

Physics of Massive Neutrinos

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- A:** Neutrino history and lepton family
- B:** Neutrino masses and flavor mixing
- C:** Description of neutrino oscillations
- D:** Implication of neutrino oscillations
- E:** θ_{13} hint and sterile neutrino puzzle
- F:** On TeV neutrino physics at the LHC
- G:** Extreme corners in the neutrino sky
- H:** Cosmic baryon number asymmetry

@Lectures at CP3, UCL, Louvain-la-Neuve, Belgium, Jan 2012

Lecture C

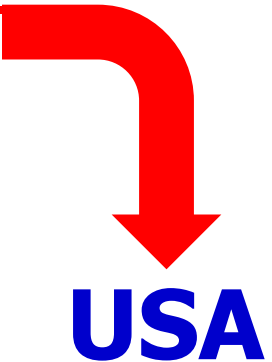
- ★ **The 3×3 Neutrino Mixing Matrix**
- ★ **Neutrino Oscillations in Vacuum**
- ★ **Neutrino Oscillations in Matter**

12 Known Flavors

3

Discoveries of lepton flavors, quark flavors and CP violation

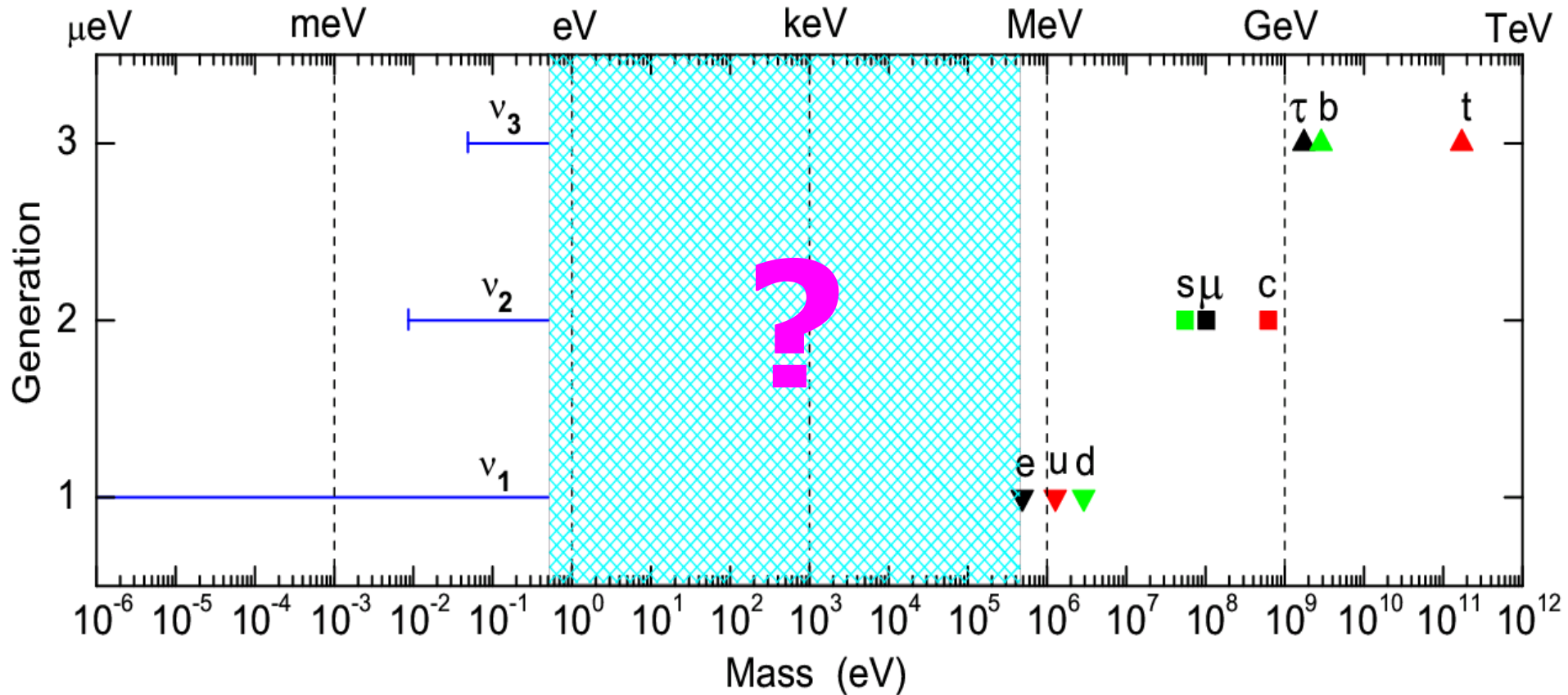
1897	electron (Thomson, 1897)
1919	proton (up and down quarks) (Rutherford, 1919)
1932	neutron (up and down quarks) (Chadwick, 1932)
1933	positron (Anderson, 1933)
1937	muon (Neddermeyer and Anderson, 1937)
1947	Kaon (strange quark) (Rochester and Butler, 1947)
1956	electron antineutrino (Cowan <i>et al.</i> , 1956)
1962	muon neutrino (Danby <i>et al.</i> , 1962)
1964	CP violation in s -quark decays (Christenson <i>et al.</i> , 1964)
1974	charm quark (Aubert <i>et al.</i> , 1974; Abrams <i>et al.</i> , 1974)
1975	tau (Perl <i>et al.</i> , 1975)
1977	bottom quark (Herb <i>et al.</i> , 1977)
1995	top quark (Abe <i>et al.</i> , 1995; Abachi <i>et al.</i> , 1995)
2000	tau neutrino (Kodama <i>et al.</i> , 2000)
2001	CP violation in b -quark decays (Aubert <i>et al.</i> , 2001; Abe <i>et al.</i> , 2001)



Flavor Puzzles

4

Hierarchy + Desert + Mixing + CP Violation



soooooooooo strange

Tiny neutrino masses should have a different origin ---- seesaws?

Flavor Mixing

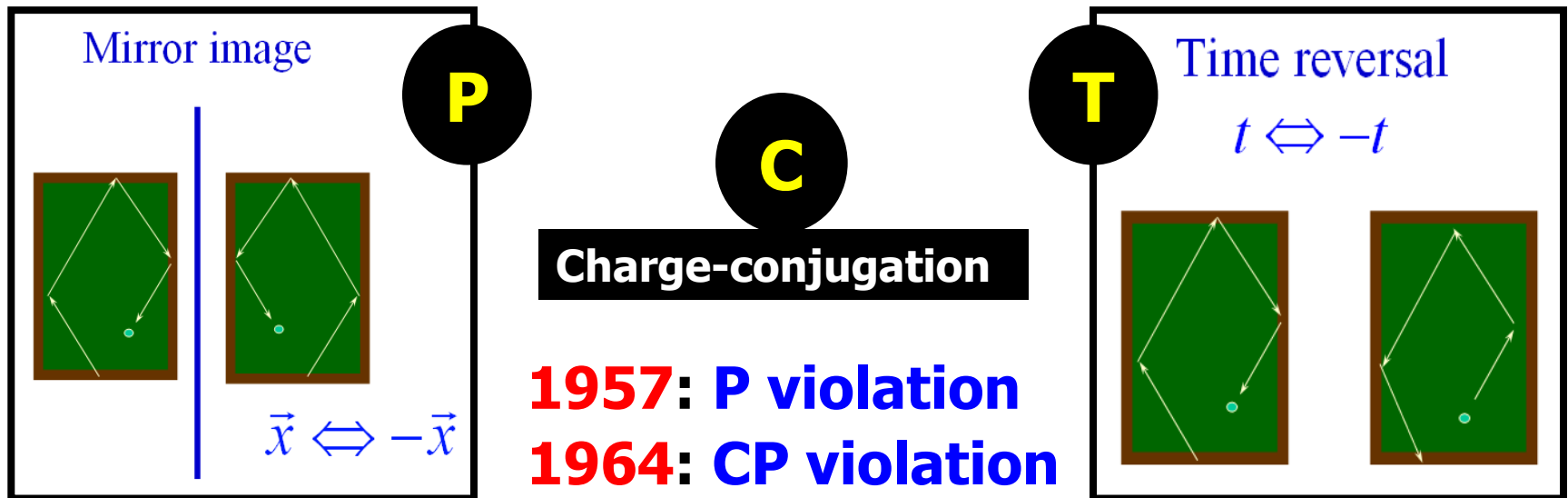
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Flavor mixing: mismatch between **weak/flavor** eigenstates and **mass** eigenstates of fermions due to coexistence of **2** types of interactions.

Weak eigenstates: members of weak isospin doublets transforming into each other through the interaction with the ***W*** boson;

Mass eigenstates: states of definite masses that are created by the interaction with the Higgs boson (**Yukawa** interactions).

CP violation: **matter** and **antimatter**, or a reaction & its CP-conjugate process, are distinguishable --- coexistence of **2** types of interactions.

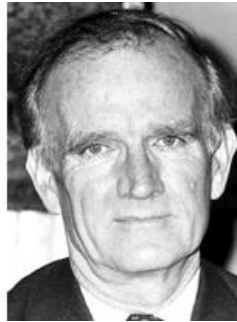


Towards the KM Paper

6

1964: Discovery of CP violation in K decays
(J.W. Cronin, Val L. Fitch)

NP 1980



1967: Sakharov conditions for cosmological
matter-antimatter asymmetry (A. Sakharov)

NP 1975



1967: The birth of the standard electroweak
model (S. Weinberg)

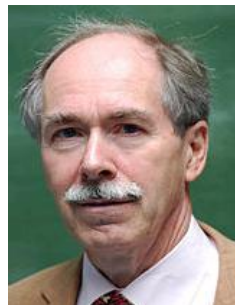
0 citation for the first 4 yrs

NP 1979



1971: The first proof of the renormalizability
of the standard model (G. 't Hooft)

NP 1999



KM in 1972

7

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)



In a framework of the renormalizable theory of weak interaction, problems of *CP*-violation are studied. It is concluded that no realistic models of *CP*-violation exist in the quartet scheme without introducing any other new fields. Some possible models of *CP*-violation are also discussed.

3 families + CP violation: Masukawa's bathtub idea!



Diagnosis of CP Violation 8

In the minimal ν SM (SM+**3** right-handed ν 's) the **Kobayashi-Maskawa** mechanism is responsible for **CP** violation.

$$\mathcal{L}_{\nu\text{SM}} = \mathcal{L}_G + \mathcal{L}_H + \mathcal{L}_F + \mathcal{L}_Y$$

$$\mathcal{L}_G = -\frac{1}{4} (W^{i\mu\nu} W_{\mu\nu}^i + B^{\mu\nu} B_{\mu\nu})$$

$$\mathcal{L}_H = (D^\mu H)^\dagger (D_\mu H) - \mu^2 H^\dagger H - \lambda (H^\dagger H)^2$$

$$\mathcal{L}_F = \overline{Q}_L i \not{D} Q_L + \overline{\ell}_L i \not{D} \ell_L + \overline{U}_R i \not{D}' U_R + \overline{D}_R i \not{D}' D_R + \overline{E}_R i \not{D}' E_R + \overline{N}_R i \not{D}' N_R$$

$$\mathcal{L}_Y = -\overline{Q}_L Y_u \tilde{H} U_R - \overline{Q}_L Y_d H D_R - \overline{\ell}_L Y_l H E_R - \overline{\ell}_L Y_\nu \tilde{H} N_R + \text{h.c.}$$



Nobel Prize 2008

ν 's Dirac mass

The strategy of diagnosis:

given proper **CP** transformations of the gauge, Higgs and fermion fields, one may prove that the **1st**, **2nd** and **3rd** terms are formally invariant, and the **4th** term can be invariant only if the corresponding **Yukawa coupling matrices** are real. (**spontaneous symmetry breaking** doesn't affect **CP**.)

CP Transformations

9

Gauge fields:

$$[B_\mu, W_\mu^1, W_\mu^2, W_\mu^3] \xrightarrow{\text{CP}} [-B^\mu, -W^{1\mu}, +W^{2\mu}, -W^{3\mu}]$$

Higgs fields:

$$[B_{\mu\nu}, W_{\mu\nu}^1, W_{\mu\nu}^2, W_{\mu\nu}^3] \xrightarrow{\text{CP}} [-B^{\mu\nu}, -W^{1\mu\nu}, +W^{2\mu\nu}, -W^{3\mu\nu}]$$

$$H(t, \mathbf{x}) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow{\text{CP}} H^*(t, -\mathbf{x}) = \begin{pmatrix} \phi^- \\ \phi^{0*} \end{pmatrix}$$

Lepton or quark fields:

$$\overline{\psi}_1 \gamma_\mu (1 \pm \gamma_5) \psi_2 \xrightarrow{\text{CP}} -\overline{\psi}_2 \gamma^\mu (1 \pm \gamma_5) \psi_1$$

$$\overline{\psi}_1 \gamma_\mu (1 \pm \gamma_5) \partial^\mu \psi_2 \xrightarrow{\text{CP}} \overline{\psi}_2 \gamma^\mu (1 \pm \gamma_5) \partial_\mu \psi_1$$

Spinor bilinears:

	$\overline{\psi}_1 \psi_2$	$i\overline{\psi}_1 \gamma_5 \psi_2$	$\overline{\psi}_1 \gamma_\mu \psi_2$	$\overline{\psi}_1 \gamma_\mu \gamma_5 \psi_2$	$\overline{\psi}_1 \sigma_{\mu\nu} \psi_2$
C	$\psi_2 \psi_1$	$i\psi_2 \gamma_5 \psi_1$	$-\psi_2 \gamma_\mu \psi_1$	$\psi_2 \gamma_\mu \gamma_5 \psi_1$	$-\psi_2 \sigma_{\mu\nu} \psi_1$
P	$\overline{\psi}_1 \psi_2$	$-i\overline{\psi}_1 \gamma_5 \psi_2$	$\overline{\psi}_1 \gamma^\mu \psi_2$	$-\overline{\psi}_1 \gamma^\mu \gamma_5 \psi_2$	$\overline{\psi}_1 \sigma^{\mu\nu} \psi_2$
T	$\overline{\psi}_1 \psi_2$	$-i\overline{\psi}_1 \gamma_5 \psi_2$	$\overline{\psi}_1 \gamma^\mu \psi_2$	$\overline{\psi}_1 \gamma^\mu \gamma_5 \psi_2$	$-\overline{\psi}_1 \sigma^{\mu\nu} \psi_2$
CP	$\psi_2 \psi_1$	$-i\psi_2 \gamma_5 \psi_1$	$-\overline{\psi}_2 \gamma^\mu \psi_1$	$-\overline{\psi}_2 \gamma^\mu \gamma_5 \psi_1$	$-\overline{\psi}_2 \sigma^{\mu\nu} \psi_1$
CPT	$\overline{\psi}_2 \psi_1$	$i\overline{\psi}_2 \gamma_5 \psi_1$	$-\overline{\psi}_2 \gamma_\mu \psi_1$	$-\overline{\psi}_2 \gamma_\mu \gamma_5 \psi_1$	$\overline{\psi}_2 \sigma_{\mu\nu} \psi_1$

$\mathcal{L}_G$

$\mathcal{L}_H$

$\mathcal{L}_F$

formally invariant
under CP

CP Violation

10

The **Yukawa** interactions of fermions are **formally invariant** under **CP** if and only if

$$\begin{aligned} Y_u &= Y_u^* , & Y_d &= Y_d^* \\ Y_l &= Y_l^* , & Y_\nu &= Y_\nu^* \end{aligned}$$

If the effective **Majorana** mass term is added into the SM, then the **Yukawa** interactions of leptons can be **formally invariant** under **CP** if

$$M_L = M_L^* , \quad Y_l = Y_l^*$$

If the **flavor states** are transformed into the **mass states**, the source of flavor mixing and **CP** violation will show up in the **CC** interactions:

quarks

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \overline{(u \ c \ t)}_L \gamma^\mu U \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ + \text{h.c.}$$

leptons

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \overline{(e \ \mu \ \tau)}_L \gamma^\mu V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W_\mu^- + \text{h.c.}$$

Comment A: **CP** violation exists since fermions interact with both the **gauge bosons** and the **Higgs boson**.

Comment B: only the **CC** interactions have so far been verified.

Comment C: the **CKM** matrix ***U*** is unitary, the **MNSP** matrix ***V*** is too?

