II. The Wess and Zumino model

① The N=1 matter supermultiplet: SUSY transf. laws

- The Wess and Zumino model (PLB 49 (1974) 52; NPB 70 (1974) 39) describes the dynamics of the N=1 matter supermultiplet:
  - one massless Weyl fermion ψ (left-handed),
  - one massless complex scalar field ϕ
- The most general SUSY transformation laws can be written as:

$$\begin{cases} \delta\phi = O_\phi(\psi) \\ \delta\psi = O_\psi(\phi) \end{cases}$$

Since a scalar field has mass-dimension $[\phi] = 1$ and a Weyl fermion $[\psi] = 3/2$

$$\Rightarrow \begin{cases} [O_\phi] = -1/2 \\ [O_\psi] = +1/2 \end{cases}$$

Moreover, O_ϕ and O_ψ depend on the spinorial SUSY transf. parameter ε.

Rk: We associate to the supercharges $(Q, \bar{Q})$ the parameters $(\varepsilon, \bar{\varepsilon})$

- Due to Lorentz invariance, we can only write:

$$\boxed{\delta\phi = \sqrt{2}\ \underbrace{\varepsilon\cdot\psi}_{\text{dot product of 2 left-handed fermion}}} \quad (1)$$

▷ The $\sqrt{2}$ is purely conventional

▷ $[\varepsilon] = -1/2 \longleftrightarrow [O_\phi] = -1/2$

- For the fermion, using again Lorentz invariance:

$$\delta\psi_\alpha = \sqrt{2}\left(a\,\sigma^\mu_{\alpha\dot\alpha}\,\partial_\mu\phi + b\,\sigma^\mu_{\alpha\dot\alpha}\,\partial_\mu\phi^\dagger\right)\bar\varepsilon^{\dot\alpha} \quad (2)$$

and $[O_\psi] = 3/2$

In order to calculate the coefficients a and b, we start from the free lagrangian describing the dynamics of ϕ and ψ:

$$\mathcal{L} = \underbrace{\partial_\mu\phi^\dagger\,\partial^\mu\phi}_{\mathcal{L}_\phi} + \underbrace{i\,\bar\psi_{\dot\alpha}\,\bar\sigma^{\mu\,\dot\alpha\alpha}\,\partial_\mu\psi_\alpha}_{\mathcal{L}_\psi}$$

and we calculate its variation under the considered SUSY transformation

$$\delta\mathcal{L}_\phi = \partial_\mu(\delta\phi^\dagger)\,\partial^\mu\phi + \partial_\mu\phi^\dagger\,\partial^\mu(\delta\phi)$$

$$= \sqrt{2}\left[\bar\varepsilon\,\partial_\mu\bar\psi\,\partial^\mu\phi + \partial_\mu\phi^\dagger\,\varepsilon\,\partial^\mu\psi\right] = -\sqrt{2}\left[\bar\varepsilon\,\bar\psi\,\Box\phi + \varepsilon\cdot\psi\,\Box\phi^\dagger\right] + \text{t.d.} \quad (3)$$

where t.d stands for a total derivative

The full Lagrangian must be supersymmetric

$$\Rightarrow \quad \delta\mathcal{L} = \delta\mathcal{L}_\phi + \delta\mathcal{L}_\psi = \text{t.d.}$$

But: $\delta\mathcal{L}_\psi = i(\delta\bar\psi)_{\dot\alpha}\,\bar\sigma^{\mu\dot\alpha\alpha}\,\partial_\mu\psi_\alpha + i\underbrace{\bar\psi_{\dot\alpha}\,\bar\sigma^{\mu\dot\alpha\alpha}\,\partial_\mu(\delta\psi)}$

using (2): $= (\ldots)a\bar\psi\,\bar\varepsilon\,\phi + (\ldots)b\,\bar\psi\cdot\bar\varepsilon\,\phi^\dagger$

