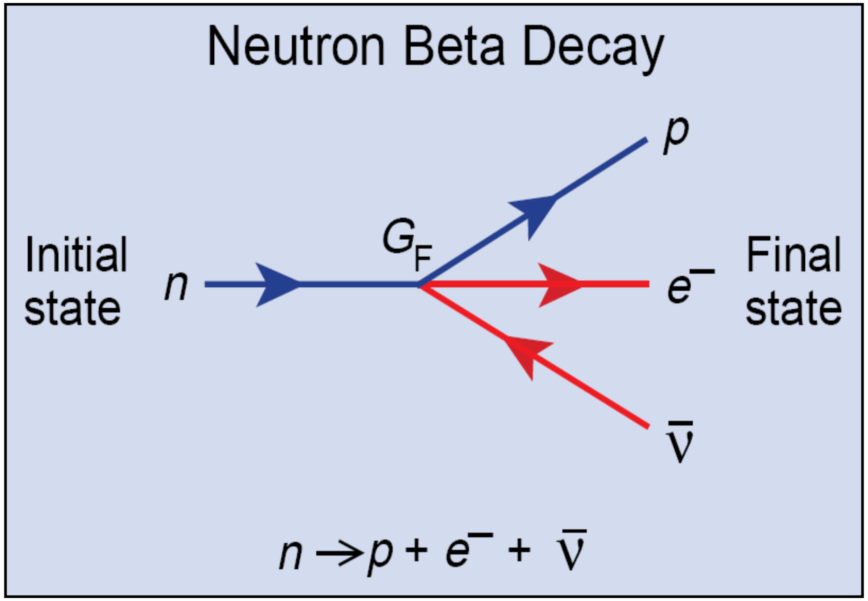
Published first in this journal and latter in Z. Phys. in 1934.

Fermi's Theory

Enrico Fermi (1933 & 1934) assumed a new force for β decay by combining 3 brand-new concepts:

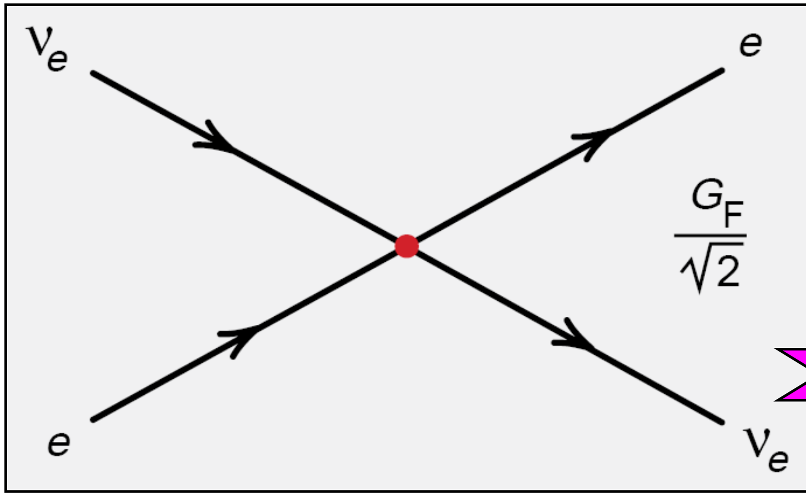
- ★ **Pauli's hypothesis: neutrinos**
- ★ **Dirac's thought: creation of particles**
- ★ **Heisenberg's idea: neutron is related to proton**

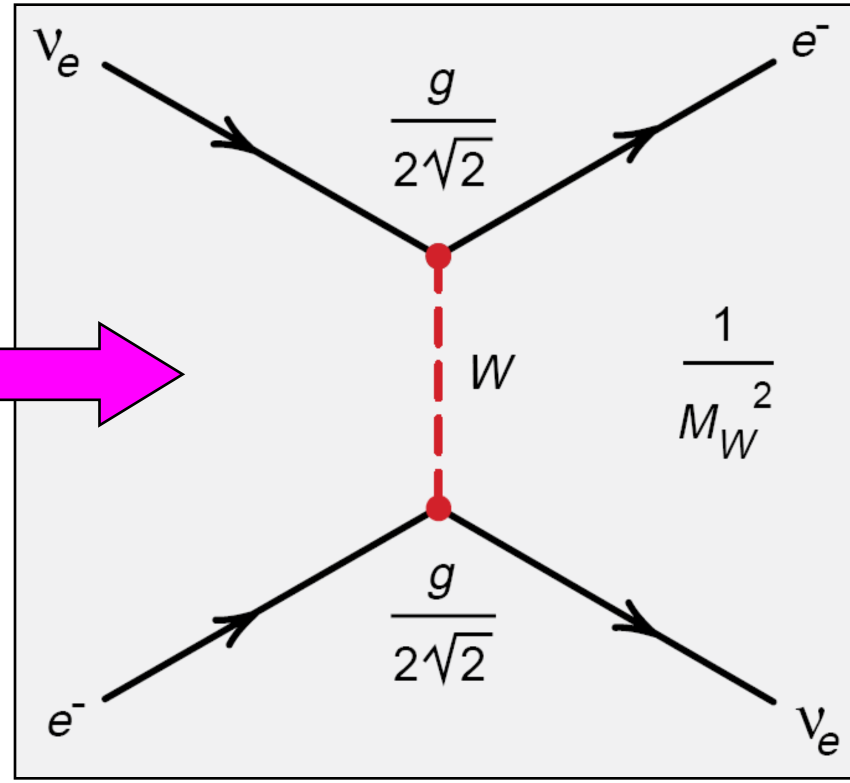




Weak Interactions

From Fermi's current-current interaction to weak charged-current gauge interactions





$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

$$M_W = 80.4\,\mathrm{GeV}$$

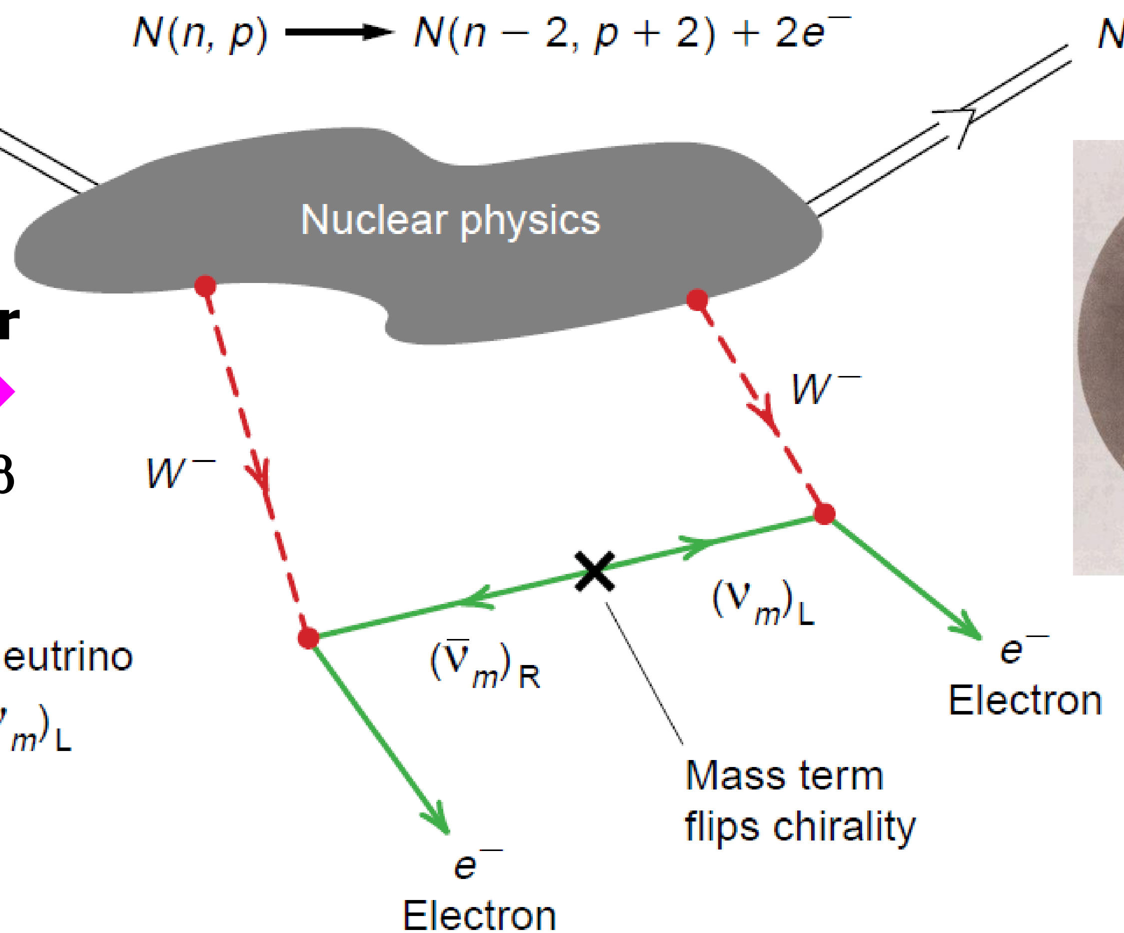
$$G_F = 1.66 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Majorana Neutrinos

Ettore Majorana, Fermi's PhD student, proposed a new idea in 1937: neutrino is its own antiparticle.

Initial state

Final state



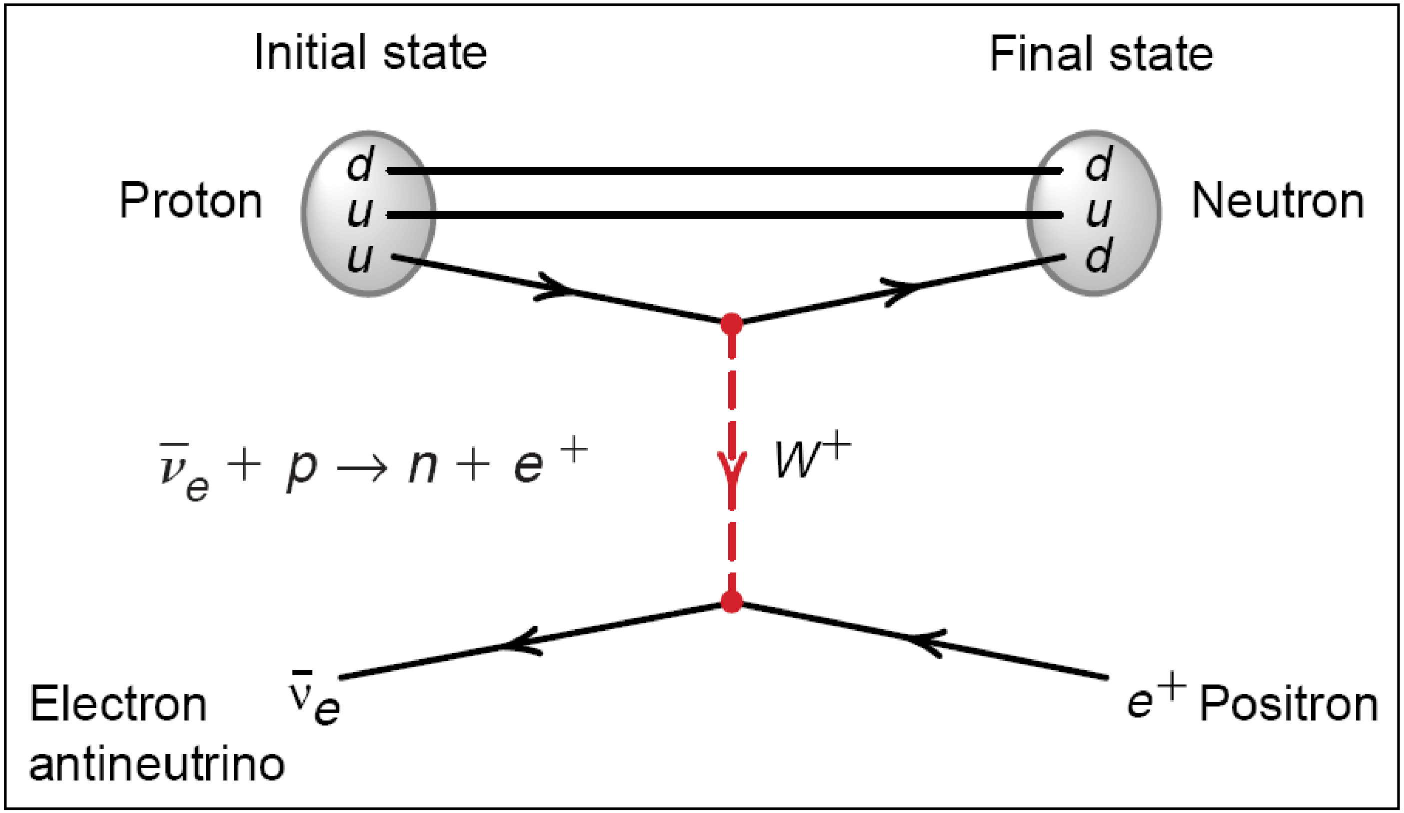
Lepton number violation ⟶ **neutrinoless ββ decay.**

