Matrix element method for differential cross section CDF vs « LHC »

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At Tevatron

Reference

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A search for resonant production of $t\bar{t}$ pairs in 4.8 fb^{-1} of integrated luminosity of $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV

- Look for narrow resonance.
- Cross section of $t\bar{t}$ production : $\sigma_{t\bar{t}} = 7.5 \pm 0.48 pb$
- $t\bar{t}$ semi-leptonic decay channel

For each candidates event, $t\bar{t}$ hypothesis is applied. The observed kinematic is mapped to parton level using the « Matrrix element » for $t\bar{t}$ production and decay.

M.E.M.

- Give a weight to each experimental event based on the probability to be produced by a particular process (Here $t\bar{t}$ hypothesis).
- Integration on the unknow variables.

$$P(x|\alpha) = \frac{1}{\sigma_\alpha} \int d\Phi |M|^2_\alpha (y) W(x,y)$$

MadGraph

MadGraph

Leading order computation

Using constraints event is reconstructed at parton level :
several configuration -> integration on it

- x : experimental event :
- α : M_{top}.
- dΦ : phase space element.
- W(x,y) : transfer function.

- Computation performs with MadWeight *

For Differential cross section

- Madweight have been adapted to estimate differential cross section.
  - Showcase with $t\bar{t}$ in dileptonic decay channel. With MG/ME generation and Delphes reconstruction (√s = 14 TeV).

- CDF M.E. tool provide the same analysis (subject of the paper).
  - Data at √s = 1.96 TeV. Semileptonic decay channel.

	CDF	MadWeight
Top mass is fixed :	172.5 GeV	173.4 GeV
Z' hypothesis	Several template	1 $M_{Z'}$ tested with several $\frac{N_{Z'}}{N_{tot}}$
	Full analysis with Background effect	Prospect for a high purity selection of $t\bar{t}$
T.F.	binned TF : 10 GeV bins in Et and 5 bins in pseudo-rapidity	Un-binned TF

Differential Matrix element method

Resonances at hadrons colliders in decay chain with invisibles particles :

CDF formulation

$$\pi(p|m_t) = \frac{1}{\sigma(m_t)} \int \underbrace{dz_a dz_b f_k(z_a) f_l(z_b)}_{} \underline{d\sigma_{kl}(p|m_t, z_a, z_b)}$$

expression of the integration on parton distribution function.

MadWeight formulation

$$P(x|\alpha) = \frac{1}{\sigma_\alpha} \int \underline{d\Phi |M|^2_\alpha(y) W(x,y)}$$

Any quantity wich can be expressed as a function of the momenta of the top decay products can be estimated as a probability density function.

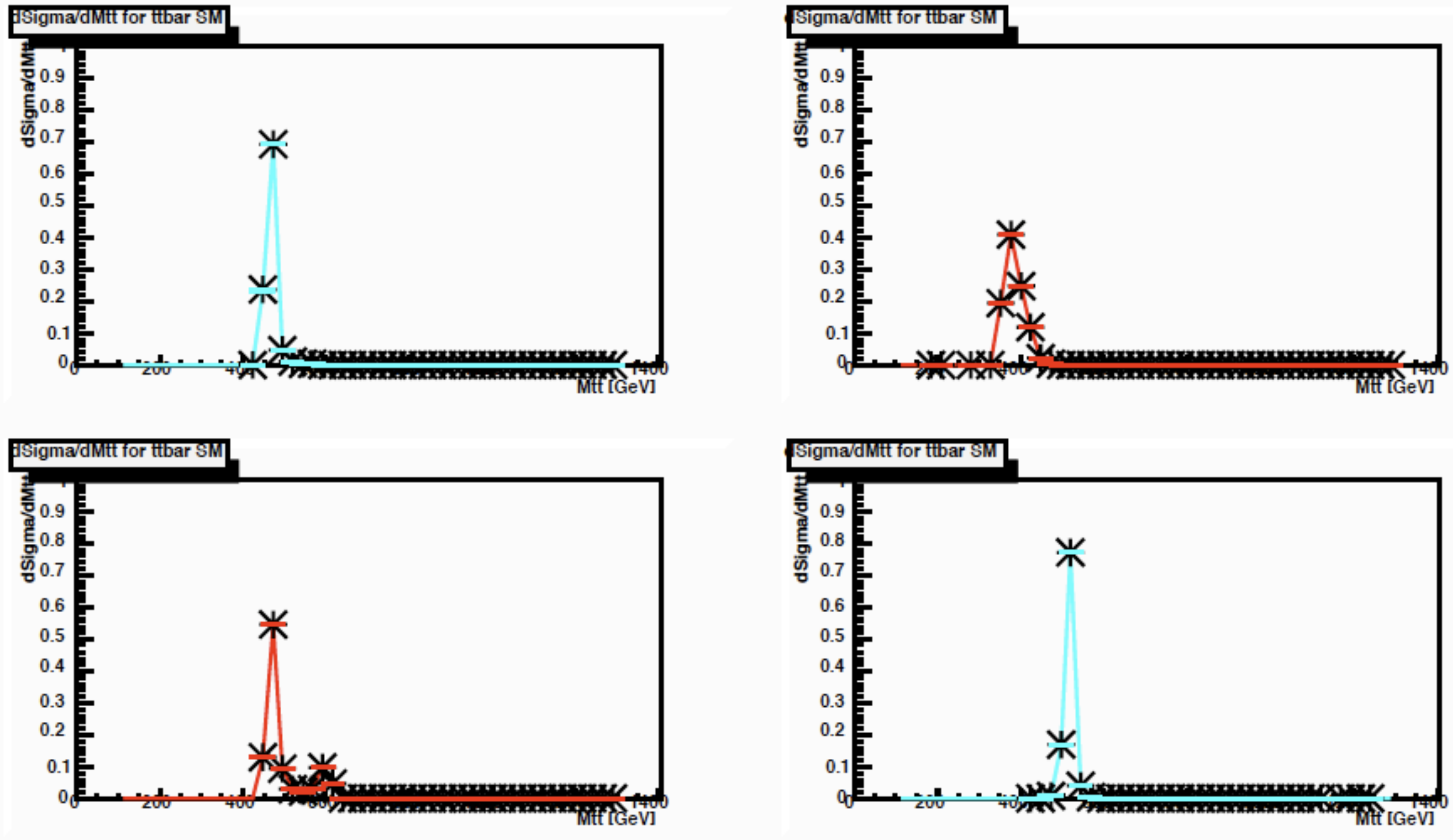
$$\rho(x) \equiv \int \sum_k \pi(p_k|m_t) W(j|p_k) \delta(x - \mathbb{M}(p_k)) dp_k$$

$$\frac{dP}{dm}(m_0) = \int d\Phi(y) |M|^2(y) W(x,y) \times \delta(m_{rec} - m_0)$$

m0 = M(pk) : invariant mass of $t\bar{t}$ system for a given partonic configuration pk .
W : is the transfer function.
In both case we make a sum on the jets permutation

P.D.F. for each event

With Madweight



For the $t\bar{t}$ system invariant mass.