$\Rightarrow$ comparing with (3): $b=0$ (we have not $\bar\psi\,\phi^\dagger$ term)

$$\Rightarrow \delta\mathcal{L}_\psi = \sqrt{2}\left(ia^*\partial_\nu\phi^\dagger\,\varepsilon^\beta\sigma^\nu_{\beta\dot\alpha}\,\bar\sigma^{\mu\dot\alpha\alpha}\,\partial_\mu\psi_\alpha + ia\bar\psi_{\dot\alpha}\,\bar\sigma^{\mu\dot\alpha\alpha}\,\sigma^\nu_{\alpha\dot\beta}\,\bar\varepsilon^{\dot\beta}\,\partial_\mu\partial_\nu\phi\right)$$

$$= \sqrt{2}\left(ia^*\,\varepsilon\sigma^\nu\bar\sigma^\mu\,\partial_\mu\psi\,\partial_\nu\phi^\dagger + ia\,\bar\psi\bar\sigma^\mu\sigma^\nu\bar\varepsilon\,\partial_\mu\partial_\nu\phi\right)$$

$$= \sqrt{2}\left(-ia^*\,\varepsilon\sigma^\nu\bar\sigma^\mu\psi\,\partial_\mu\partial_\nu\phi^\dagger + ia\,\underbrace{\bar\varepsilon\bar\sigma^\nu\sigma^\mu\bar\psi}_{\text{cf. relation (2)}}\,\partial_\mu\partial_\nu\phi\right) + \text{t.d.}$$

$$\underline{\text{But}}:\quad \bar\sigma^\mu\sigma^\nu\,\partial_\mu\partial_\nu\phi = \tfrac{1}{2}\underbrace{(\bar\sigma^\mu\sigma^\nu + \bar\sigma^\nu\sigma^\mu)}_{2\eta^{\mu\nu}}\partial_\mu\partial_\nu\phi = \Box\phi$$

$$\Rightarrow \delta\mathcal{L}_\psi = \sqrt{2}\left(-ia^*\,\varepsilon\cdot\psi\,\Box\phi^\dagger + ia\,\bar\varepsilon\cdot\bar\psi\,\Box\phi\right) + \text{t.d.}$$

Comparing with (3) and asking $\mathcal{L}_\phi + \delta_\psi = \text{t.d.}$:

$$a = -i$$

$$\Rightarrow \quad \boxed{\begin{aligned}\delta\psi &= -\sqrt{2}\,i\,(\sigma^\mu\bar\varepsilon)\,\partial_\mu\phi\\ \delta\phi &= \sqrt{2}\;\varepsilon\cdot\psi\end{aligned}} \quad (4)$$

⑨ Conventions on spinors

- In the previous section, we have introduced two types of spin indices:

  - the undotted spin indices ⟷ left-handed spinors
  - the dotted spin indices ⟷ right-handed spinors

And these indices can be upper or lower.

These indices are fundamental in supersymmetry. Care must be taken in order to avoid mistakes in calculations

- Let us summarize all the conventions we use in this course.

ⓐ Lorentz:

The metric tensor is given by:

$$\eta^{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}\,(1,-1,-1,-1)$$

and

$$\eta^{\mu\nu}\,\eta_{\nu\rho} = \delta^{\mu}{}_{\rho}$$

It can be used to raise and lower Lorentz indices

b) Pauli matrices and Levi-Civita tensor

- The Pauli matrices are

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and

$$\sigma_i = -\sigma^i$$

We can build the four-vectors

$$\sigma^\mu = (\sigma^0, \sigma^i) \quad ; \quad \bar{\sigma}^\mu = (\sigma^0, -\sigma^i)$$

with $\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

- The Levi-Civita or fully antisymmetric tensor is defined with

$$\varepsilon_{0123} = +1$$

Any cyclic permutation of (0,1,2,3) gives +1; anticyclic ones gives −1.
The other possibilities for the indices vanish.

c) Spinors

In the van der Waerden notations,

- a left-handed Weyl spinor $\lambda_L \longleftrightarrow \lambda_\alpha$
- a right-handed Weyl spinor $\bar{\chi}_R \longleftrightarrow \bar{\chi}^{\dot\alpha}$

We can raise and lower the indices through the antisymmetric tensors

$$\varepsilon^{\alpha\beta}, \varepsilon_{\alpha\beta}, \varepsilon^{\dot\alpha\dot\beta}, \varepsilon_{\dot\alpha\dot\beta}$$

which are defined through

$$\varepsilon_{12} = -\varepsilon^{12} = \varepsilon_{\dot{1}\dot{2}} = -\varepsilon^{\dot{1}\dot{2}} = 1.$$

We have:

$$\varepsilon_{\alpha\beta}\,\varepsilon^{\beta\delta} = \delta_{\alpha}{}^{\delta} \quad , \quad \varepsilon_{\dot{\alpha}\dot{\beta}}\,\varepsilon^{\dot{\beta}\dot{\delta}} = \delta_{\dot{\alpha}}{}^{\dot{\beta}}$$

and thus

$$\begin{cases} \lambda^{\alpha} = \varepsilon^{\alpha\underline{\underline{\beta}}}\,\lambda_{\underline{\underline{\beta}}} \\ \bar{\chi}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\underline{\underline{\dot{\beta}}}}\,\bar{\chi}^{\underline{\underline{\dot{\beta}}}} \end{cases}$$