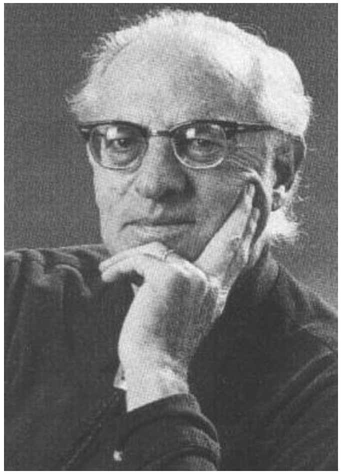
ν_m = Majorana neutrino

$= (\bar{\nu}_m)_R + (\nu_m)_L$

Impossible Challenge

An inverse β decay to detect neutrinos (Hans Bethe 1936)





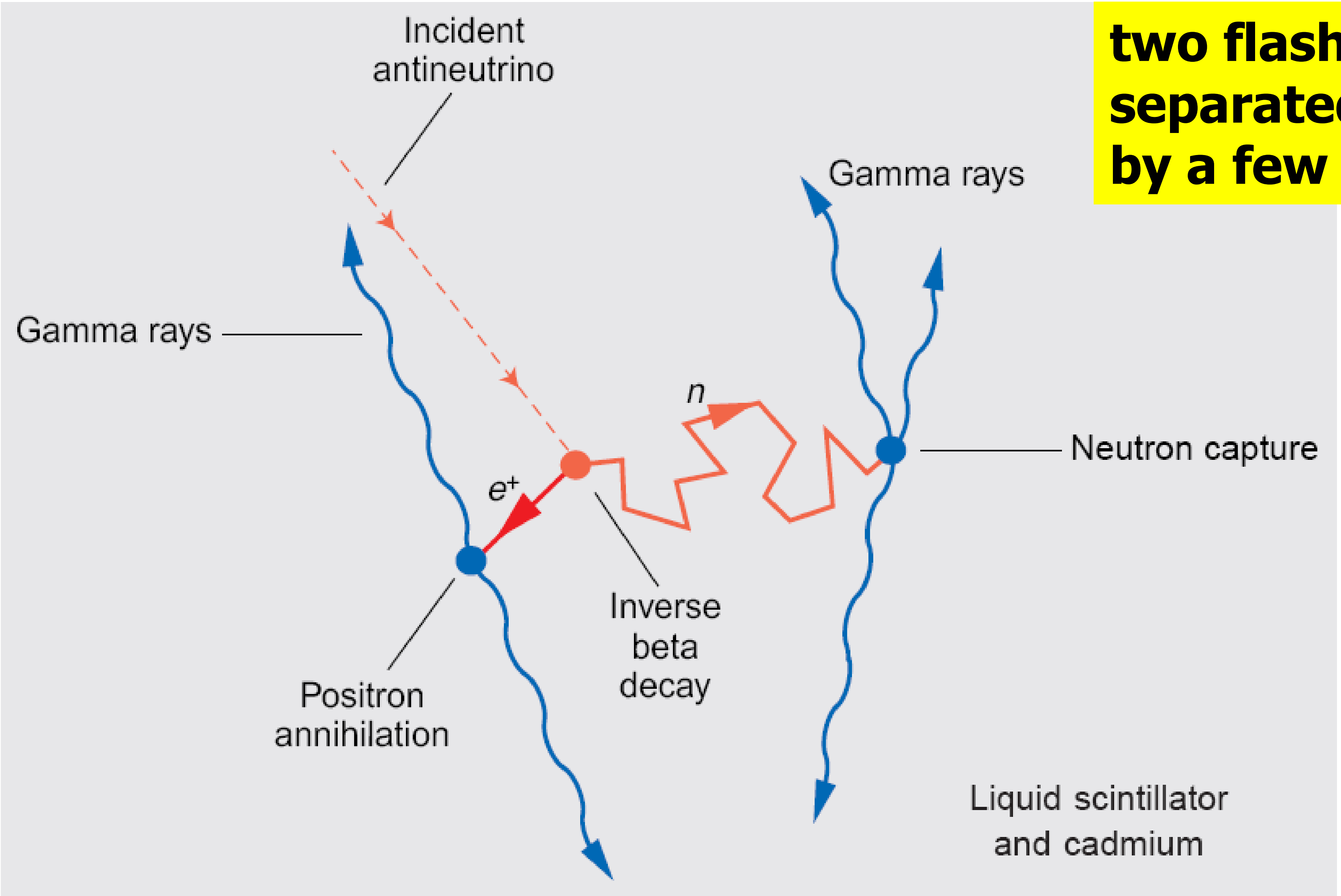
Very intense sources of neutrinos (1950's): fission bombs and fission reactors.

Frederick Reines & Clyde Cowan's Project (1951).

Reactor Neutrinos

Decision in 1952: neutrinos from a fission reactor.

two flashes separated by a few μs



Positive Result?

Reines and Cowan's telegram to Pauli on 14/06/1956:

We're happy to inform you that we've definitely detected neutrinos from fission fragments by observing inverse β decay of protons. Observed cross section agrees well with expected $\underline{6\times10^{-44}\,\mathrm{cm}^2}$.

Such a theoretical value was based on a parity-conserving formulation of the β decay with 4 independent degrees of freedom for ν's.

This value doubled after the discovery of parity violation in 1957, leading to the two-component ν theory in 1957 and the V–A theory in 1958.

Nobel Prize

A new paper on this experiment published in Phys. Rev. in 1960 reported a cross section twice as large as that given in 1956.

Reines (1979): our initial analysis grossly overestimated the detection efficiency with the result that the measured cross section was at first thought to be in good agreement with [the pre-parity violation] prediction.

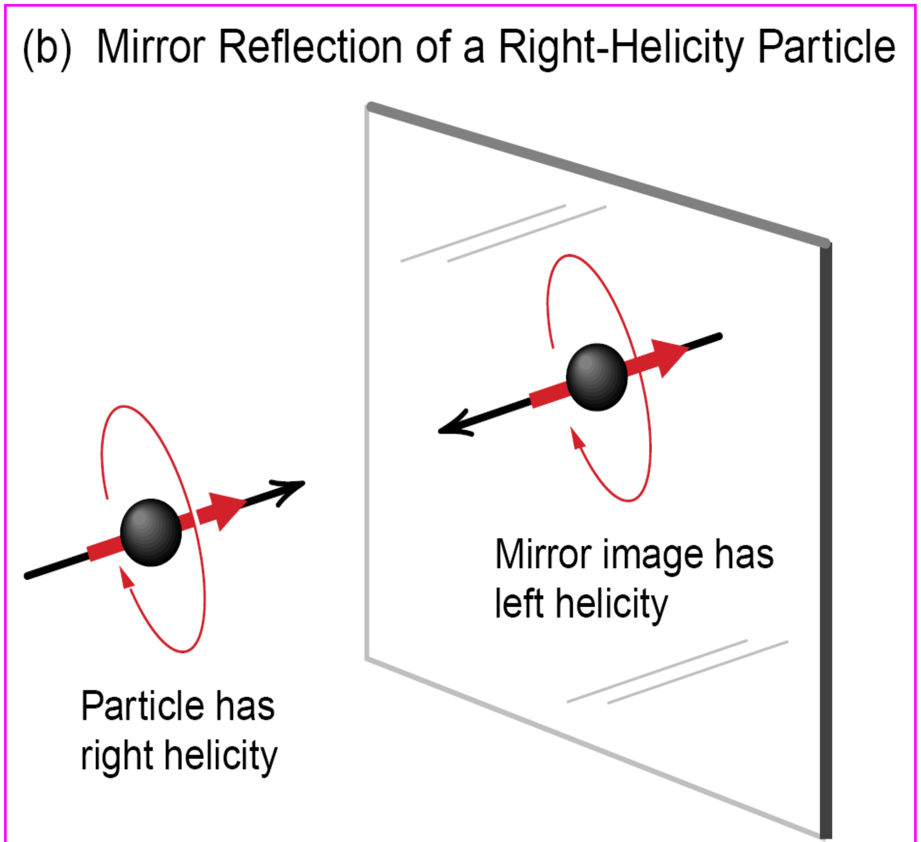
Skepticism

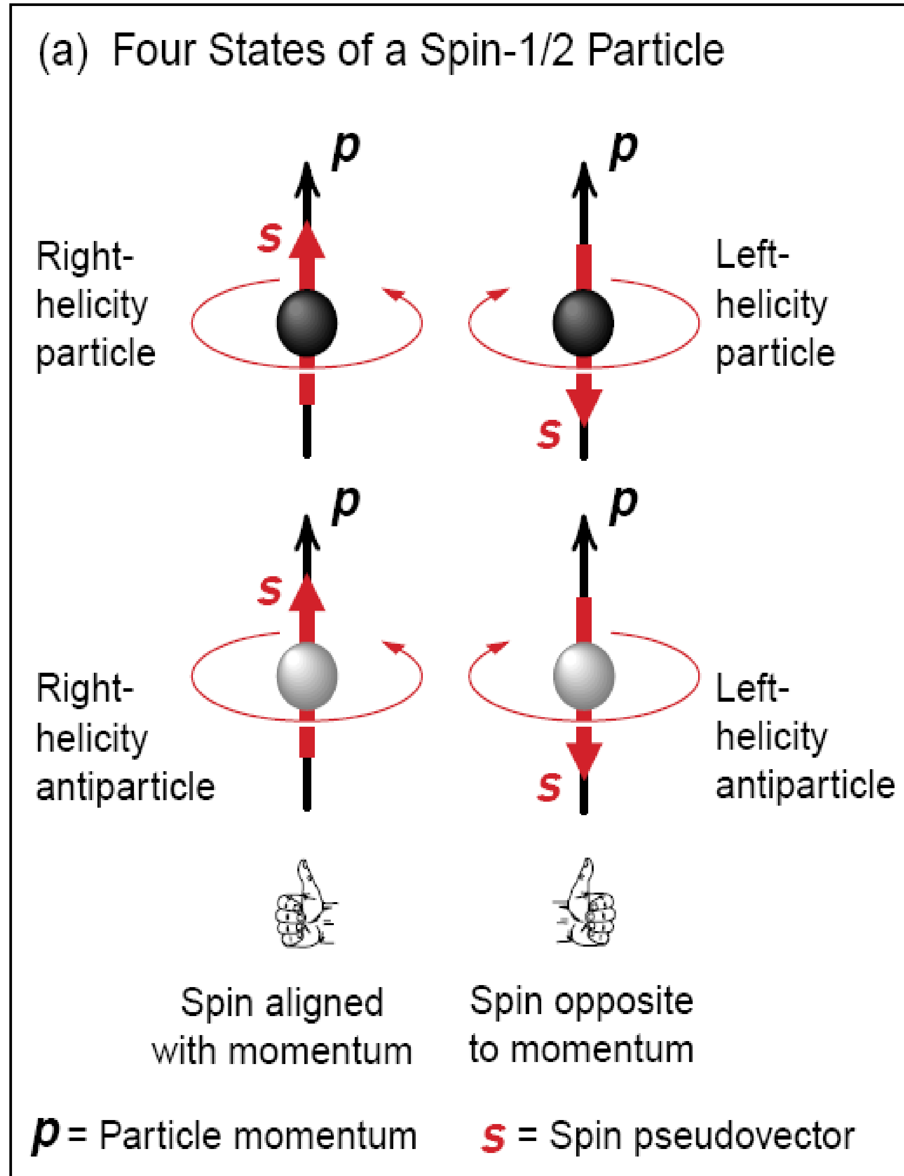
{over 39 years}

The Nobel Prize finally came to Frederick Reines in 1995!

Parity and Helicity

Parity and **Helicity**: if parity were conserved, a spin-1/2 particle would exist in both left- & right-helicity states.