Plot not available from CDF

Normalisation at 1 for each event.

At Tevatron (CDF) I

For each event : PDF of the reconstrucetd invariant mass.

	CEM	CMUP	CMX
4 jets, 1 tag	480	221	131
4 jets, ≥ 2 tags	110	33	21
≥ 5 jets, 1 tag	164	81	41
≥ 5 jets, ≥ 2 tags	47	21	16

TABLE II: Number of events observed in each subsample for data corresponding to an integrated luminosity of 4.8 fb^{-1}.

12 samples, according to different selection criteria.

CEM : electron reconstruction.

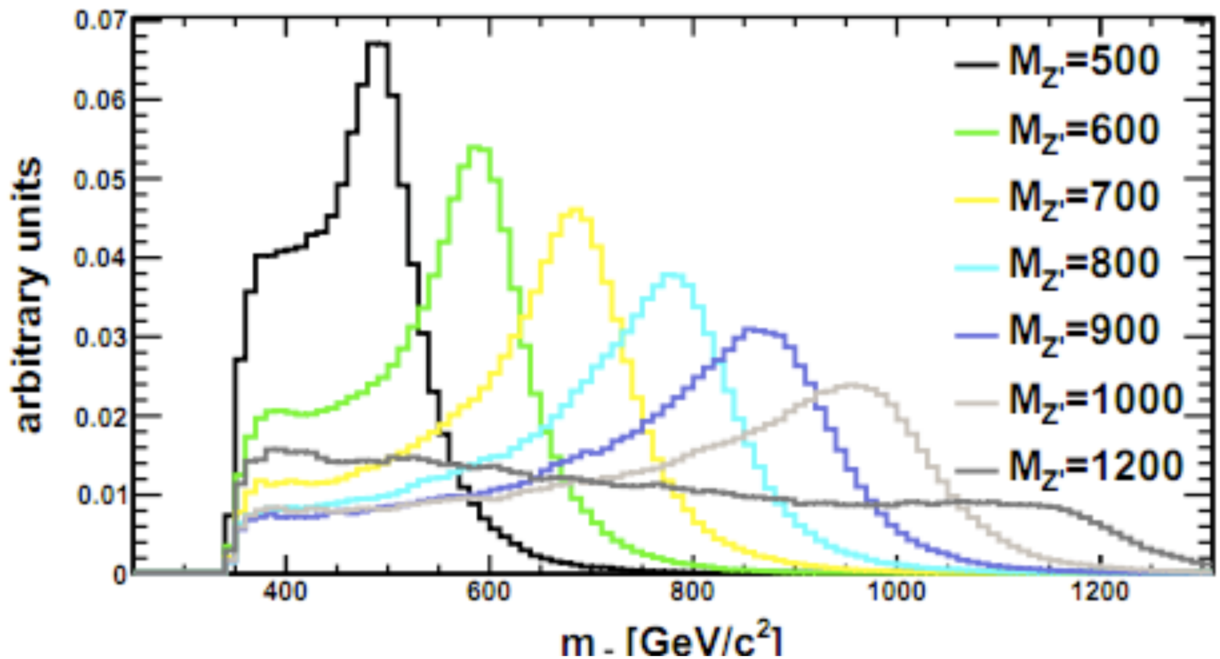
CMUP / CMX muons reconstruction.

each event is also normalized to 1 : $\int \rho(x)dx \equiv 1$

- Templates for several values of Z' pole mass :
- Mass peak generally prominent.

- Width = 1.2% of the pole mass

- Good separation with background shape.



At Tevatron (CDF) II

For each model (each background, signal)
Combination of event as (sum event-by-event pdfs) :

$$T_{c,i}(x) = \frac{1}{N_c} \sum_{j=1}^{N_c} \rho_j(x)$$

templates are normalized to unit area.

component	4 jets	$\geq$ 5 jets
non-W	46.1 ± 35.7	15.7 ± 12.2
Z+light flavor	6.4 ± 0.5	1.6 ± 0.1
W+light flavor	32.9 ± 8.5	7.4 ± 3.1
$Wb\bar{b}$	51.5 ± 12.6	12.4 ± 3.7
$Wc\bar{c}$	27.7 ± 6.6	7.3 ± 2.1
Wcj	14.0 ± 3.3	3.0 ± 0.9
single top	8.9 ± 0.4	1.4 ± 0.0
diboson	9.1 ± 0.6	2.4 ± 0.1
total non–$t\bar{t}$	196.6 ± 39.5	51.2 ± 13.3
SM $t\bar{t}$	667.1 ± 61.8	225.2 ± 21.0

TABLE I: Estimate of sample composition in terms of signal and background.

Background satisfying the selection are derived form cross section analysis

Probability for an event to be signal in sample i

$$P_i(f_{\text{sig}}) = f_{\text{sig}} P_{\text{sig},i} + (1 - f_{\text{sig}}) P_{\text{bg},i}.$$

- Signal : Z'-> $t\bar{t}$
- Background : SM $t\bar{t}$ and not $t\bar{t}$ process

$$f_{\text{sig}} = \frac{1}{n} \sigma \sum_{i=1}^{N_s} L_i A_i$$

L_i = Luminosity of the sample
A_i = acceptance (rec/selec efficiency)

With

$$P_{\text{sig},i} = \int T_{\text{sig},i}(x)\rho(x)\mathrm{d}x,$$

$$P_{\text{bg},i} = \int T_{\text{bg},i}(x)\rho(x)\mathrm{d}x$$

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The reconstructed invariant mass observed in data :