Parameter Counting

11

The **3**×**3** unitary matrix **V** can always be parametrized as a product of **3** unitary rotation matrices in the complex planes:

$$\begin{aligned} O_1(\theta_1, \alpha_1, \beta_1, \gamma_1) &= \begin{pmatrix} c_1 e^{i\alpha_1} & s_1 e^{-i\beta_1} & 0 \\ -s_1 e^{i\beta_1} & c_1 e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{i\gamma_1} \end{pmatrix} \\ O_2(\theta_2, \alpha_2, \beta_2, \gamma_2) &= \begin{pmatrix} e^{i\gamma_2} & 0 & 0 \\ 0 & c_2 e^{i\alpha_2} & s_2 e^{-i\beta_2} \\ 0 & -s_2 e^{i\beta_2} & c_2 e^{-i\alpha_2} \end{pmatrix} \\ O_3(\theta_3, \alpha_3, \beta_3, \gamma_3) &= \begin{pmatrix} c_3 e^{i\alpha_3} & 0 & s_3 e^{-i\beta_3} \\ 0 & e^{i\gamma_3} & 0 \\ -s_3 e^{i\beta_3} & 0 & c_3 e^{-i\alpha_3} \end{pmatrix} \end{aligned}$$

where $s_i \equiv \sin \theta_i$ and $c_i \equiv \cos \theta_i$ (for $i = 1, 2, 3$)

Category A: **3** possibilities

$$V = O_i O_j O_i \quad (i \neq j)$$

Category B: **6** possibilities

$$V = O_i O_j O_k \quad (i \neq j \neq k)$$

Phases

12

For instance, the standard parametrization is given below:

V

$$\begin{aligned}
 &= \begin{pmatrix} e^{i\gamma_2} & 0 & 0 \\ 0 & c_2 e^{i\alpha_2} & s_2 e^{-i\beta_2} \\ 0 & -s_2 e^{i\beta_2} & c_2 e^{-i\alpha_2} \end{pmatrix} \begin{pmatrix} c_3 e^{i\alpha_3} & 0 & s_3 e^{-i\beta_3} \\ 0 & e^{i\gamma_3} & 0 \\ -s_3 e^{i\beta_3} & 0 & c_3 e^{-i\alpha_3} \end{pmatrix} \begin{pmatrix} c_1 e^{i\alpha_1} & s_1 e^{-i\beta_1} & 0 \\ -s_1 e^{i\beta_1} & c_1 e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{i\gamma_1} \end{pmatrix} \\
 &= \begin{pmatrix} c_1 c_3 e^{i(\alpha_1 + \gamma_2 + \alpha_3)} & s_1 c_3 e^{i(-\beta_1 + \gamma_2 + \alpha_3)} & s_3 e^{i(\gamma_1 + \gamma_2 - \beta_3)} \\ -s_1 c_2 e^{i(\beta_1 + \alpha_2 + \gamma_3)} - c_1 s_2 s_3 e^{i(\alpha_1 - \beta_2 + \beta_3)} & c_1 c_2 e^{i(-\alpha_1 + \alpha_2 + \gamma_3)} - s_1 s_2 s_3 e^{i(-\beta_1 - \beta_2 + \beta_3)} & s_2 c_3 e^{i(\gamma_1 - \beta_2 - \alpha_3)} \\ s_1 s_2 e^{i(\beta_1 + \beta_2 + \gamma_3)} - c_1 c_2 s_3 e^{i(\alpha_1 - \alpha_2 + \beta_3)} & -c_1 s_2 e^{i(-\alpha_1 + \beta_2 + \gamma_3)} - s_1 c_2 s_3 e^{i(-\beta_1 - \alpha_2 + \beta_3)} & c_2 c_3 e^{i(\gamma_1 - \alpha_2 - \alpha_3)} \end{pmatrix} \\
 &= \begin{pmatrix} e^{ia} & 0 & 0 \\ 0 & e^{ib} & 0 \\ 0 & 0 & e^{ic} \end{pmatrix} \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix} \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix}
 \end{aligned}$$

$$a = (\alpha_1 - \beta_1) - (\alpha_2 + \beta_2 - \gamma_2) - \gamma_3, \quad b = -\beta_2 - \alpha_3, \quad c = -\alpha_2 - \alpha_3;$$

$$x = \beta_1 + (\alpha_2 + \beta_2) + (\alpha_3 + \gamma_3), \quad y = -\alpha_1 + (\alpha_2 + \beta_2) + (\alpha_3 + \gamma_3), \quad z = \gamma_1.$$

Physical Phases

13

If neutrinos are **Dirac** particles, the phases **x**, **y** and **z** can be removed. Then the neutrino mixing matrix is

Dirac neutrino mixing matrix

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

If neutrinos are **Majorana** particles, left- and right-handed fields are correlated. Hence only a common phase of three left-handed fields can be redefined (e.g., **z = 0**). Then

Majorana neutrino mixing matrix

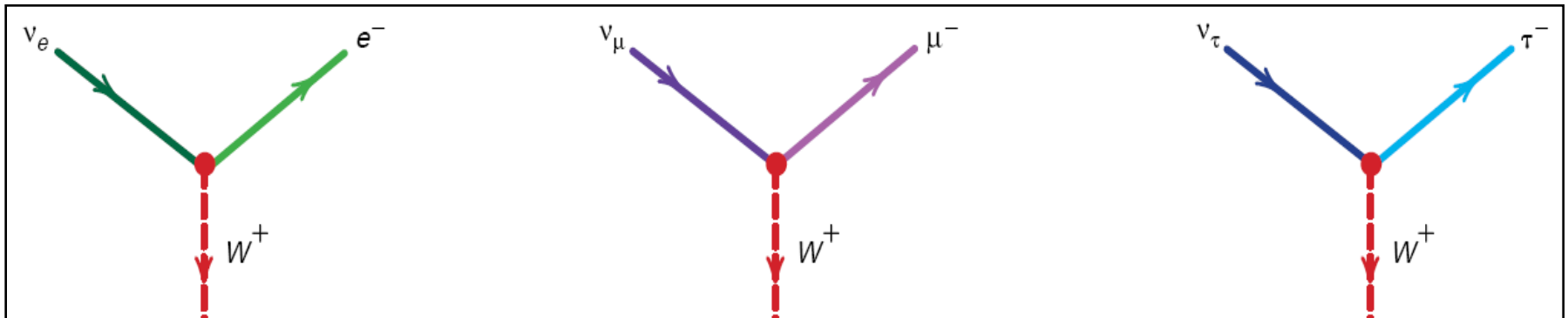
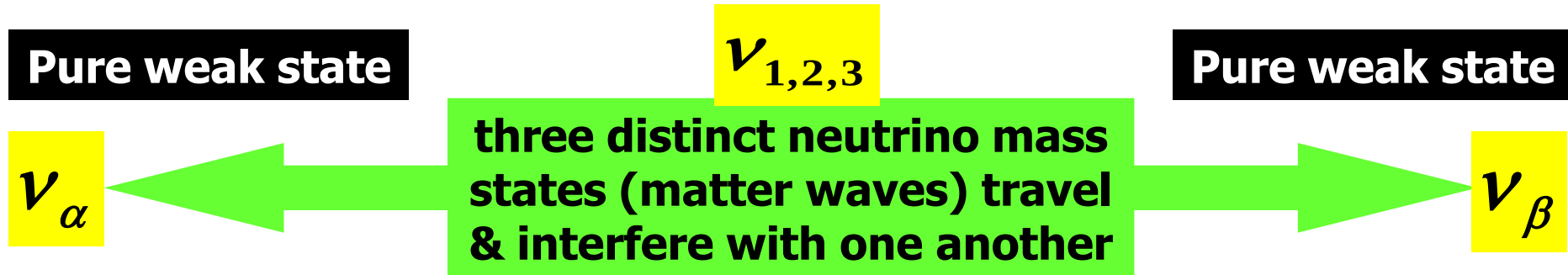
$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

What is Oscillation?

14

Oscillation — a spontaneous periodic change from one neutrino flavor state to another, is a spectacular quantum phenomenon. It can occur as a natural consequence of neutrino mixing.

In a neutrino oscillation experiment, the neutrino beam is produced and detected via the weak **charged-current interactions**.



For example : $\bar{\nu}_e$ beam : β decay; ν_μ beam : π decay; ν_τ beam : D decay

How to Calculate?

15

Boris Kayser ([hep-ph/0506165](#)): This change of neutrino flavor is a quintessentially quantum-mechanical effect. Indeed, it entails some quantum-mechanical **subtleties** that are still debated to this day. However, there is little debate about the **"bottom line"** ----- the expression for the flavor-change probability.....

Typical References:

- ♣ Giunti, Kim, "Fundamentals of Neutrino Physics and Astrophysics" (2007)
- ♣ Cohen, Glashow, Ligeti: "Disentangling Neutrino Oscillations" (0810.4602)
- ♣ Akhmedov, Smirnov: "Paradoxes of Neutrino Oscillations" (0905.1903)

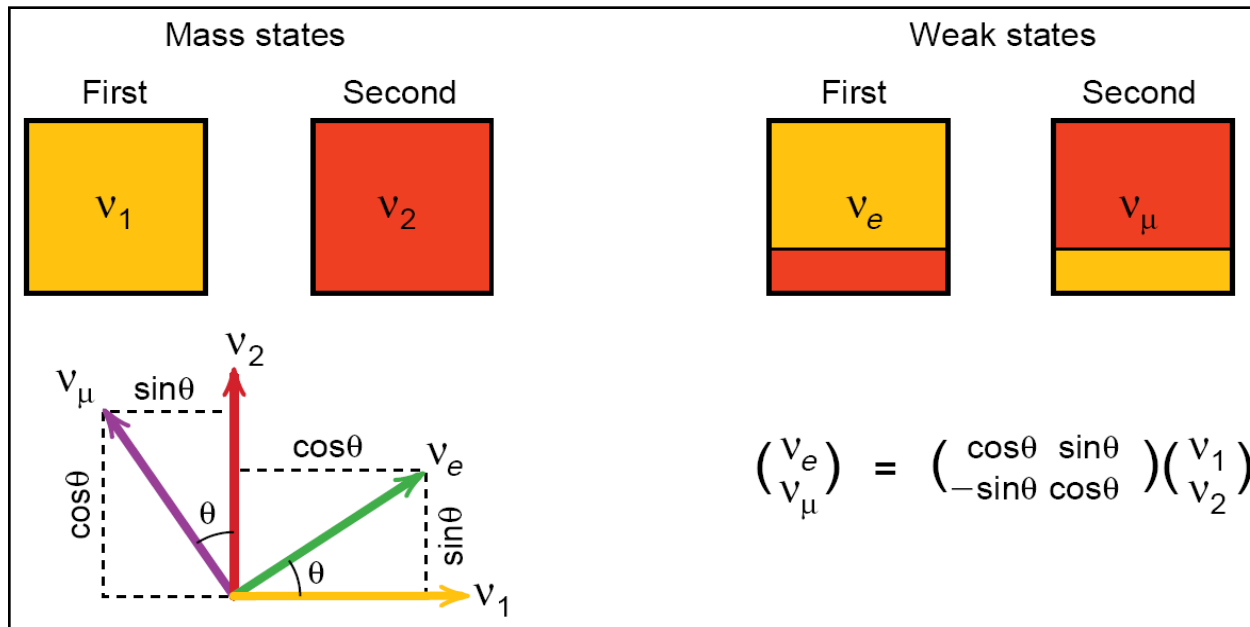
Our strategy: follow the simplest way (which is conceptually ill) to derive the **"bottom line" of neutrino oscillations:** the leading-order formula of neutrino oscillations in phenomenology.



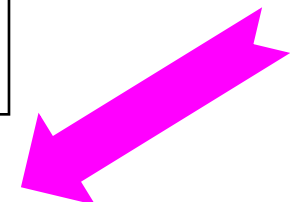
2-flavor Oscillation (1)

16

For simplicity, we consider **two-flavor** neutrino mixing and oscillation:



Approximation:
a plane wave
with a common
momentum for
each mass state



$$\begin{aligned}
 |\nu_\mu(0)\rangle &= |\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle \\
 |\nu_\mu(t)\rangle &= -\sin\theta e^{-iE_1 t}|\nu_1\rangle + \cos\theta e^{-iE_2 t}|\nu_2\rangle \\
 &= e^{-iE_1 t} \left(-\sin\theta|\nu_1\rangle + \cos\theta e^{-i\Delta E t}|\nu_2\rangle \right)
 \end{aligned}$$

$$\begin{aligned}
 \Delta E &\equiv E_2 - E_1 = \sqrt{p^2 + m_2^2} - \sqrt{p^2 + m_1^2} \\
 &\approx \left(p + \frac{m_2^2}{2p} \right) - \left(p + \frac{m_1^2}{2p} \right) \approx \frac{\Delta m^2}{2E}
 \end{aligned}$$

$$\Delta m^2 \equiv m_2^2 - m_1^2, \quad E \approx p \gg m_{1,2} \text{ (relativistic neutrino beam)}, \quad \hbar = c = 1 \text{ (natural units)}$$

2-flavor Oscillation (2)

17

The oscillation probability for **appearance** ν experiments:

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) &= |\langle \nu_e | \nu_\mu(t) \rangle|^2 = |(\cos \theta \langle \nu_1 | + \sin \theta \langle \nu_2 |) (-\sin \theta |\nu_1\rangle + \cos \theta e^{-i\Delta Et} |\nu_2\rangle)|^2 \\ &= |\sin \theta \cos \theta (1 - e^{-i\Delta Et})|^2 = 2 (\sin \theta \cos \theta)^2 \left(1 - \cos \frac{\Delta m^2 t}{2E}\right) \\ &= \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E} \end{aligned}$$

The **conversion** and **survival** probabilities in realistic units:

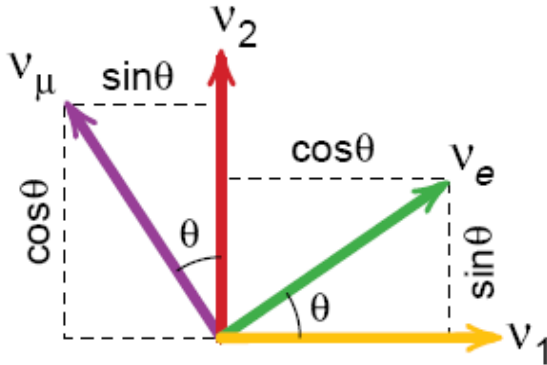
$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) &= \sin^2 2\theta \sin^2 \frac{1.27 \Delta m^2 L}{E} \\ P(\nu_\mu \rightarrow \nu_\mu) &= 1 - \sin^2 2\theta \sin^2 \frac{1.27 \Delta m^2 L}{E} \end{aligned}$$

Due to the smallness of (1,3) mixing, both **solar & **atmospheric** neutrino oscillations are roughly the 2-flavor oscillation.**

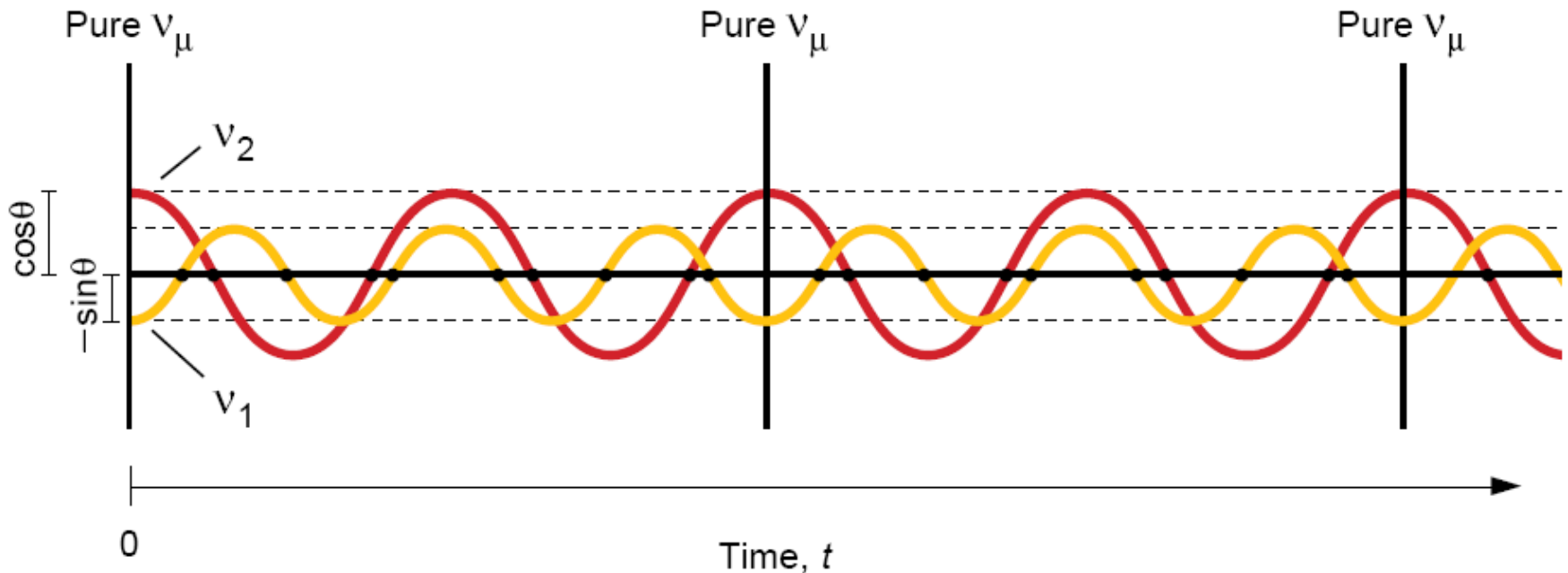
Δm^2 in unit of eV^2 , L in unit of km , E in unit of GeV

2-flavor Oscillation (3)

18

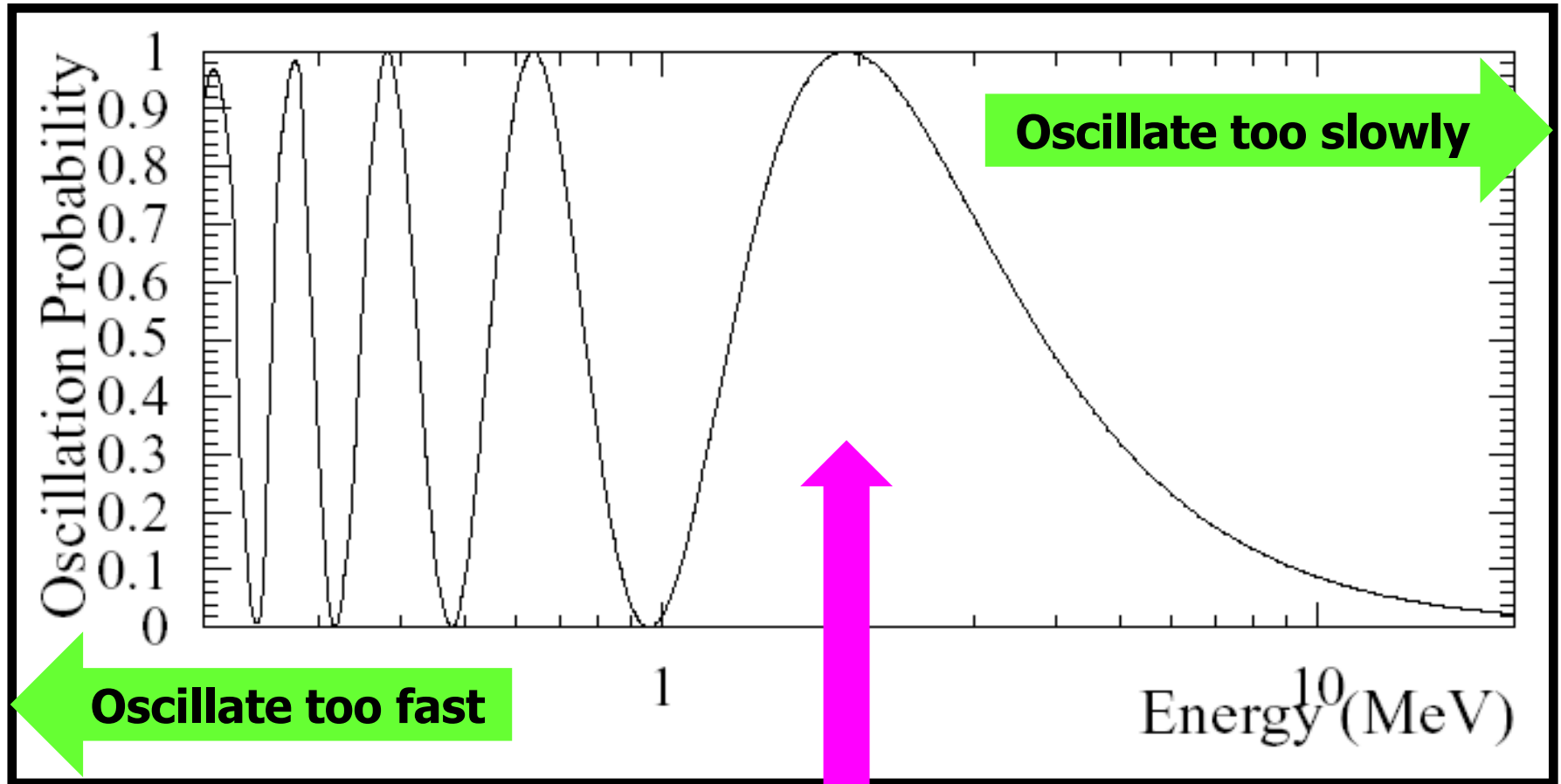


$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2\left(\frac{1.27 \Delta m^2 x}{E_\nu}\right)$$



2-flavor Oscillation (4)

19



$$P(\nu_e \rightarrow \nu_\mu) = |\langle \nu_\mu | \nu(t) \rangle|^2 = \sin^2 2\theta \sin^2 \left(\frac{1.27 \Delta m^2 L}{E} \right)$$

Exercise: Why 1.27 ?