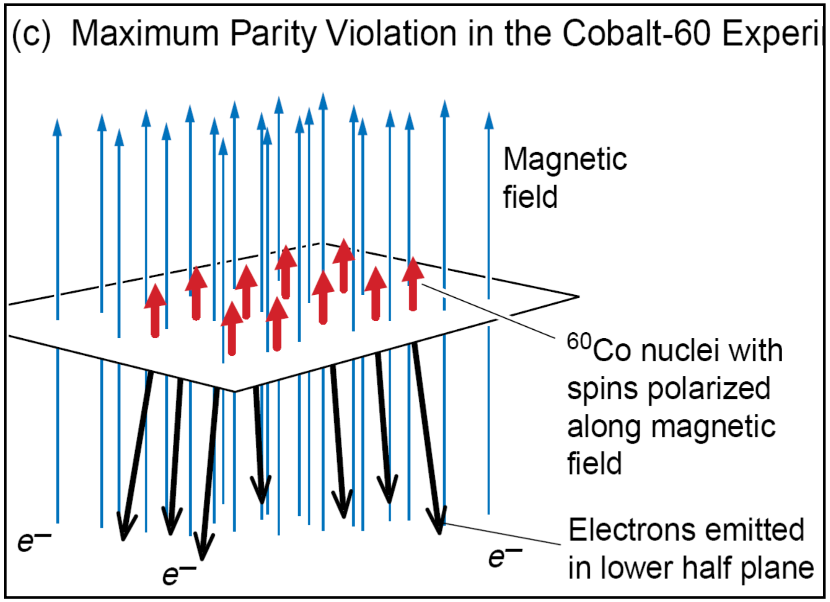


Wu's Experiment

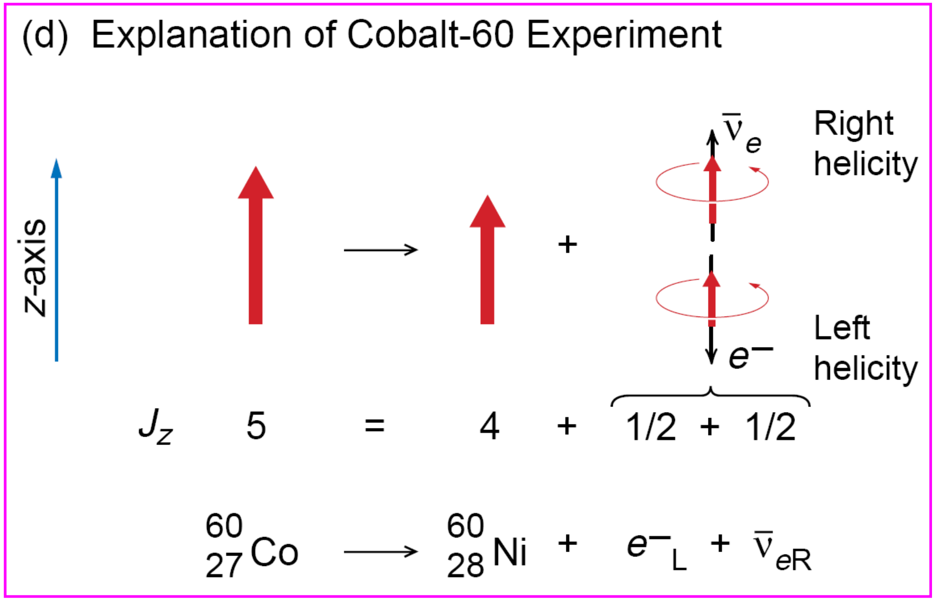
C.S. Wu aligned the spins of Cobalt-60 nuclei along an external magnetic field and then measured the directions of the electrons emitted by those nuclei. She observed maximal parity violation!

$$^{60}\text{Co}_{27} \rightarrow {}^{60}\text{Ni}_{28} + e^- + \bar{\nu}_e$$

Lee & Yang's two-component neutrino theory explained the data but implied: neutrinos are left-handed & massless!



(c) Maximum Parity Violation in the Cobalt-60 Experi



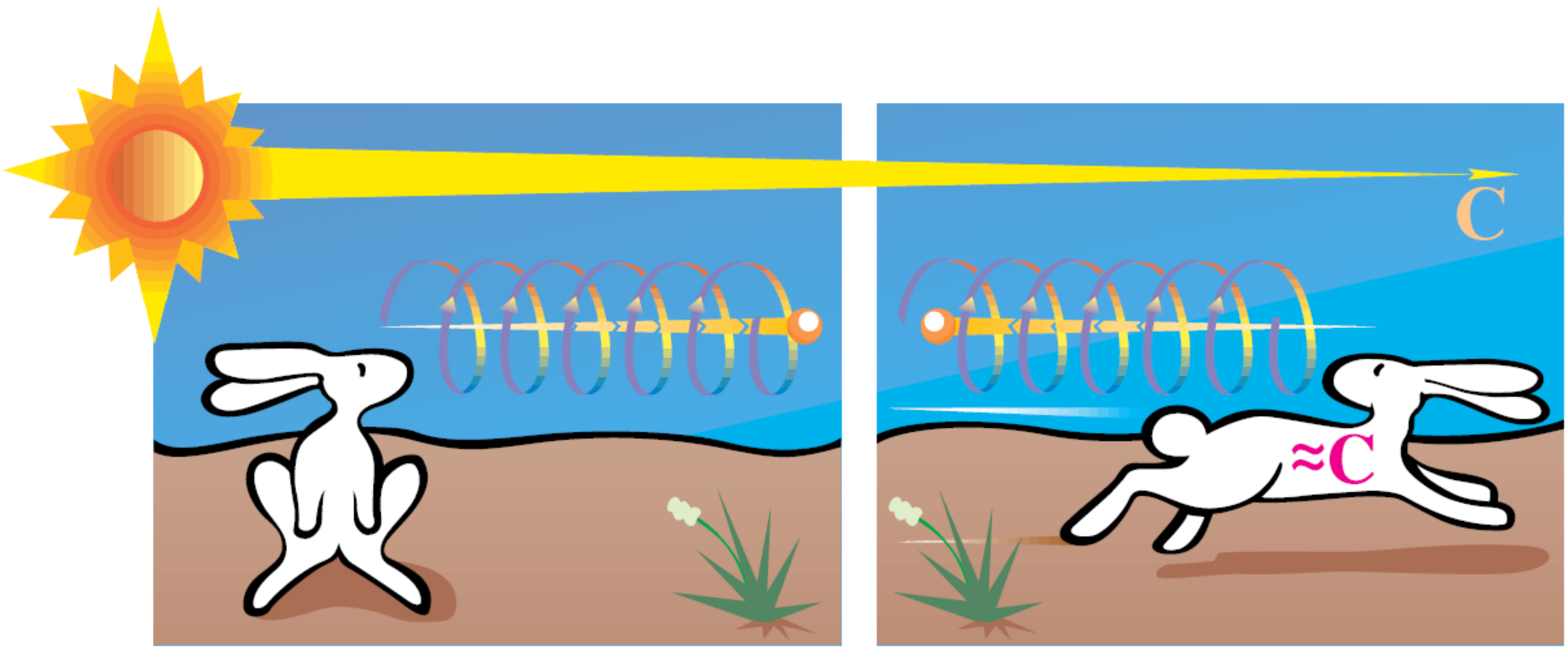
(d) Explanation of Cobalt-60 Experiment

Helicity

The Helicity of a fermion relates its spin to its direction of motion

$$\lambda = \boldsymbol{s} \cdot \boldsymbol{p}/|\boldsymbol{p}| = \pm 1/2$$

A massive particle's helicity isn't relativistically invariant.



Looks like a left-handed corkscrew.   No—like a right-handed corkscrew!

The GGS Experiment

A proof of the negative helicity of a neutrino was done in 1958.

$$ {}^{152}_{63}Eu(0^-) + e^- \to {}^{152}_{62}Sm^*(1^-) + \nu $$

In this reaction, the *Sm nucleus recoils from the neutrino. It decays into the ground state simply by emitting a γ particle. Then the polarization of this γ particle measures the neutrino's helicity in a rather non-trivial way.**

$$ \lambda = -1/2 $$

Helicity of Neutrinos*

M. Goldhaber, L. Grodzins, and A. W. Sunyar
Brookhaven National Laboratory, Upton, New York

(Received December 11, 1957)

A COMBINED analysis of circular polarization and resonant scattering of γ rays following orbital electron capture measures the helicity of the neutrino. We have carried out such a measurement with Eu^{152m}, which decays by orbital electron capture. If we assume the most plausible spin-parity assignment for this isomer compatible with its decay scheme,[1] 0−, we find that the neutrino is "left-handed," i.e., $\boldsymbol{\sigma}_\nu \cdot \hat{p}_\nu = -1$ (negative helicity).

Handedness

The **Handedness** of a spin-1/2 particle is a relativistically invariant quantity to describe this particle's spin states.

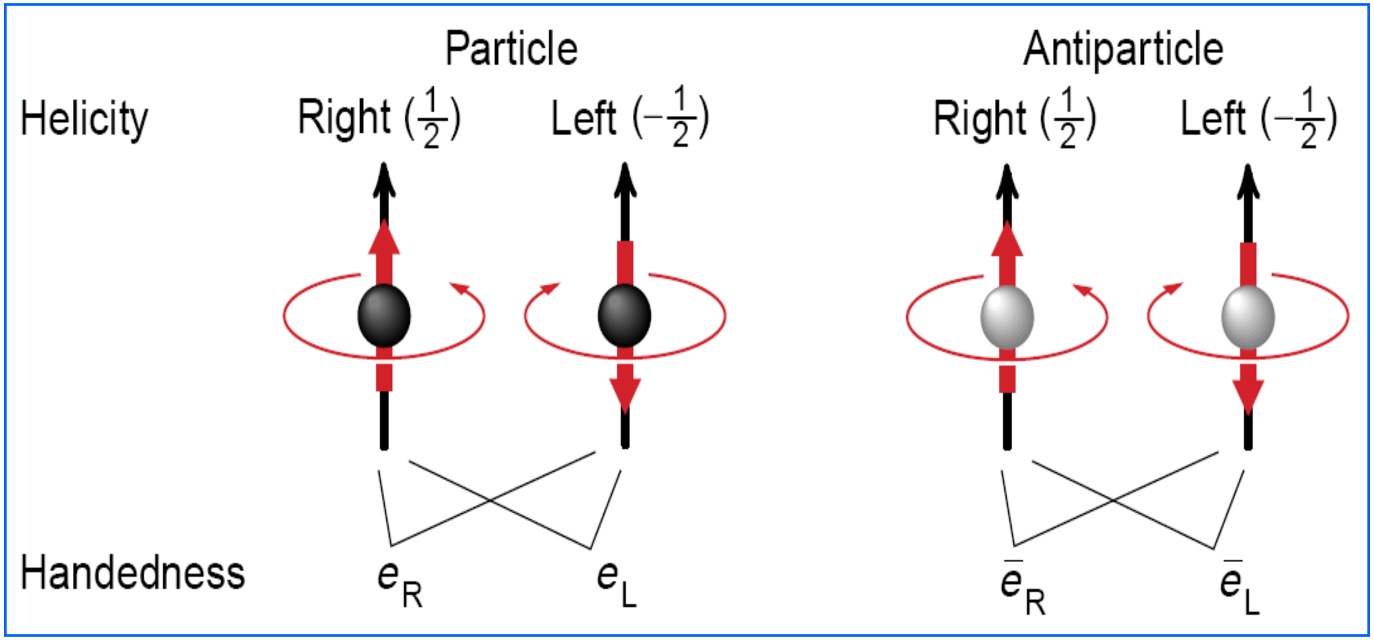
2 independent handedness or **chirality** states: **left** & **right**.

A massless or massive particle can always be decomposed into two independent components: **left-** and **right-handed**.

$$\psi = \psi_L + \psi_R$$

$$\psi_L = \frac{1+\gamma_5}{2}\psi$$

$$\psi_R = \frac{1-\gamma_5}{2}\psi$$

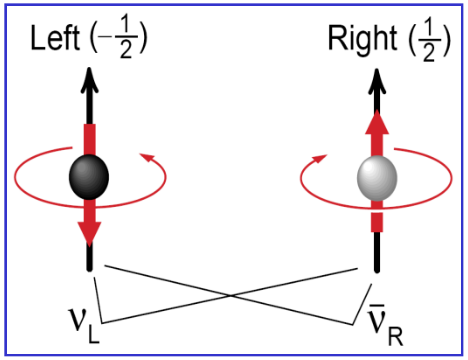


Weak States

The Handedness is an essential quantity for describing the weak force (see the table for the 1st-family in the SM).