Rk: $\varepsilon_{\alpha\underline{\underline{\beta}}}\,\varepsilon^{\underline{\underline{\beta}}\delta} = -\,\varepsilon_{\alpha\underline{\underline{\beta}}}\,\varepsilon^{\delta\underline{\underline{\beta}}}$

In addition, we note that

$$(\lambda^{\alpha})^{\dagger} = \bar{\lambda}^{\dot{\alpha}} \quad \text{and} \quad (\bar{\chi}^{\dot{\alpha}})^{\dagger} = \chi^{\alpha} \qquad (\text{etc}\ldots)$$

d) Scalar products:

We can now define scalar products of spinors.

$$\lambda \cdot \lambda' = \lambda^{\alpha} \cdot \lambda'_{\alpha} \qquad \text{for left-handed spinors}$$

$$\bar{\chi} \cdot \bar{\chi}' = \bar{\chi}_{\dot{\alpha}}\, \bar{\chi}'^{\dot{\alpha}} \qquad \text{for right-handed spinors}$$

The position of the indices is again fundamental. It can be shown that:

$$\lambda^{\alpha}\lambda_{\alpha} = -\lambda_{\alpha}\lambda^{\alpha}$$

c) Pauli matrices with the van der Waerden notations

It simply follows that:

$$\sigma^\mu \longleftrightarrow \sigma^\mu{}_{\alpha\dot\alpha} \qquad \sigma^{\mu\nu} \longleftrightarrow (\sigma^{\mu\nu})_\alpha{}^\beta$$

$$\bar\sigma^\mu \longleftrightarrow \bar\sigma^{\mu\dot\alpha\alpha} \qquad \bar\sigma^{\mu\nu} \longleftrightarrow (\bar\sigma^{\mu\nu})^{\dot\alpha}{}_{\dot\beta}$$

and the Lorentz transformations of spinors are given by:

$$\psi_\alpha \longrightarrow \psi'_\alpha = (M)_\alpha{}^\beta \psi_\beta \quad \text{with} \quad M = e^{-\frac{i}{2}\omega_{\mu\nu}\sigma^{\mu\nu}}$$

$$\bar\chi^{\dot\alpha} \longrightarrow \bar\chi'^{\dot\alpha} = (\bar M)^{\dot\alpha}{}_{\dot\beta}\, \bar\chi^{\dot\beta} \quad \text{with} \quad \bar M = e^{-\frac{i}{2}\omega_{\mu\nu}\bar\sigma^{\mu\nu}}$$

f) Ex II, 1 - useful identities to proof

* $\psi\cdot\lambda = \lambda\cdot\psi$
* $\bar\psi\cdot\bar\lambda = (\psi\cdot\lambda)^\dagger$
* $\bar\sigma^{\mu\dot\beta\beta} = \sigma^\mu{}_{\alpha\dot\alpha}\, \varepsilon^{\alpha\beta}\, \varepsilon^{\dot\alpha\dot\beta}$
* $\bar\psi\bar\sigma^\mu\lambda = (\bar\lambda\sigma^\mu\psi)^\dagger = -\lambda\sigma^\mu\bar\psi$
* $\psi\sigma^\mu\bar\sigma^\nu\lambda = \lambda\sigma^\nu\bar\sigma^\mu\psi$
* $\chi_\alpha(\lambda\cdot\psi) + \lambda_\alpha(\chi\cdot\psi) + \psi_\alpha(\lambda\cdot\chi) = 0$ (Fierz identity)
* $\sigma_{\mu\nu} = \frac{i}{2}\varepsilon_{\mu\nu\rho\sigma}\sigma^{\rho\sigma}$ self-duality of $\sigma_{\mu\nu}$
* $\bar\sigma_{\mu\nu} = -\frac{i}{2}\varepsilon_{\mu\nu\rho\sigma}\bar\sigma^{\rho\sigma}$ antiself-duality of $\bar\sigma_{\mu\nu}$
* $\sigma^\mu_{\alpha\dot\alpha}\,\bar\sigma_\mu^{\dot\beta\beta} = 2\,\delta_\alpha{}^\beta\,\delta_{\dot\alpha}{}^{\dot\beta}$
* $\sigma^\mu{}_{\alpha\dot\alpha}\,\sigma_{\mu\beta\dot\beta} = 2\,\varepsilon_{\alpha\beta}\,\varepsilon_{\dot\alpha\dot\beta}$