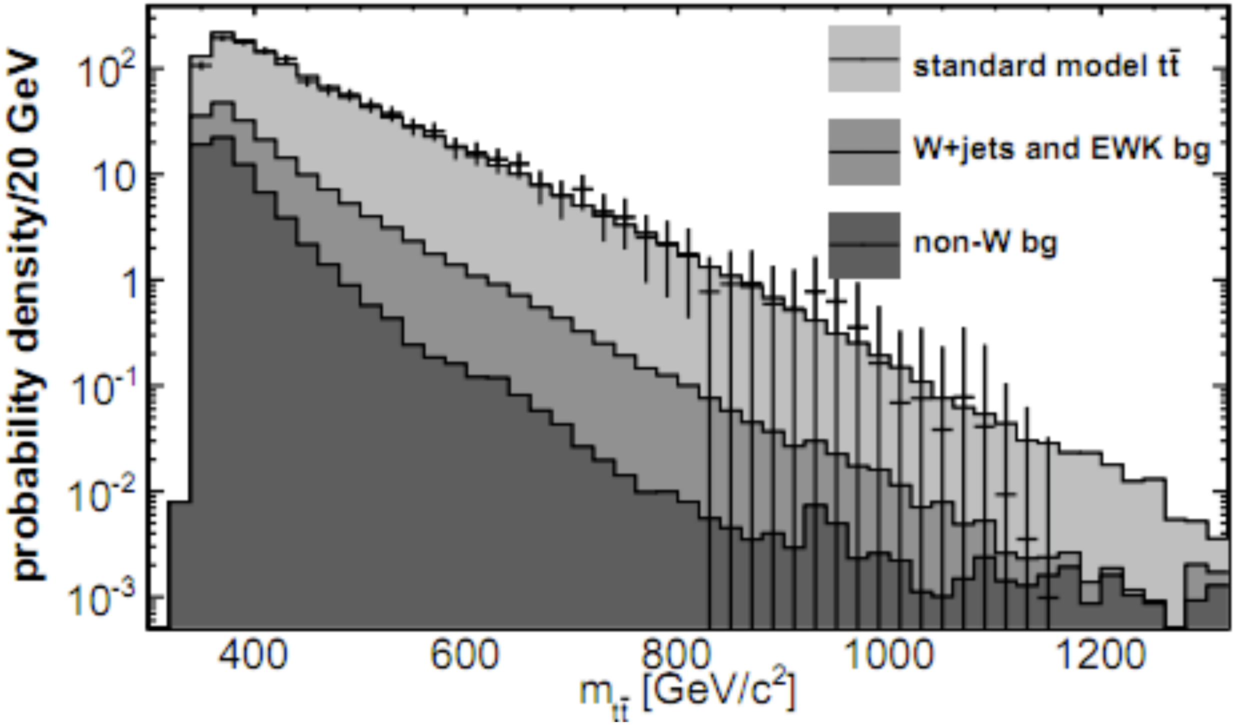

FIG. 2: The histogram of total probability density for the 1366 $t\bar{t}$ candidate events observed in $\mathcal{L} = 4.8\ \mathrm{fb}^{-1}$.

No indication of resonant $t\bar{t}$ production.

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Likelihood definition is the product (over the event i):

$$L(\sigma) = \prod_{i,j} P_i(\sigma) \cdot G(\nu_j | \overline{\nu}_j, \sigma_j)$$

ν_i : nuisance parameters with $\overline{\nu}_i$ expectation and σ_j uncertainty.

σ = Z' cross-section (Z' model).

They introduce nuisance with a gaussian distribution :
- to incorporate systematics effect to the likelihood.
- It's systematics due to acceptance effect (leptons ; b-tag)

Systematics uncertainties :

- ISR/FSR (generated by PYTHIA for M.C.)
- JES contribute significantly
- PDF negligible

-Background model syst. uncertainties are estimated by evaluating sample test (modifying the experimental condition).

Z' pole mass [GeV]	$\Delta\sigma$ [pb]
450	0.210
500	0.183
550	0.155
600	0.153
650	0.087
700	0.058
750	0.044
800	0.030
850	0.025

Z' pole mass [GeV]	$\Delta\sigma$ [pb]
900	0.020
950	0.012
1000	0.014
1100	0.016
1200	0.011
1300	0.029
1400	0.036
1500	0.065

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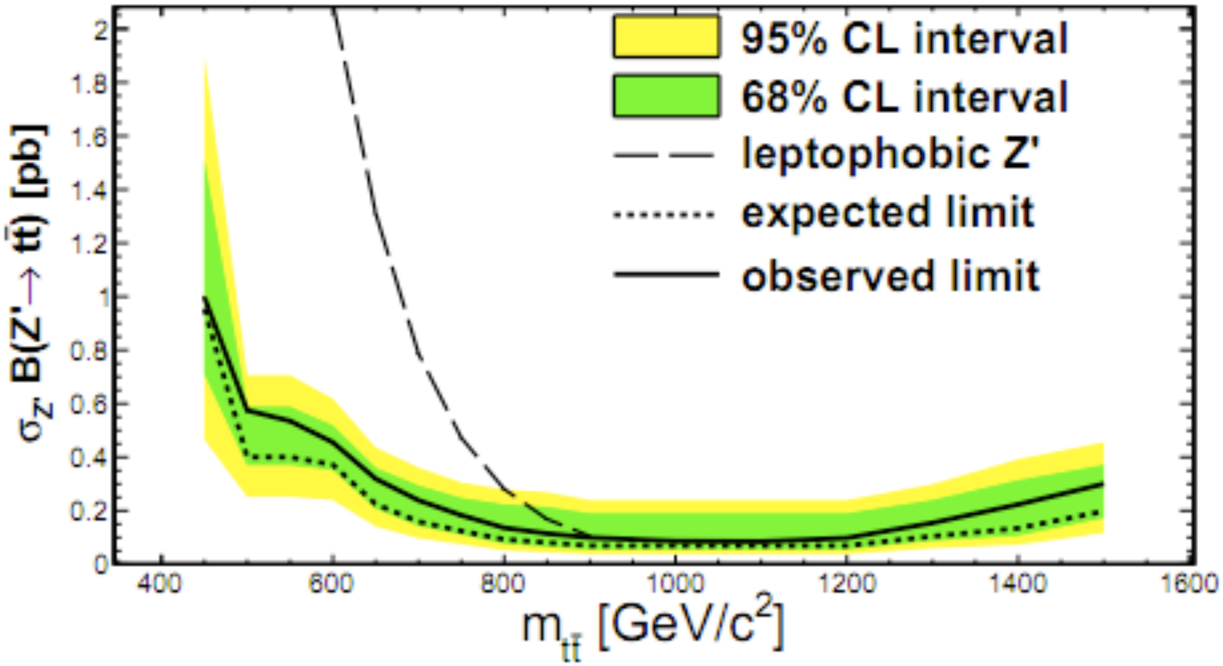
These errors are assuming to be gaussian. Likelihood definition become :

$$L'(\sigma) = \int_0^\infty G(\sigma, \sigma', \delta\sigma) L(\sigma') d\sigma'.$$

These quantity is used to compute a 95% C.L. upper limit FOR EACH Z' pole mass. (upper limit on the crosse section times branching ratio).

The limit correspond to the integral of the likelihood covering 95% of the area under the likelihood curve (starting from 0).