20

	Natural units	Realistic units
Phase factors	$\exp(-iE_{1,2}t)$	$\exp\left(-i\frac{E_{1,2}}{\hbar}t\right)$
Energies and momentum	$E_{1,2} = \sqrt{p^2 + m_{1,2}^2}$	$E_{1,2} = \sqrt{p^2c^2 + m_{1,2}^2c^4}$
Energy difference	$\Delta E = \frac{\Delta m^2}{2E}$	$\Delta E = \frac{\Delta m^2c^3}{2p} = \frac{\Delta m^2c^4}{2E}$
Time and distance	$t = L$	$t = \frac{L}{c}$
Oscillation argument	$\frac{1}{2}\Delta Et = \frac{\Delta m^2L}{4E}$	$\frac{1}{2}\frac{\Delta E}{\hbar}t = \frac{c^3}{\hbar} \cdot \frac{\Delta m^2L}{4E}$

$$c = 2.998 \times 10^5 \text{ km s}^{-1}$$

$$\hbar = 6.582 \times 10^{-25} \text{ GeV s}$$

$$\frac{c^3}{4\hbar} \Rightarrow \frac{1}{4 \times 0.1973} = 1.267 \approx 1.27$$

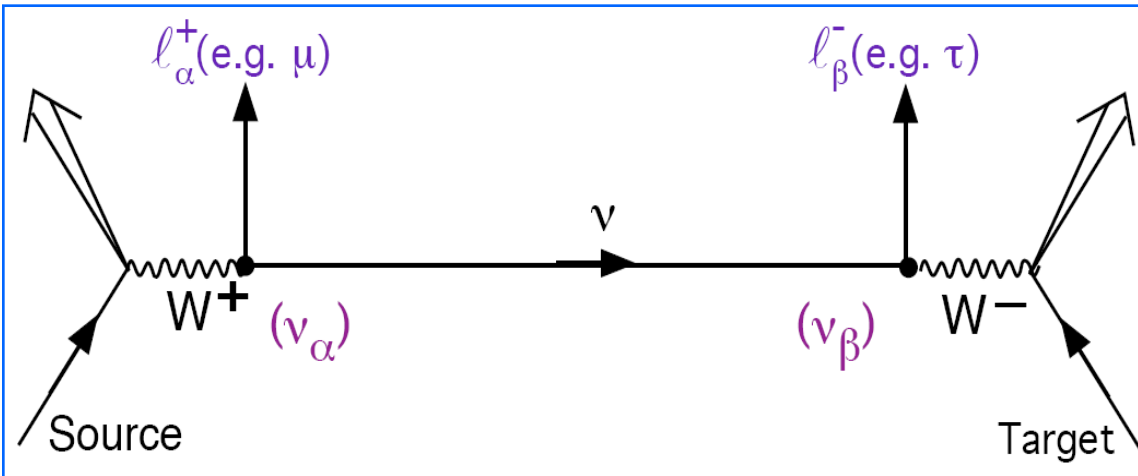
$$c = 1 \Rightarrow \hbar = 6.582 \times 10^{-25} \text{ GeV} \times 2.998 \times 10^5 \text{ km}$$

$$= 1.973 \times 10^{-19} \text{ GeV km} = 0.1973 \text{ eV}^2 \text{ GeV}^{-1} \text{ km}$$

3-flavor Oscillation (1)

21

Production and detection of a neutrino beam by **CC** weak interactions:



$$|\nu_\alpha(0)\rangle = |\nu_\alpha\rangle = \sum_{i=1}^3 V_{\alpha i}^* |\nu_i\rangle$$

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^3 V_{\alpha i}^* e^{-iE_i t} |\nu_i\rangle$$

$$\alpha, \beta, \gamma = e, \mu, \tau$$

$$i, j, k = 1, 2, 3$$

$$A(\nu_\alpha \rightarrow \nu_\beta) = \langle \nu_\beta | \nu_\alpha(t) \rangle = \left(\sum_{j=1}^3 V_{\beta j} \langle \nu_j | \right) \left(\sum_{i=1}^3 V_{\alpha i}^* e^{-iE_i t} |\nu_i\rangle \right) = \sum_{i=1}^3 V_{\alpha i}^* V_{\beta i} e^{-iE_i t}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = \left| \sum_{i=1}^3 V_{\alpha i}^* V_{\beta i} e^{-iE_i t} \right|^2$$

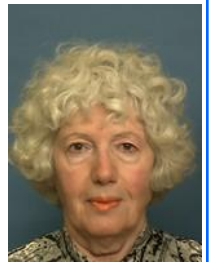
$$= \sum_{i=1}^3 |V_{\alpha i}^* V_{\beta i}|^2 + 2 \sum_{i < j} \text{Re} \left[V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^* e^{i(E_j - E_i)t} \right]$$

3-flavor Oscillation (2)

22

The formula of three-flavor oscillation probability with **CP/T** violation:

$$\begin{aligned}
 P(\nu_\alpha \rightarrow \nu_\beta) &= \sum_{i=1}^3 |V_{\alpha i}^* V_{\beta i}|^2 + 2 \sum_{i < j} \operatorname{Re}(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*) \cos \frac{\Delta m_{ji}^2 L}{2E} \\
 &\quad - 2 \sum_{i < j} \operatorname{Im}(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*) \sin \frac{\Delta m_{ji}^2 L}{2E} \\
 &= \sum_{i=1}^3 |V_{\alpha i}^* V_{\beta i}|^2 + 2 \sum_{i < j} \operatorname{Re}(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*) \\
 &\quad - 4 \sum_{i < j} \operatorname{Re}(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*) \sin^2 \frac{\Delta m_{ji}^2 L}{4E} - 2 \sum_{i < j} \operatorname{Im}(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*) \sin \frac{\Delta m_{ji}^2 L}{2E} \\
 &= \left| \sum_{i=1}^3 V_{\alpha i}^* V_{\beta i} \right|^2 - 4 \sum_{i < j} \operatorname{Re}(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*) \sin^2 \frac{\Delta m_{ji}^2 L}{4E} \\
 &\quad + 2 \sum_{i < j} \operatorname{Im}(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*) \sin \frac{\Delta m_{ji}^2 L}{2E}
 \end{aligned}$$



Jarlskog

$$\left| \sum_{i=1}^3 V_{\alpha i}^* V_{\beta i} \right|^2 = \delta_{\alpha\beta}$$

$$\operatorname{Im}(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*) = \mathcal{J} \sum_{\gamma, k} (\epsilon_{\alpha\beta\gamma} \epsilon_{ijk})$$

3-flavor Oscillation (3)

23

The **final** formula of 3-flavor oscillation probabilities with **CP** violation:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i < j}^3 \text{Re}(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*) \sin^2 \frac{\Delta m_{ji}^2 L}{4E} \\ + 8\mathcal{J} \sum_{\gamma} \epsilon_{\alpha\beta\gamma} \sin \frac{\Delta m_{21}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \sin \frac{\Delta m_{32}^2 L}{4E}$$

$$2 \sum_{i < j}^3 \text{Im}(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*) \sin \frac{\Delta m_{ji}^2 L}{2E} \\ = +2\mathcal{J} \sum_{\gamma} \epsilon_{\alpha\beta\gamma} \left(\sin \frac{\Delta m_{21}^2 L}{2E} - \sin \frac{\Delta m_{31}^2 L}{2E} + \sin \frac{\Delta m_{32}^2 L}{2E} \right) \\ = -2\mathcal{J} \sum_{\gamma} \epsilon_{\alpha\beta\gamma} \left(\sin \frac{\Delta m_{12}^2 L}{2E} + \sin \frac{\Delta m_{23}^2 L}{2E} + \sin \frac{\Delta m_{31}^2 L}{2E} \right) \\ = +8\mathcal{J} \sum_{\gamma} \epsilon_{\alpha\beta\gamma} \sin \frac{\Delta m_{12}^2 L}{4E} \sin \frac{\Delta m_{23}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E}$$

NOTE: If you have seen a different sign in front of the CP-violating part in a lot of literature, it most likely means that a complex conjugation of **✓** in the production point of neutrino beam was not properly taken into account.

Discrete Symmetries

24

Basic expression

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i < j}^3 \operatorname{Re}(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*) \sin^2 \frac{\Delta m_{ji}^2 L}{4E} \\ + 8\mathcal{J} \sum_{\gamma} \epsilon_{\alpha\beta\gamma} \sin \frac{\Delta m_{21}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \sin \frac{\Delta m_{32}^2 L}{4E}$$

CP transformation

$$V \rightarrow V^*$$

$$J \rightarrow -J$$

$$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = \delta_{\alpha\beta} - 4 \sum_{i < j}^3 \operatorname{Re}(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*) \sin^2 \frac{\Delta m_{ji}^2 L}{4E} \\ - 8\mathcal{J} \sum_{\gamma} \epsilon_{\alpha\beta\gamma} \sin \frac{\Delta m_{21}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \sin \frac{\Delta m_{32}^2 L}{4E}$$

T transformation

$$\alpha \leftrightarrow \beta$$

$$P(\nu_\beta \rightarrow \nu_\alpha) = \delta_{\alpha\beta} - 4 \sum_{i < j}^3 \operatorname{Re}(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*) \sin^2 \frac{\Delta m_{ji}^2 L}{4E} \\ - 8\mathcal{J} \sum_{\gamma} \epsilon_{\alpha\beta\gamma} \sin \frac{\Delta m_{21}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \sin \frac{\Delta m_{32}^2 L}{4E}$$

CPT invariance

$$P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha) = P(\nu_\alpha \rightarrow \nu_\beta)$$

The 1st Paper on CPV

25

Volume 72B, number 3

PHYSICS LETTERS

2 January 1978

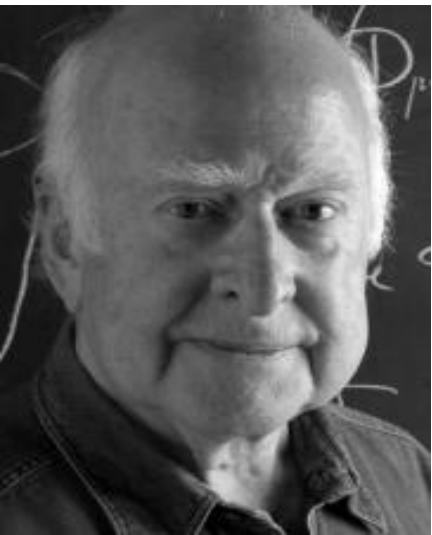
TIME REVERSAL VIOLATION IN NEUTRINO OSCILLATION

Nicola CABIBBO*

*Laboratoire de Physique Théorique et Hautes Energies, Paris, France***

Received 11 October 1977

We discuss the possibility of CP or T violation in neutrino oscillation. CP requires $\nu_\mu \leftrightarrow \nu_e$ and $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ oscillations to be equal. Time reversal invariance requires the oscillation probability to be an even function of time. Both conditions can be violated, even drastically, if more than two neutrinos exist.



Tri-maximal neutrino mixing + **maximal** CP violation:

$$A = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^* \\ 1 & a^* & a \end{pmatrix}, \quad a = \exp[2\pi i/3]$$

$$J = 1/6\sqrt{3}$$

CP & T Violation

26

Under **CPT** invariance, **CP**- and **T**-violating asymmetries are identical:

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) &= P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha) \\ &= 16\mathcal{J} \sum_\gamma \epsilon_{\alpha\beta\gamma} \sin \frac{\Delta m_{21}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \sin \frac{\Delta m_{32}^2 L}{4E} \end{aligned}$$

- Comments:
- ★ **CP** / **T** violation cannot show up in the **disappearance** neutrino oscillation experiments ($\alpha = \beta$);
 - ★ **CP** / **T** violation is a small **three-family** flavor effect;
 - ★ **CP** / **T** violation in normal **lepton-number-conserving** neutrino oscillations depends only upon the **Dirac** phase of $\mathbf{V}$; hence such oscillation experiments cannot tell us whether neutrinos are **Dirac** or **Majorana** particles.

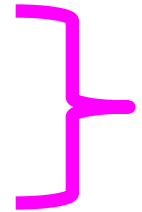
$$J = \sin \theta_{12} \cos \theta_{12} \sin \theta_{23} \cos \theta_{23} \sin \theta_{13} \cos^2 \theta_{13} \sin \delta \leq 1 / 6\sqrt{3} \approx 9.6\%$$

Disappearance

27

Most neutrino oscillation experiments are of the **disappearance** type:

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - 4 |V_{\alpha 1}|^2 |V_{\alpha 2}|^2 \sin^2 \frac{\Delta m_{21}^2 L}{4E} \\ - 4 |V_{\alpha 1}|^2 |V_{\alpha 3}|^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E} \\ - 4 |V_{\alpha 2}|^2 |V_{\alpha 3}|^2 \sin^2 \frac{\Delta m_{32}^2 L}{4E}$$



$$|\Delta m_{21}^2| = \Delta m_{\text{sun}}^2 \ll \Delta m_{\text{atm}}^2 = |\Delta m_{32}^2| \approx |\Delta m_{31}^2|$$

$$\sim 7.6 \times 10^{-5} \text{ eV}^2$$

$$\sim 2.4 \times 10^{-3} \text{ eV}^2$$

This hierarchy & the small (1,3) mixing lead to the **2-flavor** oscillation approximation for many experiments. A few upcoming experiments (long-baseline experiments) will probe the complete **3-flavor** effects.

2-flavor Approximation

28

Solar, reactor, atmospheric and accelerator ν oscillation experiments:

Experiment	Survival probability	Oscillation factor
Solar $\nu_e \rightarrow \nu_e$	$1 - \sin^2 2\theta_{12} \sin^2 \left(1.27 \frac{\Delta m_{21}^2 L}{E} \right)$	$\sin^2 2\theta_{12} = 4 V_{e1} ^2 V_{e2} ^2$
KamLAND $\bar{\nu}_e \rightarrow \bar{\nu}_e$	$1 - \sin^2 2\theta_{12} \sin^2 \left(1.27 \frac{\Delta m_{21}^2 L}{E} \right)$	$\sin^2 2\theta_{12} = 4 V_{e1} ^2 V_{e2} ^2$
Atmospheric $\nu_\mu \rightarrow \nu_\mu$	$1 - \sin^2 2\theta_{23} \sin^2 \left(1.27 \frac{\Delta m_{32}^2 L}{E} \right)$	$\sin^2 2\theta_{23} = 4 V_{\mu3} ^2 (1 - V_{\mu3} ^2)$
K2K $\nu_\mu \rightarrow \nu_\mu$	$1 - \sin^2 2\theta_{23} \sin^2 \left(1.27 \frac{\Delta m_{32}^2 L}{E} \right)$	$\sin^2 2\theta_{23} = 4 V_{\mu3} ^2 (1 - V_{\mu3} ^2)$
MINOS $\nu_\mu \rightarrow \nu_\mu$	$1 - \sin^2 2\theta_{23} \sin^2 \left(1.27 \frac{\Delta m_{32}^2 L}{E} \right)$	$\sin^2 2\theta_{23} = 4 V_{\mu3} ^2 (1 - V_{\mu3} ^2)$
CHOOZ $\bar{\nu}_e \rightarrow \bar{\nu}_e$	$1 - \sin^2 2\theta_{13} \sin^2 \left(1.27 \frac{\Delta m_{32}^2 L}{E} \right)$	$\sin^2 2\theta_{13} = 4 V_{e3} ^2 (1 - V_{e3} ^2)$

$\nu \Leftrightarrow \bar{\nu}$ Oscillations

29

Comparison: **neutrino-neutrino** and **neutrino-antineutrino** oscillation experiments.