* $\bar\sigma^\mu \sigma^\nu \bar\sigma^\rho + \bar\sigma^\rho \sigma^\nu \bar\sigma^\mu = 2\left(\eta^{\mu\nu}\bar\sigma^\rho + \eta^{\nu\rho}\bar\sigma^\mu - \eta^{\mu\rho}\bar\sigma^\nu\right)$
* $\bar\sigma^\mu \sigma^\nu \bar\sigma^\rho - \bar\sigma^\rho \sigma^\nu \bar\sigma^\mu = 2i\, \varepsilon^{\mu\nu\rho\sigma}\,\bar\sigma_\sigma$

3) Auxiliary fields

- We now get back to the Wess and Zumino model and compute the action of the commutator of two SUSY transformations of param. ε_1 and ε_2 on our scalar and fermionic fields. Since:

- $\delta_1\delta_2\phi = \sqrt{2}\ \varepsilon_2\cdot\delta_1\psi$

$$= \sqrt{2}\ \varepsilon_2\cdot\left(-\sqrt{2}\,i(\sigma^\mu\bar\varepsilon_1)\partial_\mu\phi\right)$$

$$= -2i\,(\varepsilon_2\sigma^\mu\bar\varepsilon_1)\,\partial_\mu\phi$$

- $\delta_1\delta_2\psi = -\sqrt{2}\,i\ (\sigma^\mu\bar\varepsilon_2)\,\partial_\mu(\delta_1\phi)$

$$= -\sqrt{2}\,i\,(\sigma^\mu\bar\varepsilon_2)(\sqrt{2}\,\varepsilon_1\cdot\partial_\mu\psi)$$

$$= -2i\,(\sigma^\mu\bar\varepsilon_2)(\varepsilon_1\partial_\mu\psi)$$

$$= 2i\,\varepsilon_1(\partial_\mu\psi\,\sigma^\mu\bar\varepsilon_2) + 2i\,\partial_\mu\psi\,(\varepsilon_1\sigma^\mu\bar\varepsilon_2) \qquad \text{cf. Fierz identities}$$

$$= -2i\,\varepsilon_1(\bar\varepsilon_2\bar\sigma^\mu\partial_\mu\psi) + 2i\,\partial_\mu\psi\,(\varepsilon_1\sigma^\mu\bar\varepsilon_2)$$

$$\Rightarrow \begin{cases} [\delta_1,\delta_2]\phi = -2i\,(\varepsilon_2\sigma^\mu\bar\varepsilon_1 - \varepsilon_1\sigma^\mu\bar\varepsilon_2)\,\partial_\mu\phi \\ [\delta_1,\delta_2]\psi = -2i\,(\varepsilon_2\sigma^\mu\bar\varepsilon_1 - \varepsilon_1\sigma^\mu\bar\varepsilon_2)\,\partial_\mu\psi - 2i\,(\varepsilon_1\,\bar\varepsilon_2\bar\sigma^\mu\partial_\mu\psi - \varepsilon_2\,\bar\varepsilon_1\bar\sigma^\mu\partial_\mu\psi) \end{cases}$$

- If the fermion is on-shell, its Weyl equation holds

$$\bar{\sigma}^\mu \partial_\mu \psi = 0$$

$$\Rightarrow \quad \boxed{[\delta_1, \delta_2] \begin{pmatrix} \phi \\ \psi \end{pmatrix} = -\Delta^\mu \partial_\mu \begin{pmatrix} \phi \\ \psi \end{pmatrix} \text{ with } \Delta^\mu = 2i\,(\epsilon_2 \sigma^\mu \bar{\epsilon}_1 - \epsilon_1 \sigma^\mu \bar{\epsilon}_2)} \qquad (1)$$

- From (1), it is clear that the commutator of two SUSY transformation is a translation:

$$[\delta_1, \delta_2] \begin{pmatrix} \phi \\ \psi \end{pmatrix} = i\,\Delta^\mu P_\mu \begin{pmatrix} \phi \\ \psi \end{pmatrix}$$

since we recall that:

- $P_\mu = i\partial_\mu$ generates the translations
- $\phi(x) \xrightarrow{\text{transl}} e^{i a_\mu P^\mu} \phi(x) \xrightarrow{\text{at 1st order}} (1 - a_\mu \partial^\mu)\, \phi(x)$.