Expected and observed limit

$m_{Z'}$ [GeV/c^2]	Expected limit	Observed limit
450	0.954	1.05
500	0.400	0.49
550	0.400	0.46
600	0.371	0.40
650	0.224	0.26
700	0.159	0.20
750	0.123	0.15
800	0.092	0.11
850	0.080	0.10
900	0.068	0.09
950	0.069	0.08
1000	0.069	0.07
1100	0.069	0.07
1200	0.070	0.08
1300	0.104	0.13
1400	0.134	0.18
1500	0.197	0.24

TABLE IV: Expected and observed 95% C.L. limits on the cross section times branching ratio for narrow resonance production.

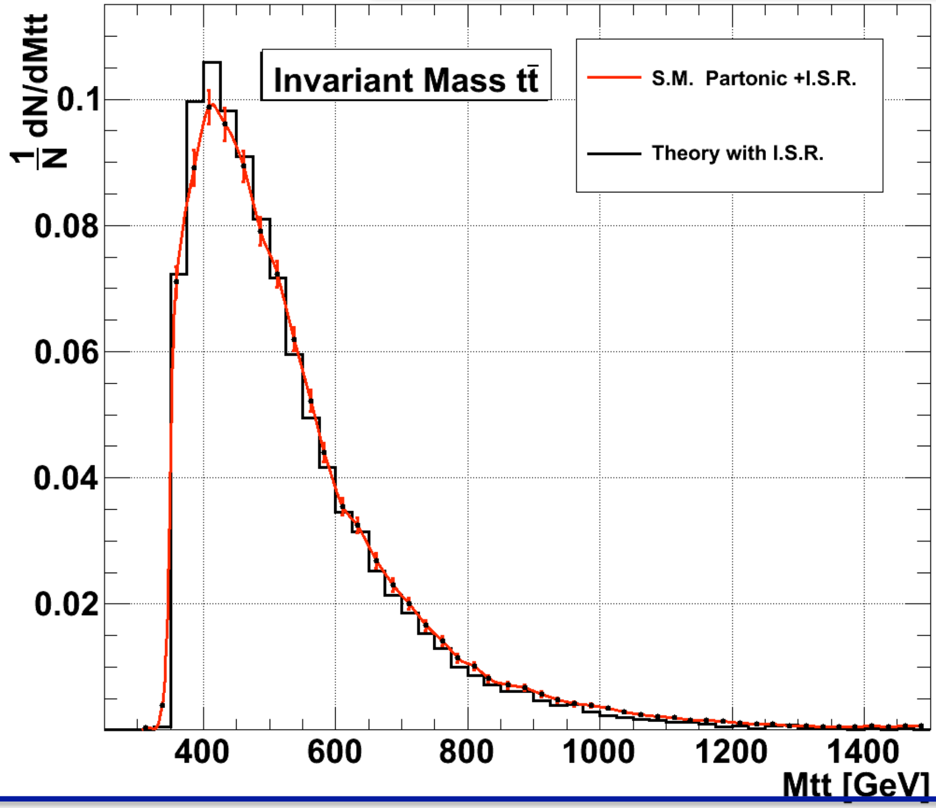
D.M.E.M. using MadWeight

All plot are done with 5000 events.

LHC expectation : Partonic level

Comparison with theory at leading order for :

- $M_{t\bar{t}}$: sensitivity to $X \rightarrow t\bar{t}$.
- $Cos(\theta^*_{t\bar{t}})$: sensitivity to the spin of X .
- 5000 events :
  - 2 isolated muons pt>20 GeV and |η|<2.4 .
  - ONLY 2 jets pt>30 GeV and |η|<2.4 and b-tagged.
  - MET > 30 GeV.



Perfect detector

transfer function :

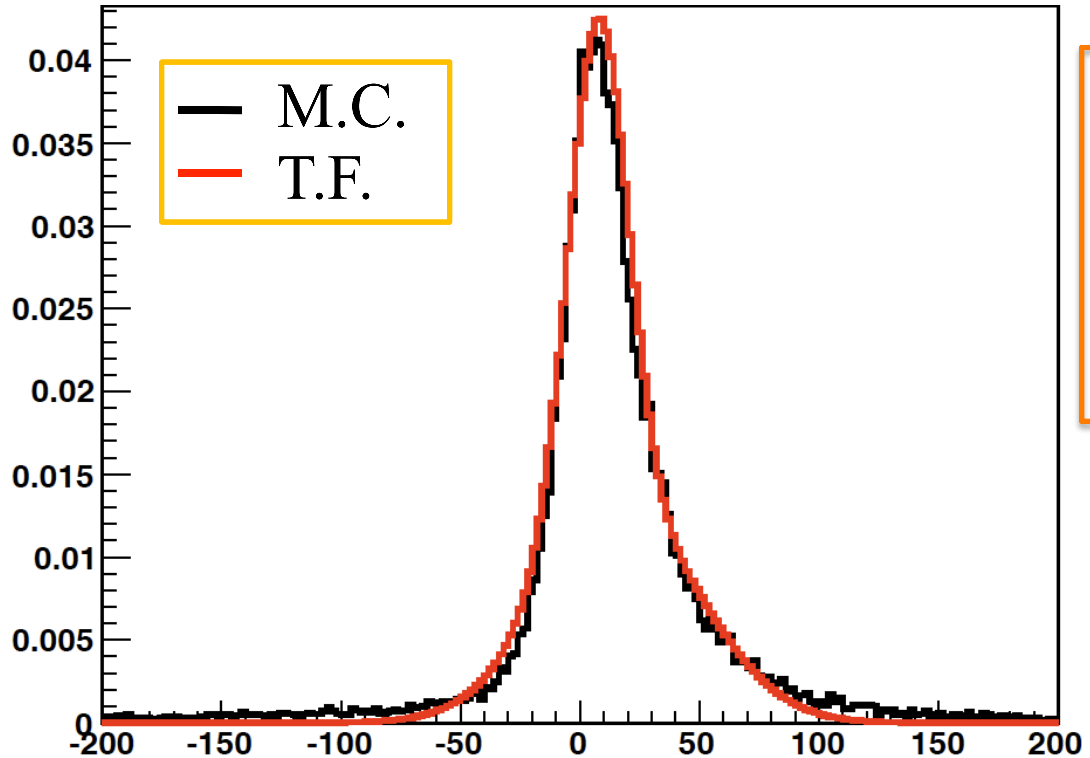
$$W = \delta \text{ function}$$

$$\sqrt{s} = 14TeV$$

LHC expectation : transfer function

Fast simulation with Delphes

•Jets energy transfer function as a double gaussian.