Neutrino-neutrino oscillation

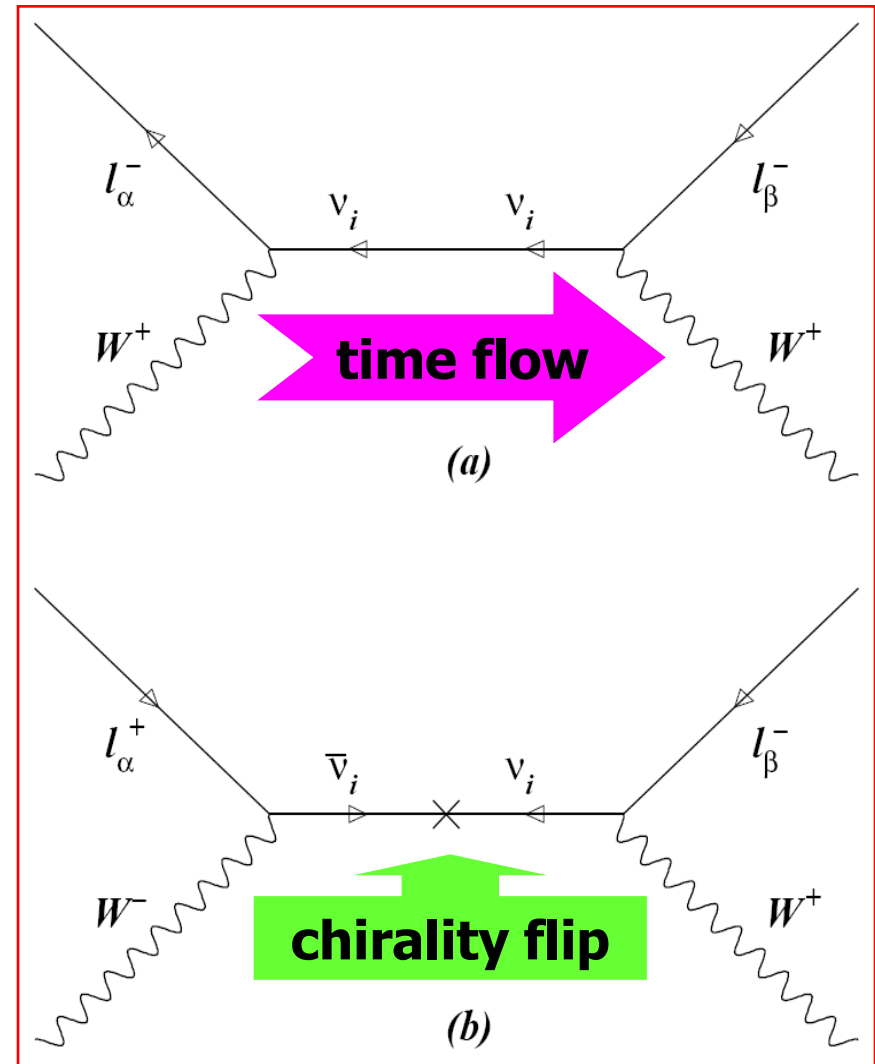
$$A = \sum_{k=1}^3 V_{\alpha k}^* V_{\beta k} e^{-iE_k t}$$

Realistic!

Antineutrino-neutrino oscillation

$$A = \frac{1}{E} \sum_{k=1}^3 V_{\alpha k} V_{\beta k} m_k e^{-iE_k t}$$

Unrealistic! (m/E too small)

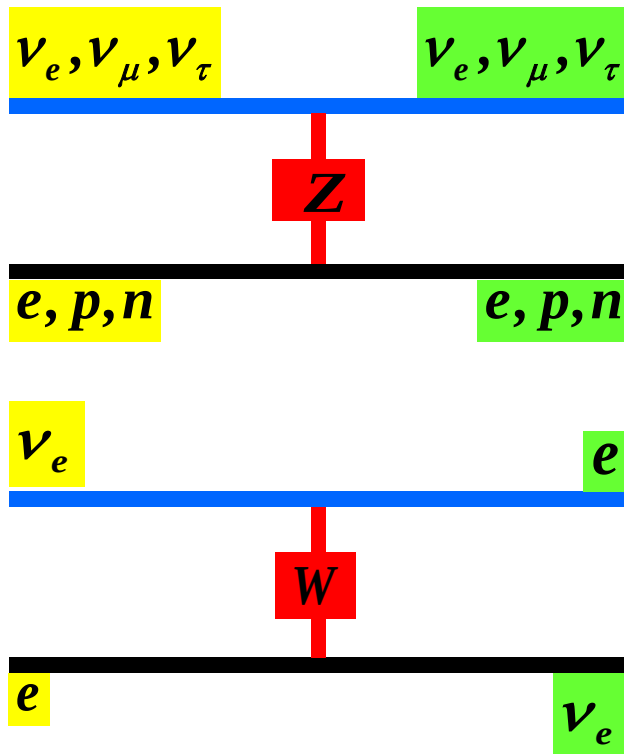


What's Matter Effect?

30

When **light** travels through a medium, it sees a **refractive index** due to **coherent forward scattering** from the constituents of the medium.

A similar phenomenon applies to **neutrino flavor states** as they travel through matter. All flavor states see a common refractive index from **NC** forward scattering, and the electron (anti) neutrino sees an extra refractive index due to **CC** forward scattering in matter.

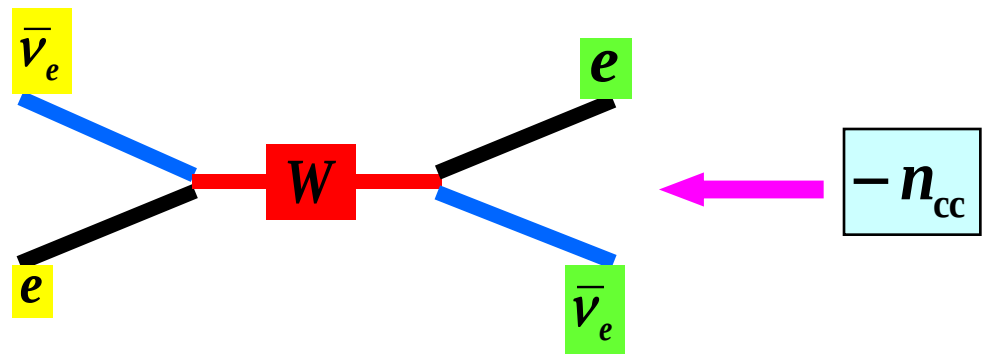


Refractive index

$$n_{\text{nc}} = 1 + \frac{2\pi N_e}{p^2} f_{\text{nc}}$$

$$n_{\text{cc}} = \frac{2\pi N_e}{p^2} f_{\text{cc}}$$

$$n_{\text{cc}} = \frac{\sqrt{2} G_F N_e}{p}$$

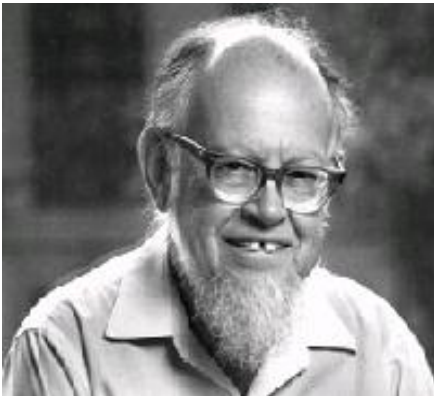


Matter May Matter

31

In travelling a distance, each neutrino flavor state develops a “matter” phase due to the refractive index. **The overall NC-induced phase** is trivial, while **the relative CC-induced phase** may change the behaviors of neutrino oscillations: **matter effects** — **L. Wolfenstein** (1978)

$$\begin{aligned}\nu_e &: \exp[ipx(n_{\text{nc}} + n_{\text{cc}} - 1)] \\ \nu_\mu &: \exp[ipx(n_{\text{nc}} - 1)] \\ \nu_\tau &: \exp[ipx(n_{\text{nc}} - 1)]\end{aligned}$$



Matter effect inside the Sun can enhance the solar neutrino oscillation (**S.P. Mikheyev** and **A.Yu. Smirnov** 1985 — **MSW effect**); matter effect inside the Earth may cause a **day-night effect**. Note that matter effect in long-baseline experiments might result in **fake CP-violating** effects.

MSW Resonance

32

Neutrino oscillation in matter:

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + \sqrt{2} G_F N_e & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$P(\nu_e \rightarrow \nu_\mu)_v = \sin^2 2\theta \sin^2 \left(\frac{1.27 \Delta m^2 L}{E} \right)$$
$$P(\nu_e \rightarrow \nu_\mu)_m = \sin^2 2\tilde{\theta} \sin^2 \left(\frac{1.27 \Delta \tilde{m}^2 L}{E} \right)$$

The matter density changes
for **solar neutrinos** to travel
from the core to the surface

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos \tilde{\theta} & \sin \tilde{\theta} \\ -\sin \tilde{\theta} & \cos \tilde{\theta} \end{pmatrix} \begin{pmatrix} |\tilde{\nu}_1\rangle \\ |\tilde{\nu}_2\rangle \end{pmatrix}$$

$$\Delta \tilde{m}^2 = \sqrt{(\Delta m^2 \cos 2\theta - 2\sqrt{2} G_F N_e E)^2 + (\Delta m^2 \sin 2\theta)^2}$$
$$\tan 2\tilde{\theta} = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2\sqrt{2} G_F N_e E}$$

resonance

MSW

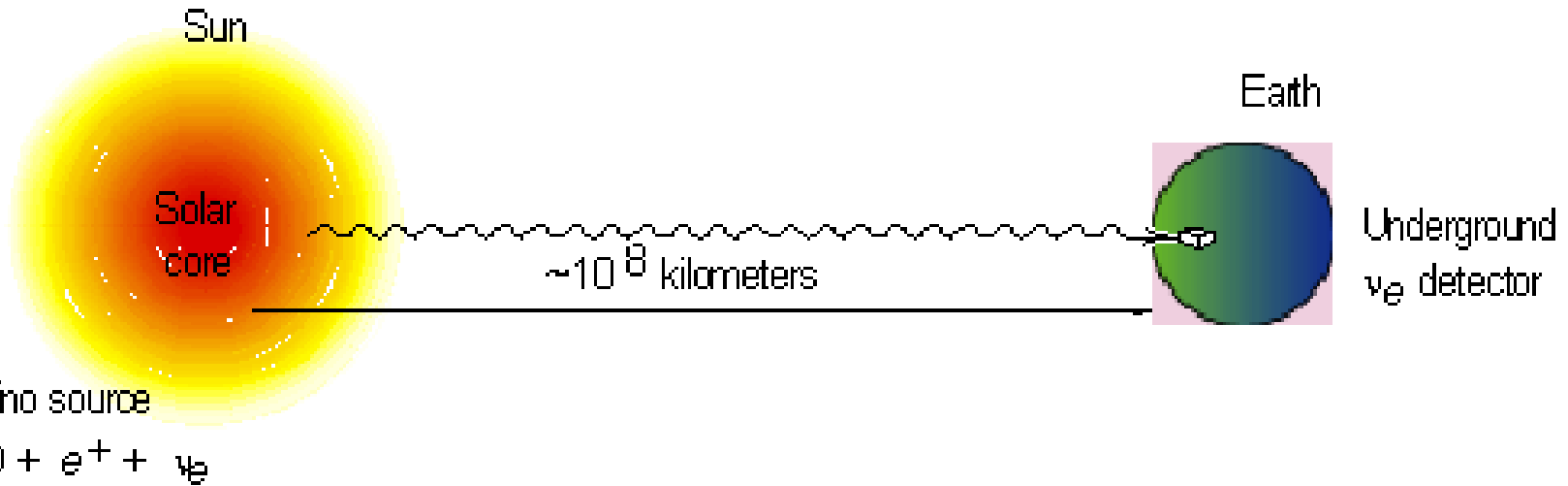
$$\tilde{\theta} = 45^\circ$$

Lecture D

- ★ Evidence for Neutrino Oscillations
- ★ Lessons from Oscillation Data
- ★ Comparing Leptons with Quarks

1968: Solar Neutrinos

34

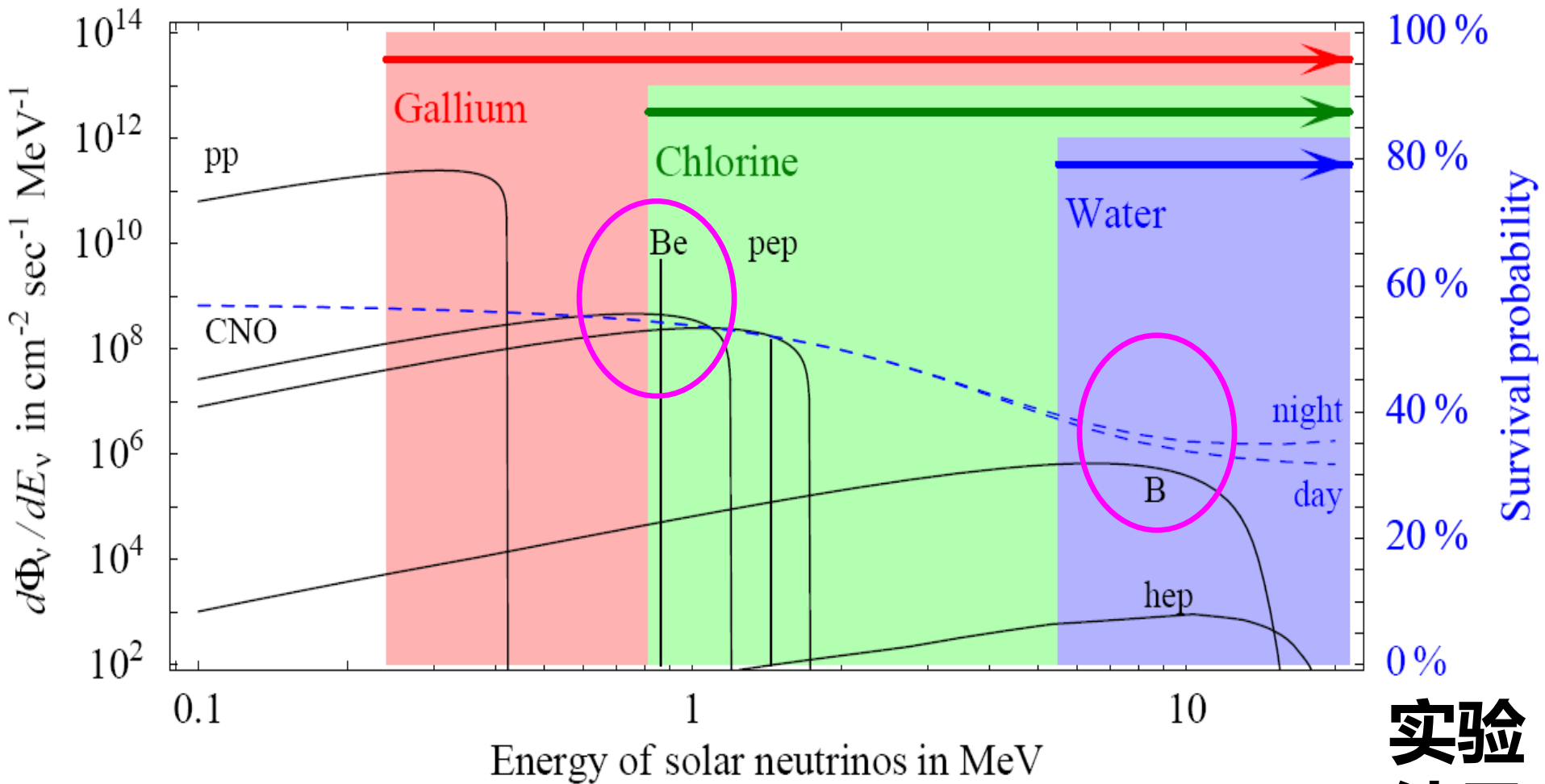


Ray Davis made the first observation of a solar neutrino shortfall (compared to **John Bahcall**'s prediction for the ν -flux) at the Homestake Mine in **1968**.

The simplest solution to this problem is **neutrino oscillation**!

Energy Spectrum

35



实验
结果

Strumia and Vissani, hep-ph/0606054

Examples: Boron (硼) ν 's $\sim 32\%$, Beryllium (铍) ν 's $\sim 56\%$

MSW Solution

36

In the two-flavor approximation:

$$N_e(0) \approx 6 \times 10^{25} \text{ cm}^{-3}$$

$$\mathcal{H}_{\text{eff}} = \frac{\Delta m_{21}^2}{4E} \begin{bmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12} \\ \sin 2\theta_{12} & \cos 2\theta_{12} \end{bmatrix} + \begin{bmatrix} \sqrt{2}G_F N_e(r) & 0 \\ 0 & 0 \end{bmatrix}$$

$7.6 \times 10^{-5} \text{ eV}^2$

$0.75 \times 10^{-5} \text{ eV}^2 / \text{MeV (at } r = 0)$

Be-7 ν 's: $E \sim 0.862 \text{ MeV}$. The vacuum term is dominant. The survival probability on the earth is (for $\theta_{12} \sim 34^\circ$):

$$P(\nu_e \rightarrow \nu_e) \approx 1 - \frac{1}{2} \sin^2 2\theta_{12} \approx 0.56$$

B-8 ν 's: $E \sim 6 \text{ to } 7 \text{ MeV}$. The matter term is dominant. The produced ν is roughly $\nu_e \sim \nu_2$ (for $V > 0$). The ν -propagation from the center to the outer edge of the Sun is approximately **adiabatic**. That is why it keeps to be ν_2 on the way to the surface (for $\theta_{12} \sim 34^\circ$):