- However, it is important to note that the SUSY algebra does not close off-shell!

This observation can be strengthened by the degrees of freedom counting:

- a complex scalar field $\equiv$ 2 bosonic degrees of freedom (on-shell <u>and</u> off-shell)
- a left-handed Weyl spinor $\equiv$ 2 fermionic degrees of freedom on-shell
  4 fermionic degrees of freedom off-shell

$\Rightarrow$ $\boxed{\text{off-shell, we are missing two bosonic degrees of freedom (we recall that } n_B = n_F\text{).}}$

- We can bypass this issue by adding a new complex scalar field F, auxiliary and vanishing on-shell. Its equations of motion are thus

$$F = F^\dagger = 0$$

We can hence derive the lagrangian density

$$\mathcal{L}_F = F F^\dagger$$

from which we get $[F] = 2$ $\Rightarrow$ this confirms the auxiliary nature of F since the usual mass-dimension for a scalar field (propagating) is $[\phi] = 1$.

The new complete lagrangian is thus:

$$\boxed{\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi + i \bar{\psi} \bar{\sigma}^\mu \partial_\mu \psi + F F^\dagger}$$

- Let us now calculate the new SUSY transformation laws.

  ▷ $\delta\phi$ cannot be F-dependent because of mass dimension

  $\Rightarrow$ $\delta\phi = \sqrt{2}\ \varepsilon . \psi$ holds.

- $\delta\psi$ can contain a term proportional to εF since $[\varepsilon] = -\frac{1}{2}$ and $[F] = 2$.

$$\Rightarrow \delta\psi = -\sqrt{2}\, i\, (\sigma^\mu \bar\varepsilon)\, \partial_\mu \phi + a\, \varepsilon . F$$

The first term cancels the variation of the scalar lagrangian while the second one induces:

$$\delta \mathcal{L}_\psi = i a^* F^\dagger\, \bar\varepsilon \bar\sigma^\mu \partial_\mu \psi + i a\, \bar\psi \bar\sigma^\mu \varepsilon\, \partial_\mu F$$

$$= i a^* F^\dagger\, \bar\varepsilon \bar\sigma^\mu \partial_\mu \psi - i a\, \partial_\mu \bar\psi \bar\sigma^\mu \varepsilon\, F \quad + t.d.$$

This must be compensated by the variation of the auxiliary lagrangian:

$$\delta \mathcal{L}_F = \delta F\, F^\dagger + F\, \delta F^\dagger$$

$$\Rightarrow \delta F = -i a^*\, \bar\varepsilon \bar\sigma^\mu \partial_\mu \psi$$

By convention, we chose $a = -\sqrt{2}$.

$$\Rightarrow \boxed{\begin{aligned} \delta\phi &= \sqrt{2}\ \varepsilon . \psi \\ \delta\psi &= -\sqrt{2}\, i\, \sigma^\mu \bar\varepsilon\, \partial_\mu \phi - \sqrt{2}\ \varepsilon . F \\ \delta F &= \sqrt{2}\, i\ \bar\varepsilon \bar\sigma^\mu \partial_\mu \psi \end{aligned}}$$

- We can check (exo II.2) that

$$\boxed{[\delta_1, \delta_2] \begin{pmatrix} \phi \\ \psi \\ F \end{pmatrix} = -\Delta^\mu \partial_\mu \begin{pmatrix} \phi \\ \psi \\ F \end{pmatrix}}$$

The SUSY algebra now closes on-shell and off-shell.

- One remark:

$$\delta F = \partial_\mu(\ldots) \equiv \text{total derivative}$$

This is related to the fact that F has the highest dimension in the supermultiplet $\Rightarrow$ its SUSY transf. law must contain a derivative.
This will also be used in the superspace formalism to build SUSY invariant Lagrangians.

(4) Interactions

- We consider a collection of chiral supermultiplets (ϕ^a, ψ^a, F^a) $(a = 1, 2, \ldots, n)$ and an holomorphic function $W(\phi)$ depending only on the scalar fields (and not on ϕ^+_a).