For example:

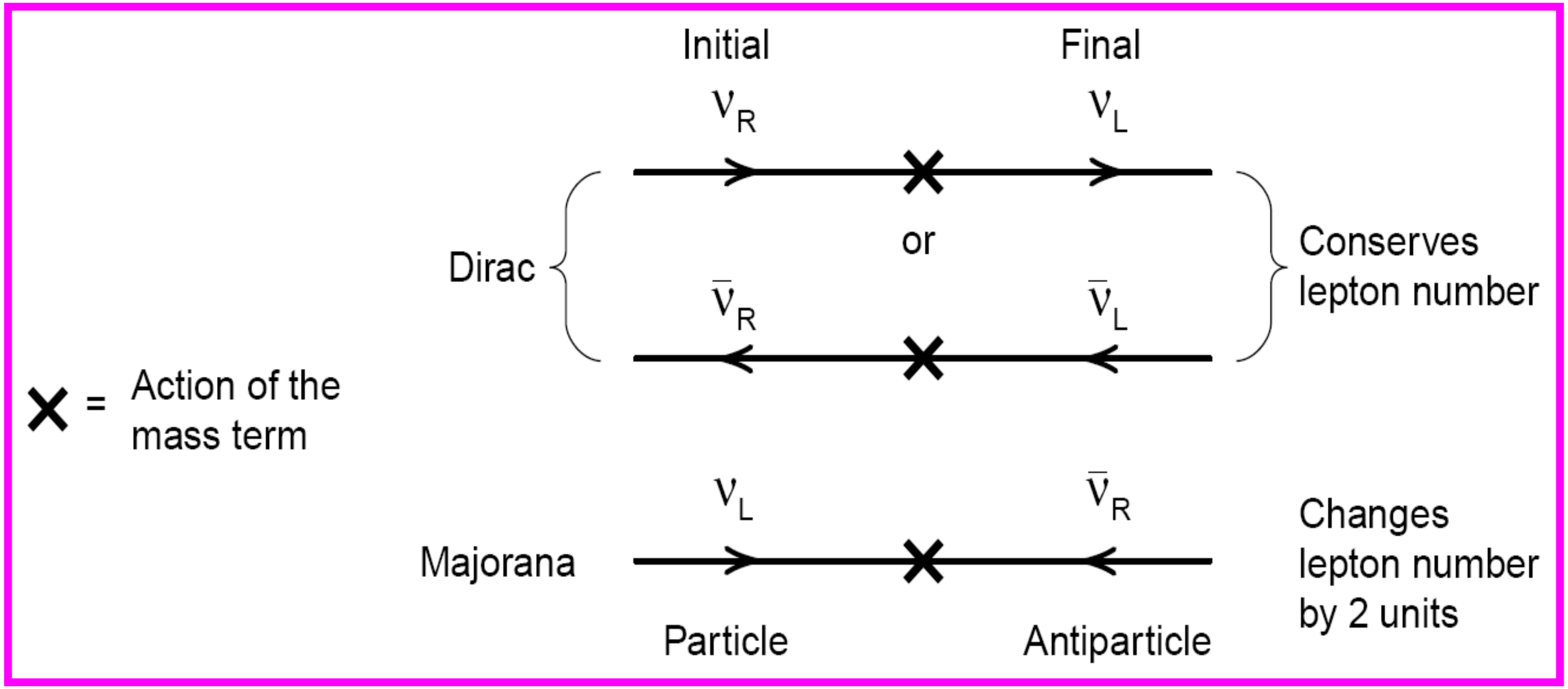
The right-handed electron is a weak isospin singlet: it does not interact with the W boson, but it does couple to the photon and to the Z boson.



Particle Number N	Handedness N_x	Particle States	Weak Isotopic Charge I_3^W	Weak Hypercharge Y^W	Electric Charge Q
+1	L	e_L	−1/2	−1	−1
+1	L	ν_L	+1/2	−1	0
−1	R	$\bar{e}_R$	+1/2	+1	+1
−1	R	$\bar{\nu}_R$	−1/2	+1	0
+1	R	e_R	0	−2	−1
−1	L	$\bar{e}_L$	0	+2	+1
+1	R	ν_R	0	0	0
−1	L	$\bar{\nu}_L$	0	0	0

Mass Terms

The Handedness is also a crucial concept for the origin of particle masses. Any interaction giving a spin-1/2 particle a nonzero mass must flip its left- and right-handed states.



$$|e_L\rangle \propto |e_{-1/2}\rangle + m/E|e_{+1/2}\rangle$$
$$|e_R\rangle \propto |e_{+1/2}\rangle - m/E|e_{-1/2}\rangle$$

2 handed states in terms of 2 helicity states: if m = 0, handedness = helicity; if m << E, it holds approximately (in a relativistic limit).

Helicity + Handedness

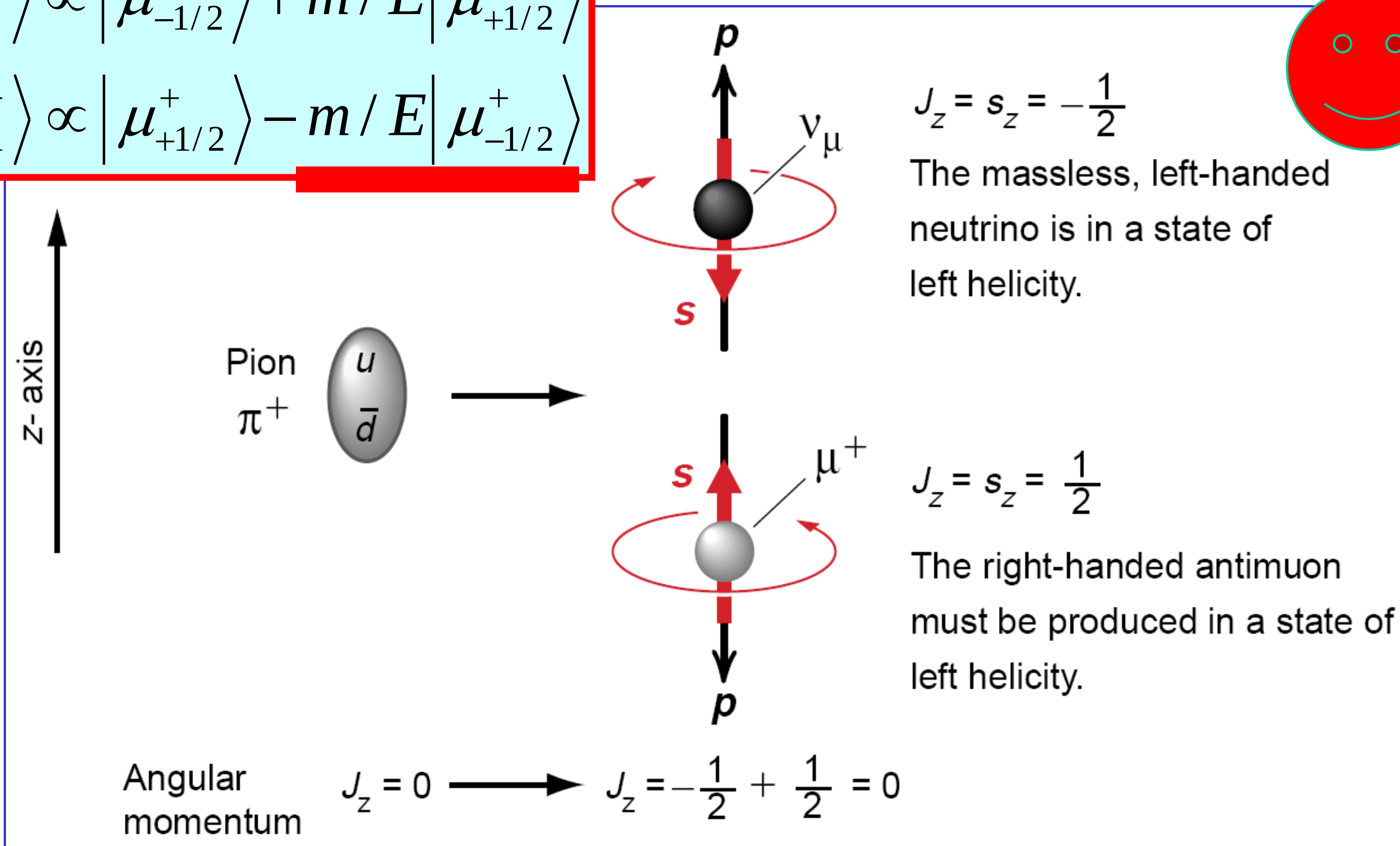
Which decay is more likely?

$\pi^+ \rightarrow e^+ + \nu_e$

$\pi^+ \rightarrow \mu^+ + \nu_\mu$

$$\left|\mu_L^+\right\rangle \propto \left|\mu_{-1/2}^+\right\rangle + m/E\left|\mu_{+1/2}^+\right\rangle$$

$$\left|\mu_R^+\right\rangle \propto \left|\mu_{+1/2}^+\right\rangle - m/E\left|\mu_{-1/2}^+\right\rangle$$

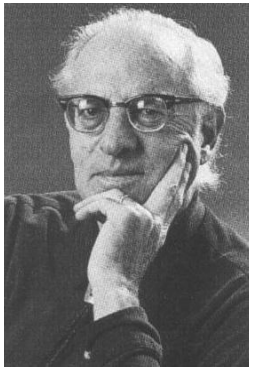


Electron & Its Neutrino

The **electron** was discovered in 1897 by **Joseph Thomson**.

The electron's anti-particle, **positron**, was discovered in 1932 by **Carl Anderson**.

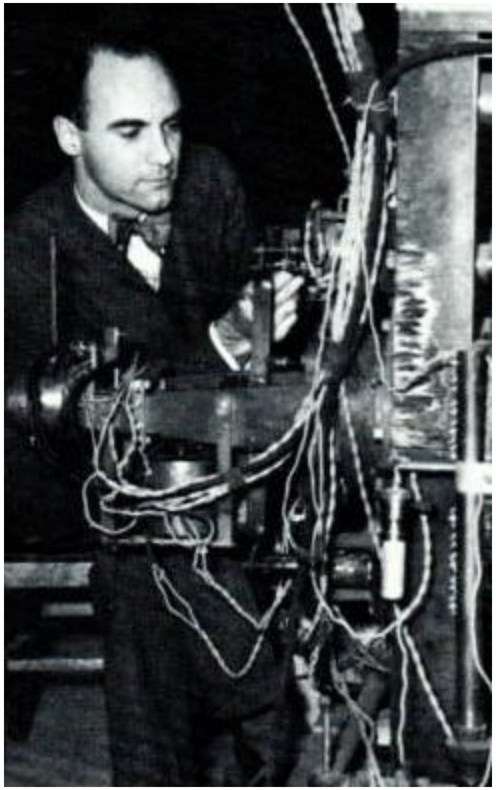
The first scientist deemed to have captured **positrons** via electron-positron annihilation was **Chung-Yao Chao** (赵忠尧 1902–1998) at Caltech in 1930.



In 1956 **Clyde Cowan** and **Frederick Reines** discovered the positron's partner, **electron antineutrino**.

Muon

The **muon** particle, a sister of the electron, was discovered in 1936 by **Carl Anderson** and his first student **S. Neddermeyer**; and independently by **J. Street** *et al*.



It was not **Hideki Yukawa**'s "pion". And it was the first flavor puzzle.

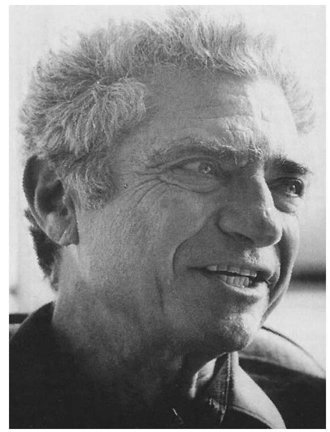
Isidor Rabi famously asked:

Who ordered that?

Isidor Isaac Rabi

Muon Neutrino

The muon neutrino was discovered by Leon Lederman, Melvin Schwartz and Jack Steinberger in 1962.



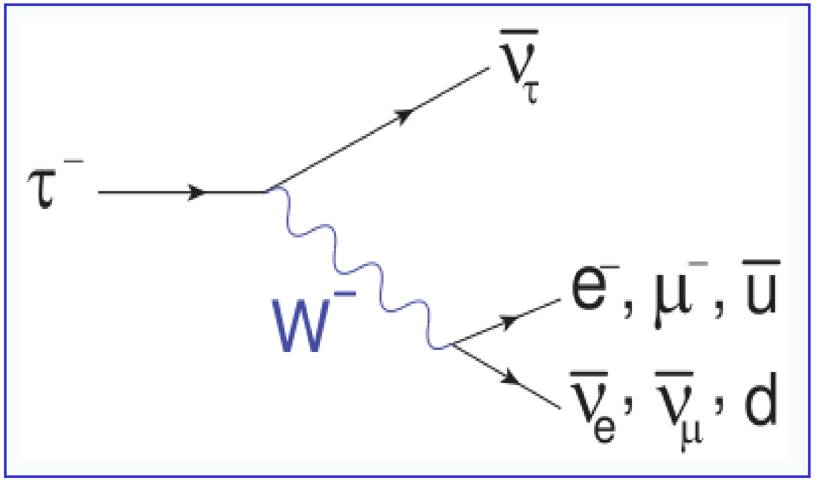
Neutrino flavor conversion was proposed by Z. Maki, M. Nakagawa and S. Sakata in 1962.

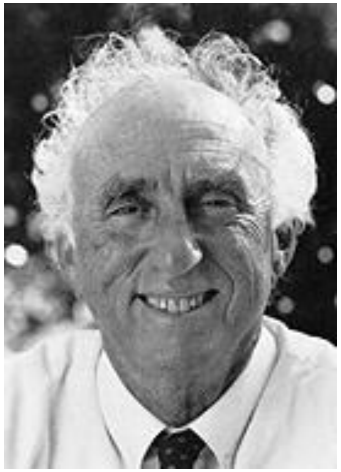
Neutrinos oscillate into antineutrinos: first proposed by Bruno Pontecorvo in 1957.