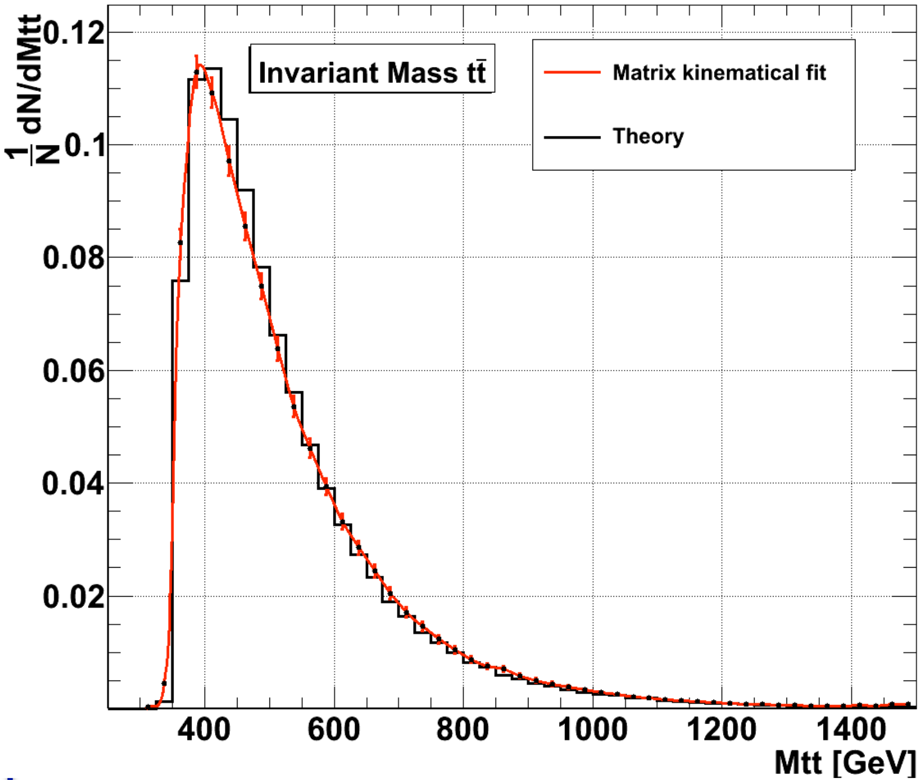


- 150.000 pairs of jets/partons
- 10 parameters :

$$P_i = a_{i0} + a_{i1}.E$$

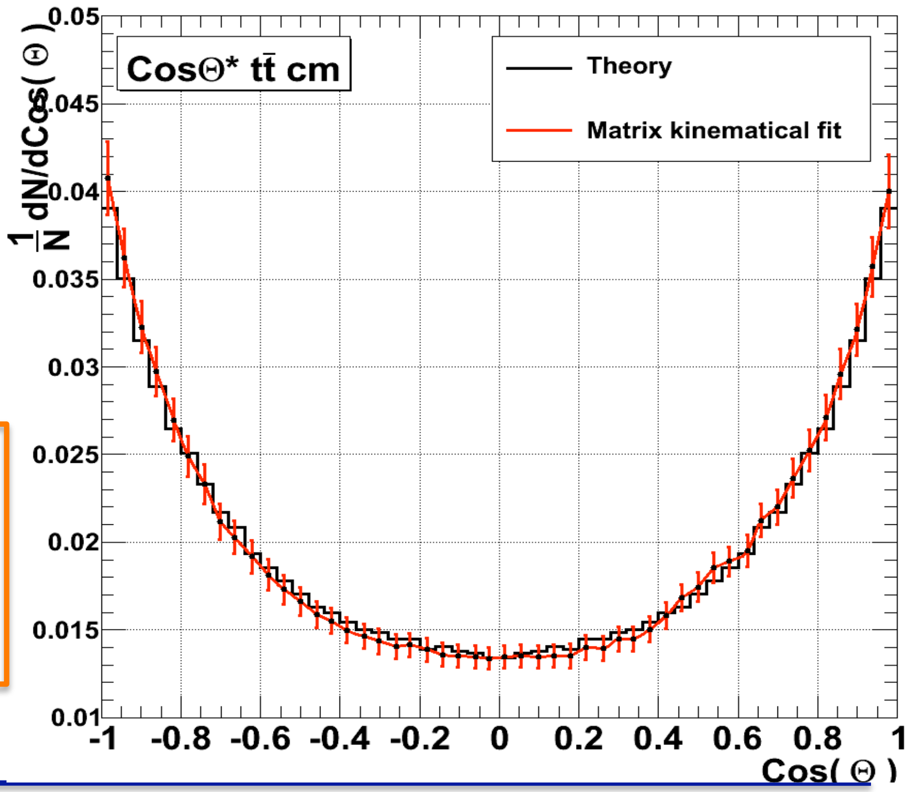
$$W(E_j, E_{part}) = \frac{1}{\sqrt{2}(P_2 + P_3.P_5)} . \left(e^{-\frac{[(E_{part}-E_j)-P_1]^2}{2.P_2^2}} + P_3.e^{-\frac{[(E_{part}-E_j)-P_4]^2}{2.P_5^2}} \right).$$