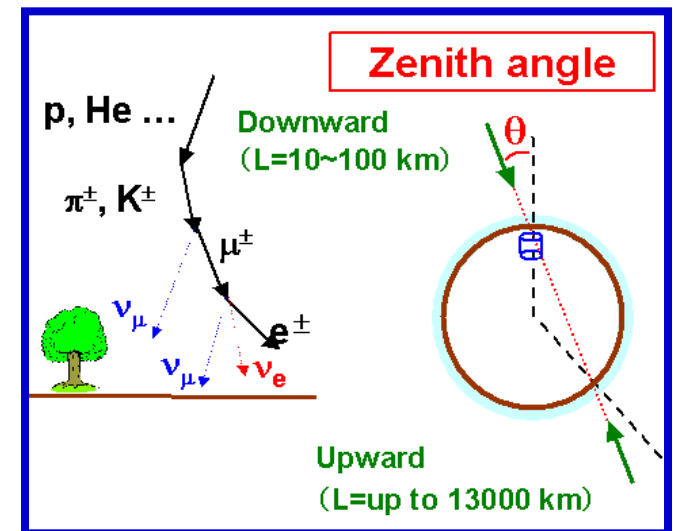
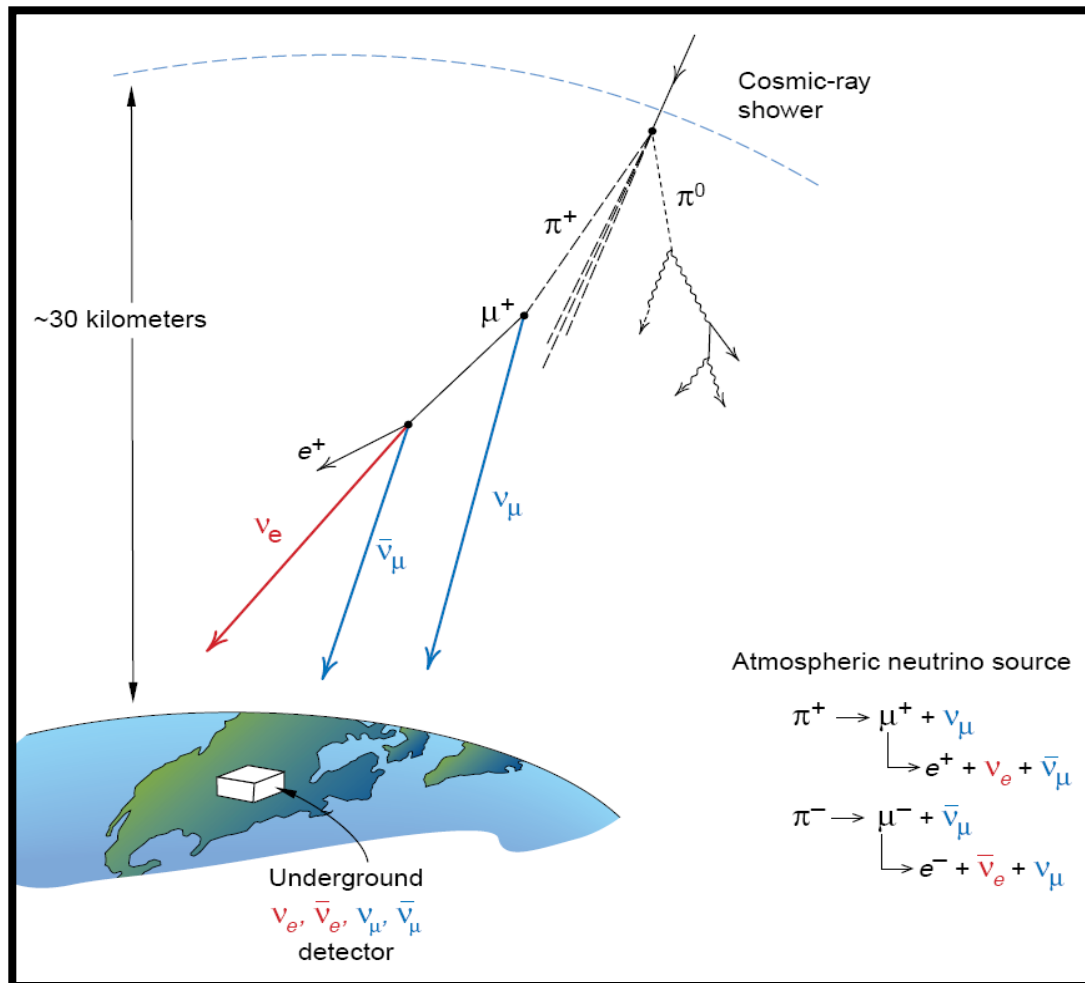
$$|\nu_2\rangle \approx \sin \theta_{12} |\nu_e\rangle + \cos \theta_{12} |\nu_\mu\rangle$$

$$P(\nu_e \rightarrow \nu_e) = |\langle \nu_e | \nu_2 \rangle|^2 = \sin^2 \theta_{12} \approx 0.32$$

1998: Atmospheric ν 's

37

Atmospheric **muon neutrino deficit** was firmly established at Super-Kamiokande (Y. Totsuka & T. Kajita 1998).



The Detector

38



Super-Kamiokande (SK)

Atmospheric neutrino, Solar neutrino, LBL neutrino

1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
------	------	------	------	------	------	------	------	------	------	------	------

Discovery of
atmospheric
 ν -oscillation

SK-I
1489 days
11146 PMTs

accident

SK-II
804 days
5182 PMTs

reconstruction

SK-III
11129 PMTs

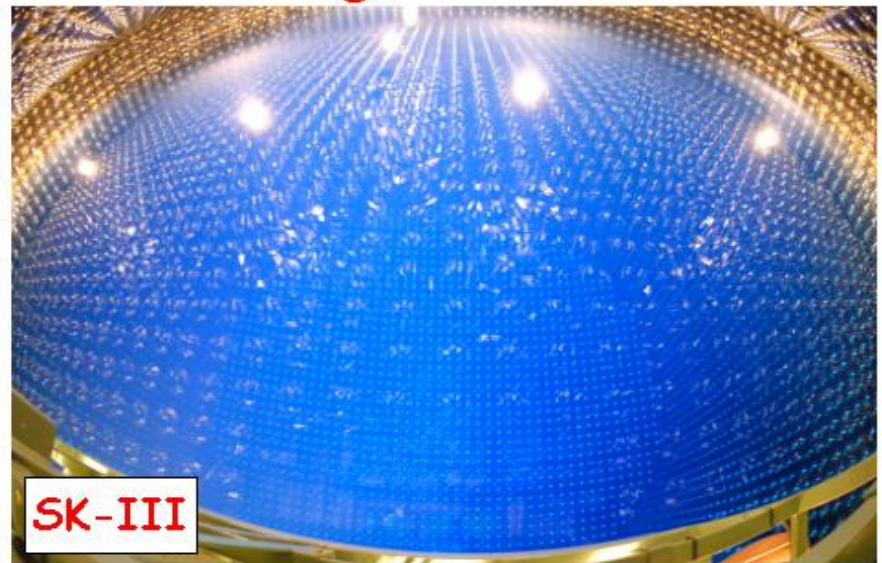


SK-III

Full reconstruction

SK-III physics run has
started on 11 July, 2006

A 50k tons cylindrical water
Cherenkov detector located
at 1km underground



SK-III

Zenith angle distributions

C. Sagi/ICHEP04

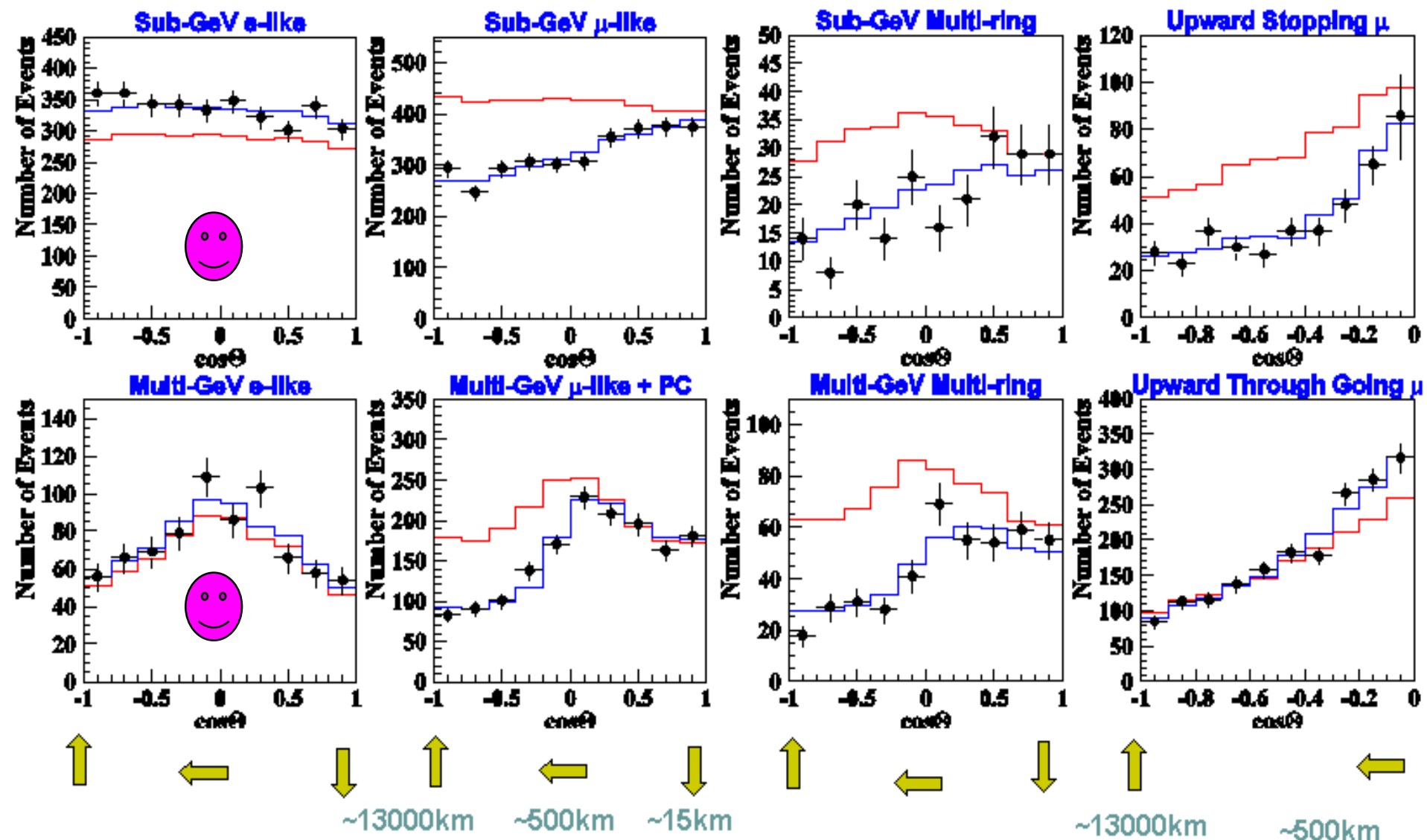
$\nu_\mu \leftrightarrow \nu_\tau$

2-flavor oscillations

Best fit

$\sin^2 2\theta = 1.0, \Delta m^2 = 2.1 \times 10^{-3} \text{ eV}^2$

Null oscillation



L/E Analysis: SK-I + SK-II

J. Raaf / Neutrino08

Datasets

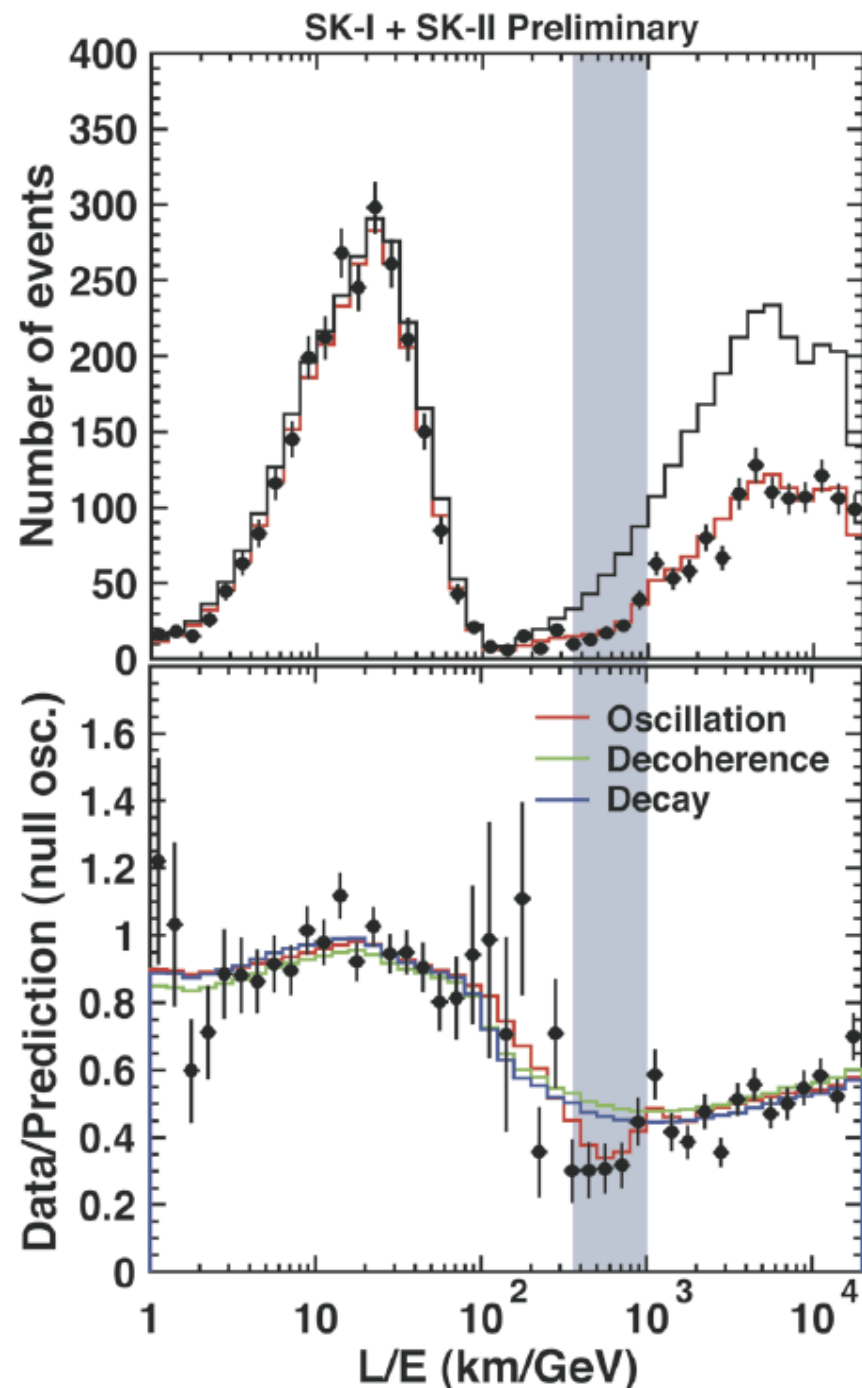
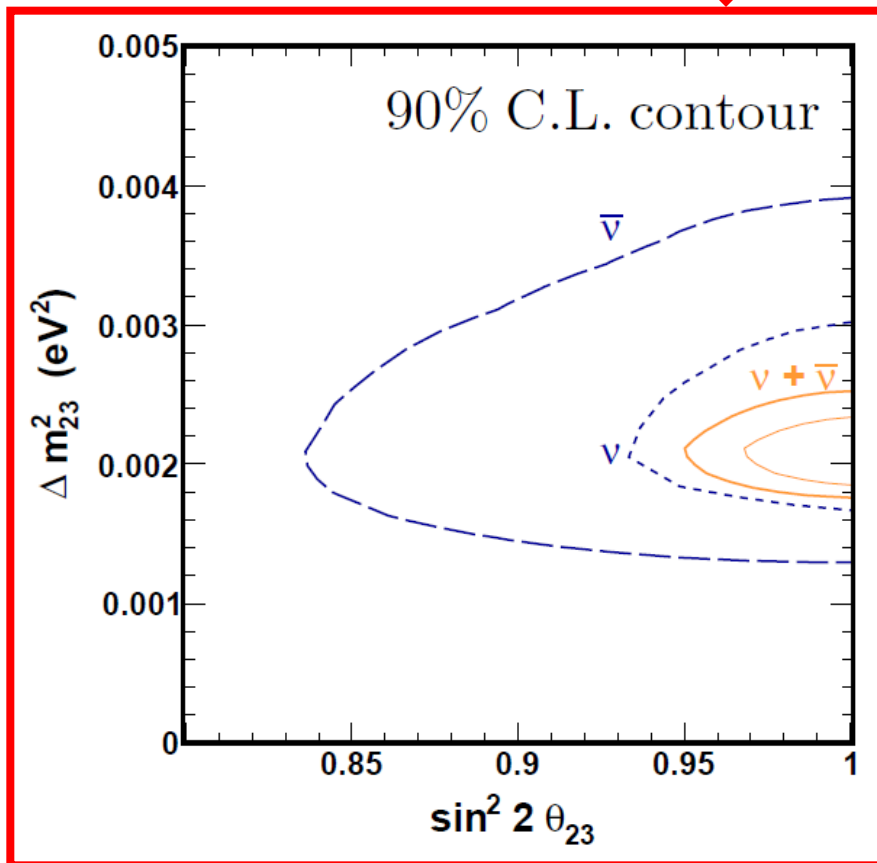
SK-I FC/PC μ -like: 1489 days

SK-II FC/PC μ -like: 799 days



Phys. Rev. Lett. 107, 241801 (2011)

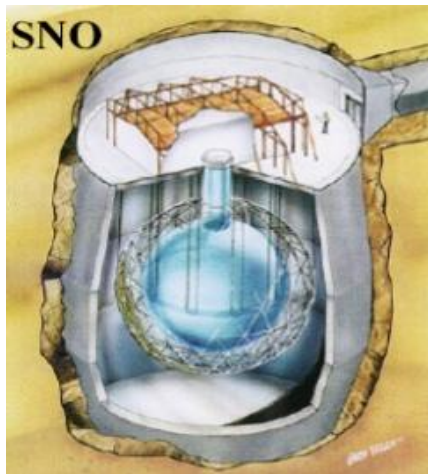
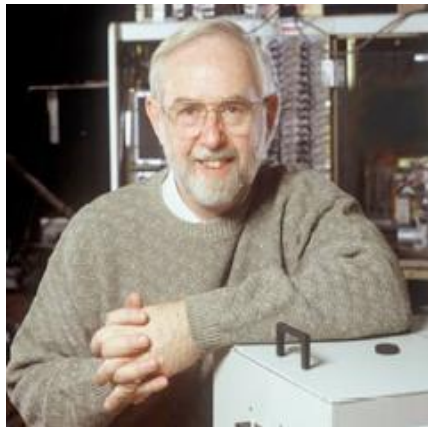
SK-I+II+III data set



2001: Solar Neutrinos

41

The **heavy water** Cherenkov detector at SNO confirmed the solar neutrino flavor conversion (A.B. McDonald 2001)



The Salient features:

Boron-8 e -neutrinos

Flux and spectrum

Deuteron as target

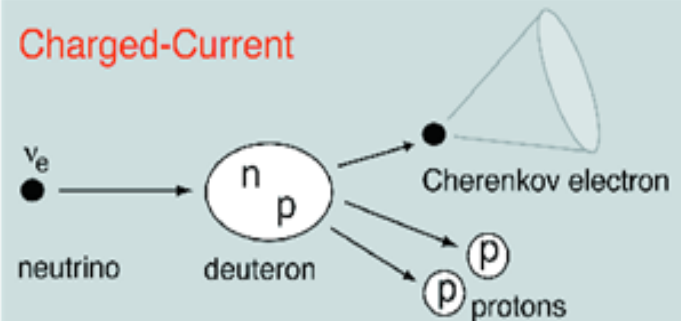
3 types of processes

Model-independent

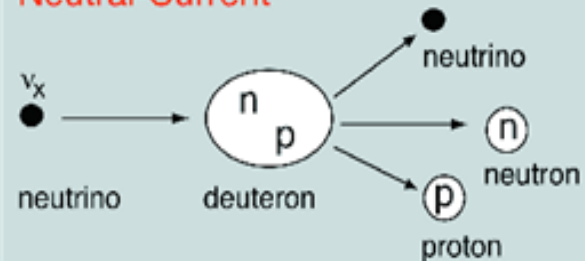
At Super-Kamiokande
only elastic scattering
can happen between
solar neutrinos & the
ordinary water.