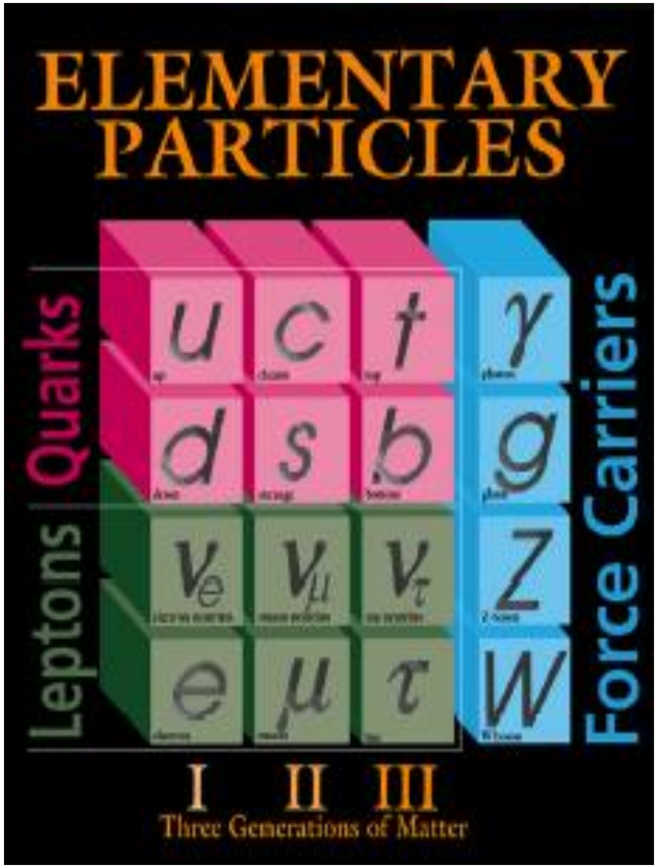
Tau & Tau Neutrino

The tau particle was discovered by Martin Perl in 1975 via:

$$e^+ + e^- \rightarrow e^\pm + \mu^\mp + \text{undetected particles}$$







In 2000, the tau neutrino was finally discovered at the Fermilab.

The lepton family is complete!

Leptons & Nobel Prizes

e	J.J. Thomson 1897	J.J. Thomson 1906 (NP)
ν_e	C.L. Cowan et al. 1956	F.J. Reines 1995 (NP)
μ	J.C. Street et al. C.D. Anderson 1936	1975 – 1936 = 1936 – 1897 = 39
ν_μ	G. Danby et al. 1962	M. Schwartz, L.M. Lederman, J. Steinberger 1988 (NP)
τ	M.L. Perl et al. 1975	M.L. Perl 1995 (NP)
ν_τ	K. Kodama et al. 2000	

Antimatter: Positron.

Predicted by P.A.M. Dirac in 1928.

Discovered by C.D. Anderson in 1932; Nobel Prize in 1936.

Sarma-Xing Theorem

In 1995 it was an Indian theorist who first discovered the 39-year gap of charged leptons.

2) **NOBEL LEPTONS.**
By K.V.L. Sarma (Tata Inst.),. TIFR-TH-95-56, Dec 1995. 13pp.
Submitted to Curr. Sci.
e-Print Archive: **hep-ph/9512420**

1975 + 39 = 2014

A summary of the discoveries made in the world of leptons is given in Table 1. We see that the third generation has started getting Nobel prizes. It is amusing that the charged-leptons crop up with a 39-year gap and may be the 4th one would show up in the year 2114. For the present, the available experimental information implies that there are no charged leptons which are heavier than tau and lighter than 45 GeV.

My contribution: corrected 2114 to 2014, so the discovery would be possible 100 years earlier (two years later)!

Lecture B

- ★ Massless Neutrinos in the SM
- ★ Dirac and Majorana Mass Terms
- ★ The Seesaw Mechanisms

Origin of "Flavor"

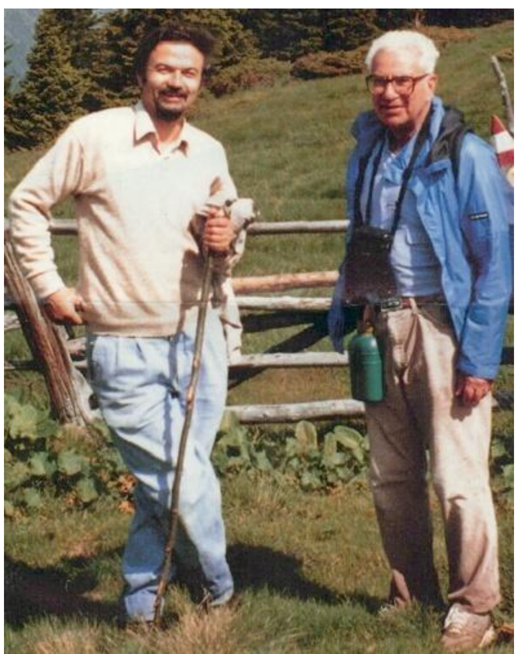
The term Flavor was coined by Harald Fritzsch and Murray Gell-Mann at an ice-cream store in Pasadena in 1971.

Flavor issues:

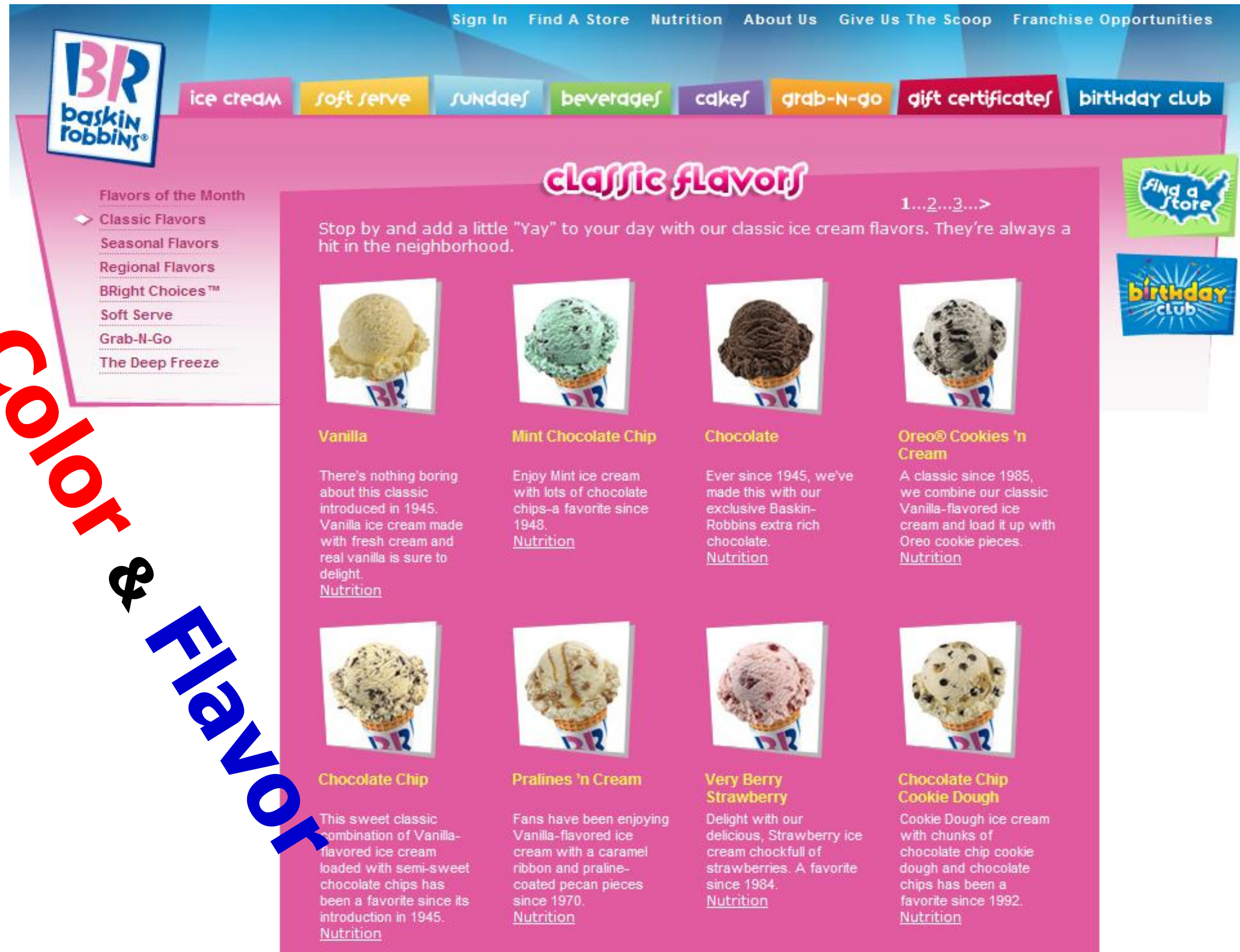
fermion mass

flavor mixing

CP violation

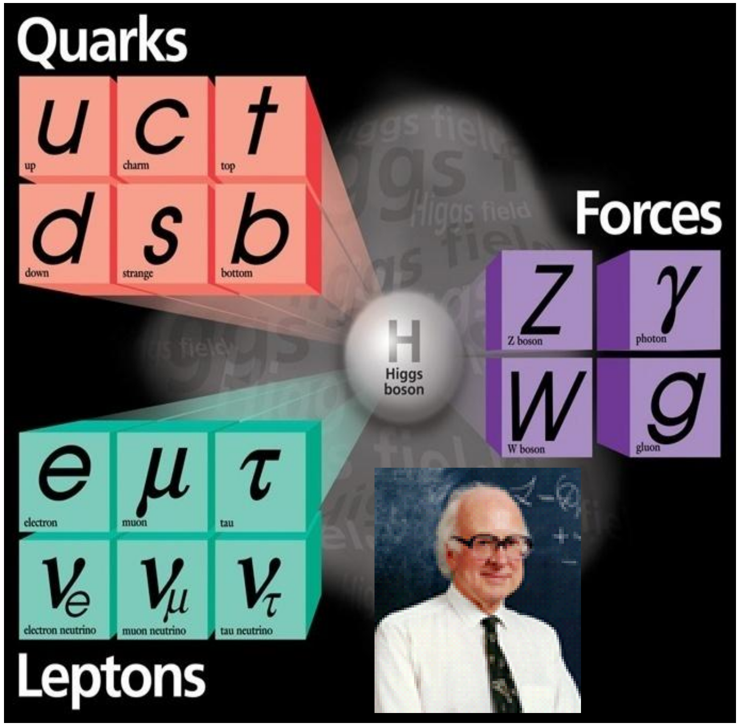


Color & Flavor



What is Mass?

Higgs is the God particle in the SM: it gives masses to other particles.



A lesson learnt from history:

All known bosons were discovered in Europe and most fermions were discovered in America.

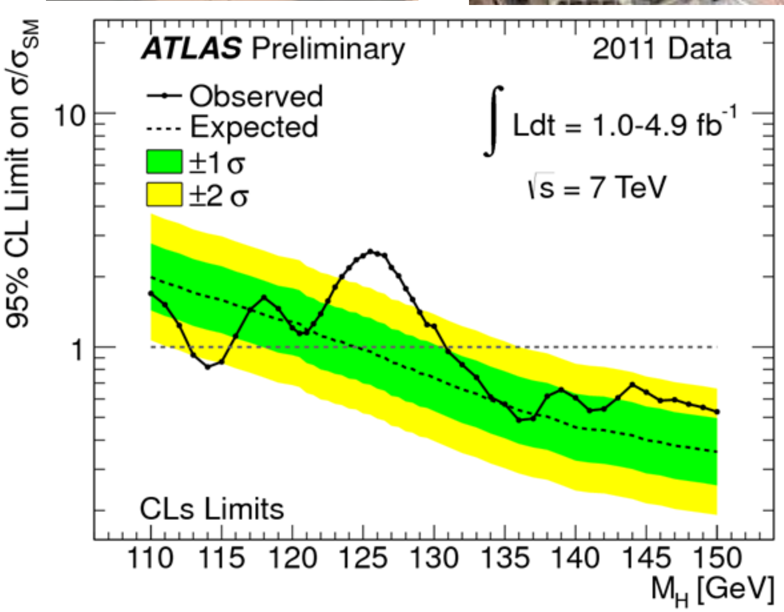
The LHC is very likely to see Higgs.

Mass is the inertial energy of a particle existing at rest.