LHC expectation : Delphes reconstruction



- Good agreement with the theory.
- No bias due to the transfer function

Reconstruction of other variables as $Cos(\theta^{*}_{t\bar{t}})$



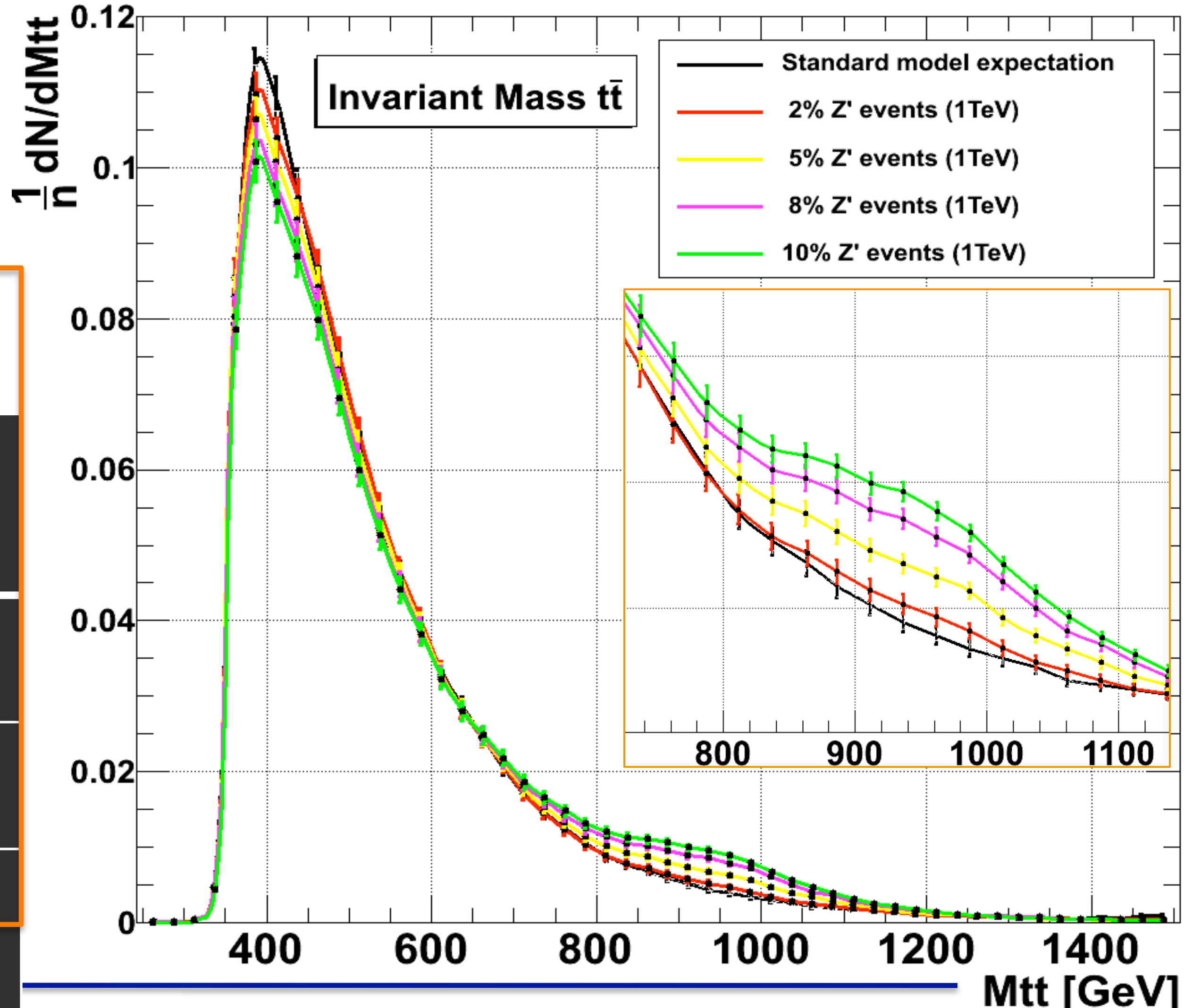
LHC expectation : search of new physics

Deviation from the standard model introducing a resonance decaying in $t\bar{t}$.

- Spin 1 particule (Z') [3] .
- 1 TeV
- Various $\frac{N_{Z'}}{N_{tot}}$: 2%, 5%, 8% 10%

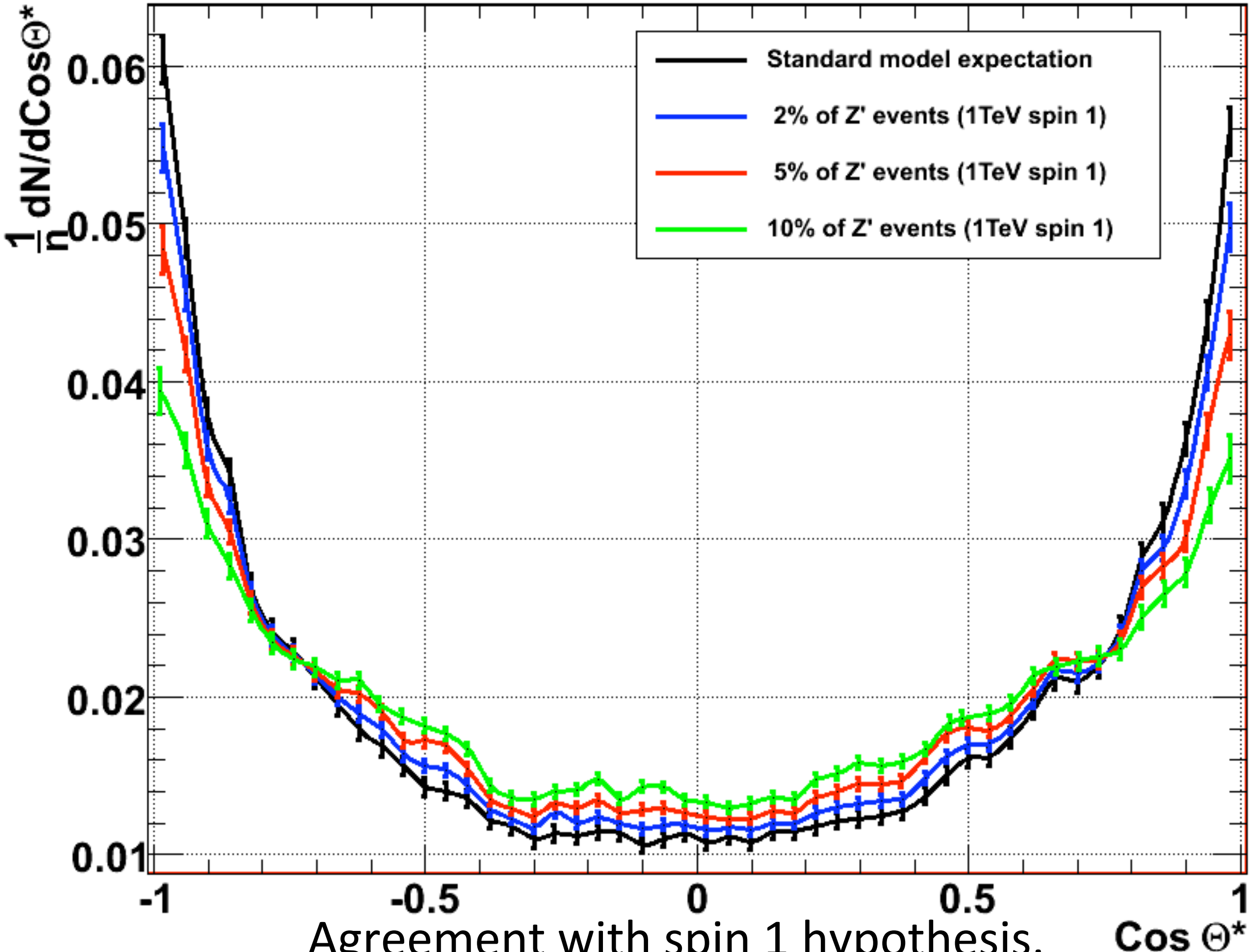
Compatibility with the S.M.

$N_{Z'}/N_{tot}$	NOT S.M. C.L.
2%	72 %
3%	95.6 %
5%	> 99.998%
8%	
10%	



LHC expectation : search of new physics

$Cos(\theta^*_{t\bar{t}})$ reconstructed for events with $M_{t\bar{t}}$ reconstructed above 700 GeV.



Agreement with spin 1 hypothesis.