Neutrino Reactions on Deuterium

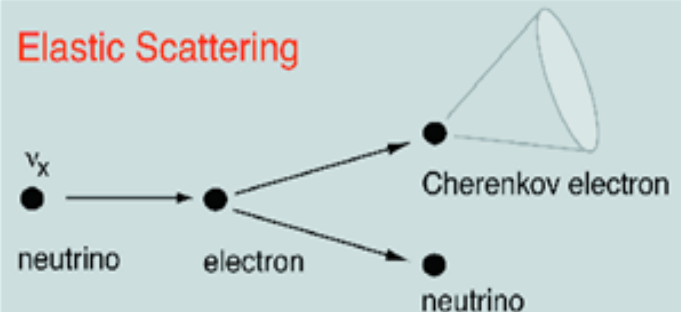
Charged-Current



Neutral-Current



Elastic Scattering



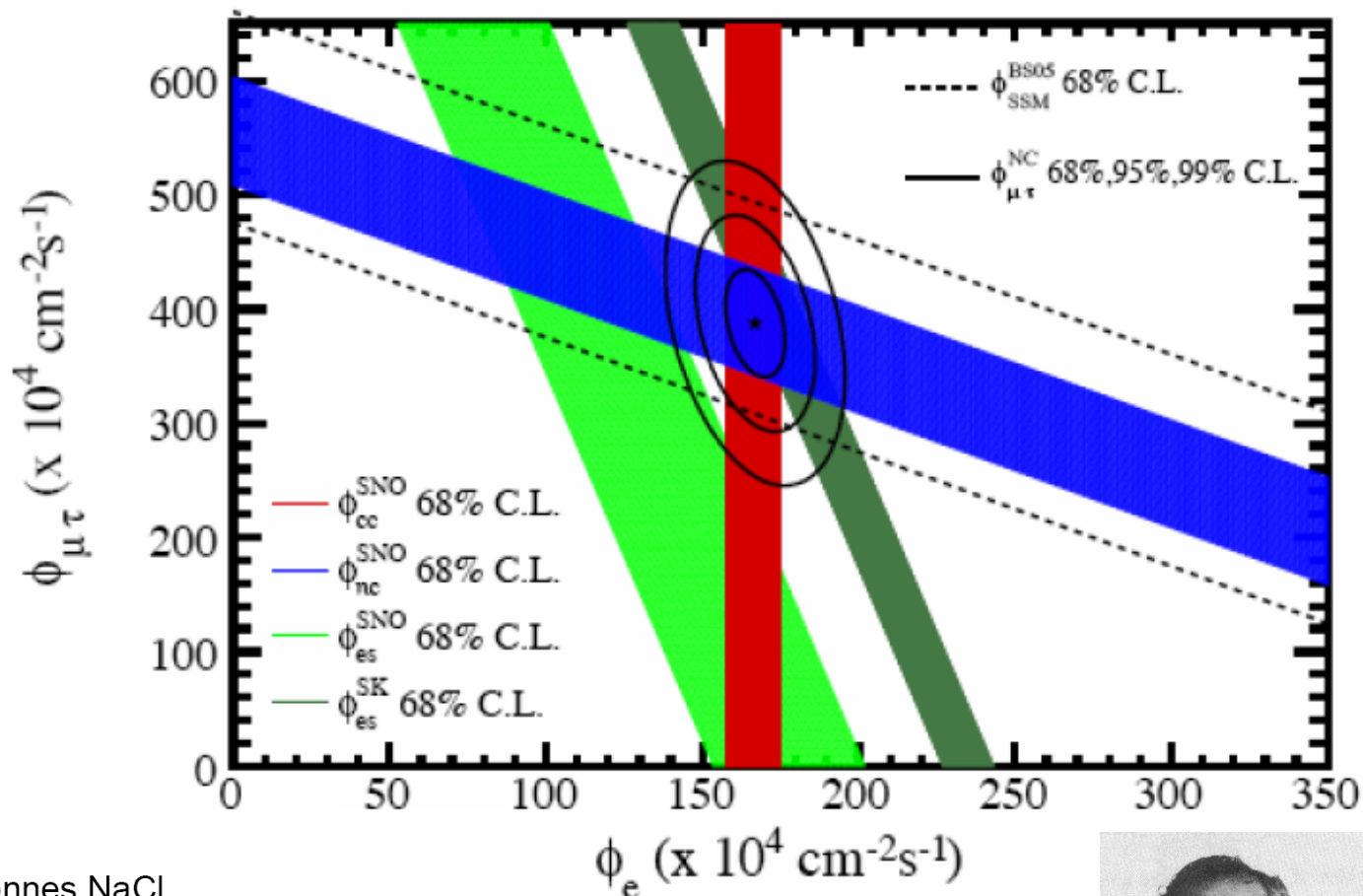
Results from SNO NCD Phase & Super-K

Preliminary

Fluxes

($10^4 \text{ cm}^{-2} \text{ s}^{-1}$)

ν_e :	167(9)
ν_{ES} :	177(26)
ν_{total} :	554(48)
ν_{SSM} :	569(91)



3 Phases:

- Just D₂O
- D₂O + 2 tonnes NaCl
- D₂O + ³He Proportional Counters ("NCDs")

$\phi_{\text{SSM}} = 569(1 \pm 0.16) \times 10^4 \text{ cm}^{-2} \text{ s}^{-1}$ (BSB05-OP:
Bahcall, Serenelli, Basu Ap. J. 621, L85, 2005).

Super-K: PRD 73, 112001, 2006

R. Robertson / Neutrino08



Nobel Prize in 2002

43



The Nobel Prize in Physics 2002

"for pioneering contributions to
astrophysics, in particular for the
detection of cosmic neutrinos"

"for pioneering
contributions to
astrophysics,
which have led to
the discovery of
cosmic X-ray
sources"

55-88-92

A lesson?



**Raymond
Davis Jr.**

🕒 1/4 of the
prize
USA



**Masatoshi
Koshiba**

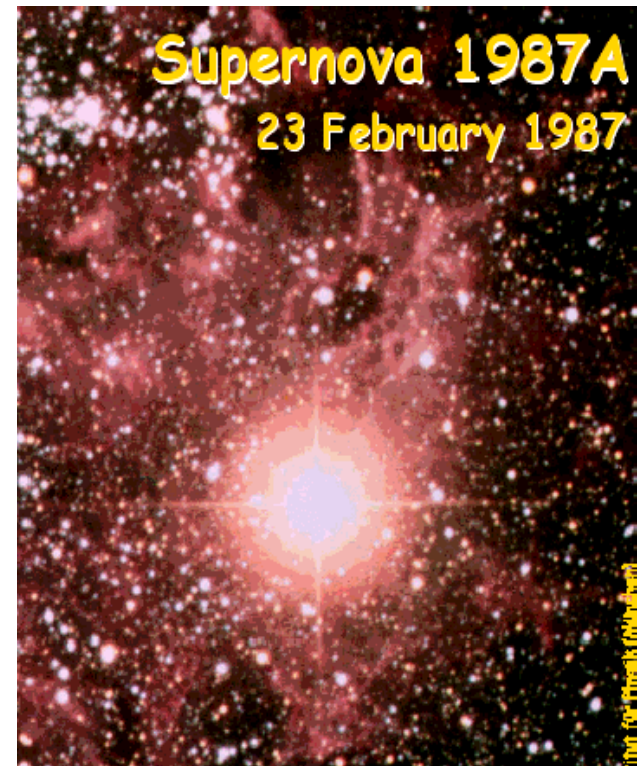
🕒 1/4 of the
prize
Japan



**Riccardo
Giacconi**

🕒 1/2 of the
prize
USA

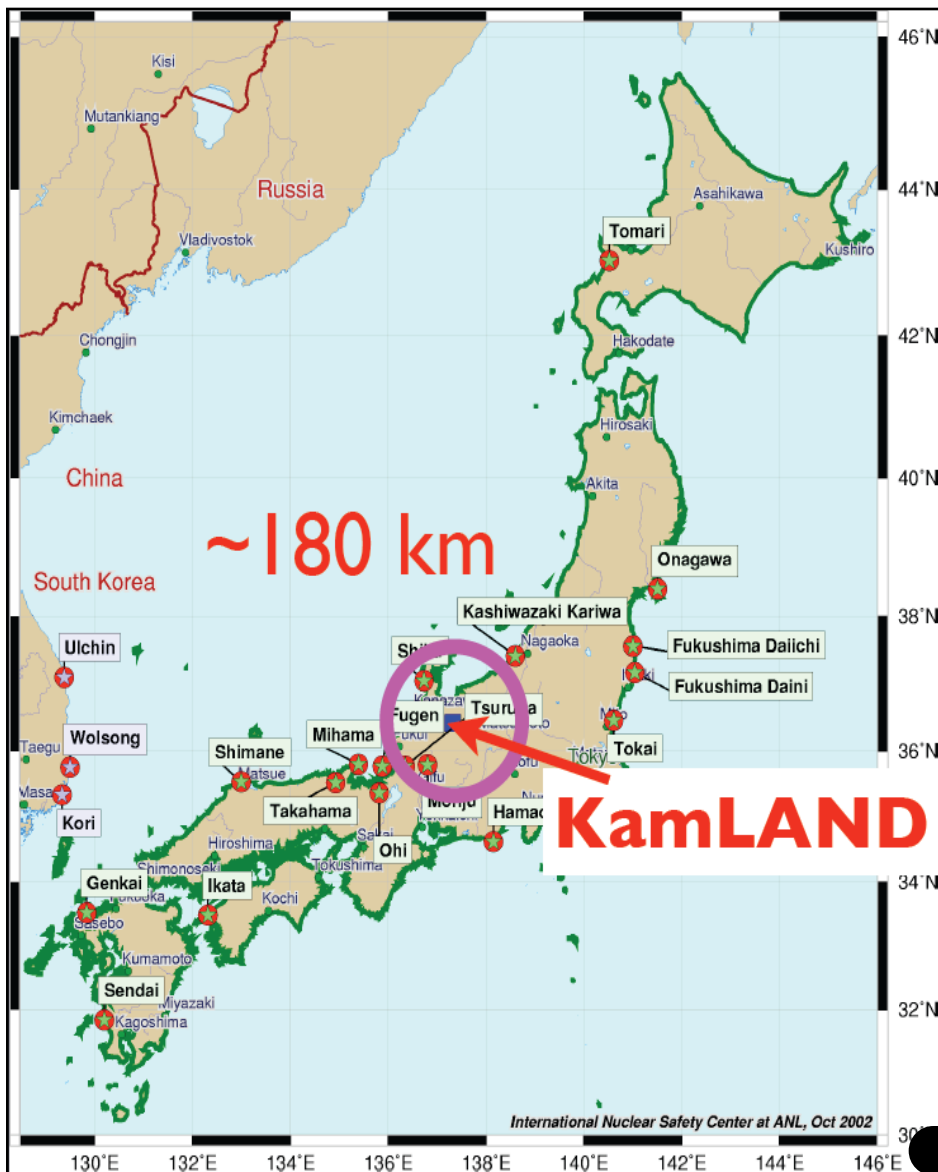
M. Koshiba: the first
detection of Supernova
neutrinos in 1987.



**New Prize
is Hopeful**

2002: KamLAND

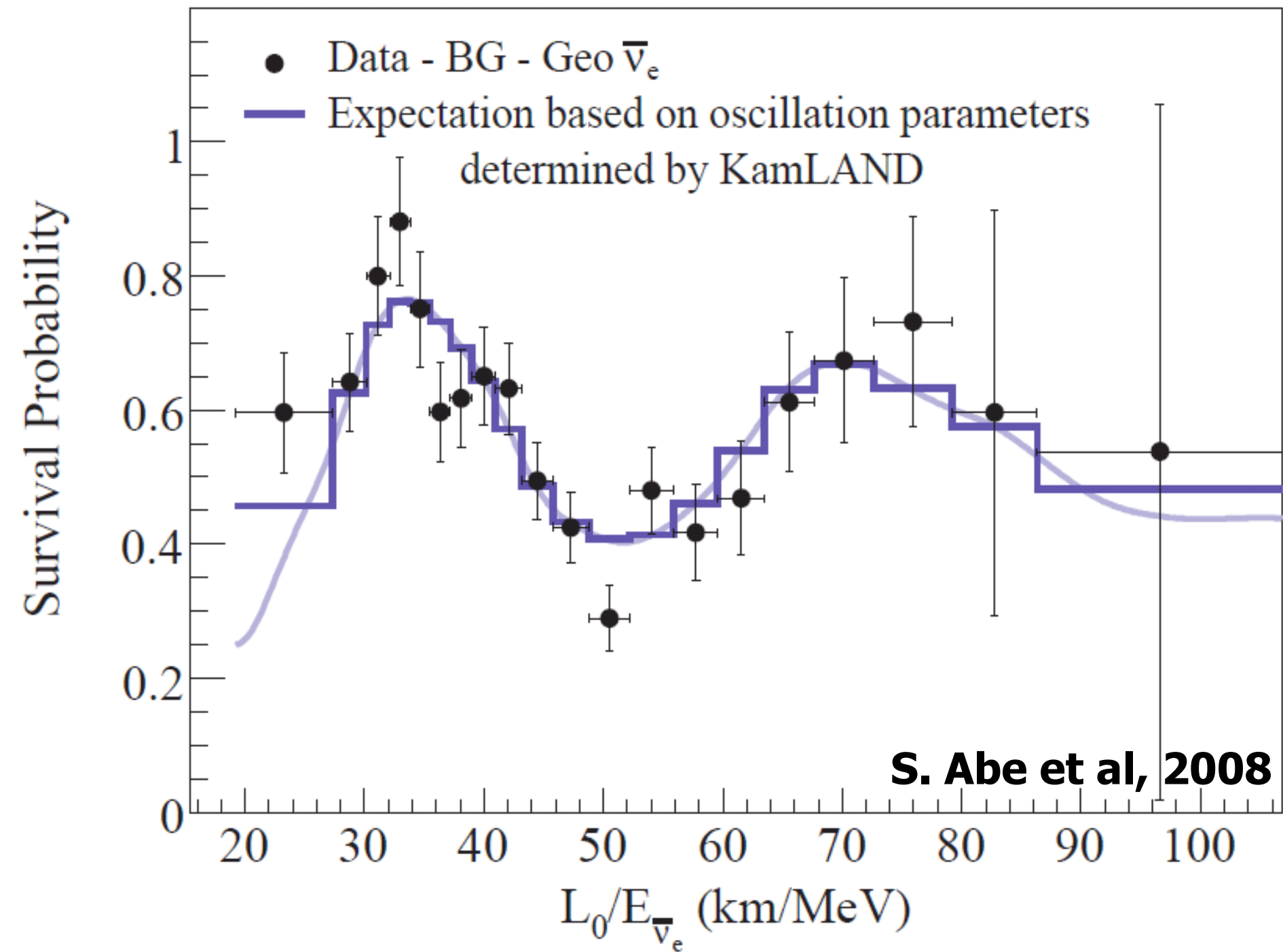
44



**Verify
the
large
angle
MSW
solution
to
the
solar
neutrino
Problem**

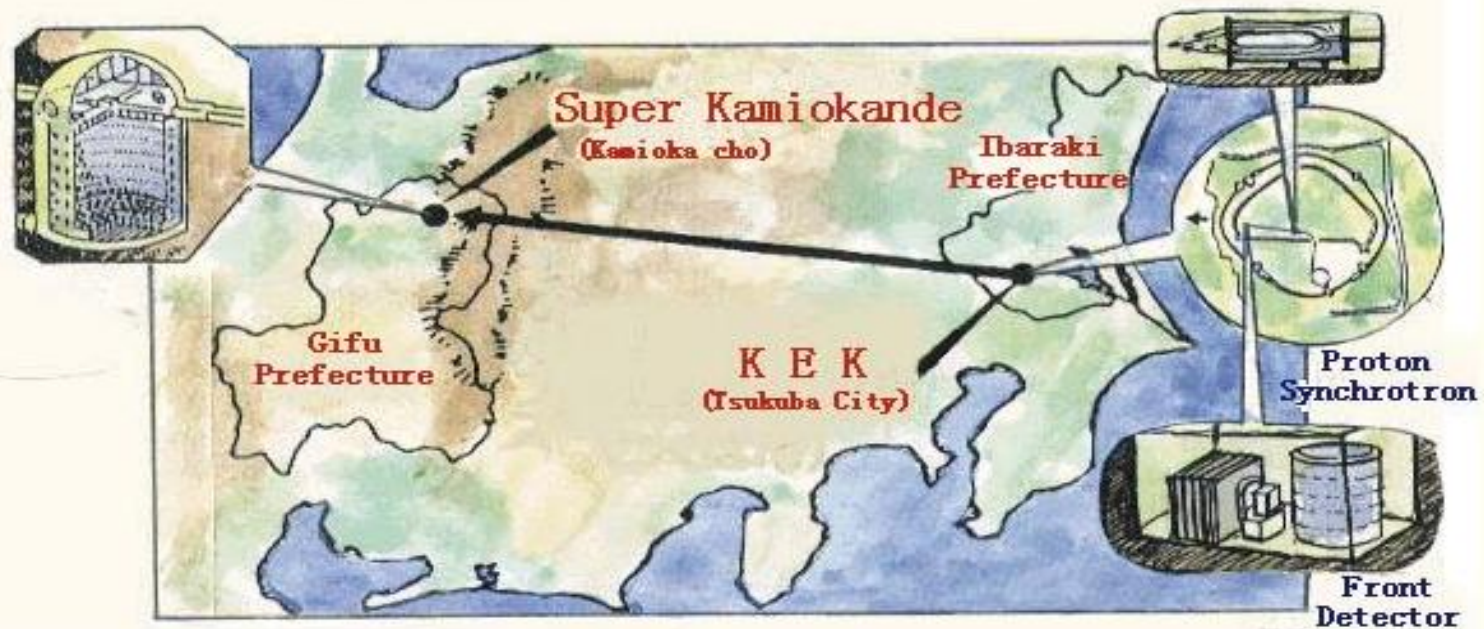
大学共同利用機関法人
KEK
高エネルギー加速器研究機構
last update: 06/04/1