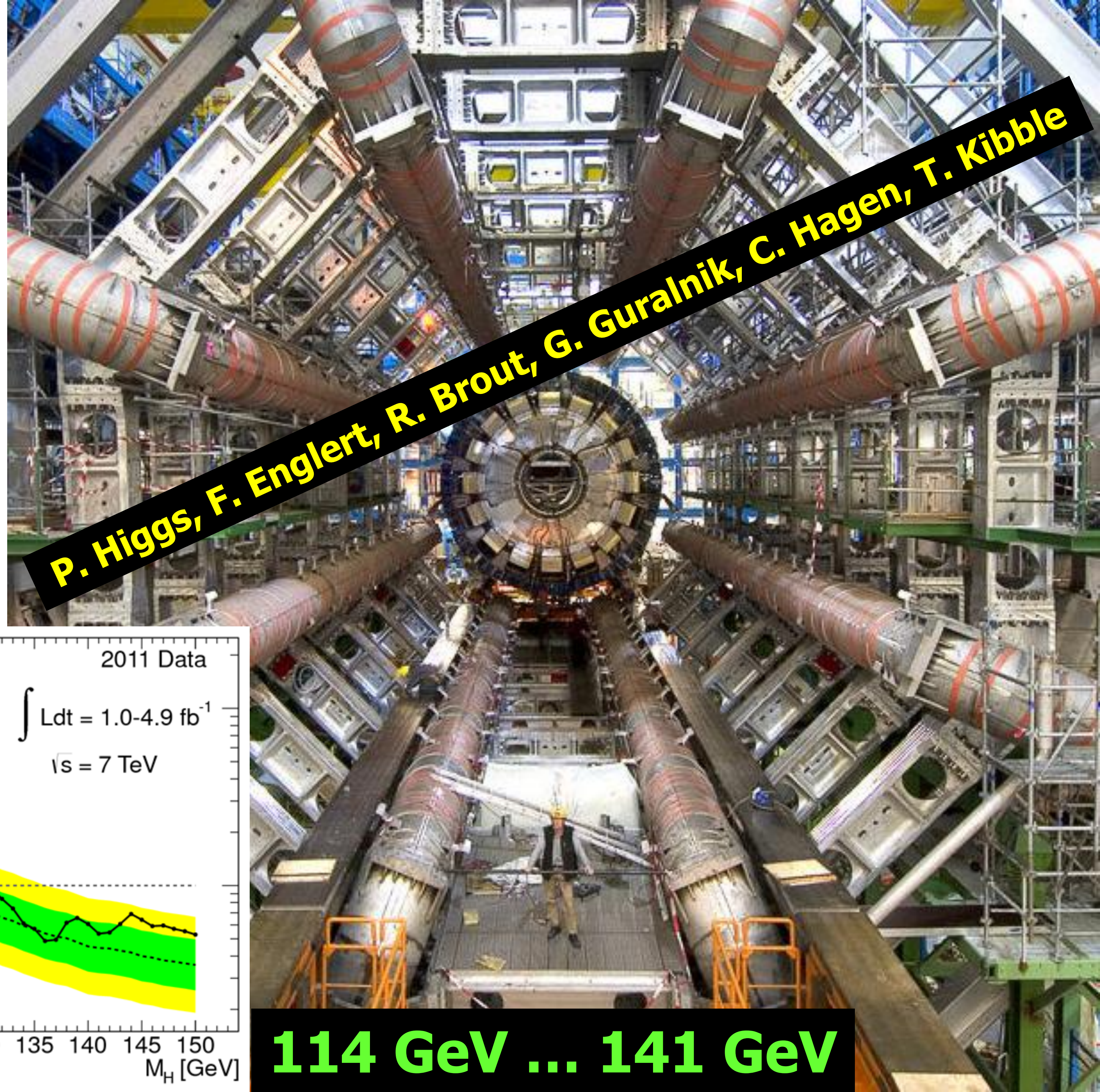
- A massless particle has no way to exist at rest. It must always move at the speed of light.
- A massive fermion (lepton or quark) must exist in both the left- and right-handed states.

2012

Higgs



ATLAS Preliminary
2011 Data
Observed
Expected
±1 σ
±2 σ
$\int Ldt = 1.0\text{-}4.9\ fb^{-1}$
$\sqrt{s} = 7$ TeV
95% CL Limit on σ/σ_{SM}
10
1
CLs Limits
110 115 120 125 130 135 140 145 150
M_H [GeV]



P. Higgs, F. Englert, R. Brout, G. Guralnik, C. Hagen, T. Kibble
114 GeV … 141 GeV

In the SM

All ν's are massless due to the model's simple structure:

---- SU(2)×U(1) gauge symmetry and Lorentz invariance

Fundamentals of a quantum field theory

---- Economical particle content:

No right-handed neutrino; only a single Higgs doublet

---- Mandatory renormalizability:

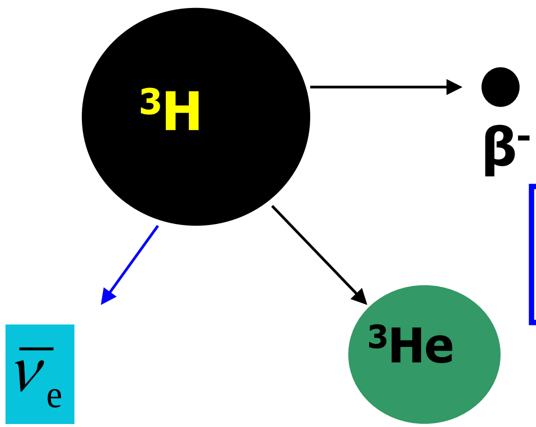
No dimension ≥ 5 operator (*B*-*L* conserved in the SM)

Neutrinos are massless in the SM: Natural or not?

YES: It's toooooooo light and almost left-handed;
NO: No fundamental symmetry/conservation law.

Constraints on Masses

Way 1: Tritium beta decay

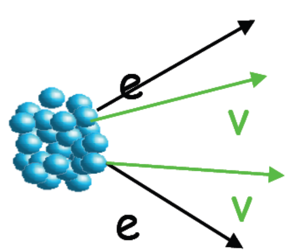


$$\langle m \rangle_e^2 \equiv \sum_i |V_{ei}|^2 m_i^2$$

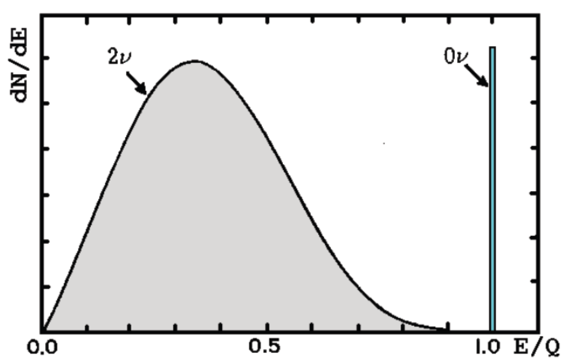
< 1 eV

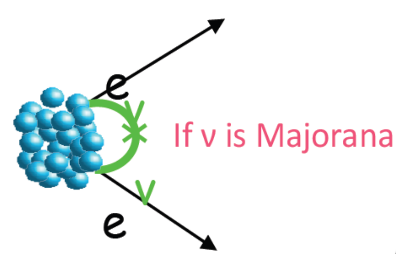
Way 3: Cosmology

Way 2: Neutrinoless ββ decay



double β decay





ν-less double β decay

$$\langle m \rangle_{\beta\beta} \equiv \left| \sum_{i=1}^{3} V_{ei}^2 m_i \right|$$

$$\langle m \rangle_{\beta\beta}^2 = m_e^2 [C_{mm} T_{1/2}^{0\nu}]^{-1}$$

Some Notations

Define the left- and right-handed neutrino fields:

$$\nu_{\rm L} = \begin{pmatrix} \nu_{e{\rm L}} \\ \nu_{\mu{\rm L}} \\ \nu_{\tau{\rm L}} \end{pmatrix}$$ [redacted] $$N_{\rm R} = \begin{pmatrix} N_{1{\rm R}} \\ N_{2{\rm R}} \\ N_{3{\rm R}} \end{pmatrix}$$

Extend the SM's particle content

$$\psi_{\rm L} \equiv \frac{1-\gamma_5}{2}\psi$$
$$\psi_{\rm R} \equiv \frac{1+\gamma_5}{2}\psi$$

Their charge-conjugate counterparts are defined below and transform as right- and left-handed fields, respectively:

$$(\nu_{\rm L})^c \equiv \mathcal{C}\overline{\nu_{\rm L}}^T \, , \qquad (N_{\rm R})^c \equiv \mathcal{C}\overline{N_{\rm R}}^T$$

$$\overline{(\nu_{\rm L})^c} = (\nu_{\rm L})^T \mathcal{C} \, , \qquad \overline{(N_{\rm R})^c} = (N_{\rm R})^T \mathcal{C}$$

$$(\nu_{\rm L})^c = (\nu^c)_{\rm R} \text{ and } (N_{\rm R})^c = (N^c)_{\rm L} \text{ hold}$$

(can be proved easily)

Properties of the charge-conjugation matrix:

$$\mathcal{C}\gamma_\mu^T \mathcal{C}^{-1} = -\gamma_\mu \, , \qquad \mathcal{C}\gamma_5^T \mathcal{C}^{-1} = \gamma_5 \, , \qquad \mathcal{C}^{-1} = \mathcal{C}^\dagger = \mathcal{C}^T = -\mathcal{C}$$

They are from the requirement that the charge-conjugated field must satisfy the same Dirac equation ($\mathcal{C} = i\gamma^2\gamma^0$ in the Dirac representation)

Dirac Mass Term

A Dirac neutrino is described by a 4-component spinor: $\nu = \nu_{\rm L} + N_{\rm R}$

Step 1: the gauge-invariant Dirac mass term and SSB:

$$-\mathcal{L}_{\rm Dirac} = \overline{\ell_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + {\rm h.c.} \Rightarrow -\mathcal{L}'_{\rm Dirac} = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R} + {\rm h.c.}$$

$$M_{\rm D} = Y_\nu \langle H \rangle \text{ with } \langle H \rangle \simeq 174 \text{ GeV}$$

Step 2: basis transformation:

$$V^\dagger M_{\rm D} U = \widehat{M}_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}$$

$$-\mathcal{L}'_{\rm Dirac} = \overline{\nu'_{\rm L}} \widehat{M}_\nu N'_{\rm R} + {\rm h.c.}$$

Mass states link to flavor states:

$$\nu'_{\rm L} = V^\dagger \nu_{\rm L} \text{ and } N'_{\rm R} = U^\dagger N_{\rm R}$$

$$\nu' = \nu'_{\rm L} + N'_{\rm R} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Step 3: physical mass term and kinetic term:

$$-\mathcal{L}'_{\rm Dirac} = \overline{\nu'} \widehat{M}_\nu \nu' = \sum_{i=1}^{3} m_i \overline{\nu_i} \nu_i$$

$$\mathcal{L}_{\rm kinetic} = i\overline{\nu_{\rm L}} \gamma_\mu \partial^\mu \nu_{\rm L} + i\overline{N_{\rm R}} \gamma_\mu \partial^\mu N_{\rm R} = i\overline{\nu'} \gamma_\mu \partial^\mu \nu' = i \sum_{k=1}^{3} \overline{\nu_k} \gamma_\mu \partial^\mu \nu_k$$

Dirac Neutrino Mixing

Weak charged-current interactions of leptons:

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^\mu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

In the flavor basis

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^\mu V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

In the mass basis

Without loss of generality, one may choose mass states=flavor states for charged leptons. So *V* is just the MNSP matrix of neutrino mixing.

Both the mass & CC terms are invariant with respect to a global phase transformation: lepton number (flavor) conservation (violation).

$$l(x) \longrightarrow e^{i\Phi} l(x)$$

$$\nu'_{\rm L}(x) \rightarrow e^{i\Phi} \nu'_{\rm L}(x)$$

$$N'_{\rm R}(x) \rightarrow e^{i\Phi} N'_{\rm R}(x)$$

		e^-	ν_e	e^+	$\overline{\nu}_e$	μ^-	ν_μ	μ^+	$\overline{\nu}_\mu$	τ^-	ν_τ	τ^+	$\overline{\nu}_\tau$
√	L	+1	+1	−1	−1	+1	+1	−1	−1	+1	+1	−1	−1
×	L_e	+1	+1	−1	−1	0	0	0	0	0	0	0	0
×	L_μ	0	0	0	0	+1	+1	−1	−1	0	0	0	0
×	L_τ	0	0	0	0	0	0	0	0	+1	+1	−1	−1

Majorana Mass Term (1)

A Majorana mass term can be obtained by introducing a Higgs triplet into the SM, writing out the gauge-invariant Yukawa interactions and Higgs potentials, and then integrating out heavy degrees of freedom (type-II seesaw mechanism):

$$-\mathcal{L}'_{\rm Majorana} = \frac{1}{2}\overline{\nu_{\rm L}} M_{\rm L} (\nu_{\rm L})^c + {\rm h.c.}$$

The Majorana mass matrix must be a symmetric matrix. It can be diagonalized by a unitary matrix

$$\overline{\nu_L} M_{\rm L} (\nu_L)^c = [\overline{\nu_L} M_{\rm L} (\nu_L)^c]^T = -\overline{\nu_L} \mathcal{C}^T M_{\rm L}^T \overline{\nu_L}^T = \overline{\nu_L} M_{\rm L}^T (\nu_L)^c$$

Diagonalization:

$$V^\dagger M_{\rm L} V^* = \widehat{M}_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}$$

$$-\mathcal{L}'_{\rm Majorana} = \frac{1}{2}\overline{\nu'_{\rm L}} \widehat{M}_\nu (\nu'_{\rm L})^c + {\rm h.c.}$$

$$\nu'_{\rm L} = V^\dagger \nu_{\rm L} \ \text{ and } \ (\nu'_{\rm L})^c = \mathcal{C}\overline{\nu'_{\rm L}}^T$$

$$\nu' = \nu'_{\rm L} + (\nu'_{\rm L})^c = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Physical mass term:

$$-\mathcal{L}'_{\rm Majorana} = \frac{1}{2}\overline{\nu'} \widehat{M}_\nu \nu' = \frac{1}{2}\sum_{i=1}^{3} m_i \overline{\nu_i} \nu_i$$

$$\text{Majorana condition } (\nu')^c = \nu'$$

Majorana Mass Term (2)

Kinetic term (you may prove $\overline{(\psi_{\rm L})^c}\gamma_\mu\partial^\mu(\psi_{\rm L})^c = \overline{\psi_{\rm L}}\gamma_\mu\partial^\mu\psi_{\rm L}$)

$$\mathcal{L}_{\rm kinetic} = i\overline{\nu_{\rm L}}\gamma_\mu\partial^\mu\nu_{\rm L} = i\overline{\nu'_{\rm L}}\gamma_\mu\partial^\mu\nu'_{\rm L} = \frac{i}{2}\overline{\nu'}\gamma_\mu\partial^\mu\nu' = \frac{i}{2}\sum_{k=1}^{3}\overline{\nu_k}\gamma_\mu\partial^\mu\nu_k$$

Question: why is there a factor **1/2** in the **Majorana** mass term?