LHC expectation : statistic

- Each event contributes to a few bins. Errors on each bin (k) are correlated.
- Correlation matrices have been computed :
  - using 100.000 events.
  - following : $COR(k,l) = \frac{Cov(k,l)}{\sigma(k)\sigma(l)}$ with $cov(k,l) = \frac{1}{N} \sum_{events} (k_i - \bar{k})(l_i - \bar{l})$

$$\sigma(k)^2 = \frac{1}{N} \sum_{events} (k_i - \bar{k})^2$$

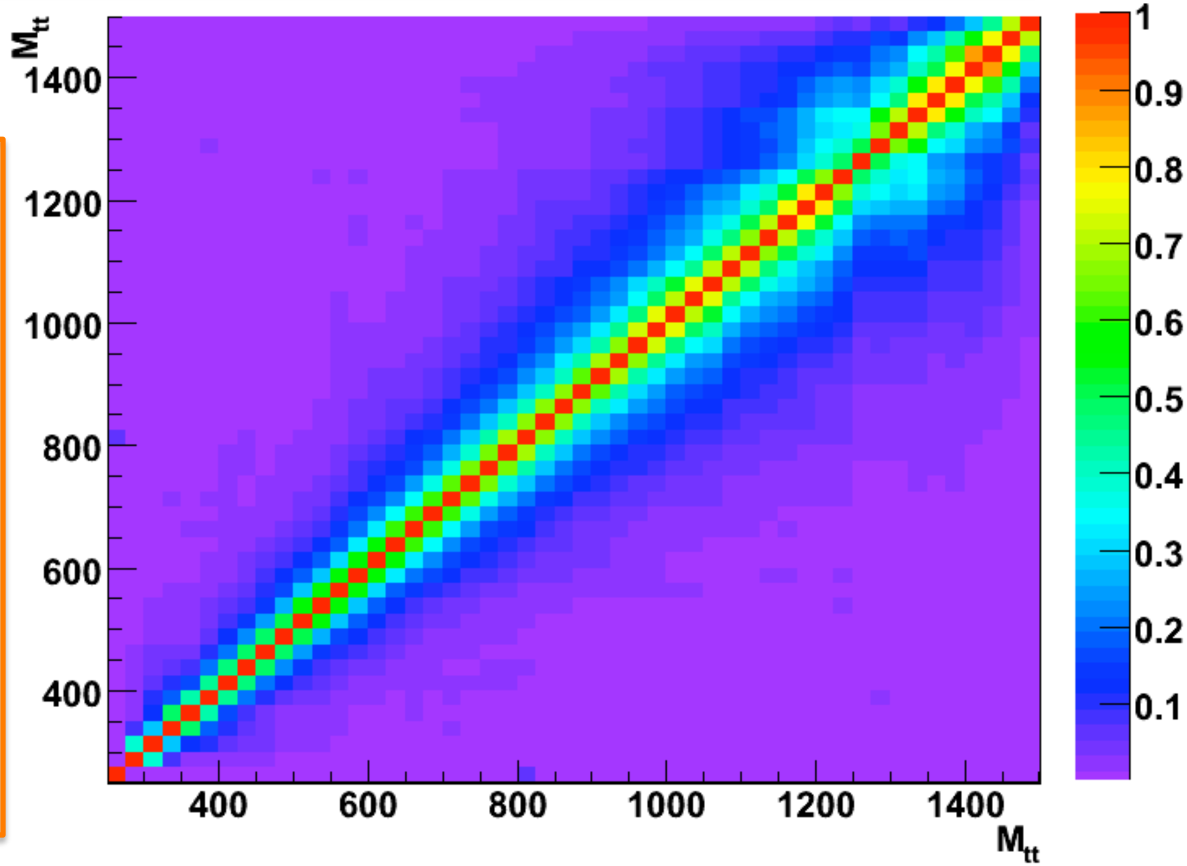
S.M. confidence level

χ^2 is defined between the standard model expectation and the observed result.

$$\chi^2 = \sum_i \frac{(x_i - x_i^{th})^2}{\sigma_i^2}$$

Taking into account the correlation :

$$\chi^2 = \sum_{ij} C_{ij}^{-1}(x_i - x_i^{th})(x_j - x_j^{th})$$

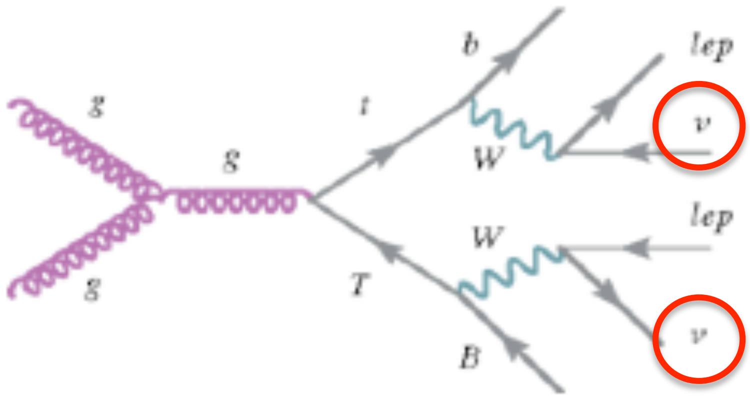


LHC expectation

LHC expectation

Matrix Element method

- Topology not fully reconstructed at detector level (ex: ttbar dileptonic)



Information has been lost :
- Neutrinos are undetected.
- b-quarks ave hadronized + showered -> jets

How to access to top quark information ?

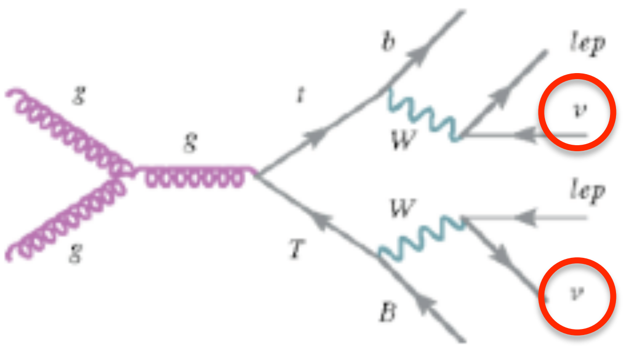
Top mass or $t\bar{t}$ system invariant mass for example !

→ Matrix Element Method

M.E.M.