Atsuto Suzuki
Director General

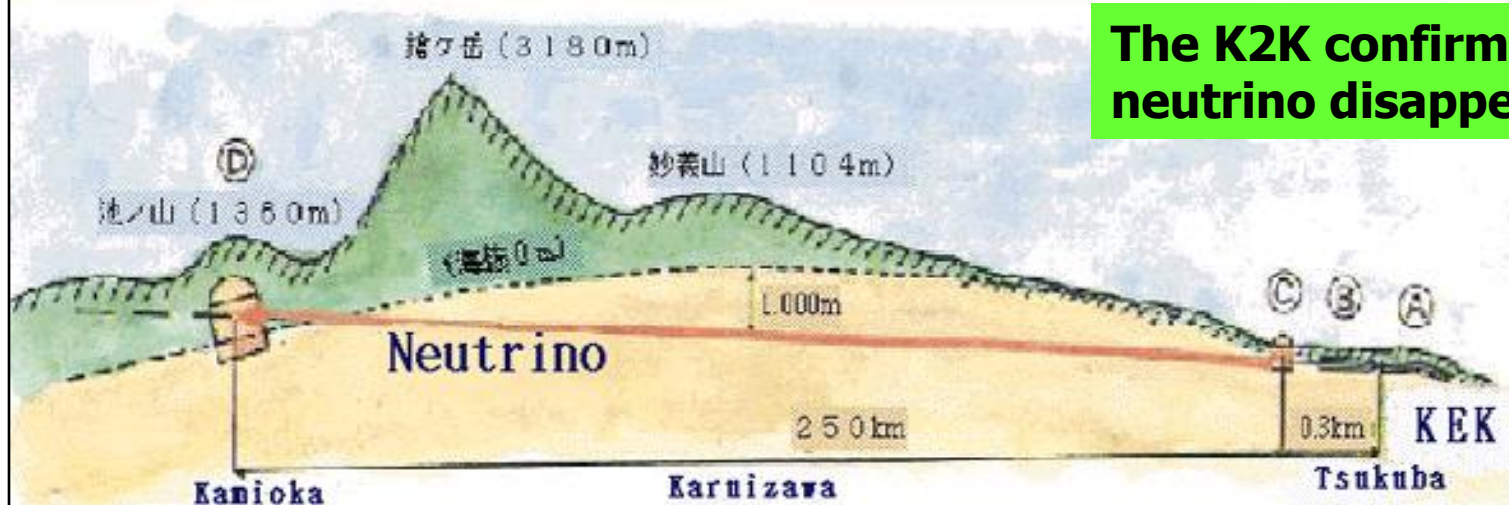


2003: K2K

46



The K2K confirms the μ -neutrino disappearance

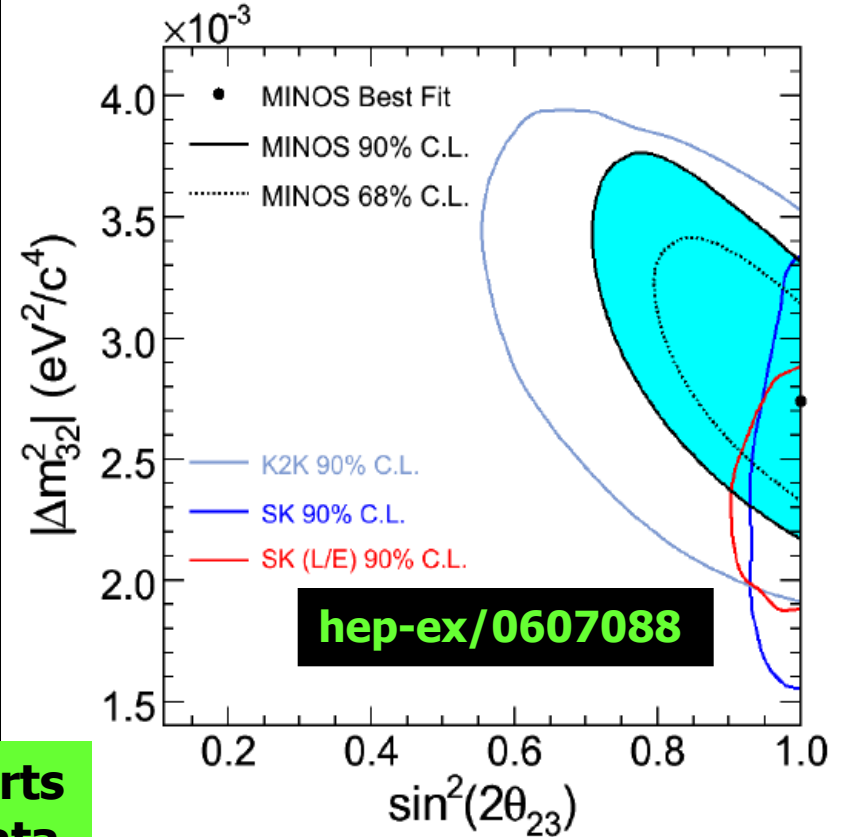


2006: MINOS

47



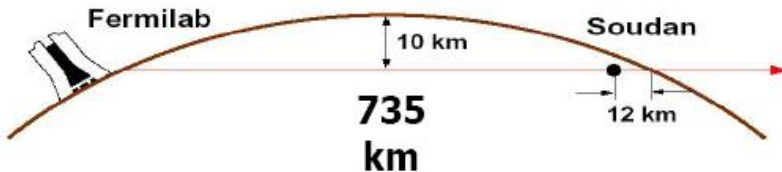
**The MINOS supports
Super-K & K2K data**



$$|\Delta m_{32}^2| = 2.74^{+0.44}_{-0.26} (\text{stat} + \text{syst}) \times 10^{-3} \text{ eV}^2$$

$$\sin^2 2\theta_{23} = 1.00_{-0.13} (\text{stat} + \text{syst})$$

Constrained to $\sin^2(2\theta_{23}) \leq 1$



Convincing?!

48

Different
neutrino sources
neutrino types
neutrino beam energies
neutrino travel distances

No need of
exotic new physics
changing SSM
new particles
fine-tuning parameters

one stone (ν oscillation) can kill all birds (anomalies)
the only new physics on solid experimental grounds



2011: T2K

49

T2K (Tokai-to-Kamioka) experiment

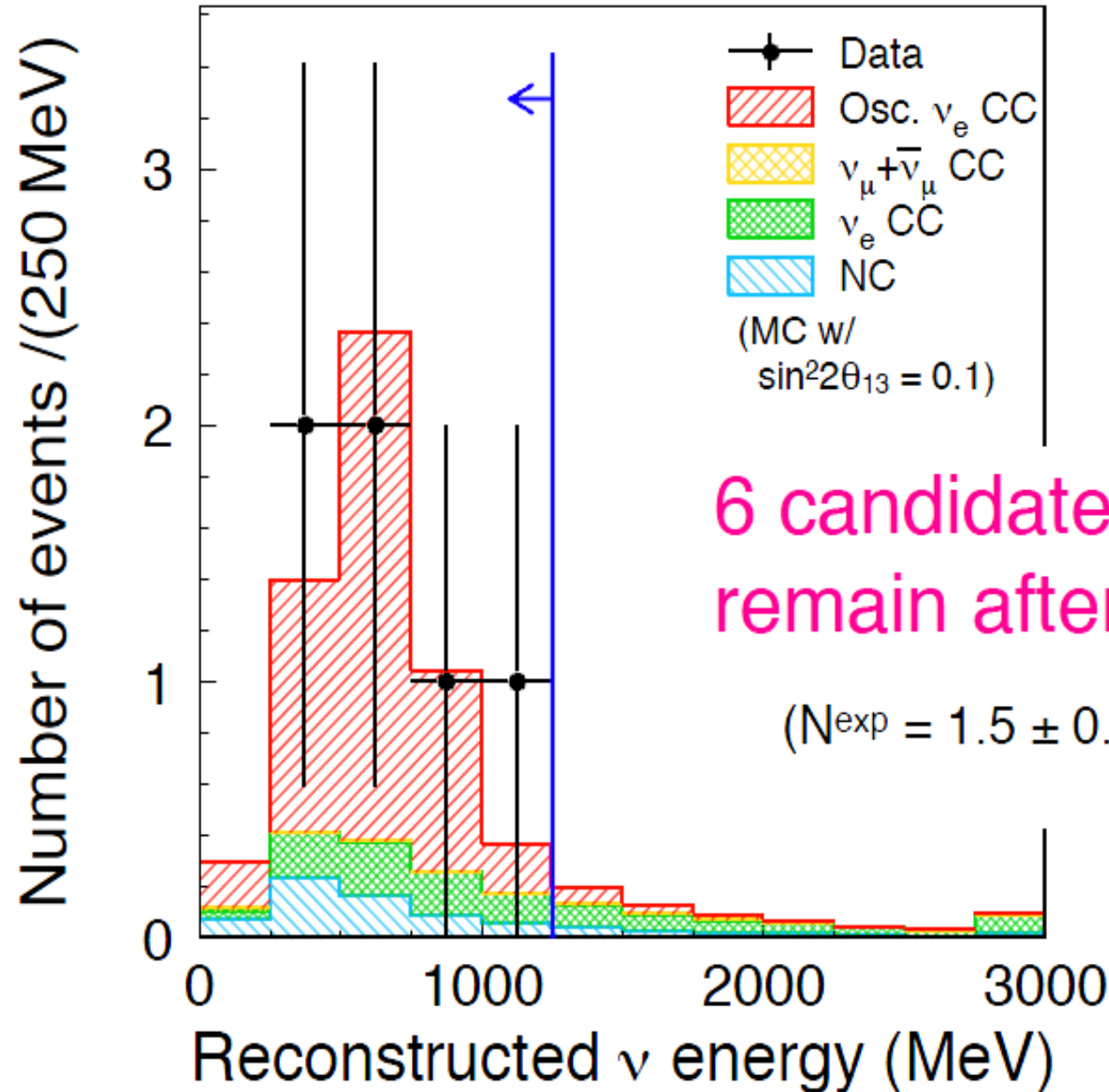


T2K Main Goals:

arXiv:1106.2822 [hep-ex] 14 June 2011

- ★ Discovery of $\nu_\mu \rightarrow \nu_e$ oscillation (ν_e appearance)
- ★ Precision measurement of ν_μ disappearance

Reconstructed ν energy cut ($E_{\text{rec}} < 1250 \text{ MeV}$) : *Final cut*



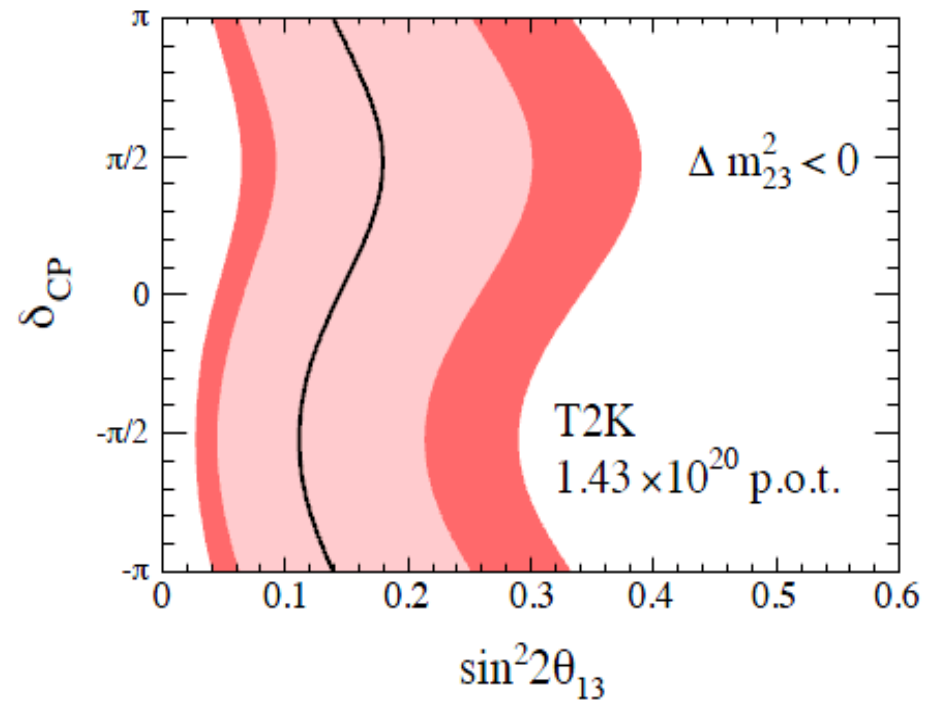
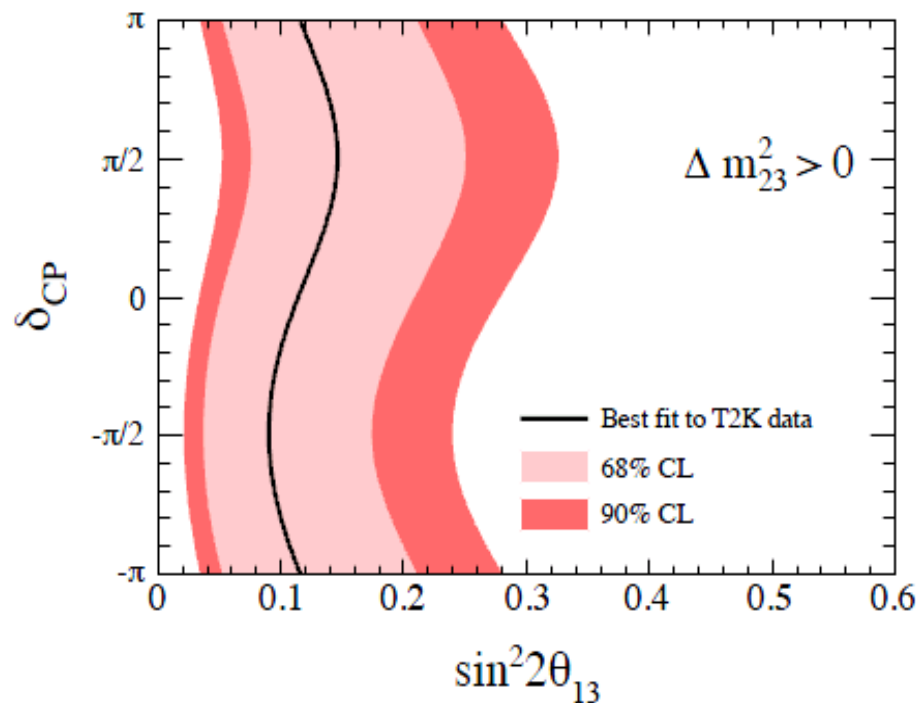
2.7 σ

6 candidate events
remain after all cuts !!

($N_{\text{exp}} = 1.5 \pm 0.3$ at $\sin^2 2\theta_{13} = 0$)

Allowed region of $\sin^2 2\theta_{13}$ as a function of δ_{CP}

(assuming $\Delta m_{23}^2 = 2.4 \times 10^{-3} \text{ eV}^2$, $\sin^2 2\theta_{23} = 1$)



90% C.L. interval & Best fit point (assuming $\Delta m_{23}^2 = 2.4 \times 10^{-3} \text{ eV}^2$, $\sin^2 2\theta_{23} = 1$, $\delta_{CP} = 0$)

$$0.03 < \sin^2 2\theta_{13} < 0.28$$

$$\sin^2 2\theta_{13} = 0.11$$

$$0.04 < \sin^2 2\theta_{13} < 0.34$$

$$\sin^2 2\theta_{13} = 0.14$$

Too Big?