Answer: it allows us to get the correct **Dirac** equation of motion.

A proof: write out the Lagrangian of free massive **Majorana** neutrinos:

$$\begin{aligned}\mathcal{L}_\nu &= i\overline{\nu_{\rm L}}\gamma_\mu\partial^\mu\nu_{\rm L} - \left[\frac{1}{2}\overline{\nu_{\rm L}}M_{\rm L}(\nu_{\rm L})^c + {\rm h.c.}\right]\\ &= i\overline{\nu'_{\rm L}}\gamma_\mu\partial^\mu\nu'_{\rm L} - \left[\frac{1}{2}\overline{\nu'_{\rm L}}\widehat{M}_\nu(\nu'_{\rm L})^c + {\rm h.c.}\right]\\ &= \frac{1}{2}\left(i\overline{\nu'}\gamma_\mu\partial^\mu\nu' - \overline{\nu'}\widehat{M}_\nu\nu'\right) = -\frac{1}{2}\left(i\partial^\mu\overline{\nu'}\gamma_\mu\nu' + \overline{\nu'}\widehat{M}_\nu\nu'\right)\end{aligned}$$

Euler-Lagrange equation:

$$\partial^\mu\frac{\partial\mathcal{L}_\nu}{\partial\left(\partial^\mu\overline{\nu'}\right)} - \frac{\partial\mathcal{L}_\nu}{\partial\overline{\nu'}} = 0$$

$\Rightarrow$

$$i\gamma_\mu\partial^\mu\nu' - \widehat{M}_\nu\nu' = 0$$

$$i\gamma_\mu\partial^\mu\nu_k - m_k\nu_k = 0$$

Majorana Neutrino Mixing

Weak charged-current interactions of leptons:

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^\mu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

In the flavor basis

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^\mu V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

In the mass basis

The MNSP matrix *V* contains 2 extra CP-violating phases.

Mass and CC terms are not simultaneously invariant under a global phase transformation --- Lepton number violation

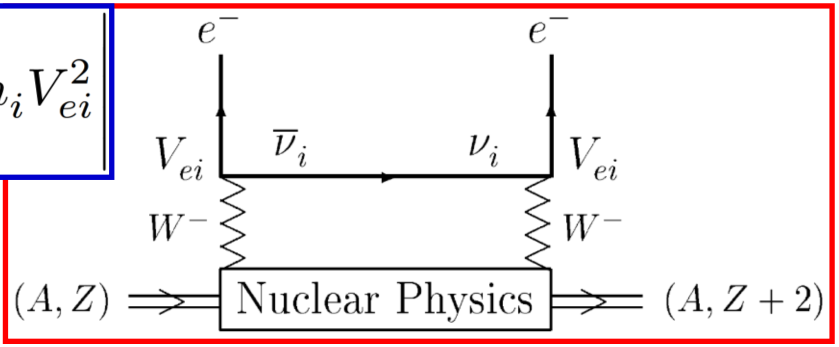
Neutrinoless double-beta decay

$$l(x) \rightarrow e^{i\Phi} l(x)$$

$$\nu'_{\rm L}(x) \rightarrow e^{i\Phi} \nu'_{\rm L}(x)$$

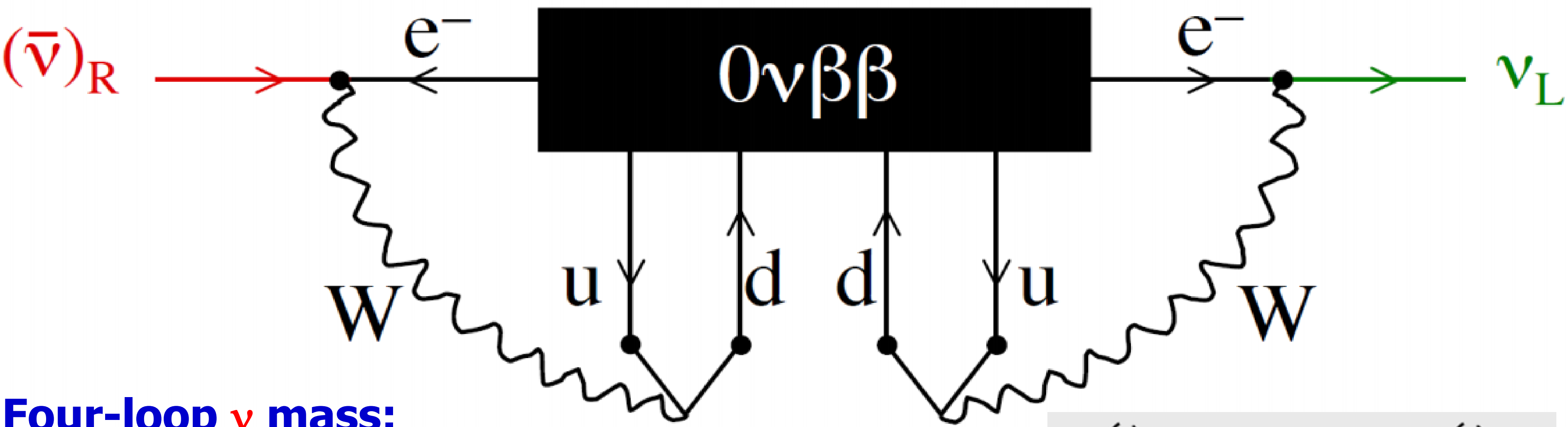
$$\overline{\nu'_{\rm L}} \rightarrow e^{-i\Phi} \overline{\nu'_{\rm L}} \text{ and } (\nu'_{\rm L})^c \rightarrow e^{-i\Phi} (\nu'_{\rm L})^c$$

$$\langle m \rangle_{ee} \equiv \left| \sum_i m_i V_{ei}^2 \right|$$



Schechter-Valle Theorem

THEOREM (1982): if a 0νββ decay happens, there must be an effective Majorana mass term.

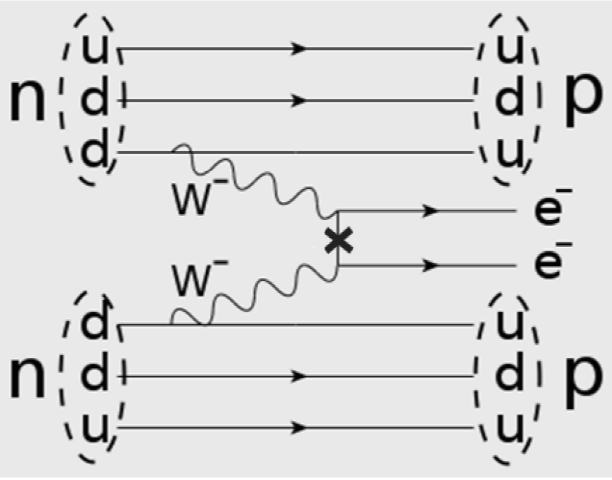


Four-loop ν mass:

$$\delta m_\nu = \mathcal{O}(10^{-24}\,\mathrm{eV})$$

(Duerr, Lindner, Merle, 2011)

Note: The black box can in principle have many different processes (new physics). Only in the simplest case, it is likely to constrain neutrino masses.



Hybrid Mass Term (1)

A hybrid mass term can be written out in terms of the left- and right-handed neutrino fields and their charge-conjugate counterparts:

$$-\mathcal{L}'_{\rm hybrid} = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R} + \frac{1}{2}\overline{\nu_{\rm L}} M_{\rm L} (\nu_{\rm L})^c + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}$$
$$= \frac{1}{2}\begin{bmatrix}\overline{\nu_{\rm L}} & \overline{(N_{\rm R})^c}\end{bmatrix}\begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}\begin{bmatrix}(\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} + {\rm h.c.}\ ,$$

type-(I+II) seesaw

Here we have used

$$\overline{(N_{\rm R})^c} M_{\rm D}^T (\nu_{\rm L})^c = \left[(N_{\rm R})^T \mathcal{C} M_{\rm D}^T \mathcal{C} \overline{\nu_{\rm L}}^T\right]^T = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R}$$

Diagonalization by means of a 6×6 unitary matrix:

$$\begin{pmatrix} V & R \\ S & U\end{pmatrix}^\dagger \begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}\begin{pmatrix} V & R \\ S & U\end{pmatrix}^* = \begin{pmatrix}\widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N\end{pmatrix}$$

$$\widehat{M}_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\},\ \widehat{M}_N \equiv {\rm Diag}\{M_1, M_2, M_3\}$$

Majorana mass states

$$\nu' = \begin{bmatrix}\nu'_{\rm L} \\ (N'_{\rm R})^c\end{bmatrix} + \begin{bmatrix}(\nu'_{\rm L})^c \\ N'_{\rm R}\end{bmatrix} = \begin{pmatrix}\nu_1 \\ \nu_2 \\ \nu_3 \\ N_1 \\ N_2 \\ N_3\end{pmatrix}$$

$$(\nu')^c = \nu'$$

It is actually a Majorana mass term!

$$-\mathcal{L}'_{\rm hybrid} = \frac{1}{2}\begin{bmatrix}\overline{\nu'_{\rm L}} & \overline{(N'_{\rm R})^c}\end{bmatrix}\begin{pmatrix}\widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N\end{pmatrix}\begin{bmatrix}(\nu'_{\rm L})^c \\ N'_{\rm R}\end{bmatrix} + {\rm h.c.}$$

$$\nu'_{\rm L} = V^\dagger \nu_{\rm L} + S^\dagger (N_{\rm R})^c$$

$$N'_{\rm R} = R^T (\nu_{\rm L})^c + U^T N_{\rm R}$$

Hybrid Mass Term (2)

Physical mass term:

$$-\mathcal{L}'_{\text{hybrid}} = \frac{1}{2}\overline{\nu'}\begin{pmatrix}\widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N\end{pmatrix}\nu' = \frac{1}{2}\sum_{i=1}^{3}\left(m_i\overline{\nu_i}\nu_i + M_i\overline{N_i}N_i\right)$$