Use theoritical information of the process :
- W and top mass.
- Energy-momentum conservation

Degree of freedom	Constraints
PDF : 2 dof	E-P conservation : 4
neutrino 1 : 3 dof	M_W : 2
neurtino 2 : 3 dof	M_t : 2
+ leptons and b-quark	+ detector informations
TOTAL : 20 dof	TOTAL : 20 constraints

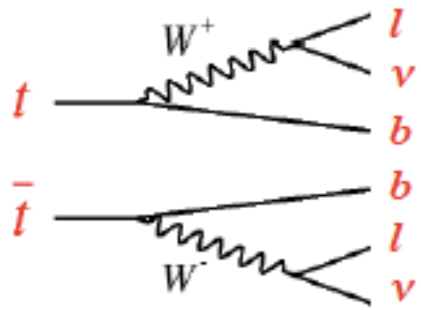


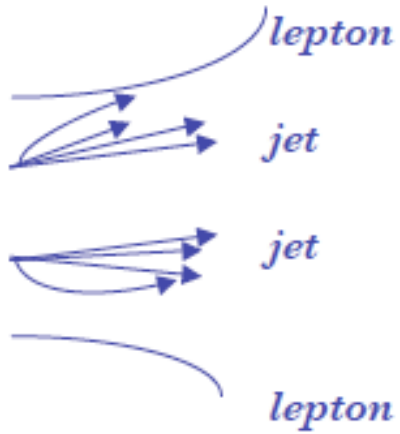
- Give a weight to each experimental event based on the probability to be produced by a particular process (Here $t\bar{t}$ hypothesis).
- Integration on the unknow variables.

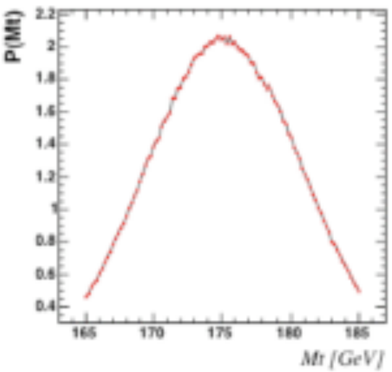
M.E.M.

- Give a weight to ttbar candidates.
- Weight in function of M_{top}.

P(parton|M_{top}) $|M|^2$ × P(exp|parton) W(exp,parton) = P(X|M_{top})





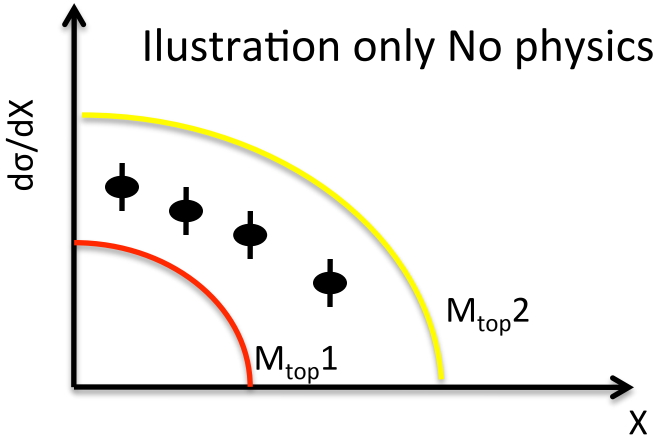


Transfer function : hadronization+showering + detector resolution effects

- All jet combinations are considered.

Why using it ?

• We have not to use constraints on reconstructed kinematical variables to built estimator of the parameter(s) we are looking for.



What's the best choice for X :

- Pt
- M_{jjb}
-
-

Matrix Element method fit ALL distributions with ALL correlations

How to estimate a parameter

e.g. top quark mass :
For each event a weight is computed for
- several top mass hypothesis (correctly normalized)

For each hypothesis results of all events are combined.

- Weight computed for different values of α (M_{top}).
- Best estimation of parameters α is the one wich maximize L .

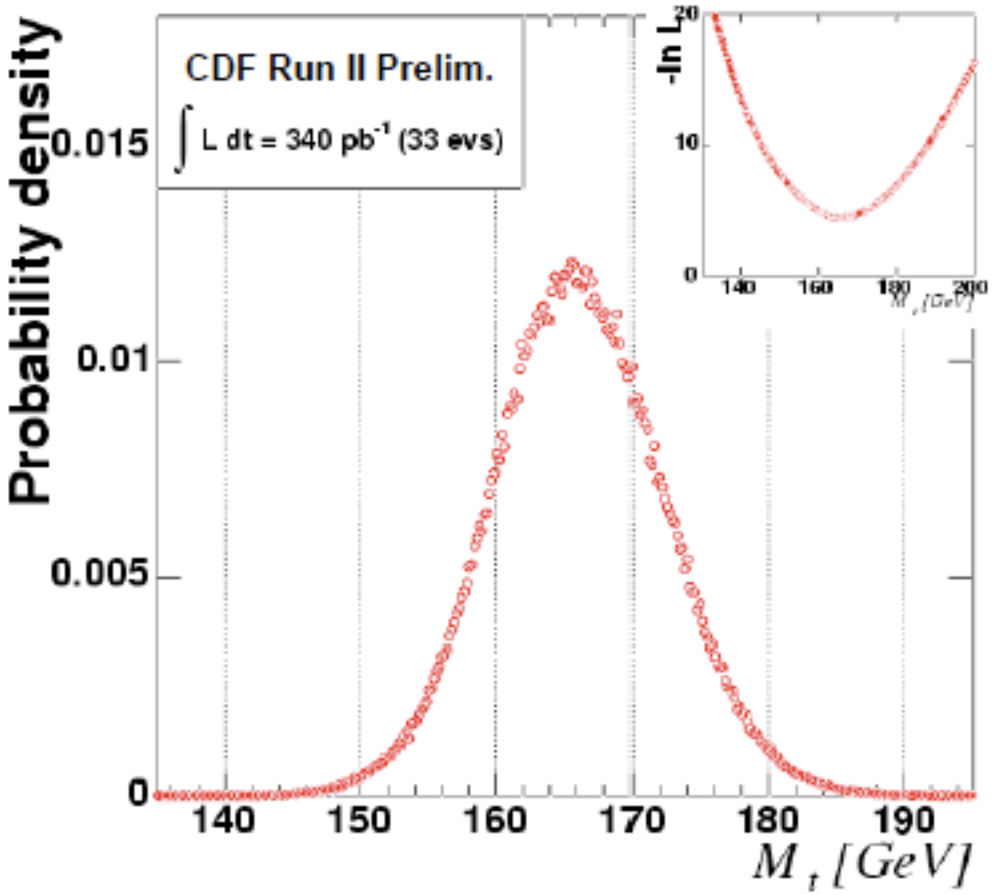
$$L(\alpha) = \prod_{i=1}^{N} \bar{P}(x_i|\alpha)$$

- we minimize :

$$-ln(L(\alpha)) = \sum_{i=1}^{N} -ln(P(x_i|\alpha))$$

At Tevatron

- At CDF :



$$M_t = 165.3 \pm 6.3\ _{stat} \pm 3.6_{syst}\ \mathrm{GeV/c^2}$$