Introducing:

$$W_a = \frac{\partial W}{\partial \phi^a} \qquad W_{ab} = \frac{\partial^2 W}{\partial \phi^a \partial \phi^b} \qquad W^*(\phi^+) = (W(\phi))^+,$$

we can show that the interaction Lagrangian

$$\boxed{\mathcal{L}_{int} = -W_a F^a - W^{*a} F^+_a - \frac{1}{2} W_{ab}\, \psi^a \cdot \psi^b - \frac{1}{2} W^{*ab}\, \bar{\psi}_a \cdot \bar{\psi}_b} \quad (1)$$

is SUSY-invariant (proof: see II.3)

$W(\phi)$ is called the superpotential

- Eq. (1) is derived by, e.g., the method of tensor calculus. Without entering into details, we start with two supermultiplets
  - (ϕ^1, ψ^1, F^1)
  - (ϕ^2, ψ^2, F^2)

and we try to build a "product supermultiplet". We can show that

empirical

$$(\phi^1\phi^2,\ \phi^1\psi^2 + \phi^2\psi^1,\ \phi^1 F^2 + \phi^2 F^1 + \psi^1\cdot\psi^2)$$

satisfies the SUSY transf. laws.

Iterating with a third supermultiplet (ϕ^3, ψ^3, F^3), one obtains

$$(\phi^1\phi^2\phi^3,\ \phi^1\phi^2\psi^3 + \phi^1\phi^3\psi^2 + \phi^2\phi^3\psi^1,\ \phi^1\phi^2F^3 + \phi^1\phi^3F^2 + \phi^2\phi^3F^1$$
$$+ \phi^1\psi^2\cdot\psi^3 + \phi^2\psi^1\cdot\psi^3 + \phi^3\psi^1\cdot\psi^2)$$

which generalizes to:

$$(W(\phi),\ W_a\psi^a,\ F^a W_a + \tfrac{1}{2}\psi^a\cdot\psi^b W_{ab})$$

where $W(\phi)$ is any fct of the scalar fields.

Since the F-component of a supermultiplet transforms as a total derivative,

$$F^a W_a + \frac{1}{2} W_{ab} \psi^a \psi^b$$

is suitable for a SUSY-conserving interaction lagrangian.

- The most general renormalizable superpotential is:

$$W = L_a \phi^a + \frac{1}{2} m_{ab} \phi^a \phi^b + \frac{1}{6} \lambda_{abc} \phi^a \phi^b \phi^c$$

and the lagrangian is given by

$$\boxed{\mathcal{L} = \partial_\mu \phi_a^\dagger \partial^\mu \phi^a + i \bar{\psi}_a \bar{\sigma}^\mu \partial_\mu \psi^a + F_a^\dagger F^a - \left(W_a F^a + \frac{1}{2} W_{ab} \psi^a \psi^b + h.c.\right)}$$

- Solving the eqs. of motion for the auxiliary fields, one gets

$$\left\{ \begin{array}{l} \dfrac{\partial \mathcal{L}}{\partial F^a} = -W_a + F_a^\dagger = 0 \\[2ex] \dfrac{\partial \mathcal{L}}{\partial F_a^\dagger} = -W^{*a} + F^a = 0 \end{array} \right. \Rightarrow \boxed{F^a = W^{*a}}$$

$$\Rightarrow \boxed{\mathcal{L} = \partial_\mu \phi_a^\dagger \partial^\mu \phi^a + i \bar{\psi}_a \bar{\sigma}^\mu \partial_\mu \psi^a - W^{*a} W_a - \frac{1}{2}\left(W_{ab} \psi^a \psi^b + h.c.\right)}$$

- The scalar potential is given by

$$V_F = W^{*a} W_a$$

which consists in a sum of positive terms

$\Rightarrow$ <u>V_F is bounded from below and non-negative.</u>

- <u>Remark 1:</u> the SUSY transformation laws can be written as:

$$\begin{cases} \delta \phi^a = \sqrt{2}\ \varepsilon . \psi^a \\ \delta \psi^a = -i\sqrt{2}\, \sigma^\mu \bar{\varepsilon}\, \partial_\mu \phi^a - \sqrt{2}\, W^{*a} \varepsilon \end{cases}$$

once on-shell. The laws are thus highly non-linear and explicitely depend on the Lagrangian, i.e. on the considered model.

$\Rightarrow$ the auxiliary fields find thus one of their other important usage.

- <u>Remark 2:</u> ◦ the scalar mass matrix is obtained from V_F:

$$\mathcal{L}_{m,\phi} = m_{ab}\, m^{*ac}\, \phi_c^{+} \phi^b$$

◦ the fermion one is simply given by

$$\mathcal{L}_{m,\psi} = -\frac{1}{2}\, m_{ab}\, \psi^a . \psi^b + h.c.$$

$\Rightarrow$ scalar and fermions have the same mass.