2011: MINOS

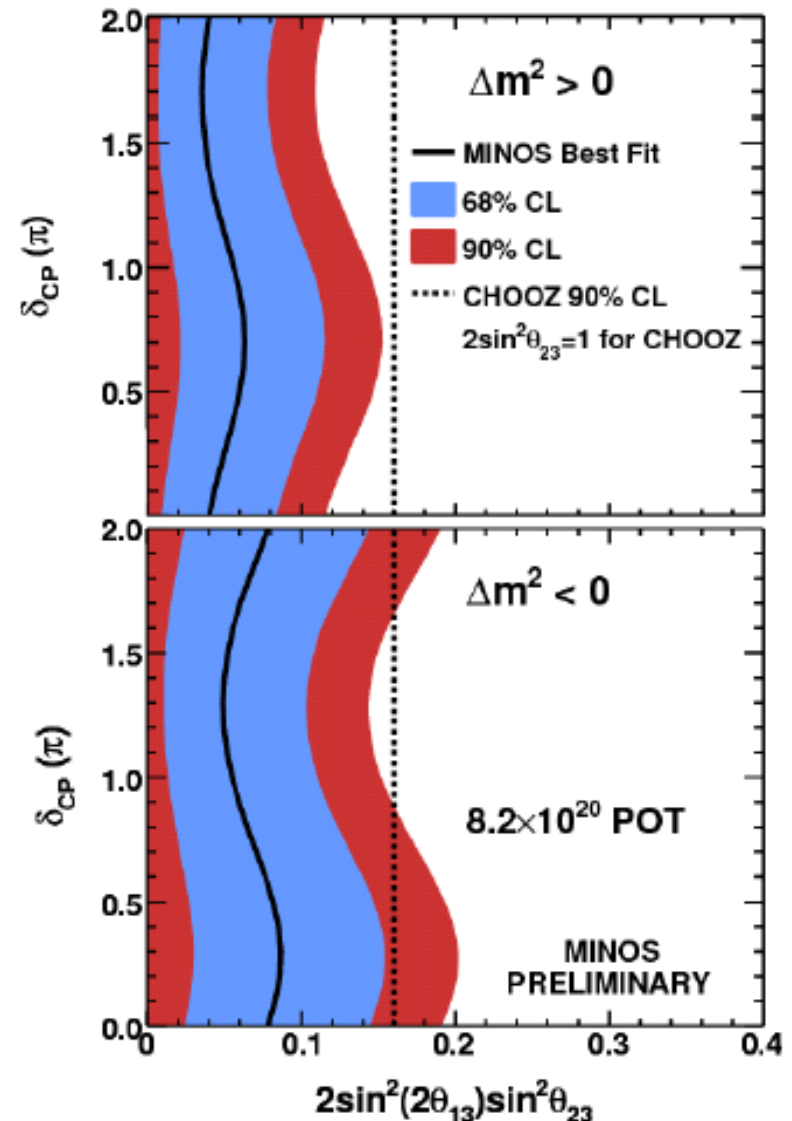
52

- MINOS has updated our electron neutrino appearance search with more data and improved analysis

$$P(\nu_\mu \rightarrow \nu_e) \approx \sin^2(2\theta_{13}) \sin^2\theta_{23} \sin^2\left(\frac{\Delta m_{atm}^2 L}{4E}\right)$$

- In the signal region, we observe 62 events with an expectation of 50

- Assuming $\delta=0$, $\theta_{23}=\pi/4$, normal (inverted) hierarchy, we set a 90% CL upper limit of $\sin^2(2\theta_{13}) < 0.12$ (0.19) and a best fit value of $\sin^2(2\theta_{13}) = 0.04$ (0.08) and exclude $\sin^2(2\theta_{13}) = 0$ @ 89% CL



2011: Double Chooz

53

First neutrino oscillation data
release of DC at LowNu11
@ Seoul (Korea) **9 Nov 2011**

Far detector data only

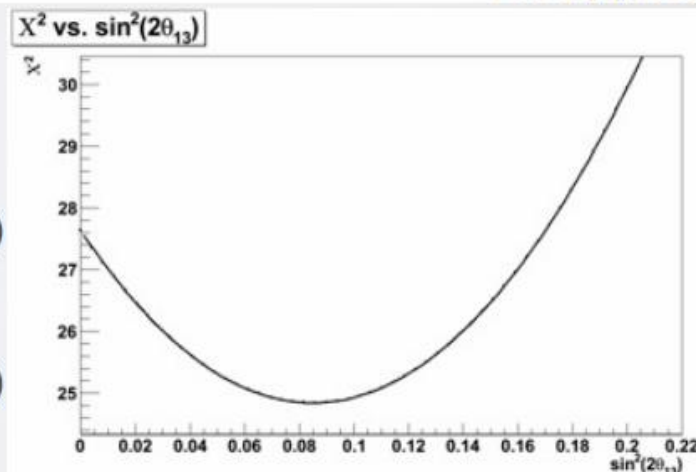
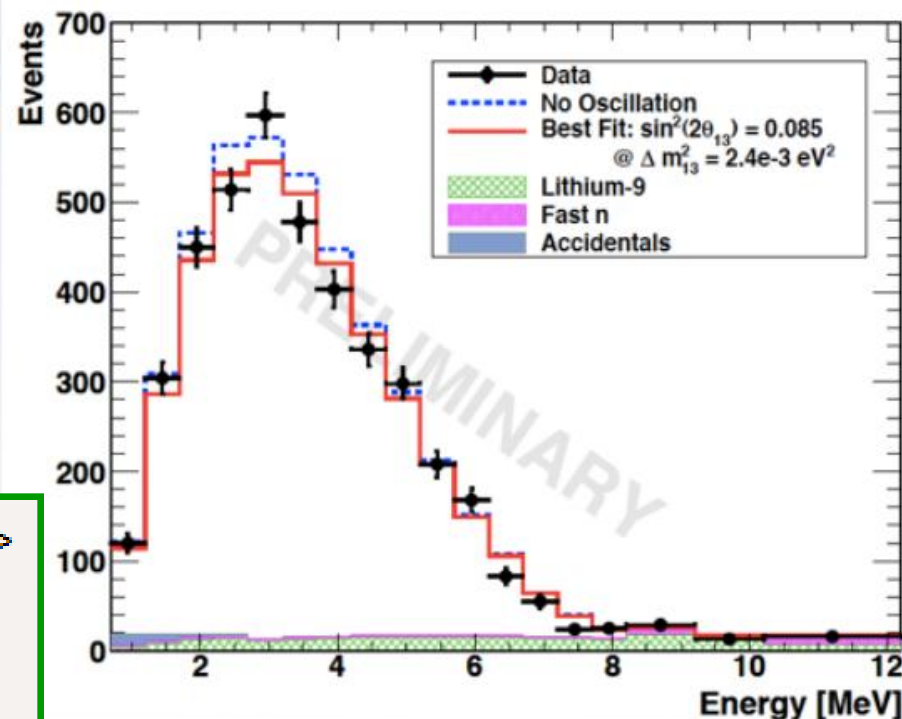
Session 2 <New reactor neutrino experiments>

11:00-12:30

Double Chooz (30): **H.De. Kerrect**

Daya Bay (30): **H. Lu**

RENO (30): **K. K. Joo**

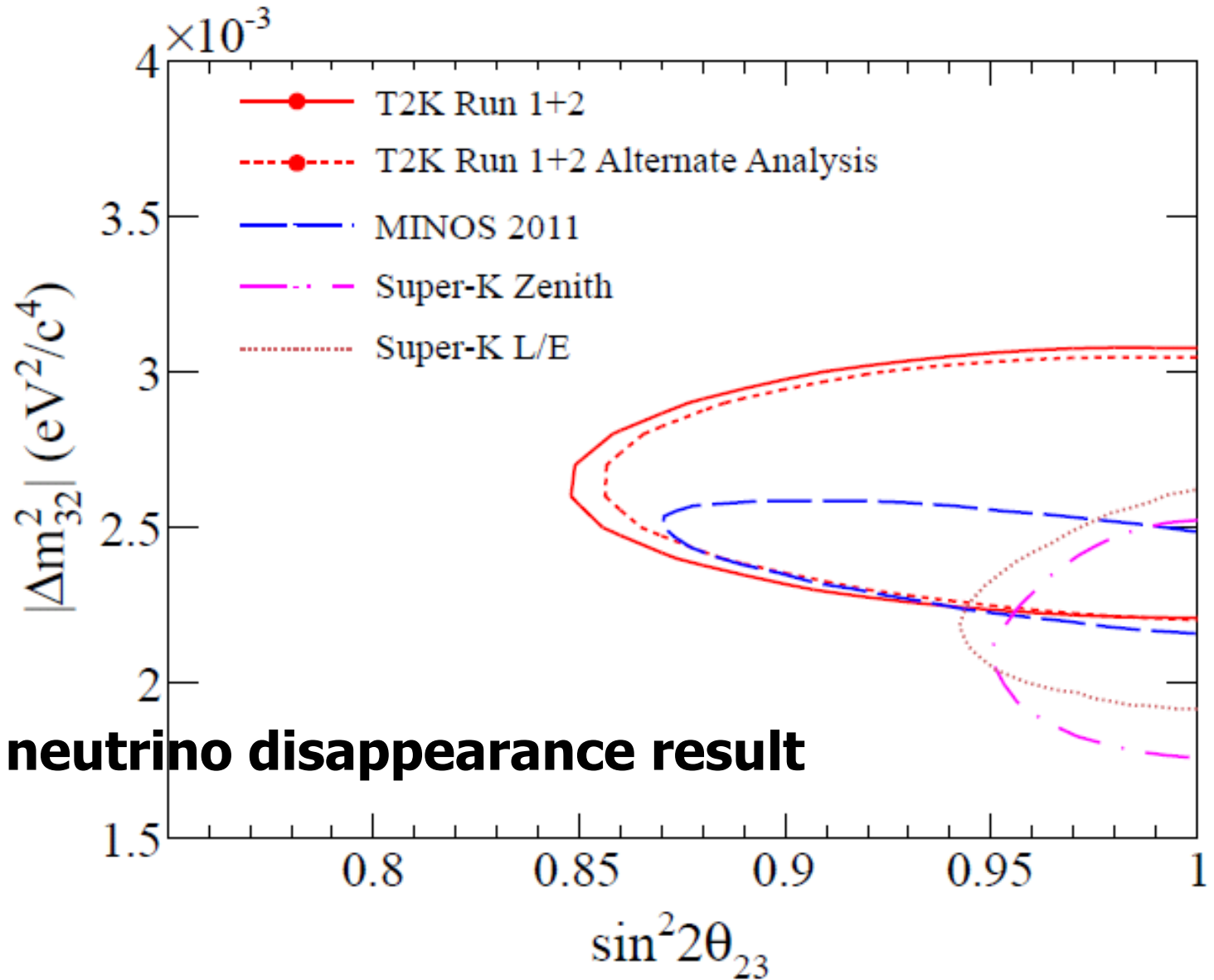


Rate + Shape Analysis:

$$\sin^2(2\theta_{13}) = 0.085 \pm 0.029(\text{stat}) \pm 0.042(\text{syst})$$

Rate Only:

$$\sin^2(2\theta_{13}) = 0.093 \pm 0.029(\text{stat}) \pm 0.073(\text{syst})$$



Muon neutrino disappearance result

3-flavor Global Fit

55

Evidence of $\theta_{13} > 0$ from global neutrino data analysis

G.L. Fogli,^{1,2} E. Lisi,² A. Marrone,^{1,2} A. Palazzo,³ and A.M. Rotunno¹

A 3-flavor global fit: arXiv:1106.6028 [hep-ph] 29 June 2011

Learnt from neutrino oscillations:

2 neutrino mass-squared differences and 3 mixing angles

TABLE I: Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the mass-mixing parameters, assuming old reactor neutrino fluxes. By using new reactor fluxes, the corresponding best fits and ranges for $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$ (in parentheses) are basically shifted by about $+0.006$ and $+0.004$, respectively, while the other parameters are essentially unchanged.

Parameter	$\delta m^2 / 10^{-5} \text{ eV}^2$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{13}$	$\sin^2 \theta_{23}$	$\Delta m^2 / 10^{-3} \text{ eV}^2$
Best fit	7.58	0.306 (0.312)	0.021 (0.025)	0.42	2.35
1σ range	7.32 – 7.80	0.291 – 0.324 (0.296 – 0.329)	0.013 – 0.028 (0.018 – 0.032)	0.39 – 0.50	2.26 – 2.47
2σ range	7.16 – 7.99	0.275 – 0.342 (0.280 – 0.347)	0.008 – 0.036 (0.012 – 0.041)	0.36 – 0.60	2.17 – 2.57
3σ range	6.99 – 8.18	0.259 – 0.359 (0.265 – 0.364)	0.001 – 0.044 (0.005 – 0.050)	0.34 – 0.64	2.06 – 2.67

Flavor Mixing Patterns

56

Quark mixing:



$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & e^{-i\delta} & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\theta_{12} \rightarrow \theta_{23} \rightarrow \theta_{13} \rightarrow \delta$$

$$\sim 13^\circ \quad \sim 2^\circ \quad \sim 0.2^\circ \quad \sim 65^\circ$$

new physics ?

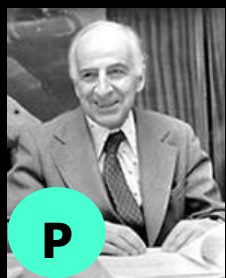
unitarity ?

turning point

Experiments:

Lepton mixing:

(in general, we consider three Majorana neutrinos)



$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & e^{-i\delta} & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\theta_{23} \rightarrow \theta_{12} \rightarrow \theta_{13} \rightarrow \delta/\rho/\sigma$$

$$\sim 45^\circ \quad \sim 34^\circ \quad < 12^\circ \quad \sim ???$$

new physics ?

unitarity ?



Naïve Understanding

57

Quark mixing

$$V_{\text{CKM}} = \begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{pmatrix}$$

Small quark mixing angles are due to **large** quark mass hierarchies?

$$m_u / m_c \sim m_c / m_t \sim \lambda^4$$

$$m_d / m_s \sim m_s / m_b \sim \lambda^2$$

$$\lambda \approx 0.22$$

3 CKM angles

$$\theta_{12} \sim \lambda$$

$$\theta_{23} \sim \lambda^2$$

$$\theta_{13} \sim \lambda^4$$

A big **CP-violating** phase in the **CKM** matrix **V** is seen.

Lepton mixing

$$|V| = \begin{matrix} & \nu_1 & \nu_2 & \nu_3 \\ \begin{matrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{matrix} & \begin{pmatrix} 0.84 \pm 0.01 & 0.54 \pm 0.02 & 0.05 \pm 0.05 \\ 0.38 \pm 0.06 & 0.60 \pm 0.06 & 0.70 \pm 0.06 \\ 0.38 \pm 0.06 & 0.60 \pm 0.06 & 0.70 \pm 0.06 \end{pmatrix} \end{matrix}$$

Large lepton mixing angles imply a **small** neutrino mass hierarchy?

$$m_e / m_\mu \sim \lambda^4 / 2$$

$$m_\mu / m_\tau \sim 4\lambda^2 / 3$$

$$\begin{matrix} \theta_{12} \sim \pi/6 \\ \theta_{23} \sim \pi/4 \end{matrix}$$

$$m_1 \sim m_2 \sim m_3$$

CP violation?

A Popular Conjecture

58

**Quark
Mixing**

large — large

small

0

θ_{12} θ_{23} θ_{13}

CKM Matrix = **Identity Matrix** + **Corrections**

Experiments

Flavor Symmetry

CP Violation

**Lepton
Mixing**

large

small

θ_{12}

θ_{23}

θ_{13}

PMNS Matrix = **Constant Term** + **Corrections**

Experiments

Flavor Symmetry

CP Violation

LO + NLO Terms

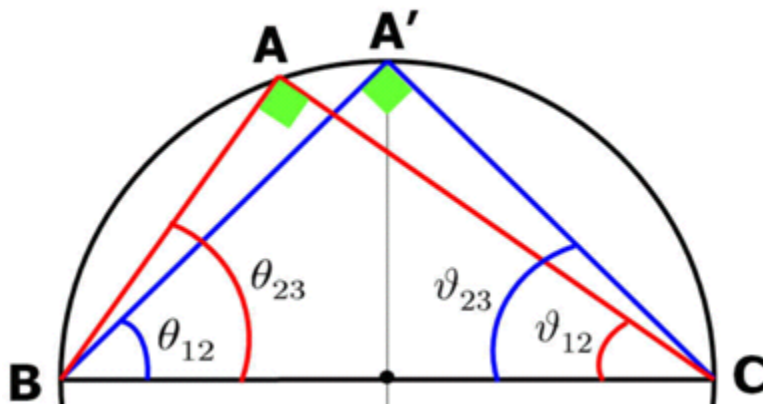
59

Democratic Neutrino Mixing Pattern (H. Fritzsch, Z.Z. Xing 1996)

$$V = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{3} \\ \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}$$

$$V = \begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

Tri-bimaximal Neutrino Mixing Pattern (P. Harrison, D. Perkins, W. Scott 2002; Z.Z. Xing 2002; X.G. He, A. Zee, 2003)



$$\begin{array}{ll} \theta_{12} = \pi/4 & \vartheta_{12} = \pi/4 - \theta_* \\ \theta_{23} = \pi/4 + \theta_* & \vartheta_{23} = \pi/4 \end{array}$$

Democratic

Tri-bimaximal

$$\theta_* = \arctan(\sqrt{2}) - \pi/4 = \pi/4 - \arctan(1/\sqrt{2}) \approx 9.7^\circ$$

Flavor Symmetries

60

Guiding Principle

Flavor / Family Symmetry is one of a few promising approaches towards deeper understanding of **fermion mass generation** and **flavor mixing** (H. Harari et al **78**; C. Froggatt and H. Nielsen **79**).

Flavor Symmetry

S_3 , S_4 , A_4 , Z_2 , $U(1)_F$, $SU(2)_F$,

Symmetry Breaking

Observed patterns of fermion masses and flavor mixing

Quantum Corrections

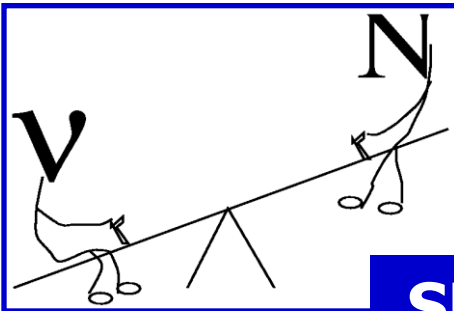
61



New Physics Scale

A mechanism of neutrino mass generation most likely works at a super-high energy scale. After integrating out heavy degrees of freedom, one is left with the **unique Weinberg** operator:

$$\frac{\mathcal{L}_{d=5}}{\Lambda} = \frac{1}{2} \kappa_{\alpha\beta} \overline{\ell_{\alpha L}} \tilde{H} \tilde{H}^T \ell_{\beta L}^c + \text{h.c.}$$



SEESAW

RGEs = Cable Car

To compare a theory with experiments, one must run the **RGEs** of the effective **v**-mass matrix down to low energies:

RGE = renormalization-group equation



Electroweak Scale

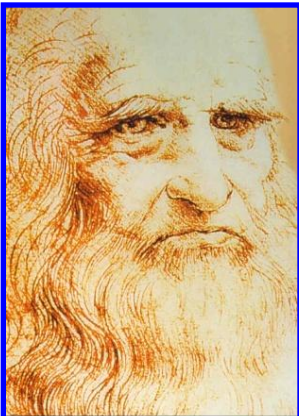
Other Possibilities?

62

What distinguishes different families of leptons or quarks?

----- they have the same gauge quantum numbers,
yet they are quite different from one another.

- ★ **Radiative Mechanism** (S. Weinberg 1972; A. Zee 1980)
- ★ **Texture Zeros** (S. Weinberg; H. Fritzsch 1977; H. Fritzsch 1978)
- ★ **Family Symmetries** (H. Harari et al 1978; C. Froggatt, H. Nielsen 1979)
- ★ **Seesaw Mechanism** (P. Minkowski 1977; T. Yanagida 1979;)
- ★ **Extra Dimensions** (K. Dienes et al; G. Dvali, A. Smirnov 1999)



LÉONARD DE VINCI

Our Philosophy

Although nature commences with reason and ends in experience, it is necessary for us to do the opposite, that is, to commence with experience and from this to proceed to investigate the reason