III. SUSY gauge Theories

- The other basic supermultiplet used in $N=1$ SUSY model building contains

  - one massless vector field
  - one massless Majorana fermion

This multiplet is associated to a given gauge group, taken abelian here (In the non-abelian case, those fields lie in the adjoint representation of the associated algebra)

- Counting the degrees of freedom, one has

	vector	fermion
on-shell	2	2
off-shell	3	4

$\Rightarrow$ as for the chiral supermultiplet, we must introduce an auxiliary field to restore supersymmetry off-shell. We have then a real scalar field D vanishing on-shell.

- The gauge supermultiplet is then denoted (A_μ, λ, D) and the associated lagrangian is given by

$$\boxed{\mathcal{L}_V = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\lambda} \bar{\sigma}^\mu \partial_\mu \lambda + \frac{1}{2} D^2} \quad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

- following the same procedure as in the Wess-Zumino model, we can derive the SUSY transformation laws of the different fields:

$$\boxed{\begin{cases} \delta A_\mu = i\,(\varepsilon \sigma_\mu \bar\lambda + \bar\varepsilon \bar\sigma_\mu \lambda) \\ \delta \lambda = i D \varepsilon + \frac{1}{2}\, \sigma^\mu \bar\sigma^\nu \varepsilon\, F_{\mu\nu} \\ \delta D = \varepsilon \sigma^\mu \partial_\mu \bar\lambda + \partial_\mu \lambda \sigma^\mu \bar\varepsilon \end{cases}} \quad (2)$$

(the normalization is conventional)

and we can show that the lagrangian of éq. (1) is SUSY-invariant (ex III.1: proof)

- We now consider a gauge transformation

$$\boxed{\delta A_\mu = \partial_\mu \alpha(x) \qquad \delta\lambda = \delta D = 0}$$

Trivially, the lagrangian is gauge invariant. We can show (exo III.2) that the SUSY algebra closes only if we account for gauge transformation in addition to the SUSY transformations. Indeed, the commutators of two SUSY transf. of parameters ε_1 and ε_2 give:

$$\boxed{\begin{aligned} [\delta_1, \delta_2]\, A_\mu &= a^\rho \partial_\rho A_\mu + \partial_\mu \alpha \\ [\delta_1, \delta_2]\, \lambda &= a^\rho \partial_\rho \lambda \\ [\delta_1, \delta_2]\, D &= a^\rho \partial_\rho D \end{aligned}}$$

exo III.2: derive a^μ and α

$\Rightarrow [\delta_1, \delta_2] = \delta_a + \delta_g$ = translation + gauge transformation

- For the non-abelian case, it is enough to replace the derivatives appearing in (2) by covariant derivatives (in the adjoint representation).