Kinetic term:

$$\begin{aligned}
\mathcal{L}_{\text{kinetic}} &= i\overline{\nu_{\rm L}}\gamma_\mu\partial^\mu\nu_{\rm L} + i\overline{N_{\rm R}}\gamma_\mu\partial^\mu N_{\rm R} \\
&= \frac{i}{2}\left[\overline{\nu_{\rm L}} \quad \overline{(N_{\rm R})^c}\right]\gamma_\mu\partial^\mu\begin{bmatrix}\nu_{\rm L} \\ (N_{\rm R})^c\end{bmatrix} + \frac{i}{2}\left[\overline{(\nu_{\rm L})^c} \quad \overline{N_{\rm R}}\right]\gamma_\mu\partial^\mu\begin{bmatrix}(\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} \\
&= \frac{i}{2}\left[\overline{\nu'_{\rm L}} \quad \overline{(N'_{\rm R})^c}\right]\gamma_\mu\partial^\mu\begin{pmatrix}V & R \\ S & U\end{pmatrix}^\dagger\begin{pmatrix}V & R \\ S & U\end{pmatrix}\begin{bmatrix}\nu'_{\rm L} \\ (N'_{\rm R})^c\end{bmatrix} \\
&\quad + \frac{i}{2}\left[\overline{(\nu'_{\rm L})^c} \quad \overline{N'_{\rm R}}\right]\gamma_\mu\partial^\mu\begin{pmatrix}V & R \\ S & U\end{pmatrix}^T\begin{pmatrix}V & R \\ S & U\end{pmatrix}^*\begin{bmatrix}(\nu'_{\rm L})^c \\ N'_{\rm R}\end{bmatrix} \\
&= \frac{i}{2}\left[\overline{\nu'_{\rm L}} \quad \overline{(N'_{\rm R})^c}\right]\gamma_\mu\partial^\mu\begin{bmatrix}\nu'_{\rm L} \\ (N'_{\rm R})^c\end{bmatrix} + \frac{i}{2}\left[\overline{(\nu'_{\rm L})^c} \quad \overline{N'_{\rm R}}\right]\gamma_\mu\partial^\mu\begin{bmatrix}(\nu'_{\rm L})^c \\ N'_{\rm R}\end{bmatrix} \\
&= i\overline{\nu'_{\rm L}}\gamma_\mu\partial^\mu\nu'_{\rm L} + i\overline{N'_{\rm R}}\gamma_\mu\partial^\mu N'_{\rm R} \\
&= \frac{i}{2}\overline{\nu'}\gamma_\mu\partial^\mu\nu' = \frac{i}{2}\sum_{k=1}^{3}\left(\overline{\nu_k}\gamma_\mu\partial^\mu\nu_k + \overline{N_k}\gamma_\mu\partial^\mu N_k\right)
\end{aligned}$$

Non-unitary Flavor Mixing

Weak charged-current interactions of leptons:

In the flavor basis

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^{\mu} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm L} W^{-}_{\mu} + {\rm h.c.}$$

$$\nu_{\rm L} = V\nu'_{\rm L} + R(N'_{\rm R})^c$$

In the mass basis

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^{\mu} \left[V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W^{-}_{\mu} + {\rm h.c.}$$

V **= non-unitary light neutrino mixing (MNSP) matrix** $VV^\dagger + RR^\dagger = 1$

R **= light-heavy neutrino mixing (CC interactions of heavy neutrinos)**

Neutrino oscillations ***V*** ⟷ ***R*** **Collider signatures**

TeV seesaws may bridge the gap between neutrino & collider physics.

Seesaw Mechanisms (1)

A hybrid neutrino mass Lagrangian may contain three distinct terms:

$$-\mathcal{L}'_{\rm hybrid} = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R} + \frac{1}{2}\overline{\nu_{\rm L}} M_{\rm L} (\nu_{\rm L})^c + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}$$
$$= \frac{1}{2}\left[\overline{\nu_{\rm L}} \quad \overline{(N_{\rm R})^c}\right] \begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M^T_{\rm D} & M_{\rm R} \end{pmatrix} \begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R} \end{bmatrix} + {\rm h.c.} \; ,$$

- ♣ **Normal Dirac mass term, proportional to the scale of electroweak symmetry breaking (~ 174 GeV);**
- ♣ **Light Majorana mass term, violating the SM gauge symmetry and having a scale much lower than 174 GeV;**
- ♣ **Heavy Majorana mass term, originating from the SU(2)_L singlet and having a scale much higher than 174 GeV.**

A strong hierarchy of 3 mass scales allows us to make approximation

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M^T_{\rm D} & M_{\rm R} \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N \end{pmatrix}$$

Seesaw Mechanisms (2)

The above unitary transformation leads to the following relationships:

$$R\widehat{M}_N = M_{\rm L} R^* + M_{\rm D} U^*$$
$$S\widehat{M}_\nu = M_{\rm D}^T V^* + M_{\rm R} S^*$$

$$M_{\rm R} \gg M_{\rm D} \gg M_{\rm L}$$
$$R \sim S \sim \mathcal{O}(M_{\rm D}/M_{\rm R})$$

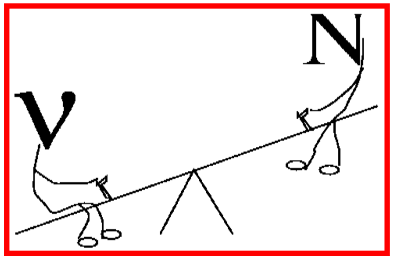
$$U\widehat{M}_N = M_{\rm R} U^* + M_{\rm D}^T R^*$$
$$V\widehat{M}_\nu = M_{\rm L} V^* + M_{\rm D} S^*$$

$$U\widehat{M}_N U^T = M_{\rm R}(UU^\dagger)^T + M_{\rm D}^T(R^* U^T) \approx M_{\rm R} \; ,$$
$$V\widehat{M}_\nu V^T = M_{\rm L}(VV^\dagger)^T + M_{\rm D}(S^* V^T) \approx M_{\rm L} + M_{\rm D}(S^* V^T)$$

$$S^* V^T = M_{\rm R}^{-1} S\widehat{M}_\nu V^T - M_{\rm R}^{-1} M_{\rm D}^T (VV^\dagger)^T \approx -M_{\rm R}^{-1} M_{\rm D}^T$$

Then we arrive at the type-(I+II) seesaw formula:

$$M_\nu \equiv V\widehat{M}_\nu V^T \approx M_{\rm L} - M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$$



Type-I seesaw limit: $M_\nu \approx -M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$ (**Fritzsch, Gell-Mann, Minkowski, 1975; Minkowski, 1977; ...**)

Type-II seesaw limit: $M_\nu = M_{\rm L}$ (**Konetschny, Kummer, 1977; ...**)

History of Seesaw

The seesaw idea originally appeared in a paper's footnote.



Seesaw—A Footnote Idea:

H. Fritzsch, M. Gell-Mann, P. Minkowski, PLB 59 (1975) 256

This idea was very clearly elaborated by Minkowski in Phys. Lett. B 67 (1977) 421 ---- but it was unjustly forgotten until 2004.

The idea was later on embedded into the GUT frameworks in 1979 and 1980:

- T. Yanagida 1979
- M. Gell-Mann, P. Ramond, R. Slansky 1979
- S. Glashow 1979
- R. Mohapatra, G. Senjanovic 1980

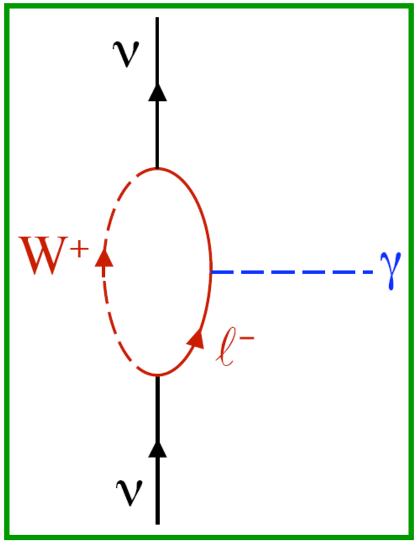
It was Yanagida who named this mechanism as "seesaw".

What is History?

History is a set of lies agreed upon

Electromagnetic Properties

A neutrino does not have electric charges, but it has electromagnetic interactions with the photon via quantum loops.



Given the SM interactions, a massive Dirac neutrino can only have a tiny magnetic dipole moment:

$$\mu_\nu \sim \frac{3eG_F}{8\sqrt{2}\pi^2} m_\nu = 3\times10^{-20} \frac{m_\nu}{0.1\,\mathrm{eV}} \mu_B$$

A massive Majorana neutrino can not have magnetic & electric dipole moments, as its antiparticle is itself.

Proof: Dirac neutrino's electromagnetic vertex can be parametrized as

$$\Gamma_\mu(p,p') = f_Q(q^2)\gamma_\mu + f_M(q^2) i\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)\left(q^2\gamma_\mu - q_\mu q^\nu\gamma_\nu\right)\gamma_5$$

Majorana neutrinos

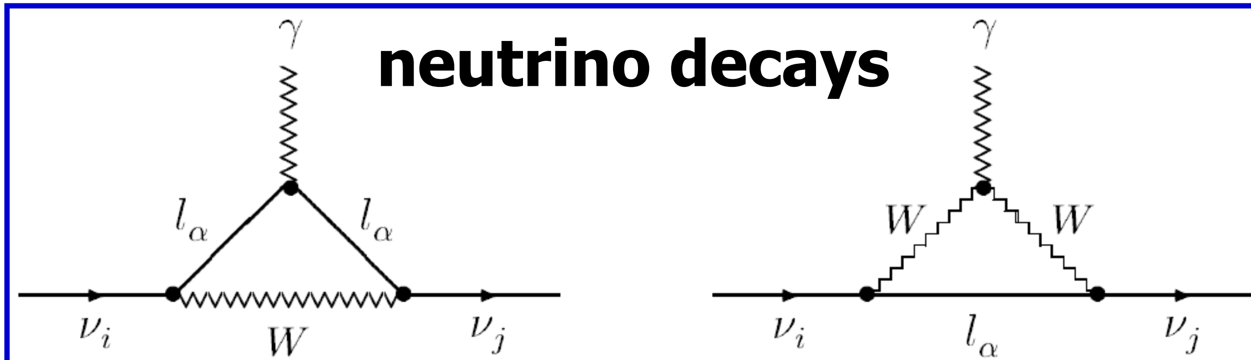
$$\overline{\psi}\Gamma_\mu\psi = \overline{\psi^c}\Gamma_\mu\psi^c = \psi^T\mathcal{C}\Gamma_\mu\mathcal{C}\overline{\psi}^T = \left(\psi^T\mathcal{C}\Gamma_\mu\mathcal{C}\overline{\psi}^T\right)^T = -\overline{\psi}\mathcal{C}^T\Gamma_\mu^T\mathcal{C}^T\psi = \overline{\psi}\mathcal{C}\Gamma_\mu^T\mathcal{C}^{-1}\psi$$

$\Rightarrow$ $f_Q(q^2) = f_M(q^2) = f_E(q^2) = 0$ **intrinsic property of Majorana ν's.**

Transition Dipole Moments

Both Dirac & Majorana neutrinos can have *transition* dipole moments (of a size comparable with μ_ν) that may give rise to neutrino decays, scattering with electrons, interactions with external magnetic field & contributions to ν masses. (Data: < a few × 10^-11 Bohr magneton).

neutrino decays

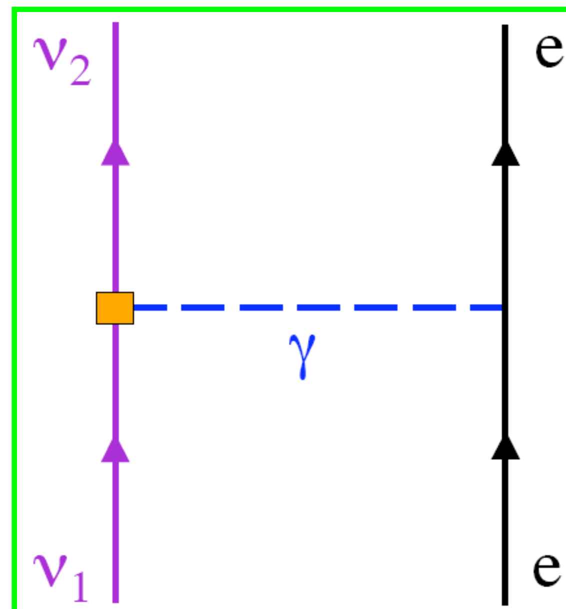


$$\mu_{\rm eff} \equiv \sqrt{|\mu_{ij}|^2 + |\epsilon_{ij}|^2}$$

$$\Gamma_{\nu_i \to \nu_j + \gamma} = 5.3 \times \left(1 - \frac{m_j^2}{m_i^2}\right)^3 \left(\frac{m_i}{1\ {\rm eV}}\right)^3 \left(\frac{\mu_{\rm eff}}{\mu_{\rm B}}\right)^2 {\rm s}^{-1}$$

$$\frac{{\rm d}\sigma'_\mu}{{\rm d}T} = \frac{\alpha^2 \pi}{m_e^2} \sum_{k=1}^{3} \left| \sum_{j=1}^{3} e^{iq_j L} V_{ej} \left(i\frac{\mu_{jk}}{\mu_{\rm B}} + \frac{\epsilon_{jk}}{\mu_{\rm B}} \right) \right|^2 \left(\frac{1}{T} - \frac{1}{E_\nu} \right)$$

scattering



Summary of Lecture B

(A) Three reasons for neutrinos to be massless in the SM.

(B) The Dirac mass term and lepton number conservation.

(C) The Majorana mass term and lepton number violation.
---- the Majorana mass matrix must be symmetric;
---- factor 1/2 in front of the mass term makes sense.

(D) The hybrid mass term and seesaw mechanisms.
---- light and heavy neutrinos are Majorana particles;
---- the 3×3 light flavor mixing matrix is non-unitary;
---- light neutrino masses: the type-(I+II) seesaw.

(E) Electromagnetic dipole moment of massive neutrinos.
---- Dirac neutrinos have magnetic dipole moments;
---- Majorana neutrinos have no dipole moments;
---- Dirac & Majorana neutrinos: transition moments.